

A model for service life control of selected device systems

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ABSTRACT

This paper presents a way of determining distribution of limit state exceedance time by a diagnostic parameter which determines accuracy of maintaining zero state. For calculations it was assumed that the diagnostic parameter is deviation from nominal value (zero state). Change of deviation value occurs as a result of destructive processes which occur during service. For estimation of deviation increasing rate in probabilistic sense, was used a difference equation from which, after transformation, Fokker-Planck differential equation was obtained [4, 11]. A particular solution of the equation is deviation increasing rate density function which was used for determining exceedance probability of limit state. The so-determined probability was then used to determine density function of limit state exceedance time, by increasing deviation. Having at disposal the density function of limit state exceedance time one determined service life of a system of maladjustment. In the end, a numerical example based on operational data of selected aircraft [weapon] sights was presented. The elaborated method can be also applied to determining residual life of shipboard devices whose technical state is determined on the basis of analysis of values of diagnostic parameters.

Keywords: reliability; unreliability; durability; service life time; destructive processes

INTRODUCTION

Many devices are equipped with systems, mechanisms which are subject to regulation processes and as a result of them the system is brought back to a required state (its nominal values). Application of an “in-service correction” to a device results from occurrence of its maladjustment. The maladjustment is caused by degradation and ageing processes taking place during device operation. Information about occurrence of such processes comes from diagnostic parameters which change their values under influence of destructive processes [7, 8, 13]. Certain types of aircraft [weapon] sights exemplify the devices subject to the above mentioned processes. Their task is a. o. to elaborate and display sight information in the form of a sight marker. The devices are required a. o. to maintain, within certain limits, zero state regarding determined sight and advance angles.

It is assumed that deviations from zero state are identified by means of determinate diagnostic parameters. When values of the diagnostic parameters exceed limit value then the device becomes not fully serviceable, i. e. it reaches a state of intermediate serviceability. In this case it is necessary to regulate the device and bring its indications to zero position. In this paper is presented a model of the determining of time interval during which quantity of a parameter increases from its initial value to limit one. The presented method makes it possible to ensure a greater working accuracy of a device and also to control process of its operation.

DETERMINATION OF DENSITY FUNCTION OF DIAGNOSTIC PARAMETER CHANGES

In the proposed model of service life estimation the following assumptions have been taken:

- Technical state of a device is determined by only one diagnostic parameter: „z” in the form of deviation from initial state (zero value).

$$z = |X - X^{\text{nom}}| \quad (1)$$

where:

- X - current value of diagnostic parameter;
- X^{nom} - nominal value (zero state) of diagnostic parameter.

- Change in value of diagnostic parameter deviation occurs during entire service time (operation and standstill);
- The parameter „z” is non-decreasing;
- Rate of change of diagnostic parameter can be described by the following relation:

$$dz/dt = c \quad (2)$$

where:

- c - random variable which characterizes susceptibility of an element to changes dependent on its features and working conditions;
- t - calendar time.

Dynamics of changes of the deflection „z” is characterized, in probabilistic sense, by the following difference equation:

$$U_{z,t+\Delta t} = PU_{z-\Delta z,t} \quad (3)$$

where:

- $U_{z,t}$ - probability of the event that in the instant t diagnostic parameter quantity takes the value of z;
- P - probability of the event that in the time interval Δt deflection value will increase by the value Δz . It is assumed that $P=1$;
- Δz - deflection increase.

In the functional form Eq. (3) is as follows:

$$U(z,t+\Delta t) = U(z-\Delta z,t) \quad (4)$$

Eq. (4) has the following sense: probability of that in the instant t deflection value was equal to $z-\Delta z$ and in the time interval Δt it increased by the value Δz .

Eq. (4) is now transformed into a partial differential equation. To this end the following approximations are assumed:

$$U(z,t+\Delta t) = U(z,t) + \frac{\partial U(z,t)}{\partial t} \Delta t \quad (5)$$

$$U(z-\Delta z,t) = U(z,t) - \frac{\partial U(z,t)}{\partial z} \Delta z + \frac{1}{2} \frac{\partial^2 U(z,t)}{\partial z^2} (\Delta z)^2$$

With the help of Eq. (5), Eq. (4) is transformed to the following form:

$$\frac{\partial U(z,t)}{\partial t} = -b \frac{\partial U(z,t)}{\partial z} + \frac{1}{2} a \frac{\partial^2 U(z,t)}{\partial z^2} \quad (6)$$

where:

- $b = E[c]$ - mean increase of diagnostic parameter deflection value per unit of time;
- $a = E[c^2]$ - mean square of increase of diagnostic parameter deflection value per unit of time.

Solution of Eq. (6) takes the form [1, 14]:

$$u(z_i,t) = \frac{1}{\sqrt{2\pi A(t)}} e^{-\frac{(z_i-B(t))^2}{2A(t)}} \quad (7)$$

where:

$$B(t) = \int_0^t b dt = bt$$

$$A(t) = \int_0^t a dt = at$$

The density function of increase of diagnostic parameter deflection value can be directly applied to estimation of reliability of a device's system.

DETERMINATION OF DISTRIBUTION OF TIME OF EXCEEDANCE OF LIMIT (PERMISSIBLE) STATE

The probability of exceedance of limit state by diagnostic parameter can be expressed with the help of the density function of changes of diagnostic parameter deflection, (7), as follows:

$$Q(t; z_g) = \int_{z_g}^{\infty} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z-bt)^2}{2at}} dz \quad (8)$$

The distribution density function of time of the first exceedance over the permissible value z_g takes the following form:

$$f(t) = \frac{\partial}{\partial t} Q(t; z_g) \quad (9)$$

Taking into account Eq. (7) one obtains:

$$f(t) = \frac{\partial}{\partial t} \int_{z_g}^{\infty} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z-bt)^2}{2at}} dz \quad (10)$$

Therefore,

$$f(t) = \int_{z_g}^{\infty} \left\{ \frac{\partial}{\partial t} \left[\frac{1}{\sqrt{2\pi at}} e^{-\frac{(z-bt)^2}{2at}} \right] \right\} dz \quad (11)$$

Using the formulation (7) one gets:

$$f(t) = \int_{z_g}^{\infty} \left\{ \frac{\partial}{\partial t} u(z,t) \right\} dz \quad (12)$$

Moreover, the time derivative of the function (7) takes the following form:

$$\frac{\partial}{\partial t} [u(z,t)] = u(z,t) \left(\frac{z^2 - b^2 t^2 - at}{2at^2} \right) \quad (13)$$

The relation (13) is now inserted into Eq. (11) to obtain the following:

$$f(t)_{z_g} = \int_{z_g}^{\infty} u(z,t) \left(\frac{z^2 - b^2 t^2 - at}{2at^2} \right) dz \quad (14)$$

Then, an anti-derivative function of the integrand of Eq. (14) is searched for. It is expected that the function of the form:

$$w(z,t) = u(z,t) \left(-\frac{z+bt}{2t} \right)$$

is the searched anti-derivative for the integrand of Eq. (14).

The test of this hypothesis is now performed:

$$\begin{aligned} \frac{\partial}{\partial z} \left[u(z,t) \left(-\frac{z+bt}{2t} \right) \right] &= -u(z,t) \left(-\frac{z-bt}{at} \right) \left(-\frac{z+bt}{2t} \right) + \\ &+ u(z,t) \left(-\frac{1}{2t} \right) = u(z,t) \left[\frac{(z-bt)(z+bt)}{2at^2} - \frac{1}{2t} \right] = \\ &= u(z,t) \left[\frac{z^2 - b^2 t^2 - at}{2at^2} \right] \end{aligned} \quad (15)$$

It can be concluded that the anti-derivative for the integrand of Eq. (14) is of the following form:

$$w(z,t) = u(z,t) \left(-\frac{(z+bt)}{2t} \right) \quad (16)$$

Now the integral (14) is calculated to get:

$$\begin{aligned} f(t)_{z_g} &= u(z_i,t) \left(-\frac{(z+bt)}{2t} \right) \Big|_{z_g}^{\infty} = \\ &= \frac{z_g + bt}{2t} \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z_g-bt)^2}{2at}} \end{aligned} \quad (17)$$

The relation (17) determines the distribution density function of time of the first exceedance of limit (permissible) state by diagnostic parameter deflection.

ESTIMATION OF SERVICE LIFE OF SELECTED STRUCTURAL SYSTEMS OF SEA-GOING SHIP OR AIRCRAFT

The formula for reliability of a ship or aircraft equipment system takes the following form:

$$R(t) = 1 - \int_0^t f(t)_{z_g} dt \quad (18)$$

where the density function $f(t)_{z_g}$ is determined by Eq. (17).

However, unreliability of a ship or aircraft equipment system can be determined by using Eq. (19) as follows:

$$Q(t) = \int_0^t \frac{z_g + bt}{2t} \cdot \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z_g - bt)^2}{2at}} dt \quad (19)$$

The integral (19) should be transformed to a simpler form. It may be observed that the integrand can be written in the form:

$$\begin{aligned} \frac{z_g + bt}{2t} \cdot \frac{1}{\sqrt{2\pi at}} e^{-\frac{(z_g - bt)^2}{2at}} &= \\ &= \frac{z_g + bt}{2t} \cdot \frac{1}{\sqrt{2\pi at}} e^{-\frac{(bt - z_g)^2}{2at}} \end{aligned} \quad (20)$$

to reduce the problem to the indefinite integral:

$$\int \frac{(z_g + bt)}{2t} \cdot \frac{1}{\sqrt{2\pi at}} e^{-\frac{(bt - z_g)^2}{2at}} dt \quad (21)$$

On substitution of:

$$\frac{(bt - z_g)^2}{2at} = u$$

the integral (21) takes the form:

$$\begin{aligned} \int \frac{z_g + bt}{2t} \cdot \frac{1}{\sqrt{2\pi at}} e^{-u} \cdot \frac{2at^2}{(bt + z_g)(bt - z_g)} du &= \\ &= \frac{1}{2\sqrt{\pi}} \int \frac{1}{\sqrt{u}} e^{-u} du \end{aligned} \quad (22)$$

Next, the subsequent substitution should be done:
 $\sqrt{u} = w$
 $du = 2w dw$

Taking into account the above given relations one can write the integral (22) in the following form:

$$\frac{1}{2\sqrt{\pi}} \int \frac{1}{w} e^{-w^2} 2w dw = \frac{1}{\sqrt{\pi}} \int e^{-w^2} dw \quad (23)$$

On substitution of:

$$\begin{aligned} w^2 &= \frac{y^2}{2} \\ dw &= \frac{y}{\sqrt{2}} \end{aligned}$$

the integral of the following form is achieved:

$$\frac{1}{\sqrt{2\pi}} \int e^{-\frac{y^2}{2}} dy \quad (24)$$

where:

$$y = \frac{bt - z_g}{\sqrt{at}}$$

Inserting the obtained results into Eq. (18) and not forgetting to assign appropriate integration limits one obtains the formula for the reliability in question:

$$R(t) = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{bt - z_g}{\sqrt{at}}} e^{-\frac{y^2}{2}} dy \quad (25)$$

The cumulative distribution function of normal standardized distribution takes the following form, [10, 12]:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{y^2}{2}} dy \quad (26)$$

Taking into account the relation (26), one can finally express the formula for reliability of a ship or aircraft structural element as follows:

$$R^*(t) = 1 - \Phi\left(\frac{b^* t - z_g}{\sqrt{a^* t}}\right) \quad (27)$$

where b^* and a^* are coefficients of the values estimated on the basis of operational data of sea – going ships or aircraft.

Therefore risk of failure of a device can be determined by means of the following relation:

$$Q^* = 1 - R^*(t) = \Phi(\gamma) \quad (28)$$

where:

$$-\gamma = \frac{b^* t - z_g}{\sqrt{a^* t}} \quad (29)$$

After assuming an unreliability level, the upper limit of the integral, γ^* , in Eq. (28), can be determined. Having γ^* from the relation (29) one can determine the service life of a device system as follows:

$$T = \frac{(2b^* z_g + (\gamma^*)^2 a^*) - \sqrt{(2b^* z_g + (\gamma^*)^2 a^*)^2 - 4b^{*2} z_g^2}}{2b^{*2}} \quad (30)$$

After the operational period T the system undergoes regulation.

To make use of the formula (30), values of the constants appearing in it should be first determined (estimated). To this end, it is assumed that the data on increasing value of diagnostic parameter deflection, obtained from observation of a tested device during its operation process, are available in the form:

$$[(z_0, t_0), (z_1, t_1), (z_2, t_2), \dots, (z_n, t_n)] \quad (31)$$

The best method for determining values of „b” and „a” from available data is that which makes use of likelihood function which can be generally expressed in the form as follows [3, 5, 18]:

$$L = \prod_{k=0}^{n-1} g(t_k, z_k, \theta_1, \theta_2, \dots, \theta_m) \quad (32)$$

where:

$g(t_k, z_k, \theta_1, \theta_2, \dots, \theta_m)$ – whole probability density function of the variable z;

- $(\theta_1, \theta_2, \dots, \theta_m)$ – density function parameters;
 z_k – measured values of wear of the parameter z in the instants $(t_1, t_2, \dots, t_{k_m})$, respectively.

Finding the estimated values $\theta_1^*, \theta_2^*, \dots, \theta_m^*$ of the unknown parameters $\theta_1, \theta_2, \dots, \theta_m$ by using the maximum likelihood method, is reduced to solving the equations of the form as follows:

$$\frac{\partial \ln L}{\partial \theta_j} = 0 \quad (33)$$

where:

$j = 1, 2, \dots, m;$

m - number of parameters which characterize a given technical object.

In this case estimation of b^* and a^* values of unknown parameters b and a by using the maximum likelihood method is reduced to solving the set of equations as follows [16, 17]:

$$\begin{cases} \frac{\partial \ln L}{\partial b} = 0 \\ \frac{\partial \ln L}{\partial a} = 0 \end{cases} \quad (34)$$

Solving the set of equations (34) one finds b^* and a^* values.

$$b^* = \frac{z_n}{t_n} \quad (35)$$

$$a^* = \frac{1}{n} \sum_{k=0}^{n-1} \frac{[(z_{k+1} - z_k) - b^*(t_{k+1} - t_k)]^2}{(t_{k+1} - t_k)} \quad (36)$$

NUMERICAL EXAMPLE AND FINAL REMARKS

One of sight system's elements is sight head by means of which sight information is displayed. Technical state of the sight head is determined with the help of diagnostic parameters which characterize sight marker position coordinates. On the basis of analysis of results of checking a certain population of sight heads it was determined that along with time of their operation, values of the parameters undergo changes as a result of influence of destructive factors. Run of change in values of one of the diagnostic parameters, recorded during operational process is exemplified in Fig. 1.

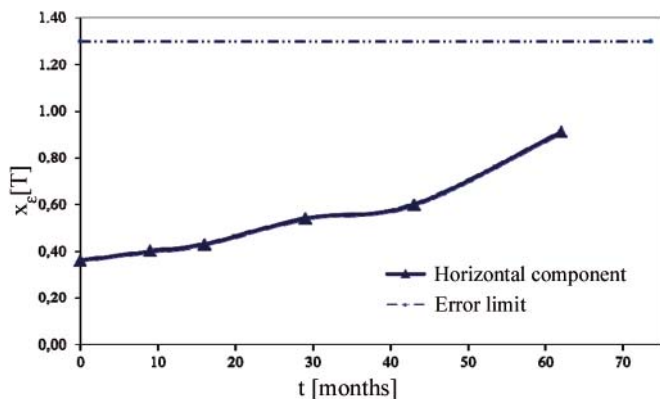


Fig. 1. Run of change in values of a diagnostic parameter, recorded during operation of a sight head

Therefore, once the data which describe diagnostic parameter values in the form of $[(z_0, t_0), (z_1, t_1), (z_2, t_2), \dots, (z_n, t_n)]$ are at disposal, the values of density function coefficients can be determined on the basis of Eq. (35) and (36):

$$\begin{aligned} b_{\varepsilon}^* &= 0.042 \\ a_{\varepsilon}^* &= 0.011 \end{aligned} \quad (37)$$

After assuming the level of maintaining zero state, the value of the parameter $\gamma^* = 2.35$ was read from normal distribution tables. Next, the parameter z_{ε} was determined from technical documentation used for maintenance operations, where information on permissible values of deviations of diagnostic parameters was included.

As the values of the parameters $b_{\varepsilon}^*, a_{\varepsilon}^*, \gamma^*, z_d$ have been already known, one inserted them into Eq. (30) to calculate the operation period after passing of which values of the diagnostic parameters will exceed limit state. For the case in question the period was found equal to:

$$T_{\varepsilon} = 12[\text{months}] \quad (38)$$

The obtained value (38) may be used for technical maintenance operations depending on an assumed maintenance strategy. On the basis of the presented method it is possible to determine successive periods in which control of diagnostic parameter of a device should be done (Fig. 2) and regulation performed in order to bring the measured quantities to zero (nominal) position.

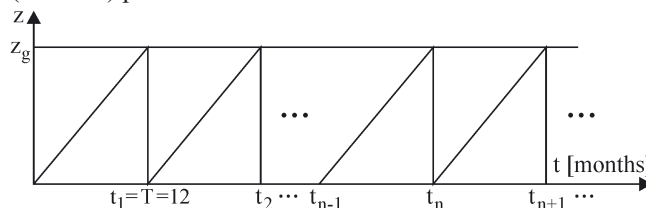


Fig. 2. Schematic diagram showing periods of device regulation to be performed to recover its nominal state

Summing up, it can be stated that the presented method seems to be correct and reasonable and makes it possible to perform technical state analysis of a device regarding character of changes of values of diagnostic parameters. The presented calculation example made it possible to verify the elaborated model as well as to reveal applicability merits of the method in question. The method may be useful in further efforts to improve both operation process and way of using sea-going ships or aircraft together with their on-board systems, by making it possible to determine the period during which the devices maintain state of serviceability.

Moreover, the presented method, in view of its general character, may be successfully applied to determination of residual service life of any technical object whose technical state can be determined on the basis of analysis of values of diagnostic parameters.

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