Efficient heuristic for non-linear transportation problem on the route with multiple ports

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ABSTRACT

We need a better transport planning tool for loading maximization and transport cost minimization on the voyage route with multiple loading/unloading (discharging) ports. The implemented heuristic algorithm is able to find out an appropriate routing sequence with maximal earnings and profit. In the same time it looks for minimal loading/discharging and transshipment costs, but with fulfillment of cargo demands in a number of ports on the route. The efficient algorithm for optimal transport of N cargo loads (e.g. contingent of containers) for ships with limited capacity is being developed. This efficient tool may significantly reduce transport costs and ensure maximal profit to freight forwarders. Also, it can be applied for supply chain management of different goods from numerous vendors. The proposed algorithm shows acceptable complexity that means that such optimization tool can be used in shipping supported with limited computing power.

Key words: Non-linear Transportation Problem; Multi-destination Routing Problem;
Minimum Cost Multi-Commodity Flow Problem; Capacity Management of Container Ships

INTRODUCTION

Pre-shipment planning is a key element for efficient cargo management and successful transport with transportation means of limited capacity (ship, train, airplane etc.). This paper deals about maritime shipping industry but it could be easily extended for another transportation system.

One of the most important problems in cargo transportation is to find the sequence of cargo distribution between multiple sources and multiple destinations which minimizes the transportation cost and better utilizes the ship capacity. The capacity management problem in shipping is extended to transportation problem of different cargo types transported by one mean on the route with multiple sources (loading ports) and multiple destinations (ports of discharge).

In example from Fig. 2 we have 5 ports. Loads of containers (contingents) are waiting to be transported as it is shown on Fig. 1. The loading amounts are given in percentage of total ship capacity. If all contingents have the same freight cost it is clear that the ship will avoid port 3, to reduce the transportation cost. The port 4 has to be on the route because the efficiency will be increased with load 4-5. The ship is barely full, only free ship capacity of 10 % is present from 1-2 and 10 % from 4-5. But if we have higher freight cost for contingent 3-4 the routing sequence will include the port 3, as it is shown with dotted line on Fig. 2. In that case the profit is more important that the idle (unused) capacity on the board. Details are shown in Fig. 6 – Fig. 8.
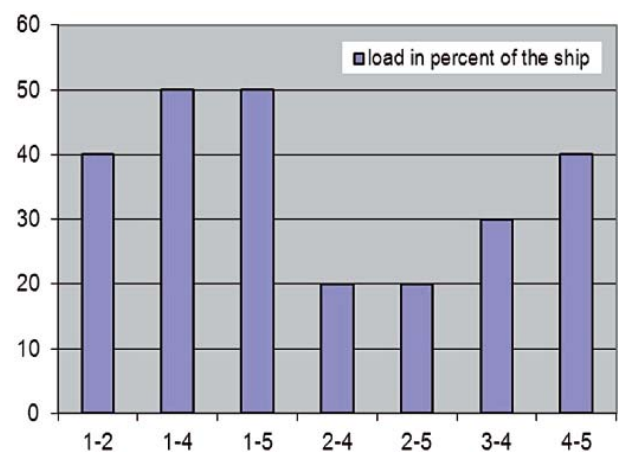

Fig. 1. Potential transfer of container contingents between ports given in percentage of the total ship capacity

Such a problem appears in route management for multi-purpose ships, tramper ships, container ships, where different contingents of cargo are transported by the ship with limited capacity; see [14]. Also, such an optimization tool can help to freight forwarder companies to select and manage the right mix of suppliers and to identify warehousing and distribution facilities best suited to customer needs. Very famous freight forwarders have extensive road and rail feeder network with links to their hubs, sub-hubs and gateways distributed all over the world; see [10, 11, 12].
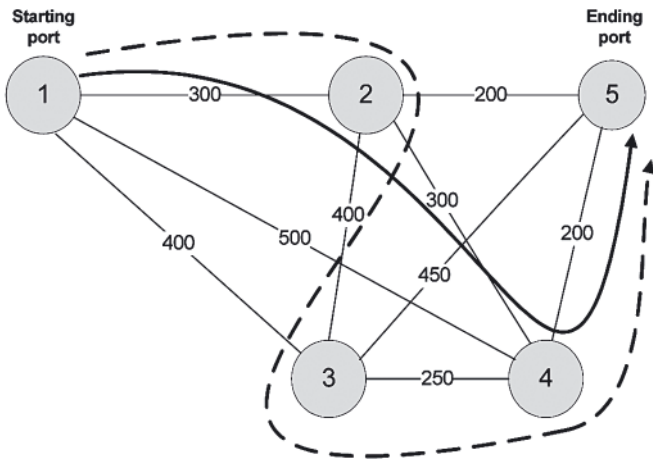
Fig. 2. Port distances. Trivial routing solution is represented with full line if all freight costs are equal for all contingents. Dotted line marks the routing solution if contingent from port 3 to port 4 is more valuable than others
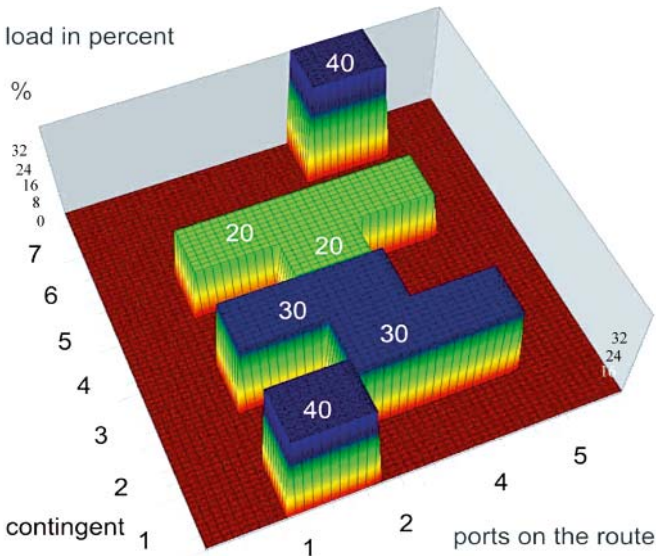

Fig. 3. Trivial routing sequence 1-2-4-5 and the load amounts on board the ship during voyage

The amount of different cargo loads (e.g. container) is in firm correlation because the total capacity of the ship is limited, e.g. in GT (Gross Tonnage) or in TEU (Twenty-foot Equivalent Unit). Taking into account cargo demands (unloading) for each cargo, and various loading ports with sufficient amount of cargo (number of containers) waiting to be loaded, we need an optimal transportation plan to minimize shipping and loading/unloading expenses, transshipment cost and cost of ship's stay in port (connected with duration of loading process). It can help in definition of ship capacity arrangement or for comparison of ships with different cargo capacity.

The non-linear transportation problem (NTP) with multiple (several) ports of loading (sources) and multiple destinations (sinks) is a very hard (NP-hard) problem so it is still the subject of many scientific papers. The problem can be solved using different techniques, see [5] and [16] In special circumstances the NTP can be seen as Minimum Cost Multiple Commodities Flow Problem (MCMCF); see [1] and [2]. In this paper we applied such network optimization approach. The mathematical model is formulated in section 2. The algorithm implementation is explained in section 3. Testing results and explanation of basic heuristic approach can be seen in section 4. Due to the presence of certain limitations the improvement of the algorithm is made in section 5. This approach, consisting of successive iterations, decreases the calculation complexity to an acceptable level.

MATHEMATICAL MODEL

Different kinds of good (e.g. container contingents) are differentiated with i for i = 1, 2,..., N. The ship with defined cargo capacity is shipping from the first to the last port marked with K M, with a possible set of intermediate (transshipment) ports. The objective is to find a loading and transshipment strategy that minimizes the total cost incurred over the whole voyage route consisting of M ports on the path (M ≤ K). We need the loading plan for various cargo/container contingents in each port to serve N cargo loads from the loading port to destinations (ports of discharge). The loading strategy consists of the load/discharge plan for each port and for each cargo contingent. The starting port on the route can be only for loading and the last port on the route can be only for discharging; other ports on the route may be for both.

The transportation problem can be represented by a flow diagram of non-oriented acyclic network. The problem can be solved using the network optimization technique as the shortest path problem; The problem can be solved using different techniques, see [13]. Figure 4 gives a network flow representation of MCMCF for N different cargo loads and M ports along the path. The common node "O" is the source of cargo for each cargo load with possible limitations. Some source ports can have limitation on charging capacity, but most of them are hub ports with capacity exceeding ship's earning capacity. Each load has the strictly defined discharging port. In Figure 4 the i-th row of nodes represents the capacity state of i-th type of cargo/container after loading in port m. Links between the nodes represent the amount of cargo transported between the ports (in TEU).

Such a transportation problem can be seen as the capacity expansion problem (CEP). For each cargo load we need appropriate ship space so it looks like expansion (load) or reduction (unload) of ship capacity in given bounds. Expansion and reduction can be done for each contingent separately but the free space can be reused for another contingent if previous load was discharged.

In the mathematical model of CEP the following notation is used:

- i, j and k = indices for cargo load. The N facilities are not ranked, just present different types of cargo/container contingents from 1, 2,..., N.
- m = indices the port of loading (charging) or discharging. The number of port of calls on the voyage including the departure port is M (m = 1,.., M).
- u, v = indices for ports in sub-problem, $1 \leq u,..., v \leq M$. All ports on the route are transshipment ports except 1 and M.
- $x_{i,m}$ = quantity of i-th load of cargo amounts (e.g. containers contingent) being loaded on board in port m (TEU). Total loading amount in port m:

$$X_m = \sum_{i=1}^{N} x_{i,m} \qquad (1)$$

- $Lx_{i,m}$ = limitations for each port and each cargo load. For convenience, the $x_{i,m}$ is assumed to be integer.
- $r_{i,m}$ = unloading of i-th cargo contingent in port m. For convenience, the $r_{i,m}$ is assumed to be integer. All unloading demands must be satisfied after discharging in last port on the route. Total discharging amount (unloading) in the port m.

$$R_m = \sum_{i=1}^{N} r_{i,m} \qquad (2)$$

- $I_{i,m}$ = the amount of cargo load i at arrival in port m (or, equivalently, at departure from port m-1). Before the first
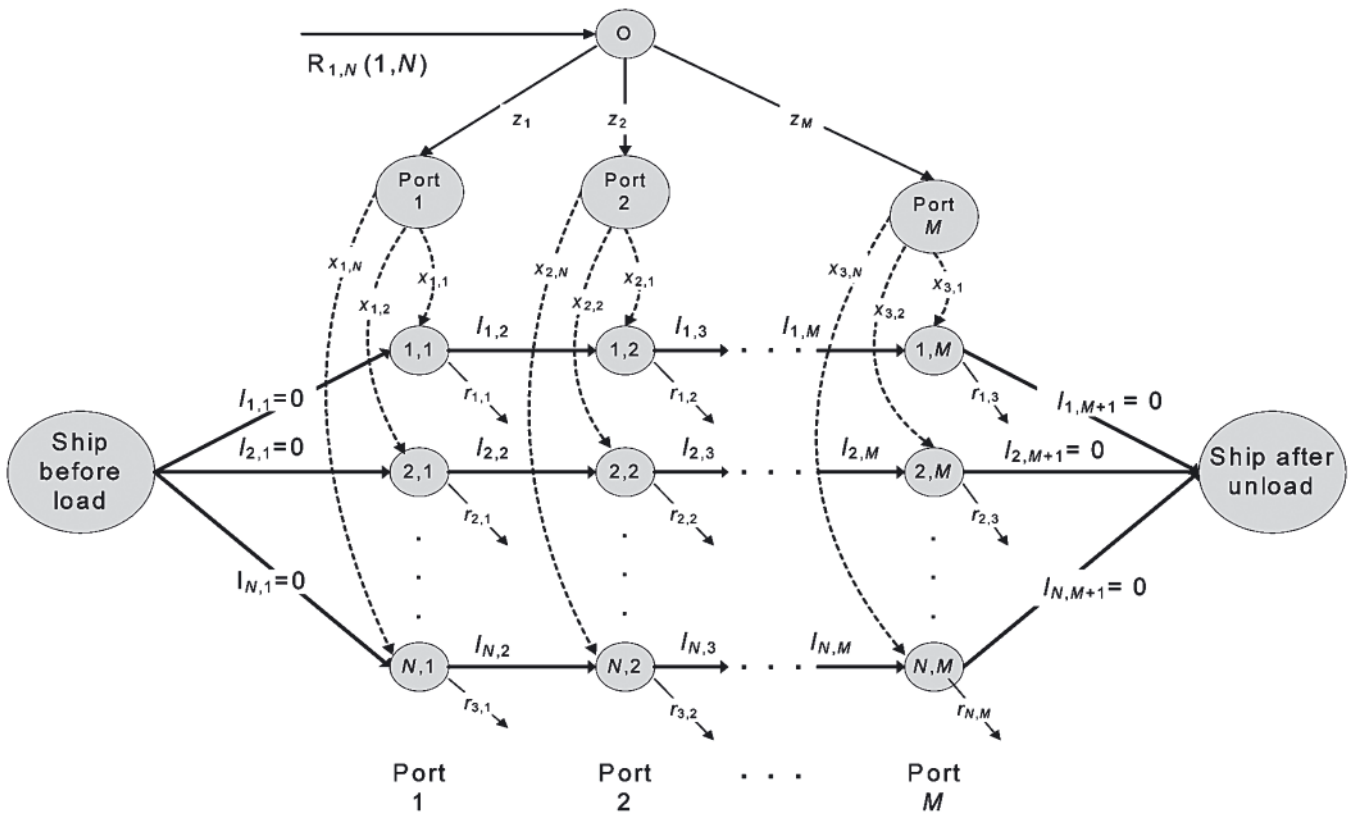
Fig. 4. *A network flow presentation of the trasnportation problem*

port of loading, $I_{i,m} = 0$. After the last port $I_{i,M+1} = 0$ for $i = 1,\ldots, N$. Capacity values cannot be negative.

- step I_i = the lowest step of possible capacity charging and discharging for capacity type i. In numerical examples it can be set, e.g. step $I_i = 10\%$ of total capacity of the ship.
- z_m = the total loading/unloading amount for all types of cargo (containers) in port m, i.e.,

$$Z_m = \sum_{i=1}^{N} x_{i,m} + r_{i,m} \qquad (3)$$

- Q = ship's deadweights in tons:

$$Q = \sum_{i=1}^{N} Q_{i,m} \qquad (4)$$

- $Q_{i,m}$ = element of ship deadweight used for i-th cargo contingent
- a_i = weight per unit of the i-th type of cargo unit (container)
- ship's transport capacity (GT or TEU) used for i-th cargo load:

$$W_{i,m} \le \frac{Q_{i,m}}{a_i} \qquad (5)$$

- ship's transport capacity (GT or TEU):

$$W = \sum_{i=1}^{N} W_{i,m} \qquad (6)$$

- used ship capacity between ports m and m + 1; for any m, m = 1, ..., M:

$$I_m = \sum_{i=1}^{N} I_{i,m} \qquad (7)$$

- shipping efficiency between ports m and m + 1:

$$e_m = \frac{I_m}{W} \qquad (8)$$

- unused ship capacity in port m:

$$IDLE_m = \sum_{i=1}^{N} W_{i,m} - \sum_{i=1}^{N} I_{i,m} \qquad (9)$$

- $lg_{i,m}$ = average number of cargo units (TEU) of i-th type of cargo (container) that can be loaded on board or discharged from board on daily basis in port m.

The total cost over time includes:

a) Transshipment cost on distance between ports m and m+1:

$$c_m = C_m \cdot d_m/s \qquad (10)$$

where:
C_m = transportation cost of the ship during voyage (per day);
d_m = distance (in nautical miles or km);
s = speed of ship (in knots). Here it is not correlated with the number of cargo units (containers) on board, the influence on speed, oil consumption, agent taxes and freight expenses, but these effects could be easily incorporated. In our examples the constant speed of the ship is incorporated in the value C_m.

b) Loading and discharging cost in port m:

$$h_m = H_m \sum_{i=1}^{N} \left(\frac{x_{i,m} + r_{i,m}}{lg_{i,m}} \right) \qquad (11)$$

where:
H_m = cost of ship stay in port m (per day).

The expenses for total duration of voyage and ship's stay in port during loading can be expressed as:

$$T = \sum_{m=1}^{M-1} \left(\frac{d_m}{s} + \sum_{i=1}^{N} \frac{x_{i,m} + r_{i,m}}{lg_{i,m}} \right) \qquad (12)$$

c) Freight cost for transshipment of cargo type i is making profit $f_{i,m}$ to forwarder company. We want to incorporate minimization of expenses with maximal profit in the same optimization process, so we have to introduce the freight cost. We can do that by the exponential cost function showing the economy of scale:

$$f_{i,m} = A_{i,m} + B_{i,m} \cdot I_{i,m}^{a_{i,m}} \quad (13)$$

where $a_{i,m}$ represents the factor of concavity for an appropriate cargo type i and for appropriate transshipment conditions on the route (m = 1, …, u, v, …, M). In some cases the constant value A_i could be used as the freight cost f_i without the influence of cargo amount on board. The optimization process will find out the most attractive cargo loads for shipping revenue.

The optimization problem can be formulated as minimization of the objective cost function - as follows:

$$\min \sum_{i=1}^{N} \sum_{m=1}^{M-1} \{c_m(d_m) + h_m(z_m) - f_{i,m}(I_{i,m})\} \quad (14)$$

so that we have:

$$I_{i,m+1} = I_{i,m} + x_{i,m} - r_{i,m} \quad (15)$$

$$I_{i,1} = I_{i,M+1} = 0 \quad (16)$$

for m = 1, 2,.., M; i = 1, 2,..., N

Generally, we try to find out the optimal loading/unloading sequence with maximal freight costs and minimal expenses; see [4] and [5]. It is a very demanding dual max/min transportation problem but it can be solved in a very simple way. The minimization of the expenses should have a strong influence on maximal profit.

ALGORITHM DEVELOPMENT

Instead of a nonlinear convex optimization that can be very complicated and time-consuming, the network optimization methodology has been efficiently applied. The main reason for such an approach is the possibility of use of discrete capacity values for a limited number of contingent loads, which improves significantly the optimization process. The multi-constrained problem (MCP) can be formulated as the Minimum Cost Multi-Commodity Flow Problem (MCMCF). Such a problem (NP-complete) can be easily represented by the multi-commodity single (common) source/multiple destination network; see [2] and [3].

The definition of the single-constrained problem for CEP is to find a path P from starting to end port such that:

$$w(P) = \min \sum_{m=1}^{M+1} \sum_{i=1}^{N} w_{i\,m}(I_{i,m}, x_{i,m}, r_{i,m}) \quad (17)$$

where:

$$I_{i,m} \leq Lx_{i,m} \quad (18)$$

satisfying the additional condition:
max. distance of:

$$P = \sum_{m_1}^{m_2} lon_i \leq LON \quad (19)$$

for i = 1, …, N; m = 1,…, M

where LON is the maximal length of the voyage in miles.

A path obeying the above conditions is said to be feasible. Note that there may be multiple feasible paths between the starting port and the ending port (node).

Generalizing the concept of the capacity states after loading/ unloading of each contingent (load) m between ports on the route we define as a capacity point - α_m.

$$\alpha_m = (I_{1,m}, I_{2,m},..., I_{N,m}) \quad (20)$$

$$\alpha_1 = \alpha_{M+1} = (0, 0,..., 0) \quad (21)$$

Let C_m be the number of capacity point values at port m (load value for each contingent after departure from the port); see Fig. 4. Only one capacity point is for the starting port and one for the ending port on the route: $C_1 = C_{M+1} = 1$. The total number of capacity points is:

$$C_p = \sum_{m=1}^{M+1} C_m \quad (22)$$

Horizontal links (branches) represent capacity flows between two neighbor ports.

Formulation (21) implies that zero values are before loading at the starting point and after unloading at the ending point.

The network optimization can be divided into two steps. At first step the minimal transportation weights $d_{u,v}$ between all pairs of capacity points (neighbor ports on the route) are calculated. It is obvious that in CEP we have to find many cost values $d_{u,v}(\alpha_u, \alpha_{v+1})$ that emanate two capacity points of neighbor links (common router), from each node (u, α_u) to node (v + 1, α_{v+1}) for v ≥ u. Calculation of this value is the capacity expansion sub-problem (CES).

The most of the computational effort is spent on computing the sub-problem values. That number depends on the total number of capacity points, see (22). The total number of all possible $d_{u,v}(\alpha_u, \alpha_{v+1})$ values representing CES between two capacity points is:

$$N_d = \sum_{m=1}^{M} C_m \cdot C_{m+1} \quad (23)$$

At second step we are looking for the shortest path in the network with the former calculated weights.

As the number of all possible $d_{u,v}(\alpha_u, \alpha_{v+1})$ values depends on the total number of capacity points it is very important to reduce that number (C_p). This can be done through imposing of appropriate capacity bounds or by introduction of additional constraints (e.g. max. shipment delay). Through numerical test-examples we'll see that many loading / unloading solutions cannot be a part of the optimal expansion sequence. It is the way how the algorithm can be significantly improved. So we can obtain the near-optimal result with significant computational savings. The objective function for CES can be formulated as follows:

$$d_{u,v} = \max \left\{ \sum_{m=u}^{v} \left[\sum_{i=1}^{N} f_{i,m}(f_{i,m}) - c_m(d_m) + \atop - h_m(z_m) - g_m(G_m) \right] \right\} \quad (24)$$

where:

$$I_{i,m+1} = I_{i,m} + D_{i,m}(x_{i,m}, r_{i,m}) \quad (25)$$

$$D_{i,m} = \sum_{i=1}^{N} (x_{k,m} - r_{l,m}); k \neq 1 \quad (26)$$

for m = 1, 2,..., M+1; k, l = 1, 2,..., N.

For every CES many different solutions can be derived depending on D_i value. Each of them represents the capacity

state of each contingent onboard the ship with loading and unloading values (amounts) in appropriate port.

In the objective function the total cost includes some different costs (weights). We want to incorporate minimization of expenses with maximization of the profit in the same optimization process. All expenses have to have negative polarity; see (24). The freight cost is denoted with $f_{i,m}(I_{i,m})$. We can differentiate the freight cost for each container load (contingent).

The transportation cost is denoted with $c_m(d_m)$, while the loading and discharging cost is denoted with $h_m(z_m)$. The idle capacity cost $g_m(I_{max}-I_m)$ could be taken in account, but only as a penalty cost to force the usage of the maximum capacity (prevention of unused/idle capacity). The port taxes can be incorporated with h_m. Costs are often represented by the fix-charge cost or by a constant value. It should be assumed that all function costs are concave and non-decreasing (some of them reflecting economies of scale) and they differ from one port to another. The objective function is necessarily the non-linear cost. With variation of cost parameters the optimization process could be easily managed, looking for benefits of the most appropriate transportation solution; see [6]. Instead of maximization of the profit we can use minimization of the reciprocal value or minimization of negative value of the objective function (24). In both cases it leads to maximization of the profit.

Suppose that all links (sub-problems) in diagram from Fig. 5. are calculated, the optimal solution for CEP can be found by searching for the optimal sequence of capacity points and their associated link state values; see Fig. 2. Then the Dijkstra's or Floyd's algorithm, or any similar algorithm, can be applied; see [7] and [13].
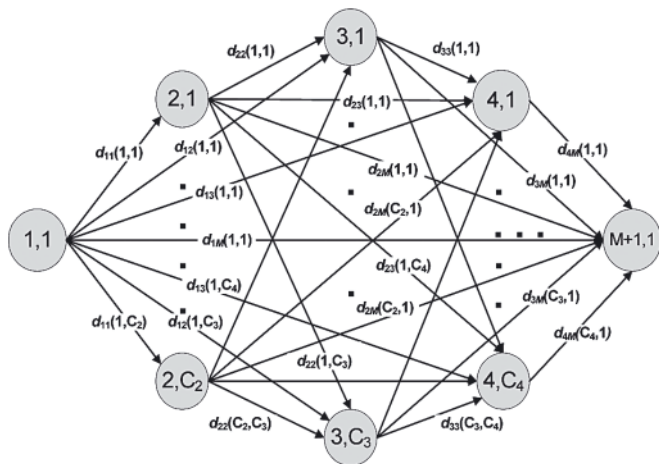

Fig. 5. The CEP problem can be seen as the shortest path problem for an acyclic network in which the nodes represent all possible values of capacity points. Links connecting neighbor ports on the route represent CES values

The complexity of the proposed algorithm is $O(C_p^2)$. As we said before C_p is in a strong correlation with the number of ports M and the number of contingents N but also with the capacity increment step I_i that can vary from contingent to contingent.

If the contingent capacity is given in TEU we have the problem with large number of capacity states. Instead, we use contingent amounts given in the percentage of the total ship capacity; see Fig. 1.

In this research the load amount on board does not influence on voyage speed neither to oil consumption but it could be easily incorporated. The loading strategy consists of loading/unloading plans for each port and for each contingent. The starting port on the route can be only for loading purpose and the last port is only for unloading; other transshipment ports may be for both.

Some limitations on the ship capacity can exist, but today most ports have loads not exceeding the ship capacity.

RESULTS OF BASIC HEURISTIC

In the route definition shown, for example in Fig. 2 we have the starting port 1 and the ending port 5, but any of three middle ports can also be included in the route.

All distances between ports are defined in miles. From Fig. 1 we can see traffic demands (possible transfer of contingents) given in the percentage of the total ship capacity. That information is gathered through market research or from statistics. From input data we can see seven contingents waiting for transport. It is obvious that the loads have to be transported on relation: 1-2 (40 %); 2-5 (50 %); 1-4 (50 %); 2-4 (20 %) 1-5 (20 %), 3-4 (30 %) and 4-5 (40 %), but we have no demands from port 2 to port 3. For simplicity all costs elements are equal ($A_{i,m} = 0$, $B_{i,m} = 15.0/\%$ of capacity; $C_{i,m} = 3.0/km$; $H_{i,m} = 1.0/\%$ of capacity; $G_{i,m} = 0$; and concavity for all costs $a_{i,m} = 0.85$.
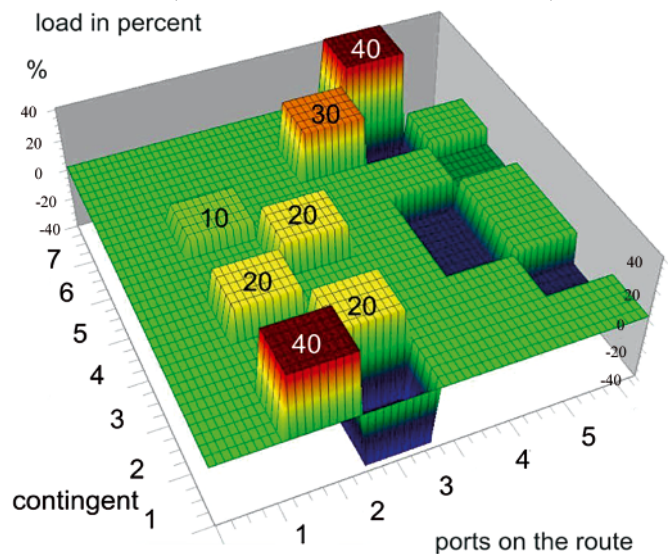

Fig. 6. Optimal solution given by loading and unloading amounts in each port on the route
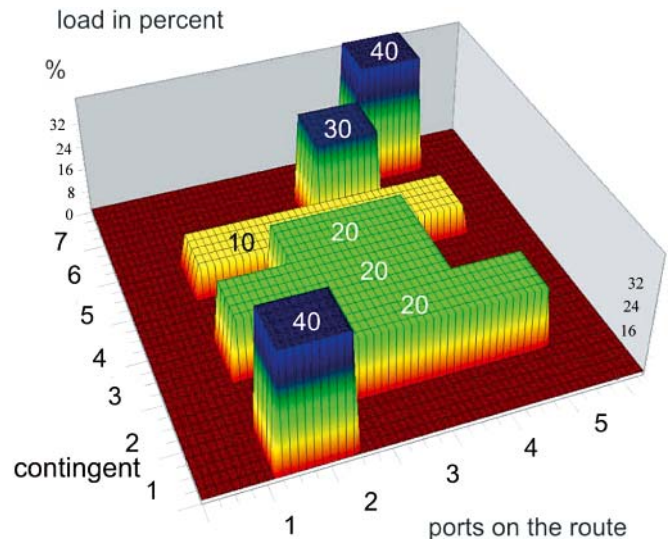

Fig. 7. Ship's occupancy on the route with particular contingents onboard

Only the freight cost for contingent 6 (3-4) is $B_{i,m} = 30,0$ that is significantly higher. In this example we force travelling across the port 3 as we have contingent from 3-4 with valuable load. According to all transport costs and the price determination (freight cost, oil consumption, transshipment cost, port taxes

etc.) we can design the route which will be the most profitable. Figures 6 and 7 present the resulting (best) route. Fig. 6. shows the loading and unloading amounts in appropriate port and Fig.7 shows the load amounts of every contingent on board the ship during the voyage. For the basic option we used the same capacity increment step I_i for all contingents and it is 10 %. We are aware that such capacity resolution is not satisfactory and, in general, we should be far away from the optimal result. In that case we have 1438 capacity states and 1438 x 1438 CES values.
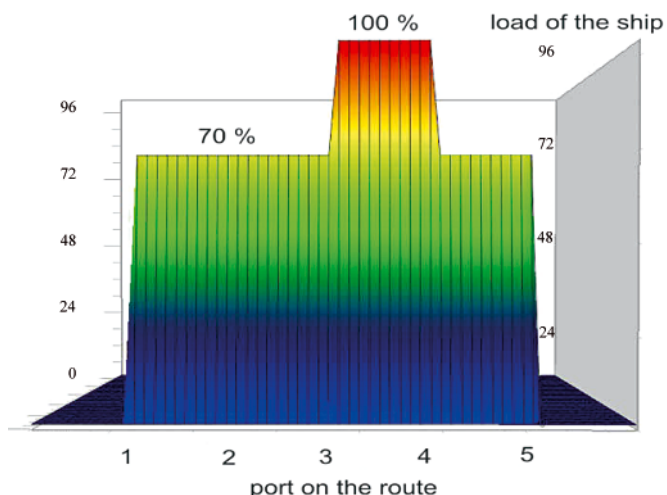

Fig. 8. *Efficiency of the ship on the route*

For our test-example the best routing option is from port 1 to port 2, to port 3, to port 4, and, finally, to port 5. The solution does not extract port 3 because it is more profitable to go this way (in spite of longer distance).

Figure 8 presents the efficiency of the ship, so idle capacity during the voyage is obvious. Only from port 3 to port 4 we have no idle capacity (100 %) on board. For this example the cost elements are similar but they can be differentiated from port to port and from contingent to contingent.

ALGORITHM LIMITATIONS AND POSSIBLE IMPROVEMENT

As we said before, the crucial element is the number of capacity points C_p. In our numerical example the starting capacity increment is 10 % and the number of capacity point is 1438. Calculation of so many CES values could be very demanding but it is still acceptable. If we decide to have smaller capacity increment step I_i the number of capacity points drastically increases.

For example, the capacity of a large container ship can be 5.000 - 10.000 TEU. The VLCS class (Very Large Container Ships) has more than 10.000 TEU. If we use the capacity increment of 10 % it means 500 - 1000 TEU. Normally we need better resolution e.g. 50 - 100 TEU or less. In that case we have to use step I_i = 1 % or 0,1 % of ship capacity. If we decide to apply for step I_i the value of 1 % instead, the number of capacity states rises up to approx. 15 000, which means that we have to calculate approx. 15 000 x 15 000 CES values. Such approach has no perspective in case of shipping supported with average computation power.

As we usually have limitation in reliable computation up power (average PC) we have to decrease that number somehow. It is clear that we need another approach to increase the resolution, but the corresponding computing complexity has to be acceptable.

So we decided to calculate it in a number of steps (with successive iterations). The first step is shown in Fig. 6 and Fig. 7. The numerical values are shown in table 1. After that we can shorten the range of capacity points using some of artificial intelligence techniques. For example, the calculated routing sequence shows that the second contingent is in amount of 20 % so we can use the range from 15 – 25 only. For some contingents we use step I_i = 5 % and for some 1 %. Capacity limitations for each contingent are given in table 1. In that case we have the number of capacity points C_p = 3483 (instead of millions). It is obvious that the problem complexity is still acceptable; see table 1. Also, we got significantly better result.

Tab. 1. *Results for the routing sequence in three successive iterations*

	1. iteration	Cost elements				2. iteration		3. iteration		
Load i	Max. capacity	$B_{i,m}$ $f_{i,m} =$ $B_{i,m}I_i^{ai,m}$	C_i $c_i =$ $C_id_m^{ai,m}$	H_i $h_i =$ $H_iZ_i^{ai,m}$	C_p	Load/Max. capacity Delta = 5	C_p	Load/Max. capacity Delta = 2	C_p	Load
1	40 (0-40) I_{step} = 10 %	15.	3.	2.		40 (35-40) I_{step} = 5 %		40 (38 – 40) I_{step} = 1 %		40
2	50 (0-50) I_{step} = 10 %	15.	3.	2.		20 (15-25) I_{step} = 5 %		20 (18-22) I_{step} = 1 %		18
3	50 (0-50) I_{step} = 10 %	15.	3.	2.	C_1 = 89 C_2 = 291 C_3 = 968 C_4 = 89 C_5 = 1	20 (15-25) I_{step} = 1 %	C_1 = 132 C_2 = 1188 C_3 = 2126 C_4 = 36 C_5 = 1	17 (15-19) I_{step} = 1 %	C_1 = 75 C_2 = 625 C_3 = 1293 C_4 = 75 C_5 = 1	17
4	20 (0-20) I_{step} = 10 %	15.	3.	2.		20 (15-20) I_{step} = 1 %		18 (16-20) I_{step} = 1 %		18
5	20 (0-20) I_{step} = 10 %	15.	3.	2.		10 (5-15) I_{step} = 1 %		15 (13-17) I_{step} = 1 %		17
6	30 (0-30) I_{step} = 10 %	30. (15.)	3.	2.		30 (25-30) I_{step} = 5 %		30 (28-30) I_{step} = 1 %		30
7	40 (0-40) I_{step} = 10 %	15.	3.	2.		40 (35-40) I_{step} = 5 %		40 (38-40) I_{step} = 1 %		40
Total CES					1438		3483		2069	
Profit						1926,33		1931,53		1932,39

In the next step (third iteration) we applied the smaller range of capacity e.g. 18 - 22. Also, we can use smaller increment step $I_i = 1$ % for all contingents. In that case we have 2069 capacity states that is still an acceptable number. It means that number of possible sub-problems Nd is about 4 million (instead of hundreds of millions). From Fig. 9 we can see that the routing sequence slightly differs from the previous result and the profit is increasing again. From Fig. 10 it is clear that the idle capacity of the ship on the route is lower. With this step by step method we can increase the resolution of the capacity states significantly and because of that we can reach much closer to the optimum.
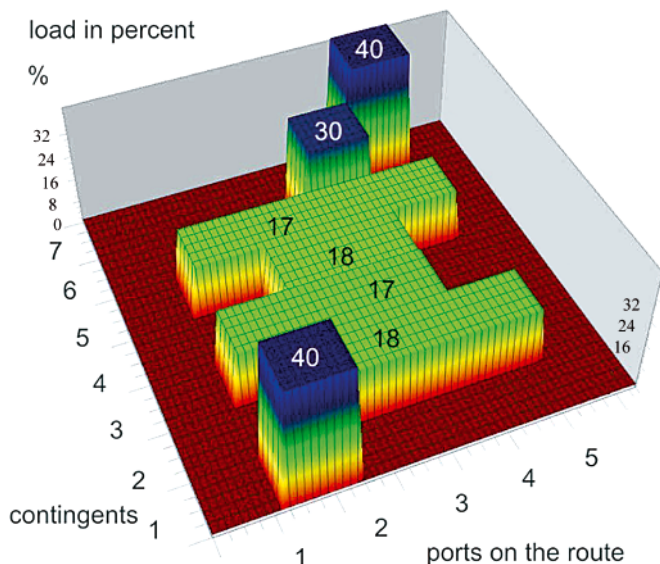

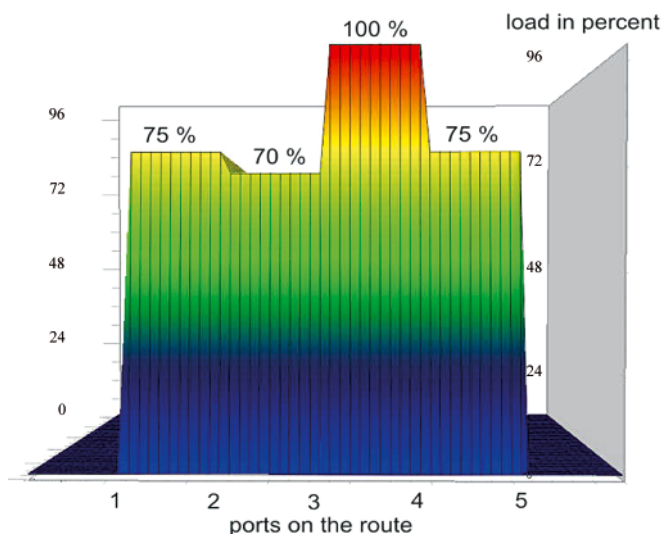**Fig. 9.** *After third iteration the profit is increased*

Fig. 10. *Efficiency of the ship is better than before*

Through many test examples it is clear that the presented approach functions very well and the complexity of the calculation process is under control. Without such step by step calculation method the complexity may be too big.

CONCLUSIONS

The proposed algorithm shows ability to solve very complex transportation problems with many loading/unloading ports and with many contingents of the load. The most important benefit is that the algorithm can solve nonlinear problems which normally occur in practice. Also, the existing limited calculation power in shipping surrounding makes the algorithms with huge complexity useless. The present approach consisting of a number of successive iterations decreases the calculation

complexity to an acceptable level. In the same time it ensures to forwarder managers very fine modulation of many input values, leading the optimization process in wanted direction.

With such optimization tool the shipping companies (freight forwarder) can ensure significant savings on multiport routes and be more profitable by following the demands and easily adapt to their changes. Also, such innovative solutions can optimize the supply chains and reduce the logistic costs.

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