**POLISH MARITIME RESEARCH** 3(83) 2014 Vol. 21; pp. 77-85
10.2478/pomr-2014-0033

# Comments on linear summation hypothesis of fatigue failures

**Grzegorz Szala,** Assoc. Prof.
West Pomeranian University of Technology, Poland

ABSTRACT

*This paper presents a comparative analysis of results of fatigue life calculations with the use of the linear summation hypothesis of fatigue failures (LHSUZ), confronted with experimental test results. The calculations and fatigue tests were performed for variable amplitude(VA), two-step and ten-step loading conditions , both in the low-cycle fatigue (LCF) and high-cycle fatigue ( HCF) range, for the case of C45 steel as an example. Experimental verification of the hypothesis LHSUZ did not revealed any significant influence of load level and form of load spectrum on conformity of results of the calculation by using the LHSUZ, to results of fatigue tests on C45 steel. However, it enabled to assess magnitude of a correction factor which appears in the considered linear hypothesis.*

**Keywords:** fatigue life calculations, summation hypothesis of fatigue failures, variable load

## INTRODUCTION

Calculations of durability of structural elements, including fatigue life of ship structures [1], [2], [3], require: to know a load spectrum expressed in the form of sinusoidal cycles of variable amplitude parameters (Sa or εac) and their mean values (Sm and εmc), cyclic properties of a material in the form of the fatigue characteristics N(S), N(Sa, Sm) or 2Nf(εac), 2Nf(εac, εmc), and to assume a failure summation hypothesis which constitutes a phenomenological description of fatigue process in structural materials. The problems are associated with some factors which affect conformity of results of fatigue life calculations to those resulting from experimental tests.

In this paper the issue of the LHSUZ hypothesis is analyzed under assumption that load spectra and material fatigue characteristics are constant elements.

The first LHSUZ hypothesis was published by A. Palmgren in 1924, who described - in a phenomenological way - the fatigue process of ball bearings. In the 1930s the hypothesis was described again by a few authors , a. o. B. F. Langren in 1937, and M. A. Miner who formulated anew the hypothesis by applying energy approach, in 1945. In the subject-matter literature the hypothesis is sometimes called Palmgren-Miner (PM) hypothesis. In its original form it was assumed that in the case of multi-step loading program, fatigue fracture will occur if the following condition is satisfied :

$$D_{N_c} = \sum_{i=1}^{k} \frac{n_i}{N_i} = 1{,}0 \qquad (1)$$

or in the case of a load spectrum given in the form of amplitude distribution :

$$D_{N_c} = \int_{S_{f(-1)}}^{S_{a\max}} \frac{dn}{N(S_a)} = 1{,}0 \qquad (2)$$

For this hypothesis the authors assumed that only these load cycles whose amplitude values exceed fatigue limit, take place in the summation process of fatigue failures:

$$S_{a_i} \geq S_{f(-1)} \qquad (3)$$

Experimental verification of PM hypothesis showed that in many cases its conformity to experimental test results is rather low. In the 1950s and 1960s, many publications dealing with modification of PM hypothesis, come out. In the modifications the following condition was assumed firstly:

$$D_{N_c} = \sum_{i=1}^{k} \frac{n_i}{N_i} = a \qquad (4)$$

and secondly - that also loads below fatigue limit have a significant effect on conformity of results of calculations to those of experimental tests. Therefore, the load cycles of the amplitudes :

$$S_{a_i} \geq k \cdot S_{f(-1)} \qquad (5)$$

were summed up, where the coefficient k is comprised in the range from 0,4 to 0,6.

In 1963, S.V. Serensen published a formula for calculation of the correction factor a in Eq. (4), depending on a form of load spectrum characterized by the spectrum diagram factor ζ and maximum load value appearing in the load spectrum : $S_{a_{\max}} = S_{a_1}$.

The factor a can be calculated by using the formula :

$$a = \frac{S_{a_{\max}} \cdot \zeta - k \cdot S_{f(-1)}}{S_{a_{\max}} - k \cdot S_{f(-1)}} \qquad (6)$$

and, the spectrum diagram factor ζ is described as follows:

$$\zeta = \frac{1}{S_{a_{\max}}} \sum_{i=1}^{k} S_{a_i} \frac{n_i}{n_c} \qquad (7)$$

In spite of these modifications, PM hypothesis has been criticized for the reason of a weak physical background concerning recognition of phenomena which occur in structural materials during fatigue process, including the fact of not taking into account the factor ζ in non-linearity of run of failure summation in fatigue process, sequence of cyclic loads and difficulties in assessing the value of the correction factor a.

According to G. Wallgren (1949), value of the factor a is comprised in the range from 0,5 to 3,0. Schütz and Zenner (1973) determined a distribution of a – values on the basis of results of 561 tests conducted on structural elements made of iron alloys, aluminium and titanium. As results from the above mention distribution, a - values are contained in the variability range from 0,1 to 10. J. Szala (1980) obtained similar results from verification of PM hypothesis on the basis of a review of 75 publications. In 1957 A. J. Bielanin made research on influence of load level ($S_{max}/S_{f(-1)}$ and $S_{min}/S_{f(-1)}$) as well as the form of load spectrum expressed by its diagram factor ζ. He conducted fatigue tests on toothed wheels and concluded from the test results that the greater the factor ζ and the higher the stresses kept in their relevant ranges: $S_{min} = (0{,}4÷1{,}2)\, S_{f(-1)}$ and $S_{max} = (1{,}4÷2{,}0)\, S_{f(-1)}$, the higher conformity between results of experiments calculations. As results from the form of Serensen's formula (6), the factor is less than 1 (a < 1,0) and is closer and closer to 1 as the factor ζ increases (the factor a = 1,0 for ζ = 1,0 which corresponds to a load of constant amplitude). The uncertainty of PM hypothesis has resulted (since 1970) in intensive searching for more effective hypotheses on fatigue failure summation, based on new definitions of fatigue failure.

Publication [4] contains a collection of formulae describing fatigue failures, based on analysis of variation of: longitudinal elasticity modulus, propagation velocity of ultra-sound wave, plastic deformation under cyclic load of constant stress variability range, stresses under conditions of constant strain variability range, plastic deformation work, micro-hardness, mean size of dislocation cell, distance between slip bands, number of micro-cracks per area unit etc. The publication in question presents 15 different formulae for calculation of the fatigue failure parameter D which, as far as PM hypothesis is concerned, is the ratio of the total number of applied cycles n and the number of cycles to fatigue failure N:

$$D_n = \frac{n}{N} \tag{8}$$

On the basis of the new definitions of fatigue failure, more than 30 hypotheses on summation of failures, applicable to fatigue life calculation, were developed.

In publication [5] they were divided into the following groups:

- linear summation hypotheses of fatigue failures,
- non- linear summation hypotheses of fatigue failures,
- hypotheses based on a concept of lines of constant fatigue failures and line of residual fatigue life.

The hypotheses were also split, in relation to fatigue failure parameter, into the groups with regard to stress, strain and energy approach, respectively.

Application of the above mentioned hypotheses in practice requires to have a comprehensive specialty knowledge because their applicability ranges are strictly limited and number of publications on their experimental verification is rather low. For these reasons they have not found any wider implementation so far.

As results from comparative analysis of the linear hypothesis and the hypotheses based on the concepts of constant failure lines as well as residual fatigue life line, the linear hypothesis is a particular case of the two remaining hypotheses.

In the 1990s and the first years of this century a few procedures for fatigue life calculation of machinery structural elements, ship structures and other objects were developed.

FITNET procedures [6] and those dealing with sea-going ship fatigue calculations [1], [2] as well as relevant requirements of ship classification societies, exemplify this kind of procedures. In the procedures, PM hypothesis with some modifications has been implemented. This fact has drawn again interest of scientists and engineers to research on verification of the hypothesis with the aim of determining a range of its applicability and to assess an impact of various factors on its conformity to experimental data.

For the first period (till the 1980s) the verifications have been conducted in rather not precisely defined conditions concerning applied loads, as well as structural features of tested objects.

The above presented observation justifies the made decision on undertaking the research project aimed at experimental verification of the linear summation hypothesis of fatigue failures, to be conducted in a broad range of variable amplitude loads for C45 steel, as an example. Scope of the research covered two-step and ten-step load programs at the stress ratio R = -1,0 (oscillatory load) and magnitude of load amplitudes both within low-cycle fatigue (LCF) and high-cycle fatigue (HCF) range, and for widely varying values of spectrum diagram factor. The test conditions was so selected as to make it possible to analyse influence of load values (both in LCF and HCF range) and a form of spectrum (the factor ζ) and to prevent against influence of any additional factors (e.g. cycle counting methods for random loads, used for preparation of load spectra, or methods for determination of local stresses and deformations in notched elements).

## FORMULATION OF THE PROBLEM

On the basis of results of the calculations performed with the use of Eq. (1) and (2) and results of the experimental tests carried out under the same conditions, the correction factor a in Eq. (4) may be calculated by means of the following formula:

$$a = \frac{N_{c_{ex}}}{N_{c_{obl}}} \tag{9}$$

For the case $N_{cex} > N_{cobl}$, a > 1,0 should be taken for fatigue life calculations, for $N_{cex} = N_{cobl}$ - a < 1,0 , and for $N_{cex} < N_{cobl}$ - a > 1,0. When the above specified values of the factor a are assumed, results of the calculations according to Eq. 4 comply with results of the experimental tests.

## Static and cyclic properties of C45 steel

The static and cyclic properties of C45 steel were determined from fatigue tests conducted in the framework of the research project No. NN 503 2221 39, titled „A hybrid method for fatigue life calculation and its verification by using results of fatigue tests of Al-alloys and steel„ ( the project has been financed by Polish Ministry of High Education and Science).

The static properties of C45 steel are the following : $R_m$ = 682 MPa, $R_e$ = 458 MPa, E = 2,15·10⁵ MPa. Its cyclic properties are described by the following formulae:

- Wöhler diagram:

$$\log S_a = -0{,}1020 \log N + 2{,}9611 \qquad (10)$$

- fatigue limit for $N_0$ = 106 cycles : $S_{f(-1)}$ = 223,5 MPa

- Manson-Coffin diagram:

$$\varepsilon_{ac} = \frac{1204}{2{,}15 \cdot 10^5}\left(2N_f\right)^{-0{,}1033} + 0{,}2179\left(2N_f\right)^{-0{,}475} \qquad (11)$$

- fatigue limit for $2N_0 = 2 \cdot 10^6$ cycles: $\varepsilon_{acf} = 1{,}472 \cdot 10^{-3}$

- Ramberg-Osgood diagram:

$$\varepsilon_{ac} = \frac{S_a}{2{,}15 \cdot 10^5} + \left(\frac{S_a}{1232}\right)^{5{,}06} \qquad (12)$$

## Variable loads

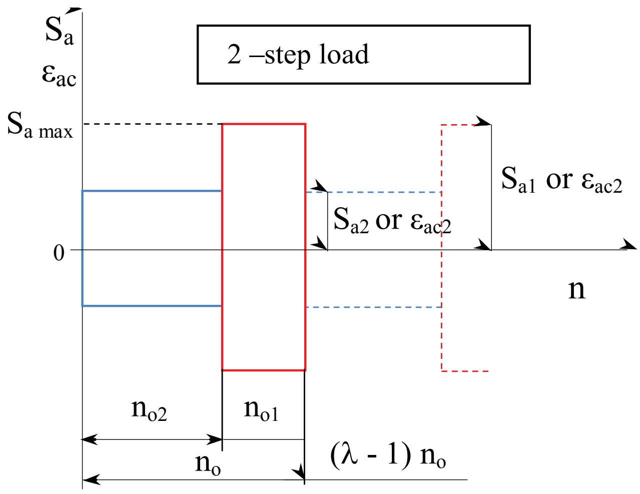



**Fig. 1a.** Schematic diagram of two-step load, with indicated relevant parameters which appear in fatigue life calculations and experimental tests.

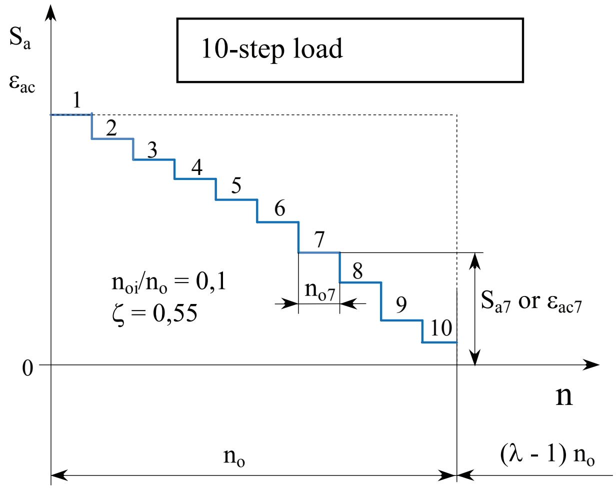



**Fig. 1b.** Schematic diagram of ten-step load, with indicated relevant parameters which appear in fatigue life calculations and experimental tests.

Fig. 1 shows schematic diagrams of two-step and ten-step loads where relevant parameters appearing in calculations and fatigue tests are also indicated. Form of two-step load spectrum is defined by the ratios: $S_{a2}/S_{a1}$ or $\varepsilon_{ac2}/\varepsilon_{ac1}$ and $n_{01}/n_0$ as well. On their basis, value of the spectrum diagram factor ζ is calculated.

The calculations were conducted for $S_{a2}/S_{a1}$ = 0,75; 0,5; 0,25, and $n_{01}/n_0$ = 0,75; 0,5; 0,25; 0,1. In the case of ten-step load spectra the same load increase per step, equal to 0,1 $S_{a1}$ ($S_{a(i+1)}$ + – $S_{ai}$ = 0,1 $S_{a1}$) and the same amount of cycles per step, $n_{0i}$ = 0,1 $n_0$, were assumed for particular steps , which led to the value ζ = 0,55 calculated according to Eq. (7).

For both the cases the following levels of Sa1 values (at stress approach): Level I - $S_{a1}$ = 615 MPa, Level II – 520 MPa, Level III – 428 MPa, and Level IV – 325 MPa, were assumed.

At strain approach, the following levels of $\varepsilon_{ac1}$ values corresponding to the above given $S_{a1}$ levels, were calculated by using Ramberg-Osgood formula (12): Level I - $\varepsilon_{ac} = 4{,}24 \cdot 10^{-2}$, Level II – $2{,}22 \cdot 10^{-2}$, Level III – $8{,}93 \cdot 10^{-3}$, Level IV – $3{,}06 \cdot 10^{-3}$.

The load levels were chosen under the assumption that in the extreme cases, i.e. Level I and $S_{a2}/S_{a1}$ = 0,75 or $\varepsilon_{ac2}/\varepsilon_{ac1}$ = 0,75, the loads are entirely contained within LCF range, and for Level IV – within HCF range. The intermediate cases , i.e. load level II and III and lower values of $S_{a2}/S_{a1}$ or $\varepsilon_{ac2}/\varepsilon_{ac1}$, contain only a part of steps within LCF range, and the remaining part of steps – within HCF range.

## Method of calculation and method of experimental tests

Detail calculation method based on PM hypothesis was described in the publication [7] where three calculation paths were assumed :

- 1st path - at stress approach - with the use of Wöhler diagram,
- 2nd path - at strain approach- with the use of Manson-Coffin diagram,

- 3rd path - hybrid calculation path covering strain - approach calculations of fatigue failures in LCF range, and stress– approach calculations of fatigue failure in HCF range.

Formulae , based on PM hypothesis, for calculation of fatigue failure resulting from one- step load program of the number of cycles $n_0$, are the following:

- for 1st path:

$$D_0 = \sum_{i=1}^{k} \frac{n_{0i}}{N_i} \tag{13}$$

- for 2nd path:

$$D_0 = \sum_{i=1}^{k} \frac{n_{0i}}{N_{fi}} \tag{14}$$

- for 3rd path:

$$D_0 = \sum_{i=1}^{l} \frac{n_{0i}}{N_{fi}} + \sum_{i=(l+1)}^{k} \frac{n_{0i}}{N_i} \tag{15}$$

Fatigue life expressed by a number of load program repetitions up to fatigue fracture, λ, is calculated from the formula:

$$\lambda = \frac{1{,}0}{D_0} \tag{16}$$

and the fatigue life expressed by a number of load cycles $N_c$, amounts to:

$$N_c = \lambda \cdot n_0 \tag{17}$$

The quantity 1,0 in the numerator of Eq. (16) shows that the calculations were conducted according to the original form of PM hypothesis described by Eq. (1). The yield point $R_e$ was assumed to be a criterion for LCF and HCF ranges.

Experimental determination of fatigue life of specimens made of C45 steel was carried out by using the method of programmed fatigue tests consisting in multifold repetition (λ times) of program's period, in compliance with Fig. 1 [8], up to fatigue fracture of specimen.

## RESULTS OF CALCULATIONS AND TESTS

### Two-step load

Tab. 1 contains results of the fatigue life calculations and results of fatigue life tests conducted on specimens of C45 steel under two-step load conditions.

The specimens were prepared in accordance with PN-EN 10002-1;2004 standard for static tests and PN-84/H-04334 standard for fatigue tests, and then tested by using INSTRON 8501 testing machine in a laboratory having accreditation of Polish Centre of Accreditation (PCA) ( Certificate No. AB 372 ).

On the basis of the results of the tests on 120 specimens, Gassner diagrams were prepared for particular load programs of different values of $S_{a2}/S_{a1}$ and $n_{01}/n_0$. 12 diagrams of the kind were prepared altogether – they are described by the formulae shown in Tab. 2. Data contained in columns: 6, 10, 14 and 18 of Tab. 1, were calculated by means of respective formulae given in Tab. 2.

Values of $N_c^S$, $N_c^{\varepsilon}$, $N_c^H$ given in Tab. 1 were calculated by using Eq. (13) through (17).

On the basis of the data contained in Tab. 1, values of the factor a were calculated with the use of Eq. (9). Results of the calculations are collected in Tab. 3, where the following notations are used, respectively:

$a_s$ - the factor determined for fatigue life calculated according to 1st path ( stress approach)

$a_\varepsilon$ - the factor determined for fatigue life calculated according to 2nd path ( strain approach)

$a_H$ - the factor determined for fatigue life calculated according to 3rd path ( hybrid method).

### Ten - step load

Programmed fatigue tests and fatigue life calculations for specimens made of C45 steel were conducted in a similar way as in the case of the above described calculations and tests for two-step load, and for the same values of $S_{a1} = S_{amax}$ or $\varepsilon_{ac1} = \varepsilon_{acmax}$.

On the basis of the tests on 15 specimens, a Gassner diagram described by the formula:

$$\log S_{a\max} = -0{,}0983 \log N_c + 3{,}0124 \tag{18}$$

was prepared.

Tab. 2. Collection of formulae describing fatigue life diagrams (acc. Gassner) for specimens of C45 steel under two- step load.

| $\frac{n_{01}}{n_0}$ | $S_{a2}/S_{a\,max}$ |
|---|---|
| | 0,75 |
| 0,75 | $\log S_{a_{\max}} = -0{,}0912 \log N_c + 2{,}9268$ |
| 0,5 | $\log S_{a_{\max}} = -0{,}0995 \log N_c + 2{,}9747$ |
| 0,25 | $\log S_{a_{\max}} = -0{,}1015 \log N_c + 2{,}9979$ |
| 0,1 | $\log S_{a_{\max}} = -0{,}0906 \log N_c + 2{,}9826$ |
| | 0,50 |
| 0,75 | $\log S_{a_{\max}} = -00977 \log N_c + 2{,}9443$ |
| 0,5 | $\log S_{a_{\max}} = -0{,}1029 \log N_c + 2{,}9837$ |
| 0,25 | $\log S_{a_{\max}} = -0{,}1013 \log N_c + 3{,}0133$ |
| 0,1 | $\log S_{a_{\max}} = -0{,}0966 \log N_c + 3{,}0189$ |
| | 0,25 |
| 0,75 | $\log S_{a_{\max}} = -0{,}1004 \log N_c + 2{,}9499$ |
| 0,5 | $\log S_{a_{\max}} = -0{,}0981 \log N_c + 2{,}9684$ |
| 0,25 | $\log S_{a_{\max}} = -0{,}0973 \log N_c + 3{,}0006$ |
| 0,1 | $\log S_{a_{\max}} = -0{,}0876 \log N_c + 3{,}0060$ |

**Tab. 1.** Results of fatigue life calculations and results of fatigue life tests conducted on specimens of C45 steel under two-step load

| No. of item | $S_{a2}/S_{a1}$ | $n_{01}/n_0$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0,75 | | | | 0,5 | | | | 0,25 | | | | 0,1 | | | |
| | | $N_c^S$ | $N_c^\varepsilon$ | $N_c^H$ | $N_c^{Ex}$ | $N_c^S$ | $N_c^\varepsilon$ | $N_c^H$ | $N_c^{Ex}$ | $N_c^S$ | $N_c^\varepsilon$ | $N_c^H$ | $N_c^{Ex}$ | $N_c^S$ | $N_c^\varepsilon$ | $N_c^H$ | $N_c^{Ex}$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| I | 0,75 | 64 | 47 | 47 | 33 | 92 | 68 | 68 | 74 | 165 | 126 | 126 | 115 | 317 | 255 | 255 | 138 |
| | 0,5 | 65 | 47 | 47 | 39 | 97 | 71 | 71 | 78 | 194 | 142 | 142 | 164 | 482 | 354 | 352 | 241 |
| | 0,25 | 65 | 43 | 47 | 40 | 98 | 71 | 71 | 68 | 195 | 142 | 142 | 150 | 487 | 355 | 355 | 298 |
| II | 0,75 | 330 | 287 | 286 | 205 | 476 | 420 | 416 | 399 | 856 | 779 | 757 | 599 | 1642 | 1599 | 1492 | 878 |
| | 0,5 | 336 | 291 | 291 | 217 | 504 | 437 | 437 | 400 | 1006 | 873 | 872 | 860 | 2498 | 2178 | 2166 | 1367 |
| | 0,25 | 336 | 291 | 291 | 213 | 505 | 437 | 437 | 374 | 1009 | 874 | 874 | 839 | 2523 | 2185 | 2185 | 2017 |
| III | 0,75 | 2223 | 2480 | 2457 | 1739 | 3210 | 3630 | 3532 | 2823 | 5777 | 6771 | 6287 | 4081 | 11066 | 14082 | 11810 | 7538 |
| | 0,5 | 2267 | 2510 | 2510 | 1589 | 3398 | 3765 | 3761 | 2653 | 6780 | 7524 | 7504 | 5880 | 16837 | 18768 | 18622 | 10261 |
| | 0,25 | 2268 | 2511 | 2511 | 1482 | 3401 | 3766 | 3766 | 2718 | 6803 | 7531 | 7532 | 6197 | 17007 | 18824 | 18830 | 18549 |
| IV | 0,75 | 33015 | 57377 | 33015 | 35640 | 47664 | 84073 | 47664 | 44947 | 85682 | 157230 | 85682 | 61471 | 164322 | 328994 | 164322 | 157532 |
| | 0,5 | 33659 | 58048 | 33659 | 26576 | 50451 | 87021 | 50451 | 38539 | 100676 | 173740 | 100676 | 89055 | 250012 | 432098 | 250012 | 177461 |
| | 0,25 | 33671 | 58049 | 33671 | 22979 | 50507 | 87021 | 50507 | 44964 | 101014 | 173740 | 101014 | 104807 | 252533 | 432098 | 252533 | 315666 |

$S_{max}$ : Level I – 615 MPa, Level II – 520 MPa, Level III – 428 MPa, Level IV – 325 MPa. $\varepsilon_{ac1}$ : Level I – $4{,}24\cdot10^{-2}$, Level II – $2{,}22\cdot10^{-2}$, Level III – $8{,}93\cdot10^{-3}$, Level IV – $3{,}06\cdot10^{-3}$.

$N_c^S$ - fatigue life calculated acc. 1st path, Eq. (13); $N_c^\varepsilon$ - fatigue life calculated acc. 2nd path, Eq. (14); $N_c^H$ - fatigue life calculated acc. hybrid method, Eq. (15); $N_c^{Ex}$ - fatigue life determined by means of programmed fatigue tests.

**Tab. 3.** Results of calculations of values of the correction factor a for C45 steel specimens under two-step load

| No. of item | $S_{a2}/S_{a1}$ | C45 steel | | | | | | | | $a = N_{ex}/N_{obl}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $n_{01}/n_0 = 0{,}75$ | | | | $n_{01}/n_0 = 0{,}5$ | | | | $n_{01}/n_0 = 0{,}25$ | | | | $n_{01}/n_0 = 0{,}1$ | | | |
| | | $\zeta$ | $a_S$ | $a_\varepsilon$ | $a_H$ | $\zeta$ | $a_S$ | $a_\varepsilon$ | $a_H$ | $\zeta$ | $a_S$ | $a_\varepsilon$ | $a_H$ | $\zeta$ | $a_S$ | $a_\varepsilon$ | $a_H$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| I | 0,75 | 0,9375 | 0,52 | 0,7 | 0,7 | 0,875 | 0,8 | 1,09 | 1,09 | 0,8125 | 0,7 | 0,91 | 0,91 | 0,775 | 0,44 | 0,54 | 0,54 |
| | 0,5 | 0,875 | 0,6 | 0,83 | 0,83 | 0,75 | 0,8 | 1,1 | 1,1 | 0,625 | 0,85 | 1,15 | 1,15 | 0,55 | 0,5 | 0,68 | 0,68 |
| | 0,25 | 0,8125 | 0,62 | 0,93 | 0,85 | 0,625 | 0,69 | 0,96 | 0,96 | 0,4375 | 0,77 | 1,06 | 1,06 | 0,325 | 0,61 | 0,84 | 0,84 |
| II | 0,75 | 0,9375 | 0,62 | 0,71 | 0,72 | 0,875 | 0,84 | 0,95 | 0,96 | 0,8125 | 0,7 | 0,77 | 0,79 | 0,775 | 0,53 | 0,55 | 0,59 |
| | 0,5 | 0,875 | 0,65 | 0,75 | 0,75 | 0,75 | 0,79 | 0,92 | 0,92 | 0,625 | 0,85 | 0,99 | 0,99 | 0,55 | 0,55 | 0,63 | 0,63 |
| | 0,25 | 0,8125 | 0,63 | 0,73 | 0,73 | 0,625 | 0,74 | 0,86 | 0,86 | 0,4375 | 0,83 | 0,96 | 0,96 | 0,325 | 0,8 | 0,92 | 0,92 |
| III | 0,75 | 0,9375 | 0,78 | 0,7 | 0,71 | 0,875 | 0,88 | 0,78 | 0,8 | 0,8125 | 0,7 | 0,6 | 0,65 | 0,775 | 0,68 | 0,54 | 0,54 |
| | 0,5 | 0,875 | 0,7 | 0,63 | 0,63 | 0,75 | 0,78 | 0,7 | 0,7 | 0,625 | 0,87 | 0,78 | 0,78 | 0,55 | 0,61 | 0,55 | 0,55 |
| | 0,25 | 0,8125 | 0,65 | 0,59 | 0,59 | 0,625 | 0,8 | 0,72 | 0,72 | 0,4375 | 0,91 | 0,82 | 0,82 | 0,325 | 1,09 | 0,99 | 0,99 |
| IV | 0,75 | 0,9375 | 1,08 | 0,62 | 1,08 | 0,875 | 0,94 | 0,53 | 0,94 | 0,8125 | 0,72 | 0,39 | 0,72 | 0,775 | 0,96 | 0,49 | 0,96 |
| | 0,5 | 0,875 | 0,79 | 0,46 | 0,79 | 0,75 | 0,76 | 0,44 | 0,76 | 0,625 | 0,88 | 0,51 | 0,88 | 0,55 | 0,71 | 0,41 | 0,71 |
| | 0,25 | 0,8125 | 0,68 | 0,4 | 0,68 | 0,625 | 0,89 | 0,52 | 0,89 | 0,4375 | 1,04 | 0,6 | 1,04 | 0,325 | 1,25 | 0,73 | 1,25 |

$S_{max}$ : Level I – 615 MPa, Level II – 520 MPa, Level III – 428 MPa, Level IV – 325 MPa. $\varepsilon_{ac1}$ : Level I – $4{,}24\cdot10^{-2}$, Level II – $2{,}22\cdot10^{-2}$, Level III – $8{,}93\cdot10^{-3}$, Level IV – $3{,}06\cdot10^{-3}$.

Tab 4 shows results of the calculations and tests for fatigue life of specimens under ten – step load. Values of the correction factor a for different methods of fatigue life calculations are given in columns: 7, 8 and 9 of this table; their notation is the same as that used in Tab. 3.

**Tab. 4.** Collected results of calculations and tests for fatigue life of specimens of C45 steel under ten – step load , and values of the correction factor a in PM hypothesis

| $S_{a\,max}$ MPa | $\varepsilon_{ca\,max}$ | $N_c^S$ | $N_c^\varepsilon$ | $N_c^H$ | $N_c^{Ex}$ | $a_S$ | $a_\varepsilon$ | $a_H$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 615 | $4{,}24\cdot10^{-2}$ | 323 | 245 | 244 | 188 | 0,58 | 0,78 | 0,77 |
| 520 | $2{,}22\cdot10^{-2}$ | 1675 | 1542 | 1515 | 1036 | 0,61 | 0,67 | 0,68 |
| 428 | $8{,}93\cdot10^{-3}$ | 11288 | 13566 | 12129 | 7506 | 0,66 | 0,55 | 0,62 |
| 325 | $3{,}06\cdot10^{-3}$ | 167612 | 325742 | 167612 | 123509 | 0,74 | 0,38 | 0,74 |

## ANALYSIS OF RESULTS OF CALCULATIONS AND TESTS

As results from the data in Tab. 3, values of the correction factor a are remarkably dispersed depending on a method used for fatigue life calculations ( acc. 1st, 2nd and 3rd path) and on load conditions. In the subject-matter literature, may be found only a few data concerning dependence of a-factor values on the spectrum diagram factor ζ (variable load intensity) and load values contained in the spectrum. Eq. (6) formulated by Serensen may serve as an example of such attempt.

The further part of this analysis presents an attempt to investigate dependence of values of the factors $a_S$, $a_\varepsilon$ and $a_H$ on load levels (I, II, III and IV) and values of the spectrum diagram factor ζ, and also to experimentally verify the formula (6).

### Influence of load level and value of the spectrum diagram factor ζ

Exemplary data for the factor as , given in Tab. 3, are presented in the coordinate frame (ζ, a), in Fig. 2.

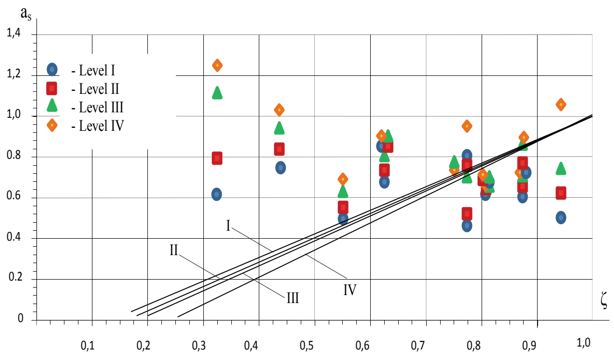



**Fig 2.** Diagrams of a – factor values calculated according to the formula (6)

As results from these data, values of the factor a are contained within the following ranges:

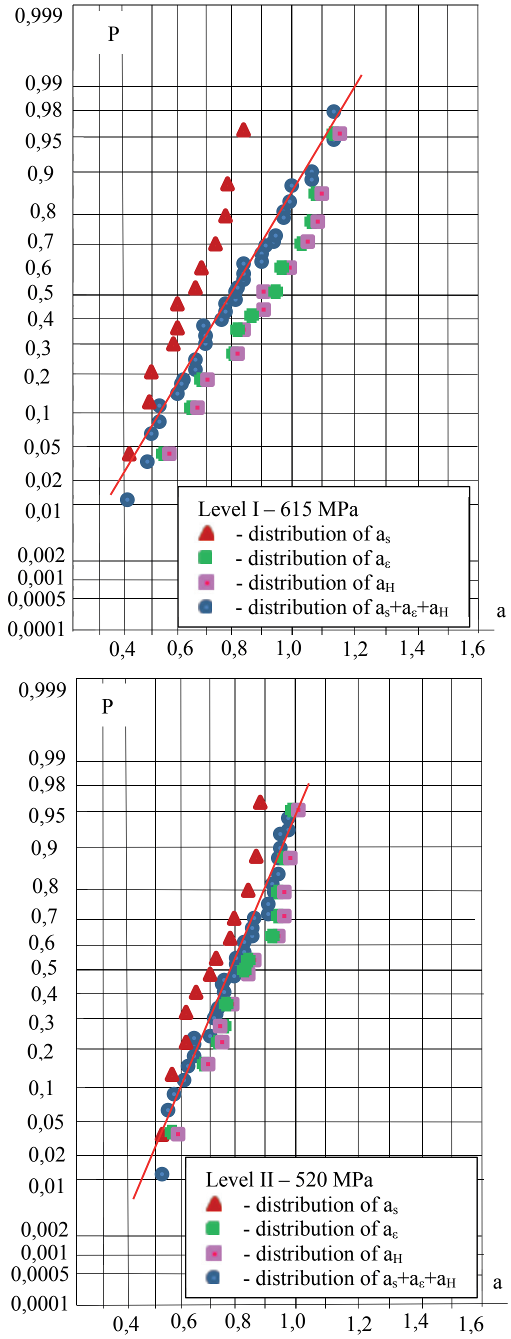
- from 0,44 to 1,15 for Level I ($S_{amax}$ = 615 MPa; $\varepsilon_{acmax}$ = $4{,}24\cdot10^{-2}$),
- from 0,53 to 0,99 for Level II ($S_{amax}$ = 520 MPa; $\varepsilon_{acmax}$ = $2{,}22\cdot10^{-2}$),
- from 0,54 to 1,09 for Level III ($S_{amax}$ = 428 MPa; $\varepsilon_{acmax}$ = $8{,}93\cdot10^{-3}$),
- from 0,4 to 1,25 for Level IV ($S_{amax}$ = 325 MPa; $\varepsilon_{acmax}$ = $3{,}06\cdot10^{-3}$).

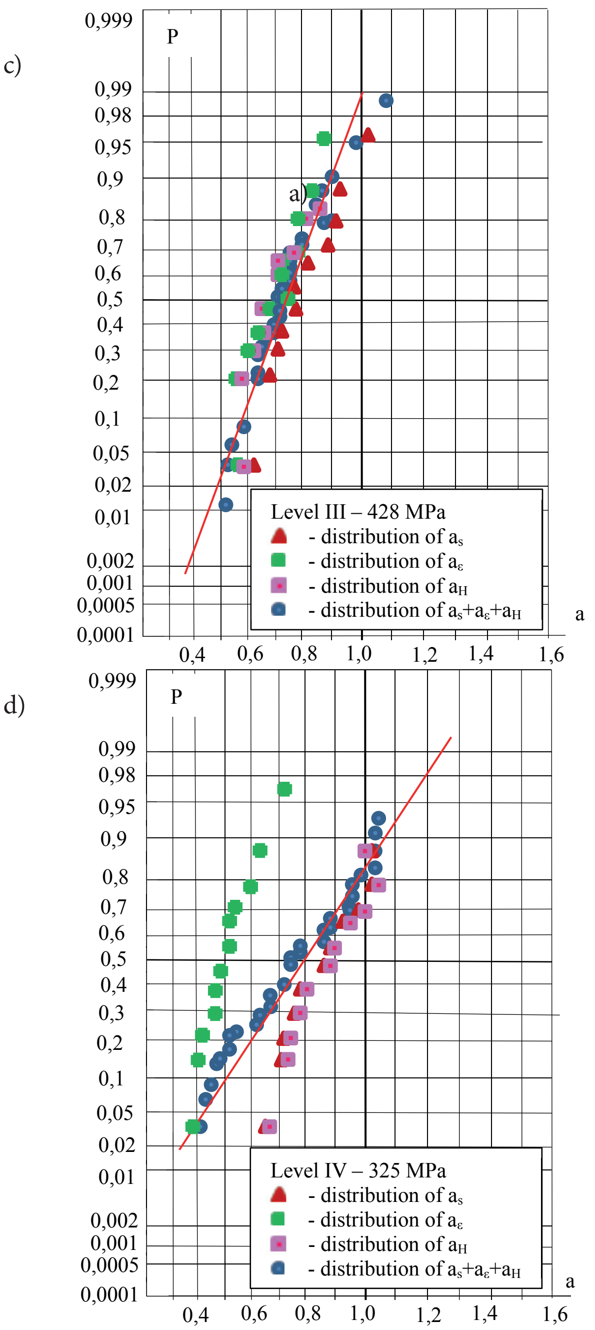
The a-factor values show no tendency to varying in function of the spectrum diagram factor ζ.

Fig. 3 presents distributions of occurrence probability of a-factor values for particular load levels, depicted in normal distribution coordinate grid, whereas parameters of the distributions are given in Tab. 5.

a)

b)






**Pic. 3.** Distributions of occurrence probability of values of the factors : $a_S$, $a_ε$ and $a_H$, and the joint distribution of ($a_S$+ $a_ε$ + $a_H$): a) – for load level I, b) – for load level II, c) – for load level III , d) - for load level IV.

As results from the data in Tab. 5 and the diagrams in Fig. 3 and 4, mean value of the factor a decreases along with stress values increasing , in the range from 0,9283 to 0,658, value of the factor aε increases along with strain values increasing, in the range from 0,53 to 0,8992. Variability of the factor $a_H$ is the lowest and only a little dependent on load level, and comprised in the range from 0,7067 to 0,9286. It shows that the hybrid calculation method is useful in the case when the loading belongs both to LCF and HCF ranges.

As results from the data given in Tab. 5 and the diagrams in Fig. 3 and 4 , the lower limit for values of the factors as, aε and $a_H$ is equal to 0,5 , that proves assumptions made for fatigue calculation procedures ( e.g. FITNET procedures [6]) to be correct.

**Tab. 5.** Parameters of probability distribution of a-factor values for particular load levels

| | Factor $a_i$ | Mean value | Standard deviation | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Level I | $a_s$ | 0,6583 | 0,1262 | -0,1509 | -1,1112 |
| | $a_ε$ | 0,8992 | 0,1815 | -0,4812 | -0,5981 |
| | $a_H$ | 0,9267 | 0,2399 | 0,7908 | 1,5169 |
| | $a=a_s+a_ε+a_H$ | 0,8167 | 0,1997 | 0,0143 | -0,9513 |
| Level II | $a_s$ | 0,7108 | 0,1090 | -0,2816 | -1,3840 |
| | $a_ε$ | 0,8117 | 0,1362 | -0,4382 | -0,9255 |
| | $a_H$ | 0,8183 | 0,1299 | -0,3338 | -1,2250 |
| | $a=a_s+a_ε+a_H$ | 0,7798 | 0,1343 | -0,1429 | -1,065 |
| Level III | $a_s$ | 0,7875 | 0,1289 | 0,8982 | 0,9138 |
| | $a_ε$ | 0,7017 | 0,1249 | 0,7902 | 0,6932 |
| | $a_H$ | 0,7067 | 0,1233 | 0,7514 | 0,7305 |
| | $a=a_s+a_ε+a_H$ | 0,7348 | 0,1283 | 0,7424 | 0,4617 |
| Level IV | $a_s$ | 0,9283 | 0,2615 | 2,1647 | 5,7703 |
| | $a_ε$ | 0,5300 | 0,1556 | 2,3016 | 6,3149 |
| | $a_H$ | 0,9283 | 0,2615 | 2,1647 | 5,7703 |
| | $a=a_s+a_ε+a_H$ | 0,7959 | 0,2987 | 1,2584 | 2,5523 |
| Results of the tests under 10-step load conditions | | | | | |
| | $a=a_s+a_ε+a_H$ | 0,6483 | 0,1077 | -1,1501 | 1,8387 |

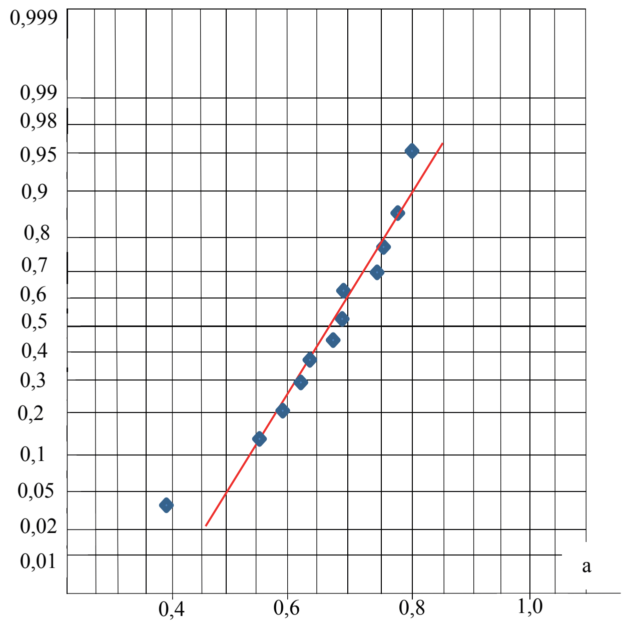



**Fig. 4.** Probability distribution of values of the factor a = $a_s$+$a_ε$+$a_H$ for ten-step load conditions

## Experimental verification of Serensen formula

The Serensen's formula (6) takes into account influence of the spectrum diagram factor ζ and load level on magnitude of the factor a in the modified PM hypothesis. As results from the form of the formula, value of the factor a increases along with the spectrum diagram factor ζ and load level increasing . These relations have not found any confirmation in the test results analyzed in this work. The conclusion is illustrated in Fig .2 where results of the tests and calculation of the factor $a_s$ were used as an example.

As results from Fig. 2, for low values of the factor ζ ( which most often occur in service loads of structural elements) the calculation results for the factor a greatly differ from these experimental, likewise in the case of high values of the factor ζ, where the calculation results of the factor a bring values much greater than experimental ones.

Another disadvantage of the discussed formula (6) is that negative values of the factor a (a < 0) may arise in the case when value of the factor $\zeta < k\frac{S_{f(-1)}}{S_{a\max}}$ , which takes place at low values of $S_{amax}$ (usually occurring in HCF range ).

## A concept of description of magnitude of the correction factor a

The below proposed concept of description of magnitude of the correction factor a in PM linear summation hypothesis of fatigue failures, consists in determining such variability ranges for the factor a, which would make it possible to delineate a domain for safe calculation results.

The empirical formula (19) which describes a lower limiting line for the set of experimental data, was developed on the basis of the test results given in Tab. 3. Values of the factor a, which lay below the limiting line, provide, as a result of calculations, a fatigue life which is lower than that experimentally obtained, therefore this result lies in the safe calculation domain.

The form of the above mentioned formula is as follows:

$$a_s = A \cdot S_{a\max}^{-1} \cdot \zeta^d + a_0 \qquad (19)$$

where the constants: A and d are experimentally determined, whereas value of $a_0$ represents a lower limit for the results predicted on the basis of experimental data or literature sources.

Such data, for the case of tests and calculations according to stress approach, described in this paper, are the following : A = 1,3; d = -4,3 and $a_0$ = 0,5.

Fig. 5 shows a graphical illustration of the formula (19) for the data taken from the calculations and tests of the factor as. Similar analysis may be performed for values of the factors aε and aH. Tab. 6 contains comparison of results of the calculations for the factor as according to the formulae (6) and (19). As results from the comparison, values of the factor as significantly differ both in relation to Samax and the factor ζ.

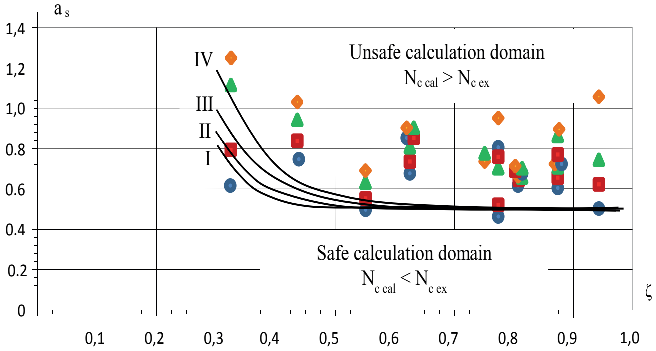



**Fig. 5.** Limiting lines for experimentally determined values of the factor $a_s$

**Tab. 6.** Collected calculation results of the factor $a_s$, according to Eq. (6) and (19)

| No. | $S_{amax}$ | ζ | | | | | Acc. Eq. |
|---|---|---|---|---|---|---|---|
| | | 0,3 | 0,4 | 0,5 | 0,7 | 0,9 | |
| 1. | 615 | 0,18 | 0,29 | 0,41 | 0,64 | 0,88 | (6) |
| 2. | | 0,88 | 0,61 | 0,54 | 0,51 | 0,50 | (19) |
| 3. | 520 | 0,15 | 0,27 | 0,40 | 0,64 | 0,88 | (6) |
| 4. | | 0,95 | 0,63 | 0,55 | 0,51 | 0,50 | (19) |
| 5. | 428 | 0,12 | 0,24 | 0,37 | 0,62 | 0,87 | (6) |
| 6. | | 1,04 | 0,66 | 0,56 | 0,51 | 0,50 | (19) |
| 7. | 325 | 0,03 | 0,17 | 0,31 | 0,59 | 0,86 | (6) |
| 8. | | 1,2 | 0,71 | 0,58 | 0,52 | 0,50 | (19) |

## CONCLUSIONS

1. No unambiguous recommendations concerning modification of LHSUZ hypothesis may be found in literature sources, including procedures for fatigue calculation of ship structures.

2. As results from experimental verification of the linear summation hypothesis of fatigue failures, the original form of the hypothesis (acc. Palmgren and Miner) provides, as a result of calculations, much higher fatigue life than that experimentally determined; such result lies in unsafe fatigue life domain.

3. Effectiveness of fatigue calculations may be improved by implementing the correction factor a in the linear summation hypothesis of fatigue failures. Lack of any method for assessing its magnitude makes its application difficult. The data searched from literature sources in which values of the factor a have been analyzed, are ambiguous, and in the extreme cases they show values from 0,1 to 10.

4. As results from the analysis presented in Subsection 4.2, the Serensen's formula (6), in case of C45 steel, does not comply with the experimental data over the whole variability range of the spectrum diagram factor ζ, and in the extreme case of low ζ - values, value of the factor a lies below zero, which is completely pointless.

5. The concept of delineation of a lower limiting line for the magnitude of the factor a (described in Subsection 4.3) makes it possible to conduct calculations in the safe fatigue life domain by choosing an appropriate value of the correction factor a.

6. Analysis of distributions of a – factor values indicates that the value a = 0,5 which is assumed in the calculation procedures [6], corresponds to a low probability ( P ≤ 0,05) of the exceeding, in calculations, of experimentally determined fatigue life.

7. On the basis of the analysis of the distributions of a - factor values achieved according to stress approach ($a_s$), strain approach ($a_ε$) and hybrid one ($a_H$), respectively, it may be concluded that results of the fatigue life calculations according to the hybrid method are the nearest to experimental test results. Moreover, the calculation results of the factor $a_H$ show the smallest spread and are most close to 1,0 within the whole variability ranges of loads and the spectrum diagram factor ζ.

**Acknowledgements**

This work has been financed by Polish Ministry of Higher Education and Science in the frame of the research project No. 2221/B/T02/2010/39.

## BIBLIOGRAPHY

1. Fricke W., Petershagen H., Paetzold H.: *Fatigue Strength of Ship Structures*, Part I: Basic Principles, Germanischer Lloyd, Hamburg, pp. 34-36, 1997.
2. Fricke W., Petershagen H., Paetzold H.: *Fatigue Strength of Ship Structures*, Part II: Examples. Germanischer Lloyd, Hamburg, pp. 20-21, 1998.
3. Polish Register of Shipping (Polski Rejestr Statków): *Fatigue strength analysis of ship steel hull ( in Polish)*, PRS Publ. No. 45/P, Gdańsk, pp. 35-43, 1998.
4. Szala J., Boroński D.: *Assessment of state of material fatigue in diagnostics of machines and devices ( in Polish)*. Publ. Institute of Operation Technology – PIB, Bydgoszcz – Radom, 2008.
5. Szala J.: *Hypotheses on summation of fatigue failures ( in Polish)*, ATR Publishers (Wydawnictwa Uczelniane ATR), Bydgoszcz, 1998.
6. Kocak M. ( Ed.), Webster S., Janosch J. J., Ainsworth R. A.,Koers R.: *FITNET Fitness-for-Service procedure-Final Drift*. MK7, 2006.
7. Szala G., Ligaj B.: *Comparative analysis of fatigue life calculation methods of C45 steel in conditions of variable amplitude loads in the low- and high- cycle fatigue ranges*, Polish Maritime Research, No 4(76), Vol 19, 2012.
8. Ligaj B.: *Influence of selected load programs generated from correlation table, on fatigue life of 18G2A steel (in Polish)*. Maintenance Problems, 3/2007, Radom, 2007.

## CONTACT WITH THE AUTOR

Grzegorz Szala, Assoc. Prof.
West Pomeranian University of Technology
Faculty of Mechanical Engineering and Mechatronics
7 Kaliskiego Street
85-789 Bydgoszcz
Poland
Email: gszala@utp.edu.pl

**Nomenclature of major notations**

| | |
|---|---|
| $D_{Nc}$ | - fatigue failure corresponding to fatigue fracture, |
| $D_0$ | - fatigue failure resulting from $n_0$ load cycles, |
| a | - correction factor in the LHSUZ – general notation, |
| $a_s$ | - correction factor in the LHSUZ – at stress approach, |
| $a_\varepsilon$ | - correction factor in the LHSUZ – at strain approach, |
| $a_H$ | - correction factor in the LHSUZ –at hybrid approach, |
| k | - number of load steps in a load program, |
| l | - number of load steps in a load program of LCF range, |
| N | - number of load cycles up to fatigue damage under sinusoidal load – general notation (fatigue life), |
| $N_c$ | - fatigue life expressed by number of load cycles, determined in programmed load conditions, |
| $N_{fi}$ | - number of load cycles up to fatigue fracture read from Manson-Coffin fatigue diagram for the total strain $\varepsilon_{aci}$, |
| $N_i$ | - number of load cycles up to fatigue fracture read from Wöhler diagram for the stress amplitude Sai, |
| $N_0$ | - basic number of load cycles corresponding to fatigue limit, |
| $n_0$ | - number of load cycles within the load program period, |
| $n_{01}$ | - number of cycles in 1st step of load program within its period, |
| $n_{0i}$ | - number of cycles in ith step of load program within its period, |
| $R = S_{min}/S_{max}$ | - stress ratio, |
| $R_e$ | - yield stress, MPa, |
| $R_m$ | - tensile stress , MPa, |
| S | - stress – general notation, MPa, |
| $S_{max}$ | - maximum stress in sinusoidal load cycle, MPa, |
| $S_{min}$ | - minimum stress in sinusoidal load cycle, MPa, |
| $S_a = 0,5(S_{max}-S_{min})$ | - stress amplitude in sinusoidal load cycle, MPa, |
| $S_m = 0,5(S_{max}+S_{min})$ | - mean stress in sinusoidal load cycle, MPa, |
| $\varepsilon_{ac}$ | - total strain, |
| $\varepsilon_{mc}$ | - mean strain, |
| $\zeta$ | - spectrum diagram factor, |
| $\lambda$ | - number of program period repetitions up to fatigue fracture, |
| LCF | - low-cycle fatigue, |
| HCF | - high-cycle fatigue, |
| $S_{f(-1)}$ | - fatigue limit determined under oscillatory sinusoidal load conditions (R = -1), |
| A, d, a | - constants in Eq. (19). |