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SENSITIVITY OF THE GAME CONTROL OF SHIP IN COLLISION SITUATIONS

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ABSTRACT

The paper introduces the application of the theory of deterministic sensitivity control systems for sensitivity analysis taking place in game control systems of moving objects, such as ships. The sensitivity of parametric model of game ship control process and game control in collision situations - sensitivity to changes in its parameters have been presented. First-order and k-th order sensitivity functions of parametric model of the process and game control are described. The structure of the game ship control system in collision situations and the mathematical model of game control process in the form of state equations are given. Characteristics of sensitivity functions of the model and game ship control process on the base of computer simulation in Matlab/Simulink software have been presented. At the end are given proposals regarding the use of sensitivity analysis to practical synthesis of computer-aided system navigator in potential collision situations.

Keywords: marine transport; safety of navigation, game control, sensitivity analysis; computer simulation

INTRODUCTION

One of the most important problems arising in design of a control system consists in predicting its probable properties. The main method of such prediction is the use of physical and mathematical models. A sufficiently complete mathematical model of a system must reflect its basic properties, including possible dependence on parameters ; it will be called parametric model of the system. A determining set of parameters is called complete if it uniquely determines all state variables of the model under investigation [1,7,8,13].

By sensitivity of control systems one usually means dependence of their properties on variation of parameters. Sensitivity theory became an independent scientific branch of cybernetics and control theory in the sixties. In major part this was connected with quick development of self-tuning systems that were constructed for effective operation under parametric disturbances [10,12,26].

Lately, sensitivity theory methods were widely used for solving various theoretical and applied problems dealing with analysis and synthesis, identification, adjustment, monitoring, testing, tolerance distribution [16,18,19,29,30].

At the same , distinction is made between the sensitivity of the model control process for changing its parameters and process optimal control sensitivity to changes in its parameters and disturbance influence [23,25,27].

The previous papers dealing with sensitivity of deterministic systems did not concern game systems [22,31].

At sea, land and air transport processes both own object and many encountered objects are involved. Control of such processes, due to the high proportion of human subjectivity in the decision-making manoeuvre, often takes the character of game control [2,3,17,20].

SENSITIVITY FUNCTIONS OF CONTROL SYSTEMS

The ship game control process as a MIMO process is described by: state equation (1), constraints of state and control (2) and quality control index (3).

$$\dot{x}(t) = \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t), t] \quad (1)$$

$$\boldsymbol{g}[\boldsymbol{x}(t), \boldsymbol{u}(t), t] \le 0 \quad (2)$$

$$\boldsymbol{I} = \int_{t_o}^{t_K} \boldsymbol{f_o}[\boldsymbol{x}(t), \boldsymbol{u}(t), t]dt \quad (3)$$

where:
x(t) – state variable,
u(t) – control variable,
t – time.

The simplest method of sensitivity analysis consists in numerical investigation of system parametric model over the whole range of variation of the determining set of parameters [31].

The main investigation method in sensitivity theory consists in using so called sensitivity functions.

Let $a_1, ..., a_m$ be a set of parameters constituting a complete set a.

Moreover, let function of state variables $F[x_i(t, a)]$ $(i=1, ..., n)$ and quality index control I be single valued functions of control variable u_k [25].

The following partial derivatives:

$$s_a^{pm} = \frac{\partial F[x(t, a)]}{\partial a} \quad (4)$$

$$s_x^{oc} = \frac{\partial I[x(u)]}{\partial x} \quad (5)$$

are called first-order sensitivity functions of the parametric process model s_a^{pm} and sensitivity functions of optimal control of the process s_x^{oc}. In automatic control literature first-order sensitivity functions are often called simply "sensitivity functions" [25].

Theoretically, we can also consider features of the sensitivity functions k-th order of parametric model $s_{k,a}^{pm}$ and sensitivity functions of optimal control $s_{k,x}^{oc}$ in the following form:

$$s_{k,a}^{pm} = \frac{\partial^k F[x(t,a)]}{\partial a_1^{k_1} ... \partial a_m^{k_m}} \qquad k_1 + ... + k_m = k \quad (6)$$

$$s_{k,x}^{oc} = \frac{\partial^k I[x(u)]}{\partial x_1^{k_1} ... \partial x_n^{k_m}} \qquad k_1 + ... + k_m = k \quad (7)$$

GAME SHIP CONTROL SYSTEM

The structure of the game control system of own ship in collisions situations at sea is presented in Fig. 1.

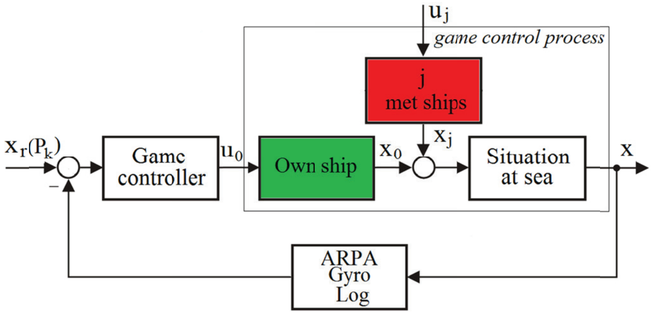

Fig. 1. The structure of game ship control system in collision situations: $\mathbf{x}_r(P_k)$– reference trajectory of own ship to the closest point of return P_k, $\mathbf{u}_0$ – control of own ship, $\mathbf{u}_j$ – control of j-th met ship, $\mathbf{x}_0$ – state variables of own ship, $\mathbf{x}_j$ – state variables of j-th met ship, $\mathbf{x}$ – state variables of game control process

The current state of the control process is determined by the co-ordinates of the own ship's position and the positions of the encountered j ships (Fig. 2):

$$x_0 = (X_0, Y_0) \quad (8)$$

$$x_j = (X_j, Y_j) \qquad j = 1, 2, ..., m \quad (9)$$

The system generates its control at the moment t_k on the basis of data received from the ARPA anti-collision system pertaining to the positions of the encountered ships:

$$x(t_k) = \begin{bmatrix} x_0(t_k) \\ x_j(t_k) \end{bmatrix} \quad j = 1, 2, ..., m \qquad k = 1, 2, ..., K \quad (10)$$

The constraints for the state co-ordinates:

$$\{ x_0(t), x_j(t) \} \in P \quad (11)$$

are navigational constraints, while control constraints:

$$u_0 \in U_0, \, u_j \in U_j \quad j = 1, 2, ..., m \quad (12)$$

take into consideration: the ships' movement kinematics, recommendations of the COLREG Rules and the condition to maintain a safe passing distance Ds satisfying the relationship:

$$D_{\min}^j = \min D_j(t) \ge D_s \quad (13)$$

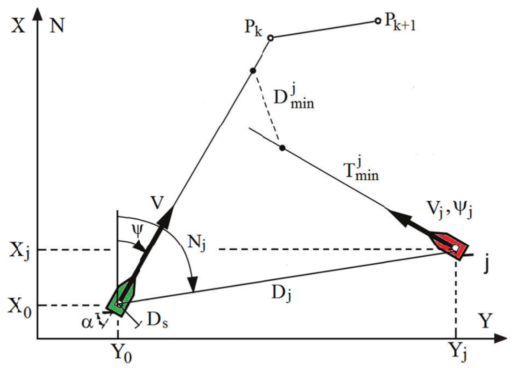

Fig. 2. Situation of relative motion of own ship and j-th met ship: V, ψ speed and course of own ship, a rudder angle of own ship, D_s – safe distance, N_j, D_j – bearing and distance to j-th ship, V_j, ψ_j - speed and course of j-th ship, D^j_{min}, T^j_{min} - min. approach distance and time of the own ship to j-th ship

GAME SHIP CONTROL PROCESS MODEL

Taking into consideration the equations of the own ship hydromechanics and equations of the own ship relative movement to the j-th encountered ship, the state process equations take the following form:

$$\left.\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_1 x_2 x_3 + a_2 x_3 |x_3| x_4 + a_3 x_3 |x_3| u_1 \\
\dot{x}_3 &= a_4 x_3 |x_3| \|x_4| x_4 + a_5 x_3 |x_3| x_4^2 + a_6 x_2 x_3 x_4 |x_4| + \\
&\quad + a_7 x_2 x_3 x_4 |x_4| + a_8 x_3 |x_3| + a_9 x_5 |x_5| x_6 + a_{10} x_3 |x_3| x_4 u_1 \\
\dot{x}_4 &= a_4 x_3 x_4 + a_5 x_3 x_4 |x_4| + a_6 x_2 x_4 + a_{11} x_2 + a_{10} x_3 u_1 \\
\dot{x}_5 &= a_{12} x_5 + a_{13} u_2 \\
\dot{x}_6 &= a_{14} x_6 + a_{15} u_3 \\
\dot{x}_{6+j} &= -x_3 + x_{7+j} x_2 + x_{9+j} \cos x_{8+j} \\
\dot{x}_{7+j} &= -x_2 x_{6+j} + x_{9+j} \sin x_{8+j} \\
\dot{x}_{8+j} &= a_{15+j} x_2 x_{9+j} u_{j1} \\
\dot{x}_{9+j} &= a_{16+j} x_{8+j} |x_{8+j}| + a_{17+j} u_{j2}
\end{aligned}\right\} \quad (14)$$

The state variables are represented by the following values:

$x_1 = \psi$ - course of the own ship,

$x_2 = \dot{\psi}$ - angular turning speed of the own ship,

$x_3 = V$ - speed of the own ship,

$x_4 = \beta$ - drift angle of the own ship,

$x_5 = n$ - rotational speed of the screw propeller of own ship,

$x_6 = H$ - pitch of the adjustable propeller of the own ship,

$x_{6+j} = X_j$ - x coordinate of the *j*-th ship position,

$x_{7+j} = Y_j$ - y coordinate of the *j*-th ship position,

$x_{8+j} = N_j$ - bearing of the *j*-th ship or Q_j – relative meeting angle,

$x_{9+j} = D_j$ - distance to *j*-th ship.

While the control values are represented by:

$u_1 = \alpha_r$ - reference rudder angle of the own ship,

$u_2 = n_r$ - reference rotational speed of the own ship's screw propeller,

$u_3 = H_r$ - reference pitch of the adjustable propeller of the own ship,

$u_{j1} = \psi_j$ - course of the *j*-th ship,

$u_{j2} = V_j$ - speed of the *j*-th ship [11].

SENSITIVITY FUNCTION OF GAME SHIP CONTROL PROCESS MODEL

As a function of state process variables $F[x(t,a)]$ is assumed the risk of collision r_j in the following form:

$$r_j(x) = \left\{ w_1 \left[\frac{D^j_{min}(x)}{D_s} \right]^2 + w_2 \left[\frac{T^j_{min}(x)}{T_s} \right] \right\}^{-\frac{1}{2}} \quad (15)$$

where:

w_1, w_2 – weight coefficients depending on the visibility at sea and dynamic length and breadth of the met *j*-th ship [4,5,6].

The sensitivity function according to (4) can be represented as:

$$s_{a_i}^{pm} = \frac{\partial F[x(t,\ a)]}{\partial a_i} = \frac{\partial r_j[x(t, a_i)]}{\partial a_i} = s_r^{a_i} \quad (16)$$

As a measure of safe control process model sensitivity to changes of process parameters it is taken a relative change of risk collision r_j caused due to deviation of the coefficient a_i in state equations by ∂a_i.

The coefficients of state equations of model process have been varied within $\pm 10\%$ from their rated value ai and each time the anti-collision manoeuvre was simulated in Matlab/Simulink software, then relative change of risk collision was calculated by using the formula (15). As a result of computer simulation investigations there were obtained the characteristics of sensitivity functions $s_r^{a_i}$ at w_1=2 and w_2=1 weight coefficients (Fig. 3), were obtained.

In Figure 4 there are additionally shown sensitivity functions of the risk collision for different values w_1 and w_2 of weight coefficients, are shown additionally

By setting different values of weight coefficients can be made multi-criteria optimization of ship safe trajectory in collision situations, taking into account the compromise between distance and time of excessive rapprochement of ships [9,14,15,21,28].

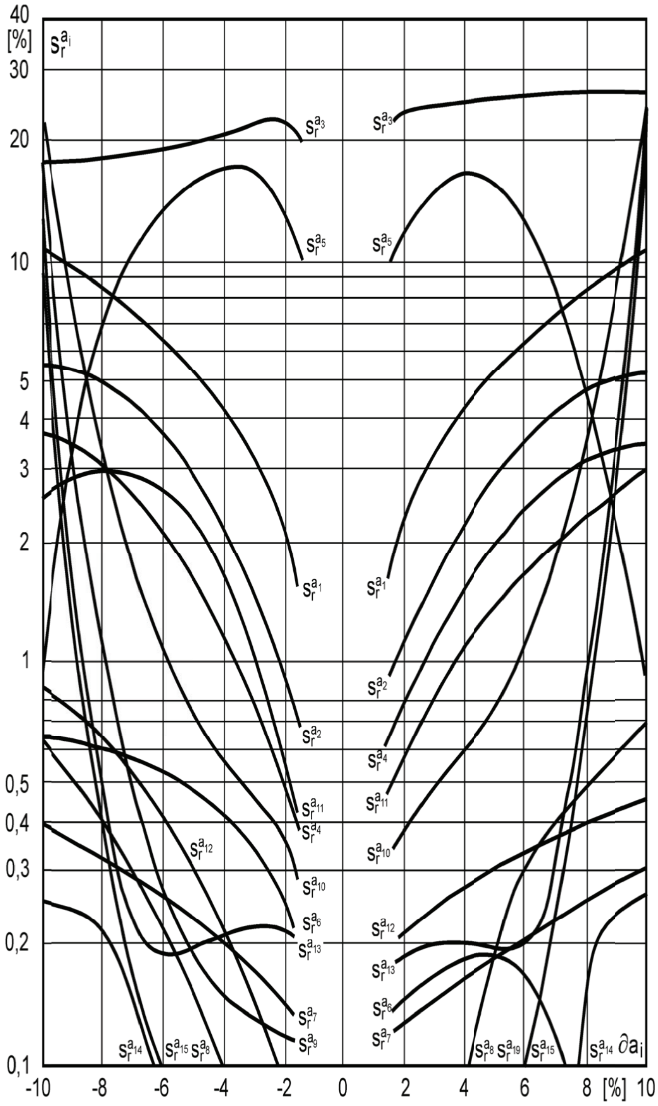

Fig. 3. Sensitivity functions of the risk of collision to the changes of state equations coefficients process model of situation motion of own ship and j-th met ship for weight coefficients w_1=2 and w_2=1

SENSITIVITY FUNCTION OF OPTIMAL AND GAME SHIP CONTROL IN COLLISION SITUATIONS

Sensitivity function of game control based on the equation (5) take the following form:

$$s_i^{gc} = \frac{\partial I_{0,j}\left[x_i(u,p)\right]}{\partial x_i} \tag{17}$$

The quality game control index $I_{0,j}$ takes the form of game payment, consisting of integral payments and final payment:

$$I_{0,j} = \int_{t_0}^{t_{Pk}} \left[x_j(t)\right]^2 + r_j(t_{Pk}) + D(t_{Pk}) \tag{18}$$

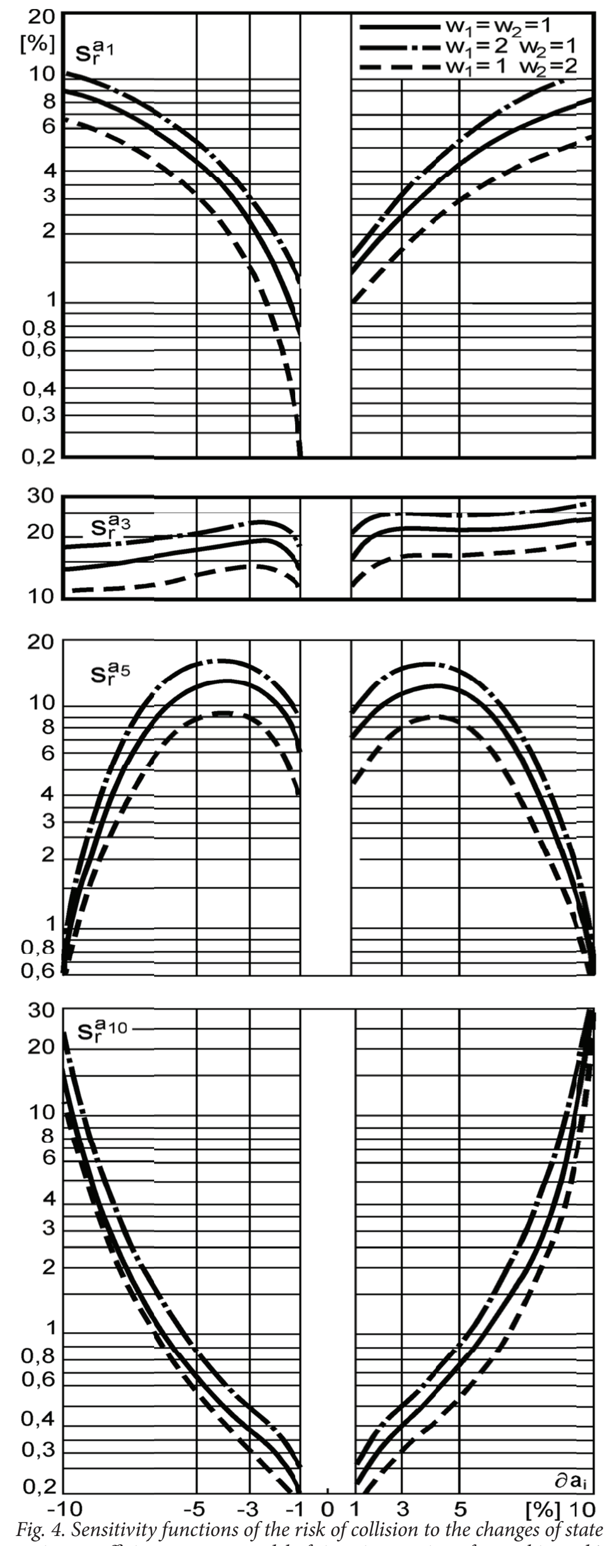

Fig. 4. Sensitivity functions of the risk of collision to the changes of state equations coefficients process model of situation motion of own ship and j-th met ship for different values of weight coefficients

The integral payment represents loss of way by the ship while passing the encountered ships and the final payment determine the final risk of collision rj(tPk) relative to the *j*-th ship and the final deviation of the own ship trajectory $D(t_{Pk})$ from the reference trajectory.

The investigation of sensitivity of the game control is applied to sensitivity analysis of the final payment $D(t_{Pk})$:

$$s_i^{gc} = \frac{\partial D(t_{Pk})}{\partial x_i} \tag{19}$$

Taking into consideration the practical application of the game control algorithm for the own ship in a collision situation it is recommended to perform the analysis of sensitivity of a safe control with regard to the accuracy degree of the information received from the anti-collision ARPA radar system in the current approach situation, from one side and also with regard to the changes in kinematical and dynamic parameters of the control process from the other side.

Admissible average errors, that can be contributed by sensors of the anti-collision system can have the following values for:

- for radar,
  - » bearing: ±0,22 deg,
  - » form of cluster: ±0,05 deg,
  - » form of impulse: ±20 m,
  - » margin of antenna drive: ±0,5 deg,
  - » sampling of bearing: ±0,01 deg,
  - » sampling of distance: ±0,01 nm,
- for gyrocompas: ±0,5 deg,
- for log: ±0,5 kn,
- for GPS: ±15 m.

The algebraic sum of all errors affecting the proper display of navigational situation, must not exceed ± 5% or ± 3 deg.

SENSITIVITY OF OPTIMAL AND GAME SHIP CONTROL TO INACCURACY OF INFORMATION FROM ARPA ANTI-COLLISION SYSTEM

Let *SC* represents such a set of state and control variables of game process information on the navigational situation that:

$$SC\left\{x_1,\ x_3,\ x_{8+j},\ x_{9+j},\ u_{j1},\ u_{j2}\right\} \tag{20}$$

Let then SCARPA represent a set of information from ARPA system containing errors of measurement and processing parameters:

$$SC_{ARPA}\left\{x_1 \pm \partial x_1, x_3 \pm \partial x_3, x_{8+j} \pm \delta x_{8+j}, x_{9+j} \pm \partial x_{9+j}, u_{j1} \pm \partial u_{j1}, u_{j2} \pm \partial u_{j2}\right\} \tag{21}$$

Relative measure of sensitivity function of the final payment in the game s_{i1} as a final deviation of the ship's safe trajectory D_{Pk} from the reference trajectory will be:

$$s_{i1}^{gc} = \left\{s_1, s_3, s_{8+j}, s_{9+j}, s_{j1}, s_{j2}\right\} \tag{22}$$

SENSITIVITY OF OPTIMAL AND GAME SHIP CONTROL TO PROCESS PARAMETERS ALTERATIONS

Let *SP* represents a set of parameters of the state process control:

$$SP\left\{p_1,\ p_2,\ p_2,\ p_3,\ p_4\right\} \tag{23}$$

where:

$p_1=t_m$ – advance time of manoeuvre with respect to the dynamic properties of own ship,

$p_2=D_s$ – safe distance, in good visibility D_s=0,1 ÷ 1,0 *nm*, in restricted visibility D_s=1,0 ÷ 3,0 *nm*,

$p_3=t_k$, - duration of one stage of ships trajectory,

$p_4=\Delta V$ – reduction of own ship speed for a deflection from course greater than 30 deg.

Let then SP_I represent a set of information containing errors of measurement and processing parameters:

$$SP_I\left\{p_1 \pm \partial p_1,\ p_2 \pm \partial p_2,\ p_3 \pm \partial p_3,\ p_4 \pm p_4\right\} \tag{24}$$

Relative measure of sensitivity of the final payment in the game si2 as a final deflection of the ship's safe trajectory DPk from the reference trajectory will be:

$$s_{i2}^{gc} = \left\{s_{p1}, s_{p2}, s_{p3}, s_{p4}\right\} \tag{25}$$

SENSITIVITY CHARACTERISTICS OF SAFE SHIP CONTROL IN GOOD VISIBILITY AT SEA

Computer simulation of the multi-step non-cooperative matrix game algorithm, as a computer software supporting the navigator manoeuvring decision, was carried out in Matlab/Simulink software on an example of the real navigational situation of passing *j* = 18 encountered ships in good visibility at sea, when D_s=0,5 *nm* (Fig. 5 and 6).

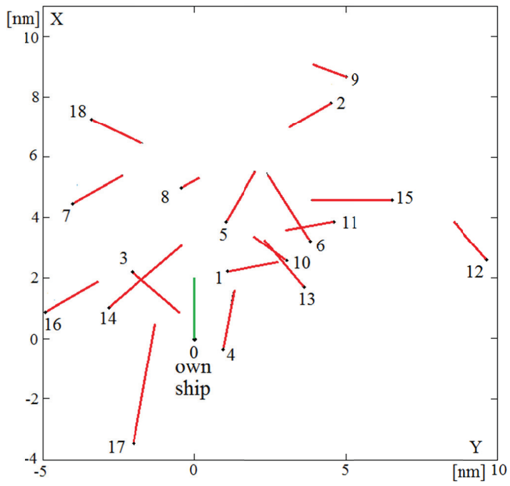

Fig. 5. The 24 minute speed vectors of the own ship and j=18 met ships

The situation was recorded in the Skagerrak Strait on board r/v HORYZONT II, a research and training vessel of the Gdynia Maritime University, on the radar screen of the ARPA anti-collision system Raytheon.

Sensitivity characteristics were determined for the alterations of values ∂x and ∂p within ±5% or ±3 deg (Fig. 7).

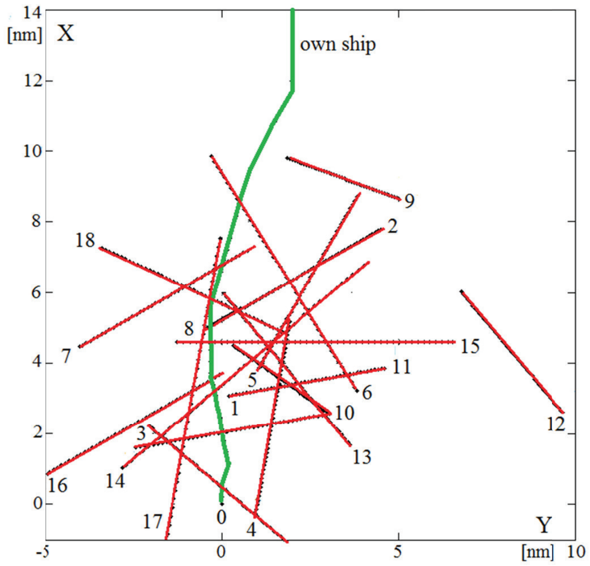

Fig. 6. The safe trajectory of the own ship for the multi-step non-cooperative matrix game algorithm in good visibility Ds=0,5 nm in situation of passing j=18 met ships, r(tk)=0, DPk=2,12 nm

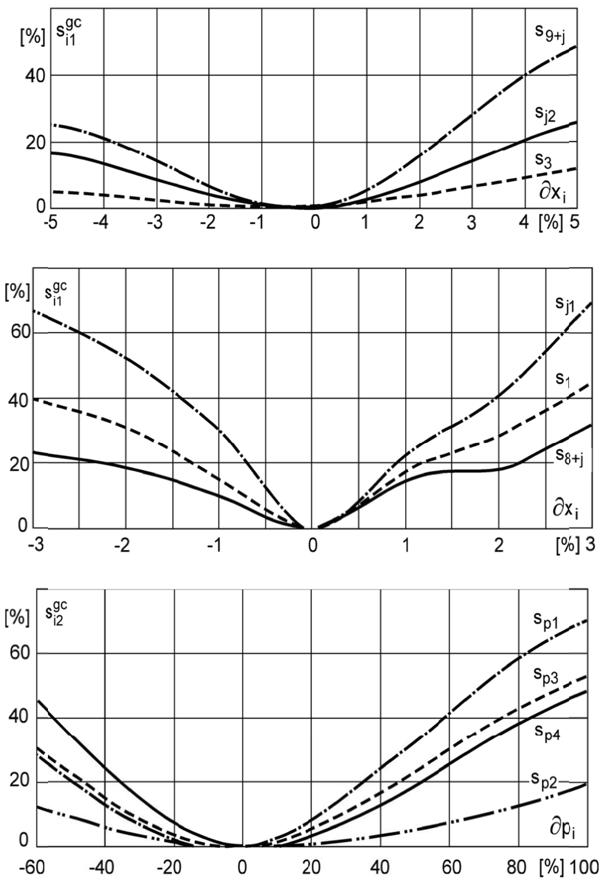

Fig. 7. Sensitivity characteristics of the game control of own ship according to the multi-step non-cooperative matrix game algorithm on an example of the navigational situation j = 18 occuring in the Skagerrak Strait

CONCLUSIONS

Analysis of the results of calculations of the sensitivity function allows to draw the following conclusions:

- in the range of sensitivity function of game ship control process model:
  - » the model process sensitivity to changes of individual coefficients of the state equations varies from 0,01% to 25%,
  - » the highest sensitivity is shown by the coefficients: a_1, a_3, a_5 and a_{15}, the lowest one by the coefficients: a_6, a_8, a_{12} and a_{14},
  - » the highest changes of sensitivity, so called second order sensitivity, have occurred for coefficients: a_9, a_{10}, a_{13} and a_{15},
  - » a greater impact on the model sensitivity is to change the weight coefficient w_1 in contrast to that of w_2,
- in the range of sensitivity function of game ship control in collision situation, considered as the sensitivity of the final game payment:
  - » it is the least for changes of the changes of safe distance and for level of reduction of own ship speed,
  - » it is the greatest for changes of the distance to *j*-th ship and changes of the course of *j*-th ship,
  - » it grows with the degree of the ships cooperation for the purpose of avoiding of collision,
  - » it grows with the number of meeting ships and with the quantity of admissible strategies for own ship and passing ships.

The sensitivity functions obtained as a result of computer simulation investigations define the requirements regarding range and accuracy of ship kinematics and dynamics identification for the model useful for the safe control system synthesis.

The future investigations should be directed to obtain further simplification of the process models taking into account their sensitivity functions.

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