MAGNETOHYDRODYNAMIC FLOW OF VISCOUS FLUID IN A SLOT BETWEEN FIXED SURFACES OF REVOLUTION

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ABSTRACT

The paper considers stationary magnetohydrodynamic flow of viscous fluid in the slot between fixed curvilinear surfaces of revolution exposed to azimuthal magnetic field. To solve the problem, the equations of boundary layer in the curvilinear coordinate system. x,θ,y , were applied. The equations of the boundary layer were solved analytically with the use of the small-parameter method. The formulas determine the field of velocity and pressure.

Keywords: laminar flow,incompressible magnetohydrodynamic flow,method of perturbation

INTRODUCTION

Magnetohydrodynamics is a section of fluid mechanics covering the aspects of the movement of fluids allowing the flow of an electrical current in magnetic and electrical field [1,11]. Especially magnetohydrodynamics (MHD) uses the laws of fluid mechanics and electrical engineering to determine the interaction of the fluid and electromagnetic field [14].

In the second half of the 20th century research was launched on the use of the phenomena of magnetohydrodynamics for the design of MHD drive of submarines.

The magnetohydrodynamic submarine drive is currently at the phase of R&D work.

The appearance of superconductors decreased the amount of energy necessary for generating the right magnetic field for designed drive systems (MHD). Superconductive magnets can generate a very strong magnetic field allowing for producing a thrust capable of moving a submarine [15].

Before appearance of controlled rheological fluids reacting to external magnetic and electric field in technology , only solid materials existed in practice. In contemporary industry the control of magnetic and electric field is used in many technical devices [14].

There are known applications of magnets and electro-magnets, controlled magnetoreological fluids not allowing the flow of an electrical current, electrorheological and Newton fluids allowing the flow of an electrical current (e.g. electrolytes, ionised seawater, non-magnetic liquid metals).

In the process of design, manufacture and assembly the application of such fluids requires the knowledge of effects which appear both as a result of external effect of the magnetic and electrical field and the effects being a result of the flow itself in the construction element.

In construction systems we can deal with a laminar flow characteristic for the so-called narrow slot and turbulent flow.

Laminar viscous fluid flows allowing the flow of an electrical current in the slots of technical devices exposed to stationary and non-stationary magnetic and electrical fields have triggered much interest [1,5,14,16]. The studies of viscous fluid flows exposed to magnetic and electrical fields as well as an effect of those fields on various physical and chemical processes have been performed for many years [4,10,11,13,].

Interestingly, research in that field is still essential for practical purposes.

A new potential offered by fluids allowing the flow of an electrical current in solving numerous design and operational issues (plain bearings, clutches, sealings, dampers etc.) in terms of friction, wear and lubrication, especially under extreme working conditions, determines the need of research of such flows [3,5,6,8,9,17].

The aim of this paper has been an analysis of the effect of magnetic and electrical field as well as inertia effects on the flow of fluid allowing the flow of an electrical current in the slot between fixed curvilinear rotating surfaces exposed to an external azimuthal magnetic and orthogonal field to the surfaces inhibiting the flow of electrical field.

BASIC EQUATIONS

The motion of electro-conductive viscous fluids flowing through a slot between the curvilinear surfaces of revolution presented in Fig. 1, is assumed to be laminar, steady and isothermal.
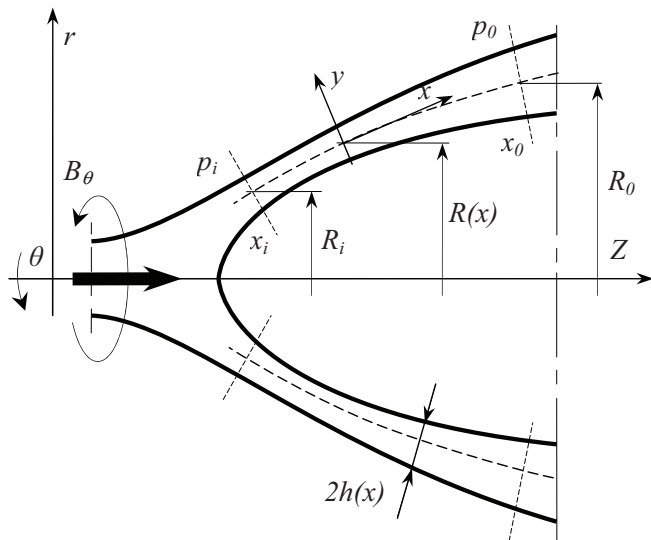

Fig. 1. Magnetohydrodynamic fluid flow area

The flow occurs in the presence of external stationary magnetic $(0, B_\theta, 0)$ and electrical $(0, 0, E_y)$ fields in a slot bounded by electrically conducting surfaces. The magnetic Reynolds number is assumed to be much less than unity, $Re_m \ll 1$, that makes it possible to disregard the effect of the magnetic field induced by the motion of the fluid.

On the base of the general laws of conservation of the mass and momentum, the equations of motion take the following form [1,2,4,7]:

The continuum equation:

$$\nabla V = 0 \tag{1}$$

The momentum equation:

$$\rho(V\nabla)V = -\nabla p + \mu \Delta V + j \times B \tag{2}$$

where: V, j, B, p, ρ, μ, – stands for the vector of viscous fluid velocity, current density vector, magnetic induction vector, pressure, density, dynamic viscosity, respectively.

In order to "close" the set of Eqs (1) and (2) the additional equations of the electromagnetic field is necessary [1]:

$$\nabla \times E = 0 \tag{3}$$

$$\nabla E = 0 \tag{4}$$

$$\nabla \times B = \mu_0 j \tag{5}$$

$$\nabla B = 0 \tag{6}$$

$$\nabla j = 0 \tag{7}$$

$$j = \sigma(E + V \times B) \tag{8}$$

where: E, μ_0, σ: electrical field vector, magnetic permeability, electrical conductivity, respectively.

Writing Eqs. (1), (2), (8) in the curvilinear system of coordinates x, θ, y and applying the estimations typical for flows occurring in thin layers, $h \ll R(x)$, we obtain [12, 17]:

$$\frac{1}{R}\frac{\partial(Rv_x)}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{9}$$

$$\rho\left(v_x\frac{\partial v_x}{\partial x} + v_z\frac{\partial v_x}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 v_x}{\partial y^2} - j_y B_\theta \tag{10}$$

$$\frac{\partial p}{\partial y} = 0 \tag{11}$$

$$j_y = \sigma\left(E_y + v_x B_\theta\right) \tag{12}$$

where: v_x, v_y, p, E_y, B_θ: velocity components, pressure, component of magnetization, component of magnetic field, respectively.

Thus, from Eq. (11) we have:

$$p = p(x) \tag{13}$$

The boundary conditions for velocity components and pressure are:
– velocity components

$$v_x(x, \mp h) = 0, \qquad v_y(x, \mp h) = 0, \tag{14}$$

– pressure
  • at the inlet:
$$p = p_i \quad \text{for} \quad x = x_i$$

- at the outlet

$$p = p_0 \quad \text{for} \quad x = x_0 \qquad \text{(15)}$$

where: x_i – denotes the inlet coordinate, x_0 – the outlet coordinate.

SOLUTION OF THE EQUATIONS OF MOTION

Introducing the following dimensionless quantities:

$$\tilde{x} = \frac{x}{R_o}, \qquad \tilde{y} = \frac{y}{h_o}, \qquad \tilde{R} = \frac{R}{R_o},$$

$$\tilde{v}_x = \frac{v_x}{v_o}, \quad \tilde{v}_y = \frac{v_y}{v_o}\frac{R_o}{h_o}, \quad \tilde{p} = \frac{ph_o}{\mu v_o}\frac{h_o}{R_o} \qquad \text{(16)}$$

we can present the equations of motion (9) ÷ (11) in the form:

$$\frac{1}{\tilde{R}}\frac{\partial(\tilde{\rho}_e \tilde{R}\tilde{v}_x)}{\partial \tilde{x}} + \frac{\partial(\tilde{\rho}_e \tilde{v}_y)}{\partial \tilde{y}} = 0 \qquad \text{(17)}$$

$$\lambda\left(\tilde{v}_x \frac{\partial \tilde{v}_x}{\partial \tilde{x}} + v_y \frac{\partial \tilde{v}_x}{\partial \tilde{y}}\right) = -\frac{\partial \tilde{p}}{\partial \tilde{x}} + \frac{\partial^2 \tilde{v}_x}{\partial \tilde{y}^2} - \tilde{J}_y \tilde{B}_\theta Ha \qquad \text{(18)}$$

$$0 = \frac{\partial \tilde{p}}{\partial \tilde{y}} \qquad \text{(19)}$$

where:

$$Ha = Bh_0\sqrt{\frac{\sigma}{\mu}}, \quad \tilde{B}_\theta = \frac{B_\theta}{B}, \quad \tilde{J}_y = j_y\frac{h_o}{v_o}\sqrt{\frac{\sigma}{\mu}}, \quad \lambda = Re\frac{h_o}{R_o}, \quad Re = \frac{2\rho v_o h_o}{\mu}.$$

The quantities marked 'zero' are average values within the discussed flow domain; λ – is the modified Reynolds number which satisfies the condition:

$$\lambda < 1 \qquad \text{(20)}$$

In Eqs. (17), (18), (19) describing motion of electro-conductive fluid, if condition (20) is satisfied, λ is a small parameter of the system.

Thus, the solution can be sought for in the form of power series with respect to λ:

$$\tilde{v}_x = \sum_{i=0}^{\infty} \lambda^i \tilde{v}_x^i, \quad \tilde{v}_y = \sum_{i=0}^{\infty} \lambda^i \tilde{v}_y^i, \quad \tilde{p} = \sum_{i=0}^{\infty} \lambda^i \tilde{p}^i \qquad \text{(21)}$$

Introducing the series (21) into Eqs. (17), (18) and grouping the terms with the same powers of λ, confining the considerations to linear approximation and returning to the previous dimensional form, we get the following equations:

$$\frac{1}{R}\frac{\partial(Rv_x^0)}{\partial x} + \frac{\partial v_y^0}{\partial y} = 0 \qquad \text{(22)}$$

$$0 = -\frac{\partial p^0}{\partial x} + \mu\frac{\partial^2 v_x^0}{\partial y^2} - j_y^0 B_\theta \qquad \text{(23)}$$

$$\frac{1}{R}\frac{\partial(Rv_x^1)}{\partial x} + \frac{\partial v_y^1}{\partial y} = 0 \qquad \text{(24)}$$

$$\rho\left[v_x^0 \frac{\partial v_x^0}{\partial x} + v_y^0 \frac{\partial v_x^0}{\partial y}\right] = -\frac{\partial p^1}{\partial x} + \mu\frac{\partial^2 v_x^1}{\partial y^2} - j_y^0 B_\theta \qquad \text{(25)}$$

The boundary conditions in accordance with Eqs. (14) and (15), have the form:
for

$$v_x^0 = v_x^1 = v_y^0 = v_y^1 = 0 \quad \text{for} \quad y = \mp h \qquad \text{(26)}$$

for

$$\begin{aligned} p^0 = p_i \quad p^1 = 0 \quad &\text{for} \quad x = x_i \\ p^0 = p_0 \quad p^1 = 0 \quad &\text{for} \quad x = x_0 \end{aligned} \qquad \text{(27)}$$

Integrating Eqs. (22)–(25) with the boundary conditions (26)–(27) we have:

$$v_x^0 = \frac{1}{2\mu Rh^3}\frac{p_i - p_0 - B_i + B_0}{(A_i - A_0) - (M_i - M_0)}(y^2 - h^2) \qquad \text{(28)}$$

$$v_y^0 = \frac{h'}{2\mu Rh^4}\frac{p_i - p_0 - B_i + B_0}{(A_i - A_0) - (M_i - M_0)}(h^2 y - y^3) \qquad \text{(29)}$$

$$p^0 = B(x) + \frac{[A(x) - A_0](p_i - B_i)[1 + Z(x)] - [A(x) - A_i](p_0 - B_0)[1 + W(x)]}{(A_i - A_0) + (M_i - M_0)} \qquad \text{(30)}$$

$$v_x^1 = \frac{1}{840}\frac{\rho C^2(Rh')}{\mu^3 R^3 h^7}(35h^2 y^4 - 7y^6 + 5h^6 - 33h^4 y^2) \qquad \text{(31)}$$

$$v_y^1 = \frac{1}{840}\frac{\rho C^2}{\mu^3 R}\left\{\left[\frac{(Rh')'}{R^2 h^7}\right]'(y^7 - 7h^2 y^5 - 5h^6 y + 11h^4 y^3) + \frac{(Rh)'h'}{R^2 h^7}(44h^3 y^3 - 14hy^5 - 30h^5 y)\right\} \qquad \text{(32)}$$

$$p^1 = D(x) - \frac{[A(x) - A_0]D_i - [A(x) - A_i]D_0}{A_i - A_0} \qquad \text{(33)}$$

where:

$$R' = \frac{dR}{dx}, \qquad h' = \frac{dh}{dx}, \qquad C = \frac{p_i - B_i - (p_0 - B_0)}{(A_i - A_0) + (M_i - M_0)},$$

$$A(x) = \int \frac{dx}{Rh^3}, \qquad A_i = A(x_i), \qquad A_0 = A(x_0),$$

$$B(x) = -\sigma E_y BR_i \int \frac{dx}{R}, \quad B_i = B(x_i), \qquad B_0 = B(x_0),$$

$$Z(x) = \frac{(M(x) - M_0)}{(A(x) - A_0)}, \quad W(x) = \frac{(M(x) - M_i)}{(A(x) - A_i)},$$

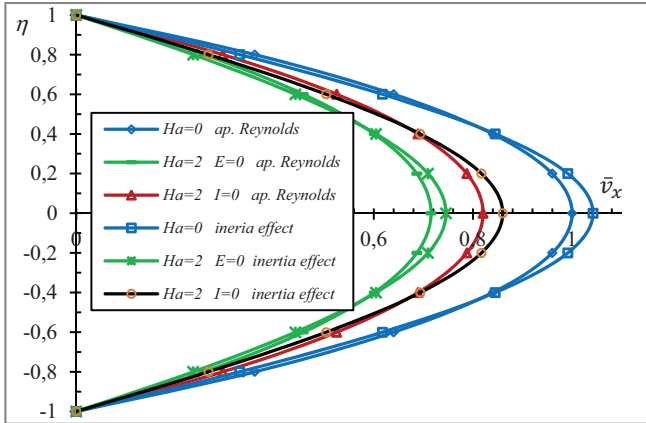$$M(x) = \frac{1}{3}Ha^2 R_i^2 \int \frac{dx}{(Rh)^3}, \quad M_i = M(x_i), \quad M_0 = M(x_0),$$

$$E_y = \frac{J}{2\pi\sigma\left[s(x) - \frac{1}{3}Ha^2 R_i^2\, ss(x)\dfrac{(A_i - A_0)}{(A_i - A_0)+(M_i - M_0)}\right]}$$
$$+\ \frac{h^2}{3\mu\sigma}HaR_i\, ss(x)\frac{(p_i - p_0)}{(A_i - A_0)+(M_i - M_0)}$$

$$s(x) = \int_{x_i}^{x_0} R\,dx, \quad ss(x) = \int_{x_i}^{x_0}\frac{dx}{R},$$

$$D(x) = \int \frac{6\rho C^2 (Rh)'}{35\mu^2 R^3 h^3}\,dx, \quad D_i = D(x_i), \quad D_0 = D(x_0).$$

The complete solution to the magnetohydrodynamic fluid flow problem inside a slot between (in general) curvilinear surfaces consists of the sum of partial solutions v^0 and v^1.

HYDROMAGNETIC FLUID FLOW BETWEEN FIXED CONICAL SURFACES

The parameters describing the geometry of the considered flow region can be written as follows (Fig. 2):

$$R(x) = x\sin(\alpha), \qquad R_i = x_i \sin(\alpha),$$
$$R_0 = x_0 \sin(\alpha), \qquad R' = \sin(\alpha)$$

Introducing the dimensionless quantities into Eqs. (28)–(32):

$$\xi = \frac{x}{R_0}, \qquad \eta = \frac{y}{h}, \qquad \bar{R}' = 1,$$
$$\bar{v}_x = \frac{v_x}{v_{xmax}^0}, \qquad \bar{v}_y = \frac{v_y}{v_{xmax}^0}\frac{R_0}{h},$$
$$v_{xmax}^0 = \frac{p_0 h^2 (\bar{p}_i - 1)}{2x\mu R_0 (a_0 - a_i)}$$

Fig. 2. Conical slot geometry

we obtain the formulae representing the motion of the MHD fluid inside the slot between the fixed conical surfaces:

$$\bar{v}_x^0 = \left(\frac{1 - \dfrac{\bar{B}_i - \bar{B}_0}{\bar{p}_i - 1}}{1 + \dfrac{m_i - m_0}{a_i - a_0}}\right)(1 - \eta^2), \tag{34}$$

$$\bar{v}_y^0 = 0, \tag{35}$$

$$\bar{p}^0 = \bar{B}(x) + \frac{[a(x) - a_0][\bar{p}_i - \bar{B}_i][1 + z(x)] - [a(x) - a_i](1 - \bar{B}_0)[1 + w(x)]}{(a_i - a_0) + (m_i - m_0)} \tag{36}$$

$$\bar{v}_x^1 = -\frac{1}{420\xi^2}\Lambda c\,\frac{a_i - a_0}{\bar{p}_i - 1}\,\frac{[\bar{p}_i - 1 - \bar{B}_i + \bar{B}_0]^2}{\left[1 + \dfrac{(m_i - m_0)}{(a_i - a_0)}\right]^2}(5 - 7\eta^6 + 35\eta^4 - 33\eta^2) \tag{37}$$

$$\bar{v}_y^1 = \frac{1}{240\xi^3}\Lambda c\,\frac{a_i - a_0}{\bar{p}_i - 1}\,\frac{[\bar{p}_i - 1 - \bar{B}_i + \bar{B}_0]^2}{\left[1 + \dfrac{(m_i - m_0)}{(a_i - a_0)}\right]^2}(\eta^7 - 7\eta^5 + 11\eta^3 - 5\eta) \tag{38}$$

$$\bar{p}^1 = \bar{D}(x) - \frac{[a(x) - a_0]\bar{D}_i - [a(x) - a_i]\bar{D}_0}{a_i - a_0} \tag{39}$$

where:

$$a(x) = \ln(\xi), \qquad a_i = a(\xi_i), \qquad a_0 = a(\xi_0),$$

$$\bar{B}(x) = -EHax_i\ln(\xi), \quad \bar{B}_i = \bar{B}(\xi_i), \quad \bar{B}_0 = \bar{B}(\xi_0), \qquad Ha = Bh_0\sqrt{\frac{\sigma}{\mu}},$$

$$E = \frac{1}{\frac{1}{2}(1 - \xi_i^2) + \frac{1}{3}\frac{Ha^2\xi_i^2\ln(\xi_i)}{1 + \frac{(m_i - m_0)}{(a_i - a_0)}}}\left[I - \frac{1}{3}Ha\xi_i\ln(\xi_i)\frac{\bar{p}_i - 1}{(a_i - a_0) + (m_i - m_0)}\right],$$

$$m(x) = -\frac{1}{6}Ha^2\frac{\xi_i^2}{\xi^2}, \quad m_i = m(\xi_i), \quad m_0 = m(\xi_0),$$

$$z(x) = 1 + \frac{(m(x) - m_0)}{(a(x) - a_0)}, \quad w(x) = 1 + \frac{(m(x) - m_i)}{(a(x) - a_i)}$$

$$I = \frac{J\sqrt{\mu}}{2\pi hp_0 x_0 \sin\alpha\sqrt{\sigma}}, \quad E = \frac{E_y x_0 \sin\alpha\sqrt{\sigma\mu}}{hp_i},$$

$$\bar{D}(x) = -\frac{3}{35}\Lambda c\,\frac{(\bar{p}_i - 1 - \bar{B}_i + \bar{B}_0)^2}{\xi^2\left[1 + \dfrac{(m_i - m_0)}{(a_i - a_0)}\right]^2},$$

$$\bar{D}_i = \bar{D}(\xi_i), \qquad \bar{D}_0 = \bar{D}(\xi_0), \qquad \Lambda c = \frac{\rho p_0 h^4}{\mu^2 R_0^2 (a_i - a_0)^2}.$$

The above given formulas have been illustrated in Fig. 3 and Fig. 6.

Fig. 3. Effect of magnetic field (Hartmann number (Ha)) and inertia effects on the profile of the velocity component $\bar{v}_x$
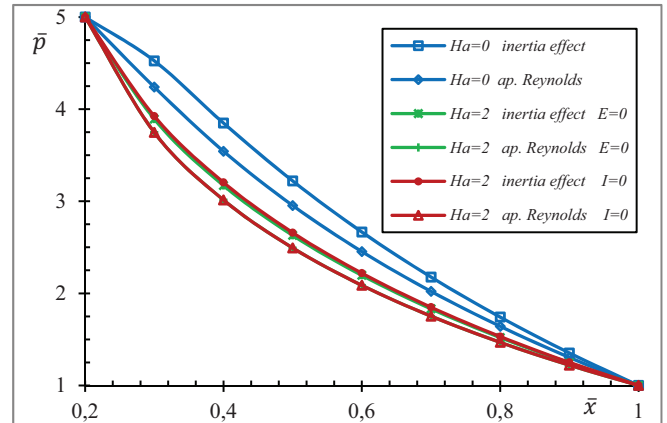
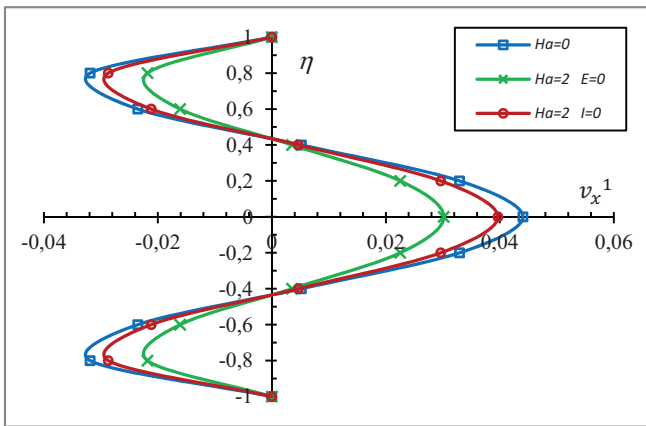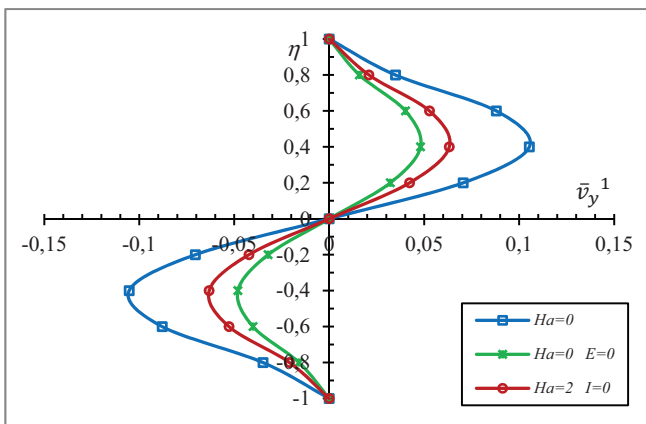$$(\; \xi = 0,6, \quad \bar{p}_i = 5, \qquad \Lambda c = 0,05 \;)$$

Fig. 4. Effect of magnetic field (Hartmann number (Ha)) and inertia effects on the profile of the velocity component $\bar{v}_x^1$

$$(\; \xi = 0,6, \quad \bar{p}_i = 5, \qquad \Lambda c = 0,05 \;)$$

Fig. 5. Effect of magnetic field (Hartmann number (Ha)) and inertia effects on the profile of the velocity component $\bar{v}_y^1$

$$(\xi_i = 0,2, \quad \xi_0 = 0,6, \quad \bar{p}_i = 5, \Lambda c = 0,05)$$

Fig. 6. Effect of magnetic field (Hartmann number (Ha)) and inertia effects on the pressure distribution $\bar{p}$

$$(\xi_i = 0,2, \quad \xi_0 = 1, \quad \bar{p}_i = 5, \qquad \Lambda c = 0,05,)$$

HYDROMAGNETIC FLUID FLOW BETWEEN FIXED SPHERICAL SURFACES

The parameters describing the geometry of the flow region can be written as follows (Fig. 7):

$$R(x) = R_k sin(\varphi), \qquad R_i = R_k sin(\varphi_i),$$

$$R_0 = R_k sin(\varphi_0),$$

$$R' = cos(\varphi), \qquad \varphi = \frac{x}{R_k}.$$

After introducing the functions used for describing the geometry of the flow area (Fig. 7) in Eqs. (28)–(32) and providing the dimensionless quantities:

$$\eta = \frac{y}{h}, \qquad \bar{R} = \frac{R(x)}{R_0}, \qquad \bar{R}' = cos(\varphi)$$

$$\bar{v}_x = \frac{v_x}{v_{xmax}^0}, \qquad \bar{v}_y = \frac{v_y}{v_{xmax}^0} \frac{R_k sin(\varphi_0)}{h},$$

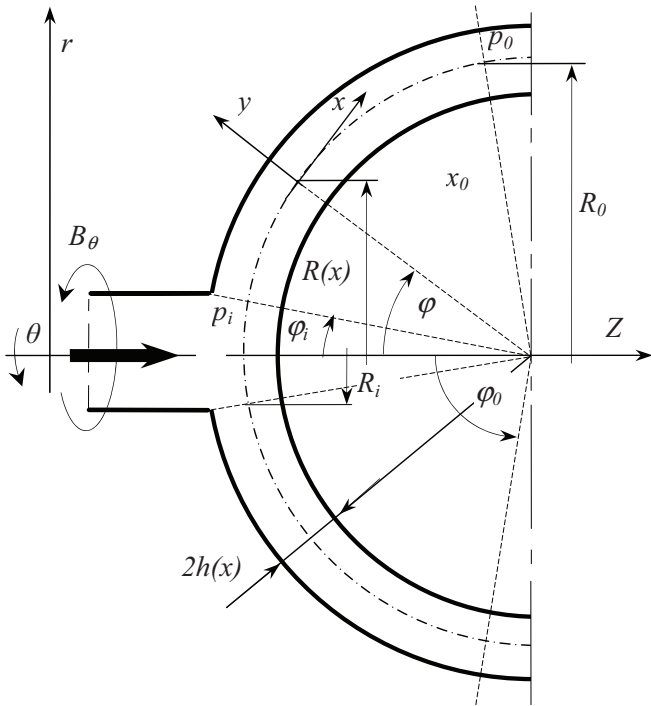$$v_{xmax}^0 = \frac{p_0 h^2(\bar{p}_i - 1)}{2\mu sin(\varphi)(a_0 - a_i)R_0}$$

Fig. 7. Spherical slot geometry

we obtain the formulas representing the motion of the MHD fluid inside the slot between the fixed spherical surfaces:

$$\bar{v}_x^0 = \left(\frac{1 - \frac{\bar{B}_i - \bar{B}_0}{\bar{p}_i - 1}}{1 + \frac{m_i - m_0}{a_i - a_0}} \right)(1 - \eta^2) \,, \tag{40}$$

$$\bar{v}_y^0 = 0 \,, \tag{41}$$

$$\bar{p}^0 = \bar{B}(\varphi) + \frac{[a(\varphi) - a_0](\bar{p}_i - \bar{B}_i)[1 + z(\varphi)] - [a(\varphi) - a_i](1 - \bar{B}_0)[1 + w(\varphi)]}{(a_i - a_0) + (m_i - m_0)} \,, \tag{42}$$

$$\bar{v}_x^1 = -\frac{1}{420} \Lambda s \frac{a_i - a_0}{\bar{p}_i - 1} \frac{[\bar{p}_i - 1 - \bar{B}_i + \bar{B}_0]^2}{\left[1 + \frac{(m_i - m_0)}{(a_i - a_0)}\right]^2} \frac{cos(\varphi)}{sin^2(\varphi)} (5 - 7\eta^6 + 35\eta^4 - 33\eta^2) \tag{43}$$

$$\bar{v}_y^1 = \frac{1}{720 sin^3(\varphi)} \Lambda s \frac{a_i - a_0}{\bar{p}_i - 1} \frac{[\bar{p}_i - 1 - \bar{B}_i + \bar{B}_0]^2}{\left[1 + \frac{(m_i - m_0)}{(a_i - a_0)}\right]^2} [1 + cos^2(\varphi)](\eta^7 - 7\eta^5 + 11\eta^3 - 5\eta) \tag{44}$$

$$\bar{p}^1 = \bar{D}(\varphi) - \frac{[a(\varphi) - a_0]\bar{D}_i - [a(\varphi) - a_i]\bar{D}_0}{a_i - a_0} \tag{45}$$

where:

$$a(\varphi) = ln \left| tan\left(\frac{\varphi}{2}\right)\right|, \quad a_i = a(\varphi_i), \quad a_0 = a(\varphi_0),$$

$$\bar{B}(\varphi) = -EHasin(\varphi_i)ln\left(tan\left(\frac{\varphi}{2}\right)\right),$$

$$\bar{B}_i = \bar{B}(\varphi_i), \quad \bar{B}_0 = \bar{B}(\varphi_0), \quad Ha = Bh_0\sqrt{\frac{\sigma}{\mu}},$$

$$E = \frac{1}{\frac{1}{2}(cos(\varphi_i) - cos(\varphi_0)) + \frac{1}{3}\frac{Ha^2 sin^2\varphi_i\left|tan\left(\frac{\varphi_0}{2}\right) - \left|tan\left(\frac{\varphi_i}{2}\right)\right|\right|}{1 + \frac{(m_i - m_0)}{(a_i - a_0)}}} \left[I - \frac{1}{3} HaEHasin(\varphi_i)_i \left[\left|tan\left(\frac{\varphi_0}{2}\right)\right| - \left|tan\left(\frac{\varphi_i}{2}\right)\right|\right] \frac{\bar{p}_i - 1}{(a_i - a_0) + (m_i - m_0)} \right],$$

$$m(\varphi) = -\frac{1}{6} Ha^2 sin^2(\varphi_i) \int \frac{d\varphi}{sin^3(\varphi)},$$

$$m_i = m(\varphi_i), \quad m_0 = m(\varphi_0),$$

$$z(\varphi) = 1 + \frac{(m(\varphi) - m_0)}{(a(\varphi) - a_0)}, \quad w(\varphi) = 1 + \frac{(m(\varphi) - m_i)}{(a(\varphi) - a_i)}$$

$$I = \frac{J\sqrt{\mu}}{2\pi hp_0 R_0 sin(\varphi_0)\sqrt{\sigma}}, \quad E = \frac{E_y R_0 sin(\varphi_0)\sqrt{\sigma\mu}}{hp_0},$$

$$\bar{D}(\varphi) = -\frac{3}{35} \Lambda s \frac{1}{sin^2(\varphi)} \frac{(\bar{p}_i - 1 - \bar{B}_i + \bar{B}_0)^2}{\left[1 + \frac{(m_i - m_0)}{(a_i - a_0)}\right]^2} \,,$$

$$\bar{D}_i = \bar{D}(\varphi_i), \quad \bar{D}_0 = \bar{D}(\varphi_0), \quad \Lambda s = \frac{\rho p_0 h^4}{\mu^2 R_0^2 (a_i - a_0)^2}.$$

Since the diagrams of the MHD fluid flow velocity distribution along the slot between the fixed spherical surfaces show no substantial differences from the magnetohydrodynamic flow velocity distribution in the slot between the fixed conical surfaces, only the formulas for pressure profiles are illustrated in the diagram (Fig. 8).
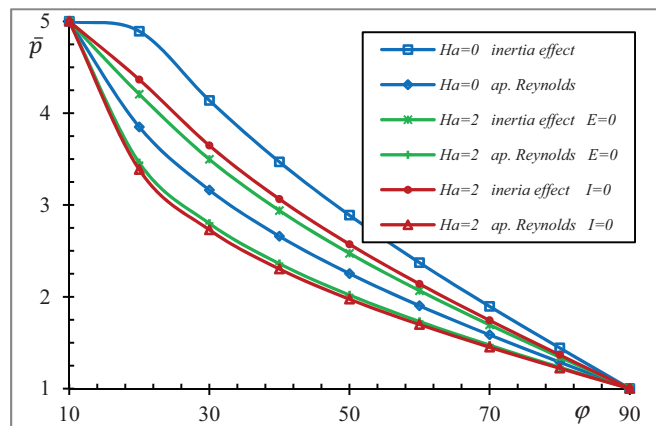

Fig. 8. Effect of magnetic field (Hartmann number (Ha)) and inertia effects on the pressure distribution $\bar{p}$

$$(\varphi_i = 10^o, \quad \varphi_0 = 90^o, \quad \bar{p}_i = 5, \quad \Lambda c = 0,05)$$

DISCUSSION OF THE RESULTS

On the basis of the diagram and equations describing the components of the field of velocity and pressure it is possible to formulate the following conclusions:

For the longitudinal velocity $\bar{v}_x$ (Fig. 3):
- the main profile of the velocity $\bar{v}_x^0$ is a parabolic profile identical to traditional Poisseuille flow profile caused by a steady gradient of pressure (constant pressure difference) characteristic for the so-called Reynolds approximation,
- the so-called secondary profile symmetric to the slot symmetry axis formed due to the occurrence of the flow inertia effects agrees with the main velocity profile $\bar{v}_x$ (Fig. 4),
- increase in the intensity of magnetic field expressed by an increase in the value of the Hartmann number, results in inhibiting the longitudinal velocity component $\bar{v}_x$,
- a short circuit in external electrical circuit ($E=0$), as compared with open electrical circuit ($I=0$), results in a greater deceleration of the velocity component $\bar{v}_x$.

For the transverse velocity $\bar{v}_y$ (Fig. 5):
- distribution of the transverse velocity $\bar{v}_y$ is the result of inertia effects occurring in the flow,
- an increase in the intensity of magnetic field expressed by an increase in the value of the Hartmann number , results in decelerating the velocity component $\bar{v}_y$,
- a short circuit in external electrical circuit ($E=0$), as compared with the open electrical circuit ($I=0$), results in a greater deceleration of the velocity component $\bar{v}_y$.

For the pressure p (Fig. 6 and Fig. 8):
- inertia effects (Λ_c, $\Lambda_s > 0$) cause a minimum pressure increase along the slot,
- an increase in the intensity of magnetic field, expressed by an increase in the value of the Hartman number Ha, results in a pressure drop along the slot, characteristic for the supply with constant pressure $\bar{p}_i$ at the inlet (it decreases the effect of inertia),
- a short circuit in the external electric circuit ($E=0$), as compared with open electric circuit ($I=0$) results in a higher drop of the pressure $\bar{p}$.

The flow through slots with curvilinear profiles is less susceptible to inertia effects.

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