POLISH MARITIME RESEARCH 2 (90) 2016 Vol. 23; pp. 14-18
10.1515/pomr-2016-0015

THE SENSITIVITY OF STATE DIFFERENTIAL GAME VESSEL TRAFFIC MODEL

Józef Lisowski, Prof.
Gdynia Maritime University, Poland

ABSTRACT

The paper presents the application of the theory of deterministic sensitivity control systems for sensitivity analysis implemented to game control systems of moving objects, such as ships, airplanes and cars. The sensitivity of parametric model of game ship control process in collision situations have been presented. First-order and k-th order sensitivity functions of parametric model of process control are described. The structure of the game ship control system in collision situations and the mathematical model of game control process in the form of state equations, are given. Characteristics of sensitivity functions of the game ship control process model on the basis of computer simulation in Matlab/Simulink software have been presented. In the end , have been given proposals regarding the use of sensitivity analysis to practical synthesis of computer-aided system navigator in potential collision situations.

Keywords: marine transport; safety of navigation, game control, sensitivity analysis

INTRODUCTION

By taking into consideration the form of the quality index the problem of optimal control of the technical processes may be split into three groups for which cost of the process course :

- is a univocal control function,
- depends on the way of control and also on a certain random event of a known statistical description,
- is defined by a choice of the control method and by a certain indefinite factor of an unknown statistical description, respectively.

The last group of the problems refers to game control systems the synthesis of which is performed by using methods of the games theory [1,2,3,13].

The following types of games can be distinguished:

- with regard to the number of players: two-person and n-person,
- with regard to the strategy sets: finite and infinite,
- with regard to the nature of co-operation: non-coalition, co-operative - through the relationships established earlier and coalition ones,
- with regard to the nature of the prize: zero-sum - closed with a saddle point determined by the optimal pure strategies and those of any sum, e.g.: international trade,
- with regard to their form of the goal function: matrix, non-continuous and convex,
- with regard to the nature of conducting the game: in the normal form - one-step static ones and in the extensive form as multi-step games determined by a sequence of movements executed alternately within the kinematical and dynamical processes, the games in their extensive form are split into: positional, stochastic and differential,
- with regard to the nature of information: with complete and incomplete information,
- with regard to the kind of an opponent: with a thinking opponent and with the nature – the environment performing random movements and not interested in the final result of the game [14].

Generally, may be distinguished three classes of the control problems which may offer possibilities to use games both for the description and synthesis of the optimal control:

- multi-layer hierarchical systems. One of the essential hierarchical languages for the steering systems of various nature and methods of determining optimal control is the theory of games including common interests and the right to the first turn [18];
- object with no information available from the disturbances operating on such object. In this case we have only the state equations of the object and a set of acceptable steering actions. Such control should be then determined as to ensure the minimal functional, under a condition that the disturbance tends to its maximum; in this case the differential game should be solved with a min- max optimum condition [10,16,17,20,22];
- object encountering a greater number of the moving objects of different quality index and final goals. An example,

in this case, may be a process of steering a ship in collision situations when encountering a greater number of the moving or non-moving objects (vessels, underwater obstructions, shore line, etc), i.e. a differential game with many participants [8,9,11,12].

This paper presents , as an example, the sensitivity of parametric model of differential game ship control process in collision situations.

DIFFERENTIAL GAME VESSEL TRAFFIC MODEL

The way of steering a ship - which is a multi-dimensional and non-linear dynamic object - depends on the range and accuracy of information on the prevailing navigational situation and on the adopted model of the process. The variety of the models to be adopted directly influences the synthesis of various algorithms supporting the navigator's work, and then on the effects of a safe control of the ship's movement.

As the process of steering the ship in collision situations, when a greater number of objects is encountered, often occurs under the conditions of indefiniteness and conflict, accompanied by an inaccurate co-operation of the objects within the context of COLREG Regulations, then the most adequate model of the process to be adopted is a model of a differential game, in general , of j tracked ships as objects of steering (Fig. 1).

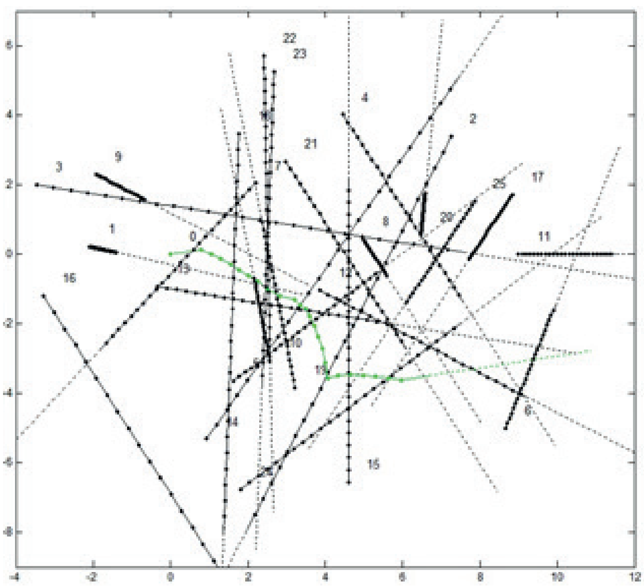


Fig. 1. Situation of relative motion of own ship (green trajectory) and j met ships

The diversity of selection of possible models directly affects the synthesis of the ship's handling algorithms which are afterwards effected by the ship's handling device directly linked to the ARPA anti-collision system and, consequently, determines the effects of the safe and optimal control.

The structure of the game control process model of own ship in collision situations at sea is presented in Fig. 2.

The control process is described by state equations:

$$\dot{x} = f(x,u,t) \tag{1}$$

constraints of state and control:

$$g(x,u,t) \leq 0 \tag{2}$$

and quality control index:

$$I = \int_{t_0}^{t_K} f_o(x,u,t)dt \tag{3}$$

where:
x – state variable,
u – control variable,
t – time.

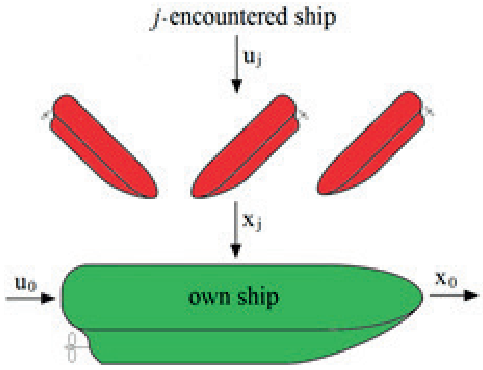


Fig. 2. Game ship control process: u_0 – control of own ship, u_j – control of j-encountered ship, x_0 – state of own ship, x_j – state of j-encountered ship

The current state of the control process is determined by the co-ordinates of the own ship's position and the positions of the encountered objects (Fig. 3):

$$\left.\begin{aligned} x_0 &= (X_0, Y_0) \\ x_j &= (X_j, Y_j) \\ j &= 1, 2, ..., m \end{aligned}\right\} \tag{4}$$

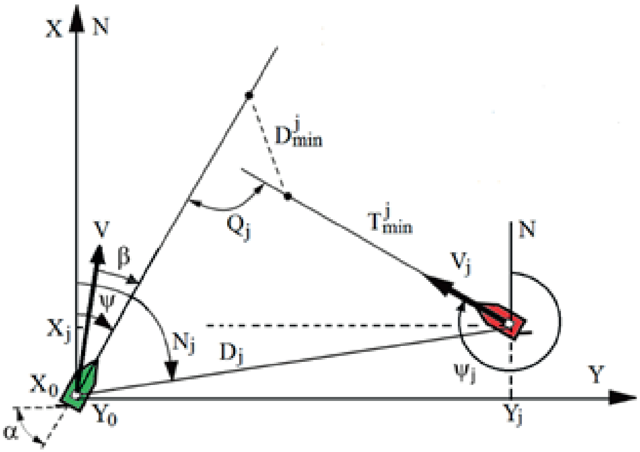


Fig. 3. Situation of relative motion of own ship and j met ship

The system generates its control at the moment t_k on the basis of data received from the ARPA anti-collision system pertaining to the positions of the encountered ships:

$$x(t_k) = \begin{bmatrix} x_0(t_k) \\ x_j(t_k) \end{bmatrix} \quad j = 1, 2, ..., m \qquad k = 1, 2, ..., K \tag{5}$$

The constraints for the state co-ordinates:

$$\left\{ x_0(t), x_j(t) \right\} \in P \tag{6}$$

are navigational constraints, while control constraints:

$$u_0 \in U_0, \; u_j \in U_j \quad j = 1, 2, ..., m \tag{7}$$

take into consideration: the ship's movement kinematics, recommendations of the COLREG Rules and the condition to maintain a safe passing distance D_s as per relationship:

$$D^j_{\min} = \min D_j(t) \geq D_s \tag{8}$$

where:

$D^j_{\min}$ - minimum approach distance of the own ship to met j-th ship.

By taking into consideration the equations of the own ship hydromechanics and equations of the own ship relative movement to the j-th encountered ship, the state process equations take the following form:

$$\left.\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_1 x_2 x_3 + a_2 x_3 |x_3| x_4 + a_3 x_3 |x_3| u_1 \\
\dot{x}_3 &= a_4 x_3 |x_3| |x_4| x_4 + a_5 x_3 |x_3| x_4^2 + a_6 x_2 x_3 x_4 |x_4| + \\
&\quad a_7 x_2 x_3 x_4 |x_4| + a_8 x_3 |x_3| + a_9 x_5 |x_5| x_6 + a_{10} x_3 |x_3| x_4 u_1 \\
\dot{x}_4 &= a_4 x_3 x_4 + a_5 x_3 x_4 |x_4| + a_6 x_2 x_4 + a_{11} x_2 + a_{10} x_3 u_1 \\
\dot{x}_5 &= a_{12} x_5 + a_{13} u_2 \\
\dot{x}_6 &= a_{14} x_6 + a_{15} u_3 \\
\dot{x}_{6+j} &= -x_3 + x_{7+j} x_2 + x_{9+j} \cos x_{8+j} \\
\dot{x}_{7+j} &= -x_2 x_{6+j} + x_{9+j} \sin x_{8+j} \\
\dot{x}_{8+j} &= a_{15+j} x_2 x_{9+j} u_{j1} \\
\dot{x}_{9+j} &= a_{16+j} x_{8+j} |x_{8+j}| + a_{17+j} u_{j2}
\end{aligned}\right\} \tag{9}$$

where state variables and control values are represented by:

$x_1 = \psi$ - course of the own ship,
$x_2 = \dot{\psi}$ - angular turning speed of the own ship,
$x_3 = V$ - speed of the own ship,
$x_4 = \beta$ - drift angle of the own ship,
$x_5 = n$ - rotational speed of the screw propeller of the own ship,
$x_6 = H$ - pitch of the adjustable propeller of the own ship,
$x_{6+j} = X_j$ - x coordinate of the j-th ship position,
$x_{7+j} = Y_j$ - y coordinate of the j-th ship position,
$x_{8+j} = N_j$- bearing of the j-th ship or Q_j – relative meeting angle,
$x_{9+j} = D_j$ - distance to j-th ship.
$u_1 = \alpha_r$ - reference rudder angle of the own ship,
$u_2 = n_r$ - reference rotational speed of the own ship's screw propeller,
$u_3 = H_r$ - reference pitch of the adjustable propeller of the own ship,
$u_{j1} = \psi_j$ - course of the j-th ship,
$u_{j2} = V_j$ - speed of the j-th ship [5,7,19].

SENSITIVITY DEFINITIONS OF PROCESS MODEL AND OPTIMAL PROCESS CONTROL

By sensitivity of control systems one usually means dependence of their properties on parameters' variation. Sensitivity theory became an independent scientific branch of cybernetics and control theory in the 1960s. This was connected in major part with quick development of self-tuning systems that were constructed for effective operation under parametric disturbances. Lately, sensitivity theory methods were widely used for solving various theoretical and applied problems with analysis and synthesis, identification, adjustment, monitoring, testing, tolerance distribution [4,6].

The same distinction is made between the sensitivity of the model control process for changing its parameters and process optimal control sensitivity to changes in its parameters and disturbance influence [15,21].

Previous papers dealt with sensitivity of deterministic systems, but not with game systems. In sea, land and air transport, processes regarding both own object and many encountered objects, occur.

Control of such processes, due to the high proportion of human subjectivity in the decision-making manoeuvre, often takes form of the control of game character.

The simplest method of sensitivity analysis consists in numerical investigation of system parametric model over the whole range of variation of the determining set of parameters.

The main investigation method in sensitivity theory consists in using so called sensitivity functions.

Let $a_1, ... ,a_m$ be a set of parameters constituting a complete set a.

Moreover, let be assumed a function of state variables $F[x_i(t,a)]$ (i=1, ... ,n).

The following partial derivative:

$$s_a^{pm} = \frac{\partial F[x(t, a)]}{\partial a} \tag{10}$$

is called first-order sensitivity function of the parametric process model s_a^{pm}.

In automatic control literature first-order sensitivity function is often called simply "sensitivity function".

Theoretically, we can also consider features of the sensitivity functions of k-th order of the parametric model $s_{k,a}^{pm}$ in the following form:

$$s_{k,a}^{pm} = \frac{\partial^k F[x(t,a)]}{\partial a_1^{k_1} \ldots \partial a_m^{k_m}} \qquad (11)$$

$$k_1 + \ldots + k_m = k$$

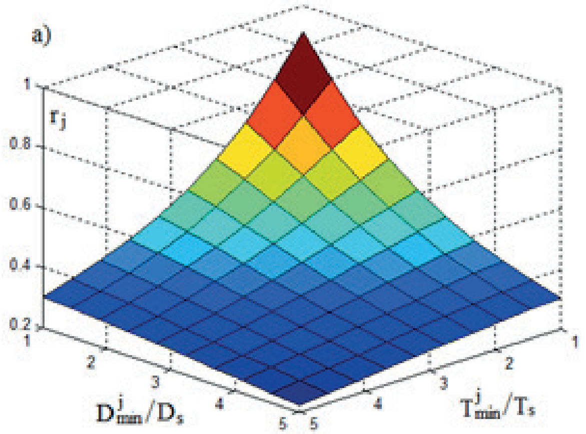
DETERMINATION OF SENSITIVITY FUNCTION OF THE DIFFERENTIAL GAME VESSEL TRAFFIC MODEL

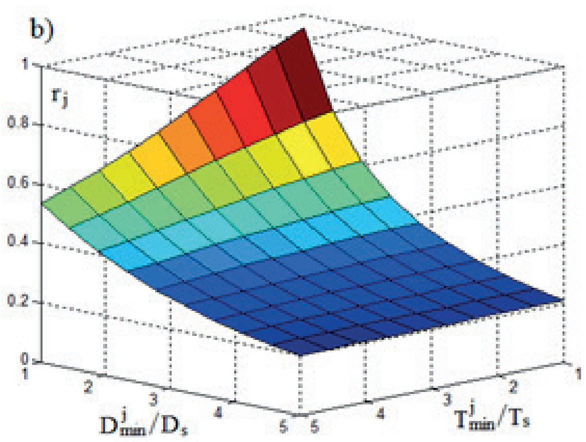
As a function of state process variables F[x(t,a)] the risk of collision r_j is assumed in the following form:

$$r_j(x) = \left[w_1 \left(\frac{D_{\min}^j}{D_s} \right)^2 + w_2 \left(\frac{T_{\min}^j}{T_s} \right)^2 + w_3 \left(\frac{D_j}{D_s} \right)^2 \right]^{-\frac{1}{2}} \qquad (12)$$

where:

w_1, w_2, w_3 – weight coefficients depending on the visibility at sea and dynamic length and breadth of the met j-th ship (Fig. 4).





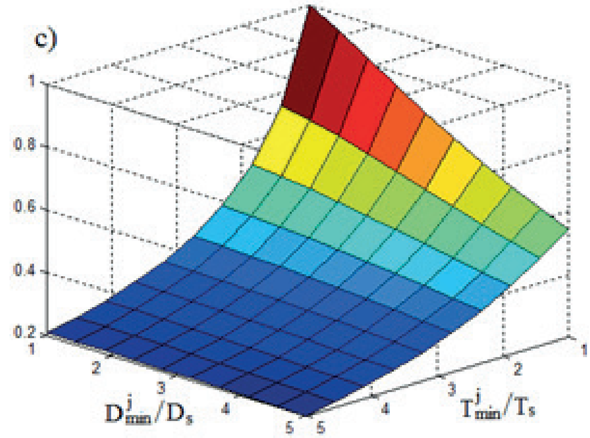


Fig. 4. Examples of risk collision function r_j: a) $w_1=w_2=w_3=0,4$ b) $w_1=0,9$; $w_2= 0,1$; $w_3=0,1$ c) $w_1=0,1$; $w_2= 0,9$; $w_3=0$

The sensitivity function according to (10) can be represented as:

$$s_{a_i}^{pm} = \frac{\partial F[x(t,a)]}{\partial a_i} = \frac{\partial r_j[x(t,a_i)]}{\partial a_i} = s_r^i \qquad (13)$$

As a measure of safe control process model sensitivity to changes of process parameters, is taken a relative change of risk collision r_j caused by deviation of the coefficient a_i in state equations by ∂a_i.

The coefficients of state equations of model process have been varied within ±10% from their rated value a_i and each time the anti-collision manoeuvre was simulated in Matlab/Simulink software, then relative change of risk collision was calculated by using the formula (13).

As a result of computer simulation investigations the characteristics of sensitivity functions s_r^i at $w_1 = w_2 = w_3 = 1$ weight coefficients (Fig. 5), were obtained.

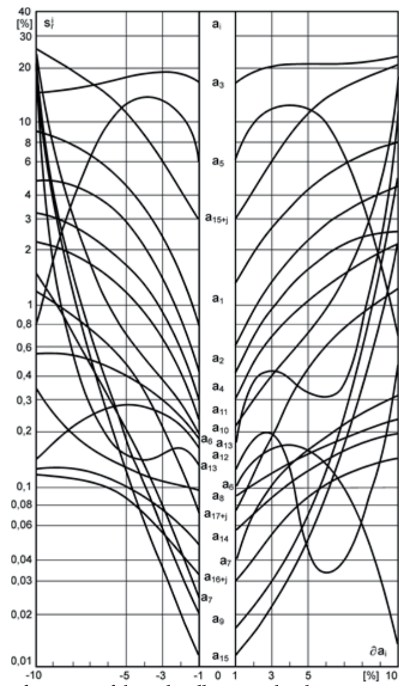


Fig. 5. Sensitivity functions of the risk collision to the changes in coefficients of state equations of process model of situation motion of own ship and j-th met ship

CONCLUSIONS

The model process sensitivity to changes of individual coefficients of the state equations varies from 0,01% to 25%.

The highest sensitivity is shown by the coefficients: a_1, a_3, a_5 and a_{15+j}, the lowest one by the coefficients: a_7, a_9, a_{15} and a_{16+j}.

The highest changes of sensitivity, so called second -order sensitivity, have occurred for the coefficients: a_7, a_9, a_{10}, a_{13} and a_{15}.

The sensitivity functions obtained as a result of computer simulation investigations define the requirements regarding range and accuracy of identification of ship's kinematics and dynamics for the model useful for the safe control system synthesis.

The future investigations should be aimed at obtaining further simplification of the process model by taking into account their sensitivity functions.

BIBLIOGRAPHY

1. Astrom K.J.: Model uncertainty and robust control. Lecture notes on iterative identification and control design. Lund Institute of Technology, Sweden, 2000, pp. 63-100.

2. Cruz J.B.: Feedback systems. Mc Graw-Hill Book Company, New York, 1972.

3. Dorf R.C. and Bishop R.H.: Modern control systems. Addison-Wesley, California, 1998.

4. Eslami M.: Theory of sensitivity in dynamic systems. Springer-Verlag, Berlin, 1994.

5. Fossen T.I.: Marine craft hydrodynamics and motion control. Wiley, Trondheim, 2011.

6. Fujarewicz K.: Structural sensitivity analysis of systems with time delay (in Polish), XVIII Krajowa Konferencja Procesów Dyskretnych (18th Domestic Conference on Discrete Systems), Zakopane, 2014.

7. Gierusz W. and Lebkowski A.: The research ship "Gdynia". Polish Maritime Research, Vol. 19, No. 74, 2012, pp. 11-18.

8. Isaacs R.: Differential games. John Wiley and Sons, New York, 1965.

9. Lazarowska A.: Safe ship control method with the use of ant colony optimization. Solid State Phenomena, Vol. 210, 2014, pp. 95–101.

10. Miller A., Rybczak M.: Model predictive control algorithm verification with the use of real time xPC target platform. Joint Proceedings, No. 84, 2014, pp. 73-84.

11. Mohamed-Seghir M.: The branch-and-bound method, genetic algorithm, and dynamic programming to determine a safe ship trajectory in fuzzy environment. 18th International Conference on Knowledge Based and Intelligent Information and Engineering Systems. Procedia Computer Science, No. 35, 2014, pp. 634-643.

12. Nisan N., Roughgarden T., Tardos E., Vazirani V.V.: Algorithmic game theory. Cambridge University Press, New York, 2007, pp. 717-733.

13. Nise N.S.: Control systems engineering. Wiley, California, 2015.

14. Osborne M.J.: An introduction to game theory. Oxford University Press, New York, 2004.

15. Rosenwasser E. and Yusupov R.: Sensitivity of automatic control systems. CRC Press, Boca Raton, 2000.

16. Rudnicki J.: Application issues of the semi-Markov reliability model. Polish Maritime Research, Vol. 22, No. 1, pp. 55-64.

17. Sanchez-Pena R.S. and Sznaier M.: Robust systems theory and applications. Wiley, New York, 1998.

18. Skogestad S. and Postlethwaite I.: Multivariable feedback control. Wiley, Chichester, 2005.

19. Szlapczynski R.: Evolutionary sets of safe ship trajectories with speed reduction manoeuvres within traffic separation schemes. Polish Maritime Research, Vol. 81, No. 1, 2014, pp. 20-27.

20. Tomera M.: Dynamic positioning system for a ship on harbour manoeuvring with different observers. Experimental results. Polish Maritime Research, Vol. 83, No. 3, 2014, pp. 13-21.

21. Wierzbicki A.: Models and sensitivity of control systems. Wydawnictwa Naukowo-Techniczne, Warszawa, 1977.

22. Zwierzewicz Z.: The design of ship autopilot by applying observer – based feedback linearization. Polish Maritime Research, Vol. 22, No. 1, pp. 16-21.

CONTACT WITH THE AUTHOR

Józef Lisowski
Gdynia Maritime University
83 Morska Str.
81-225 Gdynia
POLAND