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LATITUDE ERROR IN COMPASS DEVIATION

MATHEMATICAL METHOD TO DETERMINE THE LATITUDE ERROR IN MAGNETIC COMPASS DEVIATION

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ABSTRACT

This article aims to provide the seafarer with a tool to calculate the deviation for a righted ship in any geographical position using only the information available on board. In this way the accidental errors in the deviation card are reduced and the latitude error is made negligible. Moreover, an experimental application of this method is carried out on board a tanker to compare the latitude error in different positions at sea.

Keywords: optimal exact coefficients, average deviation, latitude error

INTRODUCTION

Today the magnetic compass is a secondary navigation system compared to other electronic or electro-mechanic compasses on board ships. However, the magnetic compass relies solely on the existence of a magnetic field and, therefore, has a advantage over the gyroscopic or satellite compasses: it works without the help of a source of energy. That is the reason why the magnetic compass is part of the necessary navigation equipment on board the current merchant ships according to SOLAS (Safety of Life at Sea) Convention.

When the magnetic compass is located in a place isolated from magnetic materials, the needle is guided only by the earth's magnetic field B_E which is split up into the horizontal and vertical components represented by the symbols B_{EH} and B_{EZ}, respectively, traditionally known as H and Z. The compass' needle will follow the component B_{EH} parallel to the magnetic meridian and the angle between the needle and the geographical meridian is called magnetic declination. If the magnetic compass is placed on board a ship, the steel of which she is built will act as a magnet creating a ship's magnetic field which makes the needle separate from the magnetic meridian an angle, called deviation. As a result, the magnetic compass course has to be corrected by the magnetic declination and deviation to obtain the true course that is parallel to the geographical meridian.

Given that the component B_{EH} changes intensity and direction depending on the geographical coordinates of the place where the compass is located, the magnetic declination will be altered when the ship is moving along the whole earth's surface. In the same way, the changes in the earth's magnetic field will affect the magnetism of the ship's steel and the deviation angle will not be the same in different positions at sea. The variations in the magnetic declination are obtained easily from nautical charts or IGRF (International Geomagnetic Reference Field) models (Zmuda 1971). However, the deviation has to be reduced by the well-known practical method called compass' adjustment since; unlike the magnetic declination, it also varies with the direction of the bow. The reduced deviations obtained after the adjustment are registered for various courses in a sheet called magnetic compass table (also deviation card) that is placed in the bridge close to the chartroom. However, the actual deviations may change from those registered in the table when the ship moves from one port to another and it is not usual for deck officers to correct this [10]. We refer to this error as latitude error or variation in deviation. Obviously, this error will increase as the ship gets closer to the magnetic poles, whose current geographical coordinates are 85° 18' N & 136° 29.4' W for North Pole and 64° 24.4' S & 137° 12.3' E for South Pole (on date 01.01.2011) [13].

The aim of this article is to develop a mathematical method to obtain the deviations in any geographical position correcting in this way the latitude error. In addition the deviations from the magnetic compass table will be compared graphically to those obtained by this method at different positions. The data for the comparison were collected from a 150,000 dwt crude tanker named "MONTE TOLEDO".

CALCULATING THE OPTIMAL EXACT COEFFICIENTS FOR A RIGHTED SHIP

The exact deviation for a righted ship (δ) may be calculated by the well-known equation of Archibald Smith and Evans [4]:

$$\tan\delta = \frac{A'+B'\cdot\sin\zeta + C'\cdot\cos\zeta + D'\cdot\sin 2\zeta + E'\cdot\cos 2\zeta}{1+B'\cdot\cos\zeta - C'\cdot\sin\zeta + D'\cdot\cos 2\zeta - E'\cdot\sin 2\zeta} \quad (1)$$

The symbols ζ and ζ' denote the magnetic and compass course, respectively, whereas A', B', C', D' and E' indicate in this article the exact coefficients that were expressed originally by Archibald-Smith as $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\mathfrak{E}$.

This equation may be also expressed as follows [3]:

$$\sin\delta = A'\cdot\cos\delta + B'\cdot\sin\zeta' + C'\cdot\cos\zeta' + D'\cdot\sin(2\zeta'+\delta) + E'\cdot\cos(2\zeta'+\delta) \quad (2)$$

On the other hand, the formula in (3) lets us calculate the deviation in degrees by a simple way although various trigonometric estimations turned it into a rough equation. Nevertheless, this approximate deviation (Δ) is normally used to adjust the magnetic compass [2].

$$\Delta = A + B\cdot\sin\zeta' + C\cdot\cos\zeta' + D\cdot\sin 2\zeta' + E\cdot\cos 2\zeta' \quad (3)$$

The symbols A, B, C, D and E are known as approximate coefficients and their values match up with the sine of the exact coefficients. Both coefficients may be considered constant for a long time. However this may not always be the case, since a bolt of lightning or a shipment of steel cargo may affect the ship magnetism [8].

If the deviation were very high (more than five degrees), it should be reduced to near zero by moving the correctors set in the binnacle. The compass adjuster is the specialized person who carries out this work. He draws up a magnetic compass table where the reduced deviations are registered for n courses. The practical method for evaluating the deviation is based on the execution of a complete compass swing circulation [7]. Nevertheless there are also other simpler methods to calculate the deviation even at single any course [11]. The magnetic and compass bearings are usually compared on 24 or 36 equidistant courses (each 15^0 or 10^0 degrees respectively). Then, by using the traditional formula in (4), the deviation (δi) is obtained for each individual course.

$$\delta_i = \zeta_i - \zeta_i', i \in \overline{1,n} \quad (4)$$

Therefore, the vectors of the magnetic course (ζi), the compass course (ζi') and the deviation sine, where i = 1 to n, can be created as it shown in (5). The subscript i denote the course to which the deviation is obtained.

$$\zeta_i = \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \vdots \\ \zeta_n \end{pmatrix} \quad \zeta_i' = \begin{pmatrix} \zeta_1' \\ \zeta_2' \\ \zeta_3' \\ \vdots \\ \zeta_n' \end{pmatrix} \quad D(i) = \begin{pmatrix} \sin\delta_1 \\ \sin\delta_2 \\ \sin\delta_3 \\ \vdots \\ \sin\delta_n \end{pmatrix} \quad (5)$$

Thus the equation in (2) may be written in matrix form as follows:

$$D(i) = K\cdot T_n(i) \quad (6)$$

Where T_n(i) and K have the forms in (7) and (8) respectively:

$$T_n(i) = \begin{pmatrix} \cos\delta_1 & \sin\zeta_1' & \cos\zeta_1' & \sin(\zeta_1'+\delta_1) & \cos(\zeta_1'+\delta_1) \\ \cos\delta_2 & \sin\zeta_2' & \cos\zeta_2' & \sin(\zeta_2'+\delta_2) & \cos(\zeta_2'+\delta_2) \\ \cos\delta_3 & \sin\zeta_3' & \cos\zeta_3' & \sin(\zeta_3'+\delta_3) & \cos(\zeta_3'+\delta_3) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \cos\delta_n & \sin\zeta_n' & \cos\zeta_n' & \sin(\zeta_n'+\delta_n) & \cos(\zeta_n'+\delta_n) \end{pmatrix} \quad (7)$$

$$K = \begin{pmatrix} A' \\ B' \\ C' \\ D' \\ E' \end{pmatrix} \quad (8)$$

CORRECTING THE DEVIATION OBTAINED BY THE SHIP'S SWING CIRCULATION

Frequently the readings of courses during the swing circulation are not very exact and consequently the deviations obtained from them and registered in the magnetic compass table may be erratic. This may be due to mistakes made by the adjuster, to Gaussin errors, to magnetic cargo carried on board, electrical devices, hoisting booms, magnetic objects near the binnacle or to any other reason that may accidentally affect the compass. Therefore the corresponding deviation curve becomes very rough and has to be smoothed by the application of least squares method [1]. The optimal exact coefficients (K) are calculated in (9) by means of the least squares method to reduce the possible deviation errors observed during the compass swing circulation [5]. All of these coefficients are constant except B' and C' which change with the geographical position.

$$K = \left[T_n^T(i)\cdot T(i)\right]^{-1}\cdot T^T(i)\cdot D(i) \quad (9)$$

Once the optimal exact coefficients are calculated, an average deviation is obtained by the formula in (6). Figure 1 shows the difference between the average deviation calculated by least squares method (soft curve) and the deviation collected from the magnetic compass table (rough curve) on board M/T "MONTE TOLEDO".

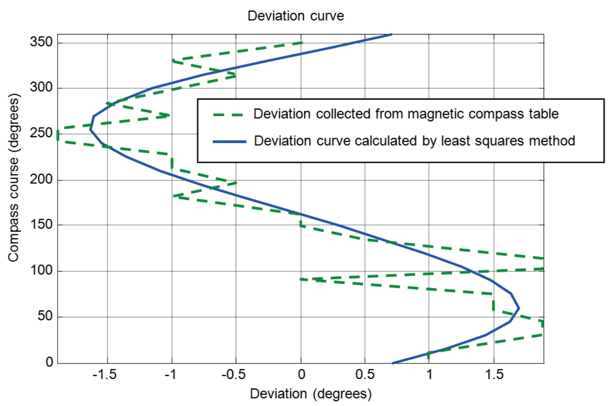

Fig. 1. *Difference between average deviation (blue continuous curve) and magnetic table deviation (green discontinuous curve) on board M/T "MONTE TOLEDO".*

PERMANENT AND INDUCED MAGNETISM IN RIGHTED SHIP

In order to study the magnetism on board, a righted ship may be plotted into a Cartesian coordinate system where the origin is placed in the centre of the rose, the axis X is on fore-and-aft line and Y on the starboard-and-port line (both on the plane of horizon), and Z on the zenith-and-nadir (vertical) line. The ship magnetism may be divided into the permanent magnetism caused by the ship's hard irons and the induced magnetism created by the ship's soft irons. The permanent magnetism (B_p) may be expressed by the vector in (10) where B_{PX}, B_{PY} and B_{PZ} are its components in the X, Y and Z axis, respectively, which have been called traditionally P, Q and R. The formula in (11) indicates the induced magnetism (B_I) where the parameters B_{EX} and B_{EY} are the components of B_{EH} in the X and Y axis, respectively, and the matrix χ indicates the susceptibility tensor of the induced magnetism, assuming the ship has an anisotropic susceptibility [9].

$$B_P = \begin{pmatrix} B_{PX} \\ B_{PY} \\ B_{PZ} \end{pmatrix} \quad (10)$$

$$B_I = \chi \cdot \begin{pmatrix} B_{EX} \\ B_{EY} \\ B_{EZ} \end{pmatrix} = \begin{pmatrix} \chi_{x,x} & \chi_{x,y} & \chi_{x,z} \\ \chi_{y,x} & \chi_{y,y} & \chi_{y,z} \\ \chi_{z,x} & \chi_{z,y} & \chi_{z,z} \end{pmatrix} \cdot \begin{pmatrix} B_{EX} \\ B_{EY} \\ B_{EZ} \end{pmatrix} \quad (11)$$

If the ship is up righted, the vertical component B_{PZ} and the coefficients $\chi_{z,x}$, $\chi_{z,y}$ and $\chi_{z,z}$ will not have an influence in the deviation. They are only noted as an additional deviation (heeling deviation) in sailing vessels with a permanent list and as compass' needle oscillations in motor ships navigating in rough seas with large rolling movements.

DETERMINING B'&C' COEFFICIENTS AND THE DEVIATION WHEN THE SHIP'S POSITION CHANGES

The exact coefficients may be expressed by the horizontal components belonging to the earth's field and permanent magnetism, as well as the horizontal coefficients from the susceptibility tensor representing the induced magnetism, λ being the shielding factor.

$$A' = \frac{\chi_{x,y} - \chi_{x,y}}{2 \cdot \lambda} \qquad B' = \frac{1}{\lambda \cdot B_{EH}} \cdot \left(B_{PX} + \chi_{x,z} \cdot B_{EZ}\right)$$

$$C' = \frac{1}{\lambda \cdot B_{EH}} \cdot \left(B_{PY} + \chi_{y,z} \cdot B_{EZ}\right) \qquad D' = \frac{\chi_{x,x} - \chi_{y,y}}{2 \cdot \lambda} \quad (12)$$

$$E' = \frac{\chi_{y,x} + \chi_{x,y}}{2 \cdot \lambda}$$

As can be seen in (12) only the coefficients B' and C' depend on B_{EX} and B_{EY}, which indicates that the value of these coefficients will change for different positions along the earth's surface due to the variations in the components of earth's magnetism. Nevertheless, the rest of the coefficients will remain constant in spite of these changes. This means that the values of B_{EX}/λ, B_{EY}/λ, $\chi_{x,z}/\lambda$ and $\chi_{y,z}/\lambda$ have to be obtained independently from each position in order to proceed to the calculation of the different coefficients B' and C'.

The values $\chi_{x,z}$ and $\chi_{y,z}$ belonging to the induced magnetism may be calculated by the traditional formulae in (13) and (14) [12] where the coefficients B_2' and C_2' are obtained respectively by steering the ship to the North or South and to the East or West in a position where the components of the earth's magnetism (B_{EH}" and B_{EZ}") are quite different from the same components in the place where the coefficients B' and C' were obtained (B_{EH} and B_{EZ}).

$$\chi_{x,z} = \lambda \cdot \frac{B_{EH} \cdot \sin B' - B_{EH}'' \cdot \sin B_2'}{B_{EZ} - B_{EZ}''}$$
$$\frac{\chi_{x,z}}{\lambda} = \frac{B_{EH} \cdot \sin B' - B_{EH}'' \cdot \sin B_2'}{B_{EZ} - B_{EZ}''} \quad (13)$$

$$\chi_{y,z} = \lambda \cdot \frac{B_{EH} \cdot \sin C' - B_{EH}'' \cdot \sin C_2'}{B_{EZ} - B_{EZ}''}$$
$$\frac{\chi_{y,z}}{\lambda} = \frac{B_{EH} \cdot \sin C' - B_{EH}'' \cdot \sin C_2'}{B_{EZ} - B_{EZ}''} \quad (14)$$

Later, the values of B_{PX}/λ and B_{PY}/λ may be calculated using the formulae in (15) and (16):

$$\frac{B_{PX}}{\lambda} = B_{EH} \cdot B' - \frac{\chi_{x,z}}{\lambda} \cdot B_{EZ} \quad (15)$$

$$\frac{B_{PY}}{\lambda} = B_{EH} \cdot C' - \frac{\chi_{y,z}}{\lambda} \cdot B_{EZ} \quad (16)$$

Once the value of B_{PX}/λ, B_{PY}/λ, $\chi_{x,z}$ and $\chi_{y,z}$ are calculated, the value of the new optimal exact coefficients B' and C' at any other position may be obtained by the application of formulae in (12). Then the deviation at any geographical position can be calculated by executing repeatedly the equation in (2) within a do-while iterative block, until the difference between both deviation values of the equation is reduced to near zero. The approximate deviation (Δ) obtained from (3) may be used as first value to be evaluated in the block.

APPLICATION ON BOARD A SHIP

In this section the mathematical method to obtain the deviation is put into practice on board a 150,000 dwt crude tanker. The deviation calculated for the 1st of January 2011 in different geographical positions is compared to the deviation at the position where the compass was adjusted the last time (27^0 N & 091^0 W). The values of B_{EH} and B_{EZ} are determined by using the on-line calculator based on the 11th generation IGRF model [6]. The compass is located 20 meters above the sea in order to input the data required by this calculator. In any case, a difference of 20 meters in height only gives an error of less than one thousandth gauss in the geomagnetic components.

The figures 2 and 3 show a geographical map in three-dimensional form concerning the components B_{EH} and B_{EZ} on the 1st of January 2011, respectively. Their values have been obtained from a geomagnetic model and local magnetic anomalies may be not shown. This occurs, for example, in the Baltic Sea where the mathematical model of the Earth's magnetic field does not consider local magnetic anomalies [14]. As can be seen in these figures, the values of these components are not only changing with the geographical latitude but also with the geographical longitude and, although the latitude affects in a greater degree, the term "latitude error" would not be considered absolutely correct. This is partly due to the variation of longitude is caused by the slight inclination of the magnetic equator with respect to the geographical equator. Another cause has to do with the fact that the earth's magnetic field is non-homogeneus.

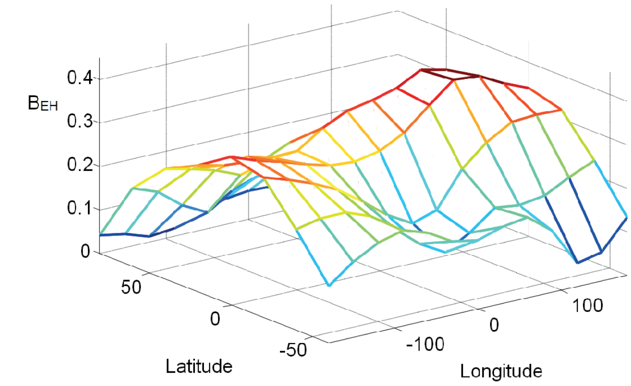

Fig. 2. Value of B_{EH} component (in gauss) on 01.01.2011.

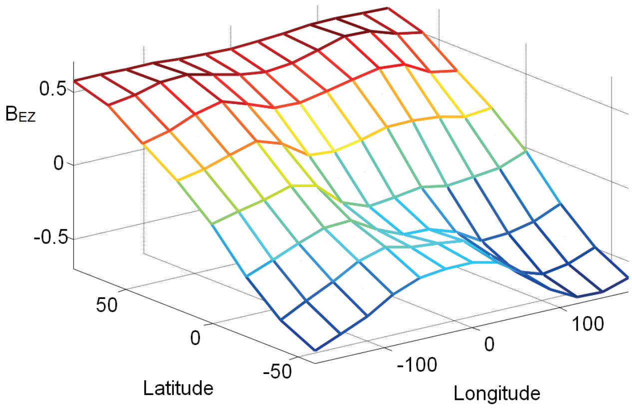

Figure 3. Value of B_{EZ} component (in gauss) on 01.01.2011.

Table 1 shows the optimal exact coefficients that result from the application of the least squares method, as well as the components of the coefficients B' and C' obtained by changing the ship's position in accordance with the method defined in section 4. As far the calculation of these components, the ship navigated a long passage to vary the latitude around 25^0 until she reached a difference close to 0.10 and 0.40 gauss in the earth's components B_{EH} and B_{EP}, respectively. The low value of the coefficient A' indicates that the binnacle is properly installed at centreline, which means that the North of the verge ring and the fore-and-aft line coincide. The existence of the E' coefficient suggests that the ship contains unsymmetrical arrangements of horizontal soft iron. On the other hand, the higher values of B' and C' indicate that the deviation curve is mainly semicircular, as shown in figure 1.

Table 1. B_{PX}, B_{PY}, $\chi_{x,z}$ and $\chi_{y,z}$ parameters and optimal exact coefficients of "MONTE TOLEDO" at the last place of compass adjustment. Date: 01.01.2011.

Parameters	Optimal exact coefficients
$\chi_{x,z}/\lambda = 0{,}0143$	$A' = 0{,}0007$
$\chi_{y,z}/\lambda = -0{,}0044$	$B' = 0{,}0271$
$B_{PX}/\lambda = 0{,}0012$	$C' = 0{,}0100$
$B_{PY}/\lambda = 0{,}0042$	$D' = 0{,}0017$
	$E' = 0{,}0018$

Figures 4 to 12 show the deviation curve at the place of the last adjustment (in blue colour) and at other positions (in red colour). These curves allow us to compare how the deviation is changing throughout all the navigational waters of the earth's surface when the ship changes her geographical position. It should be noted that the calculations were carried out on 1st January 2011.

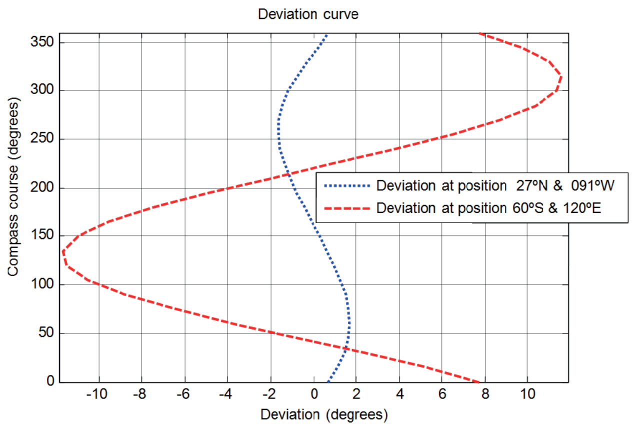

Fig. 4. *Deviations comparative at 600S & 1200E*

According to figure 4 the variation in deviation may reach a little more than 12^0 when the ship is navigating at geographical position 60^0S and 120^0E. Bearing in mind that, in this case, the ship is sailing nearest to the magnetic pole (only around 680 miles away), this variation should be the highest in the test.

This seems to be confirmed in figures 5 and 6 (position 40^0S) since the maximum variation in deviation is reduced to only 4^0. On the other hand, both deviation curves are out of phase (nearly 180^0) in these figures, which makes the variation greater.

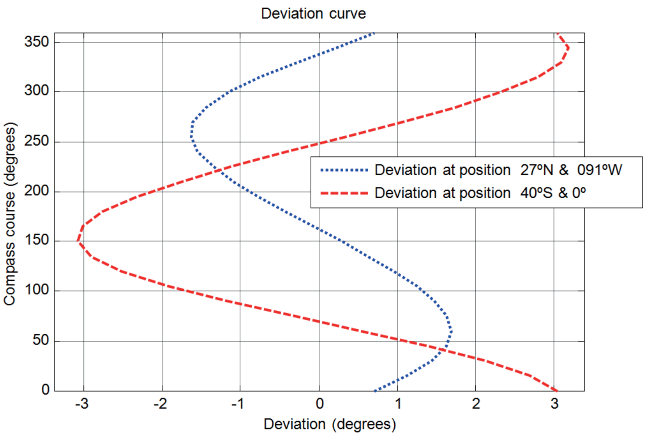

Fig. 5. Deviations comparative at 40^0S & 0^0

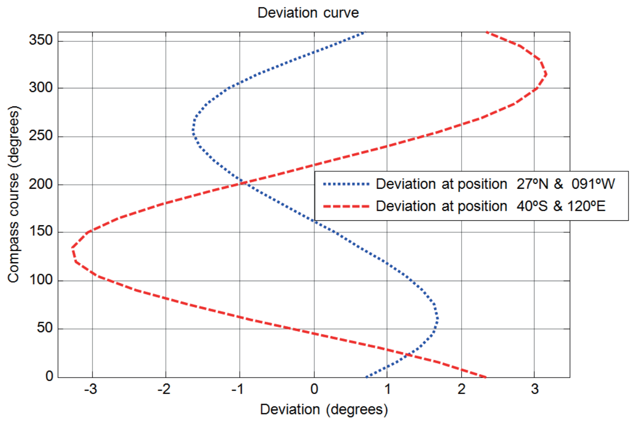

Fig. 6. Deviations comparative at 40^0S & 120^0E

As the ship gets nearer the geographical equator, the variation in deviation is gradually reduced to slightly less than 2^0 at geographical latitude 20°S and 1^0 at the geographical equator (figures 7 and 8). Moreover, it is also noted that the phases of both curves are approaching.

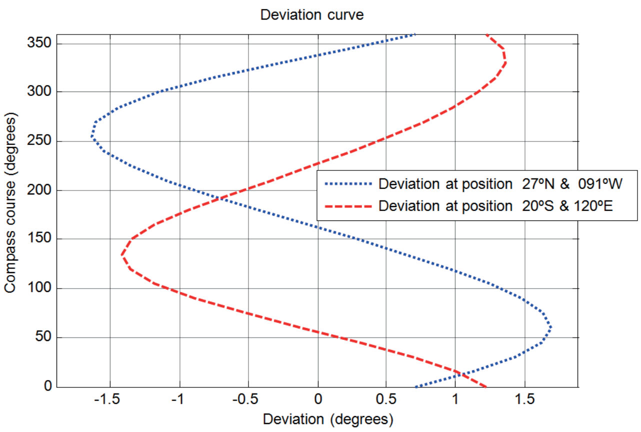

Fig. 7. Deviations comparative at 20^0S & 120^0E

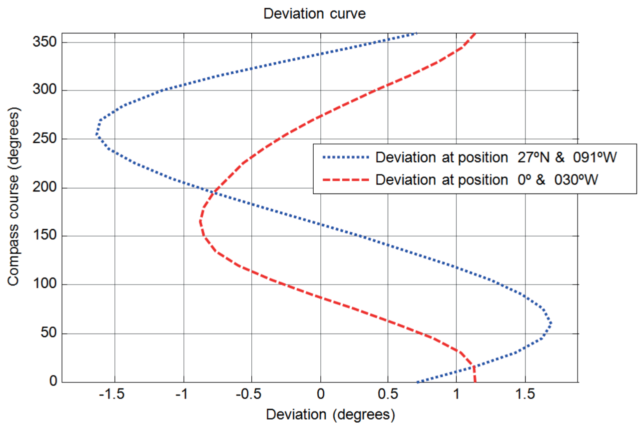

Fig. 8. Deviations comparative at 0^0S & 30^0W

In figures 9 and 10 the maximum variation is less than 1^0 due to the fact that the ship is near the position where the magnetic compass was last adjusted. By comparing both figures, it emerges that the curves in figure 10 are nearer than in figure 9 notwithstanding that the distance between positions in figure 10 is longer than in figure 9. This is due to the longitude influence, since the difference of the earth's magnetic components between the red and blue curves in figure 9 (B_{EH} = 0,04 and B_{EZ} = 0,17 gauss) are higher than in figure 11 (B_{EH} = 0,02 and B_{EZ} = 0,05 gauss).

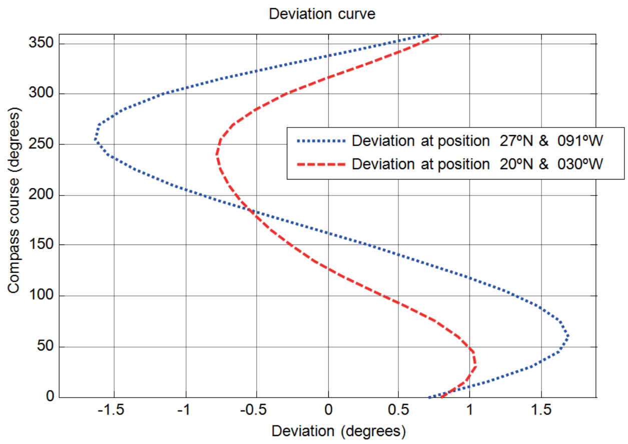

Fig. 9. Deviations comparative at 20^0N & 30^0W

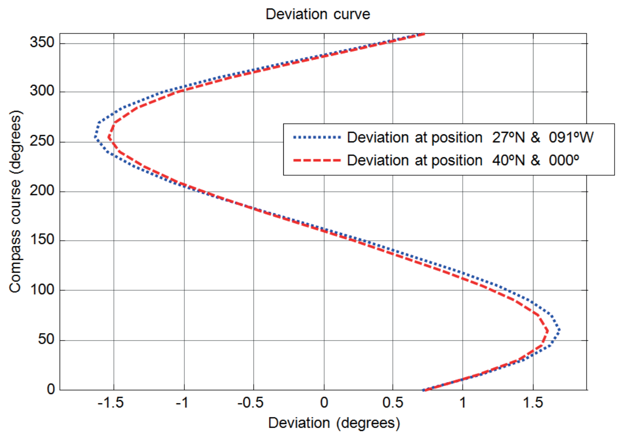

Fig. 10. Deviations comparative at 40^0N & 000^0

In figure 12 the variation in deviation reaches only 6^0 despite the fact that the ship is sailing in geographical latitude near the North Pole. This variation is not as high as in figure 4 due to the geographical longitude difference is near to 180^0. Obviously, the maximum variation in deviation in figure 11 is only near to 3^0 since the latitude is not so near the pole's geographical latitude as in figure 12.

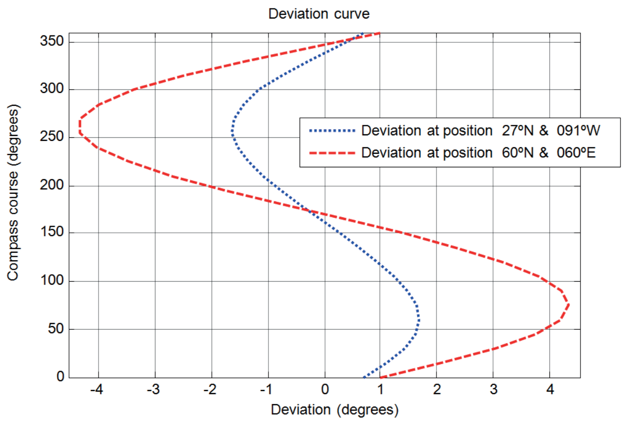

Fig. 11. Deviations comparative at 60^0N & 60^0E

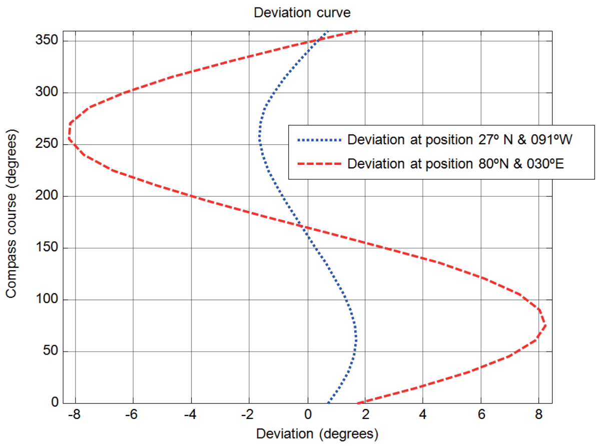

Fig. 12. Deviations comparative at 80^0N & 30^0E

CONCLUSIONS

The mathematical method presented in this article is easily programmable, and may help ship officers carry out a program of latitude error calculation in their computers. It is only necessary to be in possession of the last magnetic compass table and the deviation for the cardinal compass courses at places where the earth's magnetic components have changed considerably.

According to the results of the experimental application, the latitude error for a properly installed magnetic compass may reach 12^0 when the ship is 600 - 700 miles away from the magnetic pole. But it decreases rapidly when the ship strays from this point. Thus the error is less than 4^0 for the zone between parallels 60^0N and 40^0S covering the most of navigational waters. Moreover, it is also important to take into account the position of the port where the compass was last adjusted, since the latitude error in deviation depends on the distance to this port. In this sense, it is preferable that the compass adjustment would be carried out in a port placed at mid-latitude of waters where the ship usually navigates.

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