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ACCURACY OF THE GPS POSITIONING SYSTEM IN THE CONTEXT OF INCREASING THE NUMBER OF SATELLITES IN THE CONSTELLATION

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ABSTRACT

A possibility of utilising the GPS system for navigation and transport are fundamentally dependent on the accuracy in positioning. Two fundamental factors decisive for its value are the values of the User Range Error (URE) and Dilution of Precision (DOP), strictly related to the number of satellites forming the constellation. The nominal constellation of GPS satellites consists of 24 units which gives a possibility of identification of coordinates all over the globe. In the last few years, however, the nominal number of satellites in the constellation was much higher, and the URE value has been constantly increasing.
The authors of the paper try to estimate the impact of the changing number of GPS satellites on accuracy of position coordinates with a variable URE value. Mathematical model for estimating geometrical indicators' value, utilising data derived from the almanac files has been presented. Following a drawn-up algorithm and calculations made with Mathcad software, the authors carried out a comparative analysis of mean daily values of DOP indicators for a variable number of satellites included in the GPS constellation in the years 2001-2013. Then, the authors have established representative values of Two Distance Root Mean Square Error (2drms) 2D and 3D, and calculated a percentage increase of accuracy in the period under discussion.

Keywords: GPS, User Range Error, Dilution of Precision, GPS Almanac, Accuracy of Positioning.

Introduction

The GPS system plays a fundamental role in the process of object's navigation and geodetic measurements, utilising active geodetic satellite networks [1, 2], in which planning a campaign on the basis of the almanac data of the constellation is a key factor for carrying out accurate engineering measurements for inventorying and diagnostic purposes [3, 4]. In maritime navigation where safety is a core of the process [5], information on conditions of geometric satellite measurements, resulting in specific GPS positioning error, should be taken into consideration during the navigation at the stage of planning the construction of navigation infrastructure [6, 7] and monitoring vessel traffic [8, 9]. It is also an important element which utilises electronic navigation support tools, such as imaging – ECDIS [10, 11]. A similar situation can be found in the case of aviation positioning, where GNSS technologies are at the initial stage of implementation within the systems of precision approach and landing of the aircraft [12], or dynamically today developing solutions integrating the GIS and GNSS systems in aviation [13].

One cannot fail to mention individual navigation, i.e. the use of GPS in non-professional applications, where GNSS receivers are widely used in tourism or sports, and accuracy characteristics strictly related to the constellation of satellites available to the user at the moment of measurement, are a decisive factor for their accuracy [14, 15], and availability.

Measuring errors which can be divided into three fundamental categories, influence the accuracy of determining position coordinates in the GPS system. The first category is errors caused by signal propagation, including errors resulting from ionospheric and tropospheric delays, and errors caused by received signal's multi-path routing. These issues will not be discussed in further part of the paper. The second category includes errors resulting from spatial and ground segments' operations, i.e. satellite ephemeris errors, satellite clock errors, and errors caused by the constellation's geometry, represented by DOP indicators. The third group includes receivers' instrumental errors, which currently are not highly decisive for positioning accuracy [16]. Errors included in the second group may be assessed and included in the navigation message in a form of the User Range Accuracy (URA) [17]. According to the 2011 standard [18], the value of URA is a standard deviation of the standard User Range Error (URE). URE consist of satellite clock errors and their ephemeris errors [19]. The control segment traces a position of each individual satellite, defining a vector of errors of its position and clock, against the time standard UTC, maintained by the US Naval Observatory [20]. The US Federal Aviation Administration publishes quarterly data concerning current value of URA for the GPS system [21]. Detailed analyses connected with the value can be found in the book listed as [22]. The value of standard deviation (rms) of URE can be calculated through the function of errors in satellite location measured along-track, cross-track and radially, and satellite clock error as [23]:

$$\mathrm{URE} = \sqrt{\left(0,98\mathrm{R} - \mathrm{T}\right)^2 + 0,141^2\left(\mathrm{A}^2 + \mathrm{C}^2\right)}, \quad (1)$$

where:
URE – User Range Error [m],
R – satellite Radial Error [m],
A – satellite Along-track Error [m],
C – satellite Cross-track Error [m],
T – satellite Clock Error [m].

Tab. 1 presents typical component error value for individual satellite blocks. The data have been analysed for 2010, when the segment consisted of the total of 31 satellites, 30 in operation. Space segment was created by satellites of three blocks, IIA, IIR and IIR-M, in a quantity of 11, 12 and 8 respectively [24].

Tab. 1. Typical component values of URE value in the function of GPS satellite block [23].

Error component	**block IIA**	**block IIR**	**block IIR-M**
R – radial [m]	0,243	0,130	0,145
A – along-track [m]	1,258	0,921	1,000
C – cross-track [m]	0,675	0,575	0,594
T – clock (time) [m]	1,074	0,384	0,498
URE [m]	***1,076***	***0,418***	***0,527***

Accuracy of determining position for the GPS system depends on the value of a selected geometrical indicator (DOP) and the User Equivalent Range Error (UERE), composed of both URE and User Equipment Error (UEE). The value of UEE for equipment manufactured in 1980s was on average 5,5 m (p = 0,95), whereas at present it is 1,6 m (p = 0,95) [25]. Therefore the above-presented formula for accuracy of position determination may be presented in the following form [26]:

$$\mathrm{drms} = \mathrm{UERE} \cdot \mathrm{DOP} = \sqrt{\mathrm{URE}^2 + \mathrm{UEE}^2} \cdot \mathrm{DOP}, \quad (2)$$

where:
drms – Distance Root Mean Square Error (horizontal, vertical, spatial, clock), depending on the selected DOP indicator [m],
UERE – User Equivalent Range Error [m],
URE – User Range Error [m],
UEE – User Equipment Error [m],
DOP – suitable Dilution of Precision: GDOP, PDOP, HDOP, VDOP, and TDOP [-].

The equation for accuracy of position determination for the above presented GPS system, has a simplified form, thanks to which it is satisfactory and universal for numerous applications. It is correct, because all the calculations of pseudo-ranges are subject to normal distribution (Gaussian distribution). The following diagram (Fig. 1) presents a change in the value of the Two Distance Root Mean Square Error (2drms) 2D of the GPS system (p = 0,95) in the function of the variable URE value [22], with the assumption of UEE of 0,8m (rms) and HDOP indicator equal to 1,5.

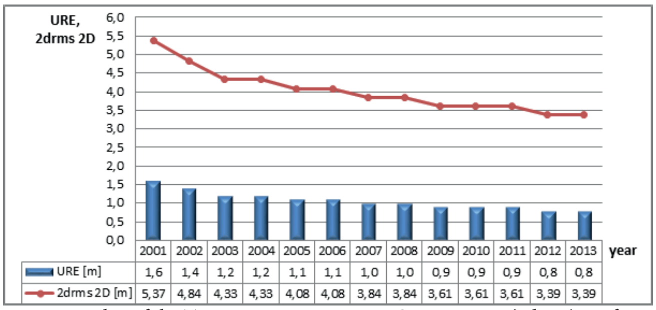

Fig. 1. Value of the Two Distance Root Mean Square Error (2drms) 2D for the GPS system, dependent on URE from 2001-2013. The URE value was estimated upon [22].

The US Department of Defence permanently monitors technical condition of equipment carrying out its tasks, and makes current data available to the users, facilitating forecasting the satellites' constellation. Current and archive data of the almanac, in various formats, are available at the website of the US Coast Guard Navigation Centre which is responsible for supplying current information on GPS to individual users. In order to stipulate a real number of satellites in the system in the years 2001-2013, all the almanac files for the period in question have been analysed (a total of 4188 files from the USCG website), and on their basis a diagram illustrating the number of satellites (both active and inactive) for the GPS constellation in the function of individual years was made (Fig. 2). Next, it will be the basis to calculate mean daily value of DOP indicators, dependent on the changing number of GPS satellites. The simplification used so far (adoption of a constant HDOP value), irrespective of the number of satellites in the constellation, must be regarded as approximation the authors of the paper would like to avoid. Even more so that the number of satellites is constantly changing, thus decreasing (in the statistical sense of the word) the DOP value.

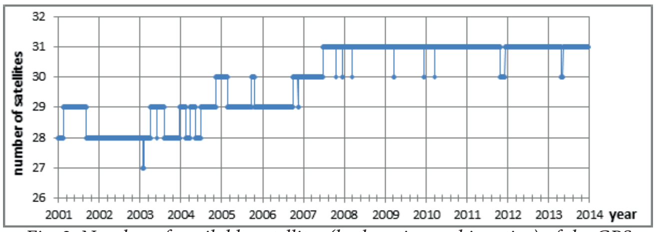

Fig. 2. Number of available satellites (both active and inactive) of the GPS system, derived from the almanac files (USCG) in the years 2001-2013.

In further part of the paper the authors carry out an analysis of the Two Distance Root Mean Square Error (2drms) 2D and 3D in the function of variable number of satellites, influencing the DOP values, and shrinking in years URE value.

Determining the geometrical DOP coefficients' values

DOP coefficients are a measure of geometric conditions for determining positions. This is a scalar value which informs about spatial distribution of elements in relation to the observer. For a detailed description of DOP coefficients see [27]. The process of calculating the value of geometric coefficients for a given moment of observation should be started with determining coordinates of GPS satellites and the receiver in a system Earth-Centred Earth-Fixed (ECEF) at the right moment (assuming that the Kepler's laws define the satellite movement). Therefore the following data should be acquired for each individual satellite from the almanac files [28]:

t_{oa} – GPS Time of Applicability [s],
e_s – Eccentricity [-],
δi – Inclination Offset [semicircles], [rad], [°],
$\dot{\Omega}$ – Rate of Right Ascension [semicircles/s], [rad/s], [°/s],
$\sqrt{A}$ – Square Root of Semi-Major Axis [ ],
Ω_0 – Longitude of Orbital Plane [semicircles], [rad], [°],
ω – Argument of Perigee [semicircles], [rad], [°],
M_0 – Mean Anomaly [semicircles], [rad], [°].

The next step is the transformation of satellites' coordinates from the ECEF system into the ENU (East, North, Up), calculation of their elevation (topocentric altitude), and disregarding satellites with the altitude's negative value, or lower than that assumed, and for the other satellites – calculation of azimuths measured from the position of the receiver. Below can be seen a matrix of transformation between ECEF-ENU systems, taking a form of [29]:

$$F = \begin{bmatrix} -\sin(L) & -\sin(B)\cdot\cos(L) & \cos(B)\cdot\cos(L) \\ \cos(L) & -\sin(B)\cdot\sin(L) & \cos(B)\cdot\sin(L) \\ 0 & \cos(B) & \sin(B) \end{bmatrix}, \quad (3)$$

where: B, L – receiver's geodetic coordinates [rad],

which facilitates calculation of satellites coordinates in the ENU system:

$$\begin{bmatrix} x_{ENU} \\ y_{ENU} \\ z_{ENU} \end{bmatrix} = F^T \cdot \begin{bmatrix} x_s - x_u \\ y_s - y_u \\ z_s - z_u \end{bmatrix}, \quad (4)$$

where:
x_{ENU}, y_{ENU}, z_{ENU} – satellite coordinates in the ENU system [m],
x_s, y_s, z_s – satellite coordinates in the ECEF system [m],
x_u, y_u, z_u – receiver coordinates in the ECEF system [m],
thus allowing to calculate on their basis the elevation (topocentric altitude) of the satellite:

$$el = \operatorname{arctg}\left(\frac{z_{ENU}}{\sqrt{x_{ENU}^2 + y_{ENU}^2}}\right) \quad (5)$$

and its azimuth:

$$Az = \begin{cases} 0 \quad \text{for} \quad x_{ENU} = 0 \wedge y_{ENU} > 0; & \operatorname{arctg}\left(\left|\frac{x_{ENU}}{y_{ENU}}\right|\right) \quad \text{for} \quad x_{ENU} > 0 \wedge y_{ENU} > 0 \\ 0{,}5\pi \quad \text{for} \quad x_{ENU} > 0 \wedge y_{ENU} = 0; & 0{,}5\pi + \operatorname{arctg}\left(\left|\frac{y_{ENU}}{x_{ENU}}\right|\right) \quad \text{for} \quad x_{ENU} > 0 \wedge y_{ENU} < 0 \\ \pi \quad \text{for} \quad x_{ENU} = 0 \wedge y_{ENU} < 0; & \pi + \operatorname{arctg}\left(\left|\frac{x_{ENU}}{y_{ENU}}\right|\right) \quad \text{for} \quad x_{ENU} < 0 \wedge y_{ENU} < 0 \\ 1{,}5\pi \quad \text{for} \quad x_{ENU} < 0 \wedge y_{ENU} = 0; & 1{,}5\pi + \operatorname{arctg}\left(\left|\frac{y_{ENU}}{x_{ENU}}\right|\right) \quad \text{for} \quad x_{ENU} < 0 \wedge y_{ENU} > 0 \end{cases} \quad (6)$$

And then, with the application of the line-of-sight matrix G:

$$G = \begin{bmatrix} \cos(el_1)\cdot\sin(Az_1) & \cos(el_1)\cdot\cos(Az_1) & \sin(el_1) & 1 \\ \cos(el_2)\cdot\sin(Az_2) & \cos(el_2)\cdot\cos(Az_2) & \sin(el_2) & 1 \\ \vdots & \vdots & \vdots & 1 \\ \cos(el_n)\cdot\sin(Az_n) & \cos(el_n)\cdot\cos(Az_n) & \sin(el_n) & 1 \end{bmatrix}, \quad (7)$$

where indexes 1-n denominate values for further individual satellites, and a covariance matrix:

$$C = \left(G^T \cdot G\right)^{-1}, \quad (8)$$

geometric coefficients may be calculated:

$$GDOP = \sqrt{C_{0,0} + C_{1,1} + C_{2,2} + C_{3,3}}, \quad (9)$$

$$PDOP = \sqrt{C_{0,0} + C_{1,1} + C_{2,2}}, \quad (10)$$

$$HDOP = \sqrt{C_{0,0} + C_{1,1}}, \quad (11)$$

$$VDOP = \sqrt{C_{2,2}}, \quad (12)$$

$$TDOP = \sqrt{C_{3,3}}. \quad (13)$$

Simulation tests

In order to assess the impact of alteration in the number of satellites on the accuracy of positioning, an algorithm allowing for determination of geometric coefficients for any location or time, was worked out with the Mathcad software. Analyses were used to determine the mean daily value of the DOP coefficients in individual years. Out of each successive year, excluding the turning off of the GPS Selective Availability (SA) by US President Bill Clinton on 2nd May 2000 [30], the almanac file was selected; it was the file representing most frequent number of GPS satellites in a given year (Tab. 2), for which, afterwards, changes in the DOP value during a full stellar day (equivalent to double orbiting the globe by the GPS constellation) were obtained.

Tab. 2. Representative number of satellites in the years 2001-2013

Year	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Number of satellites	29	28	28	29	29	29	30	31	31	31	31	31	31

Calculations were carried out for Gdynia location (54° 32'N, 18° 32' E), a minimum elevation (topocentric altitude) of satellites equal to 0°, and a period of 23h 56' 4", i.e. a period after which a geometry of GPS constellation over a given point will be identical to that at the start of the observation. In order to receive equal brackets of analyses carried out, the value of 4 seconds was adopted as a step of making DOP calculations. These short brackets of analyses permitted for taking into account changes in geometric coefficients' values, which are usually disregarded by general access software applied in geodesy and navigation for analysis of constellation's geometry, with a typical calculation bracket of 10 minutes. Then mean values of geometric coefficients HDOP and PDOP were calculated for a given period of time (a stellar day) and Two Distance Root Mean Square Errors (2drms) 2D and 3D in the function of a variable value of URE, with the assumption of UEE equal to 0,8m (Fig. 3 and Fig. 4).

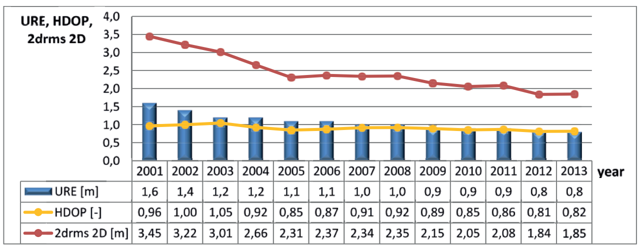

Fig. 3. Value of the Two Distance Root Mean Square Error (2drms) 2D for the GPS system, dependent on URE from 2001-2013

In order to estimate the impact of variable number of GPS satellites on accuracy of determination of position in reference to current URE value, it was suggested to adopt a relative (to the year 2001) percentage change in accuracy of determining 2D and 3D positions in the years 2001-2013, as presented in Fig. 1, with the application of equation:

$$\Delta 2\text{drms} = \frac{2\text{drms}_n - 2\text{drms}_{2001}}{2\text{drms}_{2001}} \cdot 100\%, \qquad (14)$$

where n = 2001, 2002, ..., 2013.

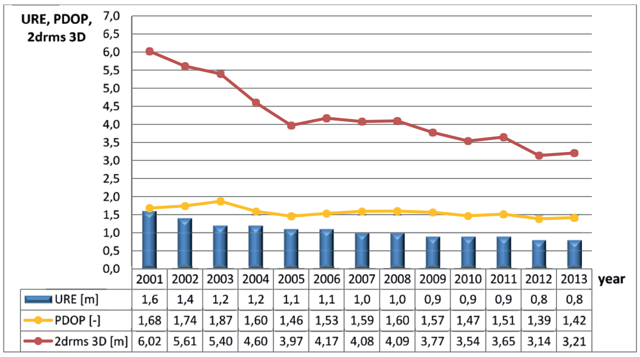

Fig. 4. Value of the Two Distance Root Mean Square Error (2drms) 3D for the GPS system, dependent on URE from 2001-2013

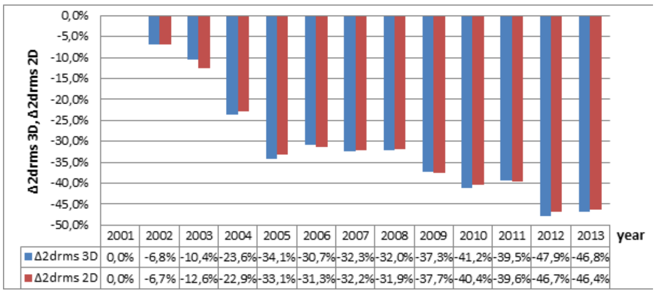

Fig. 5. Relative percentage change in accuracy of determining 2D and 3D positions in the years 2001-2013

Discussion

The research shows that from the moment the Selective Availability was switched off, the system keeps improving its accuracy of positioning, which can be observed from the periodically (2001, 2008) published statistical data concerning the accuracy in a form of a standard. In the 2001 SPS standard, accuracy of positioning in a horizontal plane should not be higher than 13 m (p = 0,95), and in vertical one, 22 m (p = 0,95); in 2008 version, however, an average global positioning accuracy (p = 0,95) should not have exceeded 9 m and 15 m respectively. This means that the Δ2drms relative value alteration, calculated according to the 2001 standard, was -30,7 %. Similar result was obtained on the basis of the analyses discussed above, and it was -32,0 %.

It should be noticed, however, that the acquired absolute results of simulation tests do not include measurement errors related to the signal's propagation (ionospheric and tropospheric delays), discussed in the introduction, therefore they differ from the above-mentioned values, derived from the standards. The volume of errors depend to a large extent on the type of receiver used for determination of position, therefore they were on purpose omitted in the analyses.

Contemporary equipment receiving C/A signals at single L1 frequencies use diversified models of compensating these errors, and taking them into consideration would not be representative in the context of this research.

The target of the paper was to estimate the accrual of accuracy, therefore the authors carried out the research on the impact of variable number of the GPS satellites on the accuracy of determination of position coordinates with the variable value of URE. Simultaneously, a constant value of the User Equipment Error (UEE) equal to 0,8 m (rms) was assumed.

Conclusions

1. The values of the User Range Error (URE) and geometric coefficients (DOP) are factors decisive for accuracy of positioning in the GPS system.
2. Although in some years the number of satellites was the same, geometric coefficients varied slightly, which was caused by the variable geometry of the GPS satellites' constellation.
3. An increase in the number of satellites from 28 to 31 resulted in a decrease in the value of the HDOP coefficient by 0,2, and PDOP - by 0,4.
4. In 2001-2013, decrease in the User Range Error had higher impact on the accuracy of positioning than the increase in the number of satellites.
5. On the average, the Two Distance Root Mean Square Error (2drms) 3D is 1,7 times higher than 2D.
6. In the period under analysis, the Two Distance Root Mean Square Errors (2drms) 2D and 3D decreased by almost 50%.

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