

# IDENTIFICATION OF SONAR DETECTION SIGNAL BASED ON FRACTIONAL FOURIER TRANSFORM

Wang Biao<sup>1,2</sup>

Tang Jiansheng<sup>2</sup>

Yu Fujian<sup>2</sup>

Zhu Zhiyu<sup>1</sup>

<sup>1</sup> School of Electronic and Information, Jiangsu University of Science and Technology, Zhenjiang, Jiangsu Prov., China

<sup>2</sup> Key Laboratory of Underwater Acoustic Warfare Technology, Beijing, China

## ABSTRACT

*Aiming at the source of underwater acoustic emission, in order to identify the enemy emission sonar source accurately. Using the digital watermarking technology and combining with the good time-frequency characteristics of fractional Fourier transform (FRFT), this paper proposes a sonar watermarking method based on fractional Fourier transform. The digital watermark embedding in the fractional Fourier transform domain and combined with the coefficient properties of the sonar signal in the fractional Fourier transform to select the appropriate watermark position. Using the different characteristics of the signals before and after embedding, an adaptive threshold was set for the watermark detection to realize the discrimination of sonar signals. The simulation results show the feasibility and has better resolution and large watermark capacity of this method, while the robustness of the watermark is better, and the detection precision is further improved.*

**Keywords:** Fractional Fourier transform, Watermark, Sonar

## INTRODUCTION

At present sonar is the main equipment of ocean exploration and target detection, with the increasing frequency of marine activities, underwater sources are filled with various sources of acoustic emission from unknown sources. These sonar source emit various signals, however, these signals are highly similar, resulting in the inability to identify their source, the importance of information security has become a key issue in underwater activities.

The latest method of the current sonar signal identification is based on the digital watermarking signal identification, by detecting whether the received signal contains a watermark to identify its identity. Mobasser B G and Lynch R S et al. proposed watermark embedding using short-time Fourier transform (STFT) and discrete cosine transform (DCT) for the identification of active sonar signals in 2008 and 2010 respectively. But the algorithm is more complex and does not fully consider the multiplicity, attenuation and Doppler shift of ocean channel. In 2011, the effect of channel-related

features on the performance of watermarking and the use of embedded watermarks to improve detection performance were proposed. In the identification of sonar signals has been studied in the algorithm, although channel attenuation and Doppler have been taken into account, but the influence of the pulse interference on the detection between the embedded watermark is still not improved, the robustness of the watermark in the channel is poor and also seriously affects identification accuracy. Visibility detection accuracy and robustness of watermark the improvement of these two aspects has the very important significance for the identification of sonar signal, this paper examines these two aspects. And found that the fractional Fourier transform (FRFT) has good time-frequency characteristics, based on this foundation, proposes a digital watermarking sonar identification algorithm based on FRFT.

The fractional Fourier transform can be regarded as a generalized form of the Fourier transform. In recent years, it has received extensive attention in signal processing, which combines the characteristics of the signal in the time domain

and frequency domain. Application of this method first appeared in the digital image watermarking, and has a good effect, the literature [5][6] is the application of watermark in image. In this article, the watermark is embedded into the sonar signal by fractional Fourier transform and the power of the FRFT (Frequency Fractional Fourier Transform) is taken as the degree of freedom in the watermark embedding. At the same time, it is also used as an embedded key, which not only enhances the security of the watermark but also increases the robustness of the watermark.

## PRINCIPLE OF DIGITAL WATERMARK EMBEDDING BASED ON FRFT

### THEORETICAL BASIS

FRFT as a generalized form of the Fourier transform, It can be interpreted as the representation of the signal in the fractional Fourier transform domain after the signal is rotated by any angle counterclockwise around the origin in the time-frequency plane. If Fourier transform of signal can be seen as a representation of counterclockwise rotation of  $\pi / 2$  on the time axis to the frequency axis, then FRFT can be regarded as a representation of the signal on the time by axis counterclockwise rotation of the angle  $\alpha$  to the  $\mu$  axis ( $\mu$  axis is called fractional Fourier domain),  $x(t)$  of the signal the fractional Fourier transform (FRFT) is defined as:

$$X_{\alpha}(u) = \{F^{\alpha}[X(t)]\}(u) = \int_{-\infty}^{+\infty} X(t)K_{\alpha}(t, u)dt \quad (1)$$

Where FRFT transform kernel  $K_{\alpha}(t, u)$  is:

$$K_{\alpha}(t, u) = \begin{cases} \sqrt{\frac{1-j\cot(\alpha)}{2\pi}} \exp(j\frac{t^2+u^2}{2} \cot(\alpha)) & \alpha \neq n\pi \\ \delta(t-u) & \alpha = 2n\pi \\ \delta(t+u) & \alpha = (2n+1)\pi \end{cases} \quad (2)$$

Formula:  $\alpha = p, \pi / 2$  is the rotation angle of FRFT;  $F^{\alpha}$  is the alpha Order Fractional Fourier operator symbol. The inverse transformation is:

$$x(t) = \int_{-\infty}^{+\infty} X_{\alpha}(u)K_{\alpha}(t, u)dt \quad (3)$$

When the  $p=1$  ( $\alpha=\pi / 2$ ), the FRFT degenerate to the traditional Fourier transform, when the transformation order is close to 1, the response is the frequency domain characteristic. When the transformation order is close to 0, the response is the time domain characteristic.

The so-called decomposition method is based on FRFT (Fractional Fourier transform) expression. FRFT is decomposed into the convolution form of the signal, which uses FFT to calculate FRFT. In this paper, we use the fast decomposition algorithm proposed by Bultheel A et al, which is very suitable for FRFT numerical calculation of the signal. However, the operation mechanism of this fast algorithm determines that the original signal must be normalized by dimension localization before the FRFT numerical calculation. The discrete fractional Fourier transform using dimension normalization is defined as: the time interval of the signal is  $[-\Delta t / 2 \Delta t / 2]$ , frequency range is  $[-\Delta f / 2 \Delta f / 2]$ , the time bandwidth product of the signal is  $N = \Delta t \Delta f$ , the scale factor of the normalized introduced dimension is S:

$$\Delta x = \sqrt{\Delta t \Delta f} \quad x = t / s \quad v = f x s \quad (4)$$

Then the interval can be normalized to  $[-\Delta x / 2 \Delta x / 2]$ , After normalized sampling interval  $1 / \Delta x$ ,  $N = \Delta x^2$ .

The fractional Fourier transform can be rewritten as:

$$X_p(u) = A_{\alpha} \exp[j\pi u^2 \cot \alpha] \int_{-\infty}^{+\infty} x(t) \exp[j\pi t^2 \cot \alpha] \exp[-j2\pi u t \csc \alpha] dt \quad (5)$$

Among  $0 < |\alpha| < \pi$

$$A^{\alpha} = \exp[j(P-1)\pi / 4] / \sqrt{|\sin \alpha|}, \alpha = P\pi / 2$$

By the above formula Fractional Fourier transform can be divided into:

- (1) The multiplication of the signal with the linear frequency modulation function
- (2) Fourier transform (the argument is multiplied by the scale factor  $\csc \alpha$ )
- (3) Multiplication with linear frequency modulation function
- (4) Multiplied by a complex factor

Finally, the normalized expression is:

$$\begin{aligned} &= \frac{A_{\alpha}}{2\Delta x} \left[ \exp\left\{-\frac{j\pi \tan \frac{\alpha}{2} m^2}{(2\Delta x)^2}\right\} X \right. \\ &\times \sum_{n=-N}^{n=N} \exp\left[\frac{j\pi(\csc \alpha)(m-n)^2}{(2\Delta x)^2}\right] \\ &\left. \times \left[ \exp\left\{-\frac{j\pi \tan \frac{\alpha}{2} m^2}{(2\Delta x)^2}\right\} x\left(\frac{n}{2\Delta x}\right) \right] \right. \\ X_p\left(\frac{m}{2\Delta x}\right) &= \frac{A_{\alpha}}{2\Delta x} \sum_{n=-N}^N \exp\left[\frac{j\pi(\cot \alpha)m^2}{(2\Delta x)^2} - \frac{j2\pi(\csc \alpha)mn}{(2\Delta x)^2}\right] \\ &+ \frac{j\pi(\cot \alpha)n^2}{(2\Delta x)^2} \left. \right] x\left(\frac{n}{2\Delta x}\right) \quad (6) \end{aligned}$$

In the specific calculation, the signal sample is firstly interpolated twice, and after the calculation of the above

equation (6), the  $N$  sample values of the fractional Fourier transform are obtained by interpolating the result by twice times, although the value of  $p$  is 0.5 to 1.5, but using the periodic and additive of the fractional Fourier transform can be generalized to all the order of the fractional Fourier transform.

## SONAR DIGITAL WATERMARK EMBEDDING SYSTEM

The FRFT coefficients are modified by embedding robust watermarks in the Fractional Fourier transform domain of the signal, the source of the robust watermark is mainly Gaussian random sequence, which combines the signal with the coefficients characteristics of the fractional Fourier transform, select the appropriate watermark position, by calculating the detection statistics to detect.

### WATERMARK GENERATION

The selection of watermark sequences generally requires good autocorrelation characteristics, Gaussian sequence satisfy this requirement and its autocorrelation function has the characteristics of impulse function. The watermarking composed of random sequences of Gaussian distributions has good robustness. Gaussian noise has the characteristics of uniform energy and strong correlation. Studies have shown that the using a random sequences as a watermark for embedding does not affect the overall performance of the signal. If there is an obvious peak at a certain offset at the time of detection, and the threshold is exceeded, so indicate the watermark exists. The robustness of the watermark can be changed by changing the set threshold, this method is more prominent robustness than other watermarking methods, and detection algorithm is also relatively simple. Therefore, the use of random sequences as a watermark sequence is a better choice. In this paper, a complex random sequence is generated. The combination of two Gaussian random sequences is a complex random sequence, which produces a Gaussian sequence with mean 0 and variance of 1, the length is  $M$ ,  $R1$  and  $R2$ , which form a complex random sequence  $W = R1 + jR2$ . This Gaussian sequence is generated by the key control, it can be through the pseudo-noise generator, select a specific seed as a key to increase the security of the watermark, this paper uses a four-digit as a "seed".

### WATERMARK EMBEDDING ALGORITHM

Preprocessing is carried out before the carrier signal is embedded in the watermark. Firstly, the sonar waveform  $x(t)$  is sampled to obtain the discrete points, and the sampling points are discrete values of  $N$  to produce the discrete signal  $X$ :

$$X = \{x(n), n = 1, 2, \dots, N\} \quad (7)$$

Using the pseudo-noise generator to produce two lengths are  $M$ , respectively subject to  $(0, \sigma_1)$  and  $(0, \sigma_2)$  random sequences  $R1$  and  $R2$ , to form a complex random sequence  $W = R1 + jR2$ . This article  $\sigma_1 = \sigma_2 = 1$

The discrete signal  $X$  is decomposed by fractional discrete Fourier transform, the degree of freedom is  $p$ , and the choice of degree of freedom is based on the range of  $[0, 1]$ , get the fractional Fourier transform coefficients  $S_i, i = 1, \dots, N$ . The embedding position is selected according to the obtained fractional Fourier transform coefficient feature, and the watermark embedding selection is close to the position of the instantaneous frequency band  $IF$  ( $IF$  refers to the region of the energy distribution of the signal in the corresponding frequency domain), due to the fractional Fourier transform will occur in the rotation of the domain, so it is generally chosen to be embedded in the middle of the fractional Fourier transform. According to the additive rule, the fractional Fourier transform coefficients of the watermark can be obtained:

$$S_i^w = S_i + aW = a(R1 + jR2), i = L + 1, \dots, L + M \quad (8)$$

The embedding strength is  $a$ .

After the embedding, the fractional Fourier transform with the degree of freedom of  $-p$  is used to obtain the signal after the watermark.

### WATERMARK EMBEDDING CRITERION

The watermark sonar does not affect the continuity and bandwidth of the sonar signal, if the embedded watermark brings unexpected energy beyond the sonar itself is undesirable, the watermarked sonar energy is expressed by the following expression:

$$E_{xw} = \int x^2(t)dt + k^2 \int w^2(t)dt + k \int x(t)w(t)dt \quad (9)$$

The watermark strength is  $K$ .

It can be seen from the expression that the embedding watermark is attached to add energy for the non-watermarked sonar, in which the SWR (signal watermark ratio) of the embedding intensity is in the range of 15dB ~ 25dB, the SNR in the simulation environment is 10dB ~ 15dB, SWR is at least 15dB, its energy is lower than the background noise, and the watermark and the sonar are irrelevant, so the embedded watermark energy has no real effect on the sonar.

The watermark embedding is also constrained by the distribution of watermark energy, the watermark embedding is mainly to improve the accuracy of the sound source identification. Signal in the time-frequency domain, the watermark can be embedded in the instantaneous frequency domain, the distribution of the instantaneous frequency domain of the LMF signal exhibits a linear characteristic, The watermark can also be embedded outside the instantaneous frequency domain, but within the time-frequency domain, the watermark is embedded in the instantaneous frequency domain, which can be effectively hidden below the instantaneous frequency domain, but requires a smaller SWR (strong watermark) for detection, embedded outside the instantaneous frequency domain if the same test results need to achieve higher SWR, watermark is relatively weak.

## WATERMARK DETECTION METHOD

The algorithm used in this paper is a fast decomposition algorithm proposed by Bultheel A et al, using this algorithm for FRFT numerical calculation will produce a certain amount of error. In order to carry out effective detection, select the appropriate detection statistics, watermark detection by calculating the detection of statistical  $d$  and with a predetermined threshold to complete. The test statistic  $d$  is calculated by the following equation:

$$\begin{aligned} d &= \sum_{i=L+1}^{i=L+M} [R1_i - jR2_i] S_i^{(a)} \\ &= \sum_{i=L+1}^{i=L+M} [R1_i - jR2_i] [S_i + a(R1_i + R2_i)] \\ &= a \sum_{i=L+1}^{i=L+M} [R1_i^2 + R2_i^2] + \sum_{i=L+1}^{i=L+M} [S_i R1_i - jS_i R2_i] \end{aligned} \quad (10)$$

$S_i^{(a)}$  indicate the FRFT coefficients of the watermarked signal may have been attacked. The attack mainly comes from the underwater noise, the simulation environment parameters can be seen in Table 1. Assuming that the watermarked signal is not attacked, since the mean values of  $R1$  and  $R2$  are zero and the variance respectively are  $\sigma_1, \sigma_2$ , the mean value of the signal detection statistic  $d$  embedded in the watermark is:

$$E(d) = aM(\sigma_1^2 + \sigma_2^2) \quad (11)$$

For no embedded watermark signal, so:

$$E(d) = 0 \quad (12)$$

In both cases, the variance of  $d$  is the same, that is, the number  $M$  of FRFT coefficients of usually embedded in the watermark is very large (up to several hundred). Therefore, the value of  $|d|$  will vary greatly between watermarked and non-watermarked cases, this method can detect watermarks well when the appropriate threshold is set. In view of the fact that FRFT inevitably brings some error to the coefficients, the actual value of  $|d|$  is smaller than the theoretical value, and the errors caused by different transformation angles are different. According to experience, in the application, the threshold is slightly less than  $|d|/2$ , the simulation results also verify its effectiveness.

## SIMULATION ANALYSIS

The simulation of this paper is carried out in the simulation environment as shown in Table 1, mainly through the watermark is tested primarily by the response of different watermark sequences and different transform order detectors, and the response curves under different SNR (signal noise ratio) are analyzed when the transform order is the same. This paper also analyzes the embedding strength and embedded

watermark ratio of two relevant variables of watermark capacity.

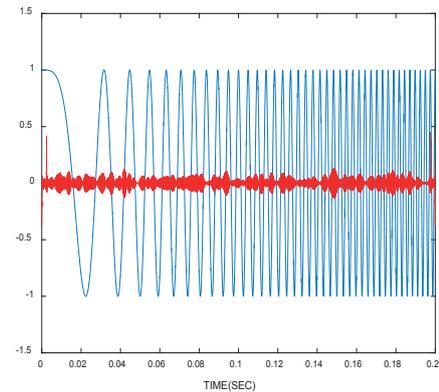
First, explain the SWR that appears in the simulation:

$$SWR = 10 \lg \frac{\text{carrier signal power}}{\text{embedding watermark power}}$$

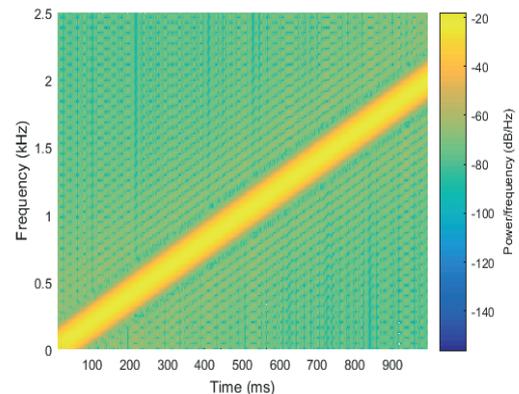
Tab. 1. Simulation parameters

Parameter name	Value
Seawater depth	200m
Transmitter / receiver depth	50m
Transmitter/ receiver horizontal distance	2000m
Channel type	Rice channel
Multi-way gain	-60dB,-65dB,-70dB
Multi-way delay	1.333(0s), 1.334(0.001s), 1.34(0.01s)
Doppler frequency shift	0Hz, 10Hz
Carrier signal waveform	Linear frequency modulation signal, time width 1s, bandwidth 2kHz

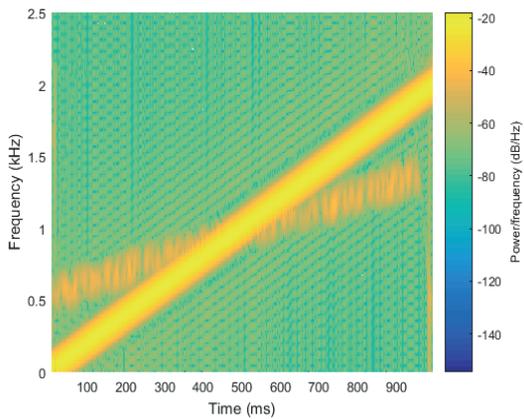
## SIGNAL TIME-FREQUENCY DIAGRAM



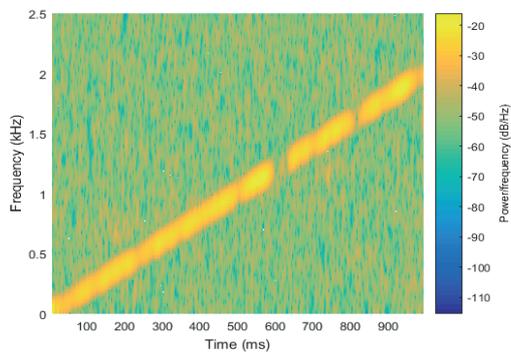
Signal plus watermark time domain diagram (a)



The original signal time-frequency diagram (b)



Embedded watermark (SWR = 15dB) time-frequency diagram (c)



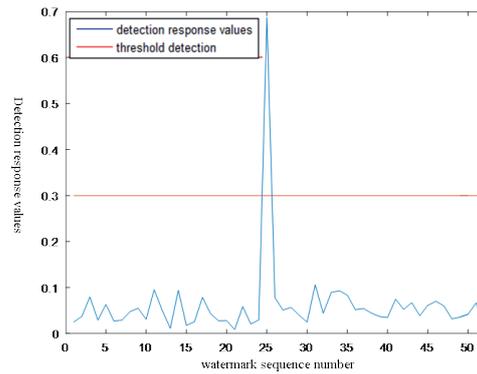
Embedded watermark plus noise (SNR = 10dB) time-frequency diagram (d)

Fig.1 Time frequency diagram

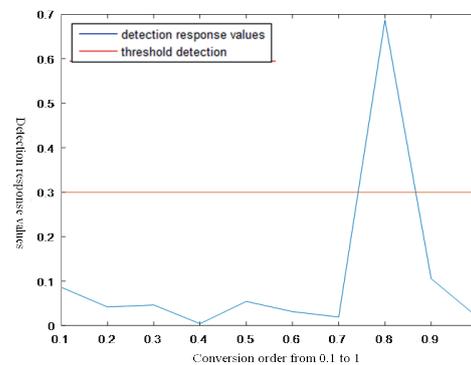
As the simulation analysis of time-frequency diagram, figure 1 (a) is a signal and watermark time-domain diagram when the number of transformations is 0.8 and the embedding intensity is 0.3. Figure 1 (b) is the original carrier time-frequency diagram, compared with Figure 1 (c) and Figure 1 (d) show that the watermark is hidden by the noise is invisible, Figure 1 (c) can be seen in the watermark embedded near Instantaneous frequency bandwidth, instantaneous frequency (IF) bandwidth is the signal exists in the time-frequency domain distribution range, through the above four graphs, it can be seen that the time-frequency characteristic of the LFM signal is not affected by the watermark.

## WATERMARK CAPACITY

According to the above parameters, the detection threshold of this paper is according to the modulus of the embedded watermark detection statistic  $d$  to selected, and the detection threshold is set to 0.3. In the simulation, 51 Gaussian random sequences are used, the 25th watermark sequence is the watermark embedded in this paper. The rest of the sequence generation keys are different from the correct watermark, and the different transformation orders are also tested, the simulation results are shown in the following figures:



The response value of different watermark sequences (a)



Detector response value of different order of transformation (b)

Fig.2 Detection of different watermark sequence and order

From Figure 2 (a) and Figure 2 (b), it can be seen that the watermark is only detected when the watermark number is 25, and the watermark is detected only under the correct condition of the conversion order, the watermark sequence based on the “seed” adds another key transform order, increase the security of the watermark.

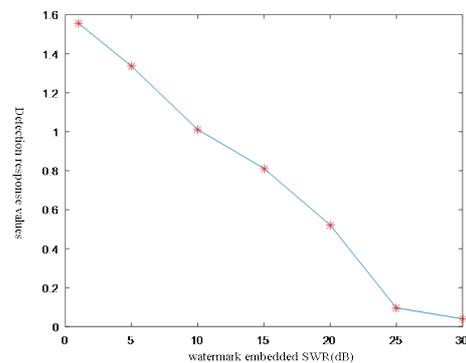


Fig.3 Detector response of different SWR

Tab. 2. Constrain data of embedding strength and swr

embedding strength K	0.07	0.1275	0.2267	0.4	0.7	1.275	2.02
SWR (dB)	30	25	20	15	10	5	1

It can be obtained from Fig. 3 that when the watermark embedding intensity is larger, through the calculation of the embedding strength is 3.03, the SWR tends to value of 0, has reached the limit value. When the watermark strength is 0.1, the SWR value reached 30, but at this time the detection response value has been lower than the detection threshold, if the embedding strength is low will appear false alarm. The above can be obtained in the SWR 10dB to 18dB is more appropriate, when the SWR is too small, as shown in Figure 1 (a) for the watermark embedded strength of 0.5 signal and watermark time domain map. The embedded watermark is Gauss sequence, it can be seen from the formula (10) that the length of the watermark sequence has a certain influence on the response value of the detector. When the length of the watermark is changed, the response value changes as follows:

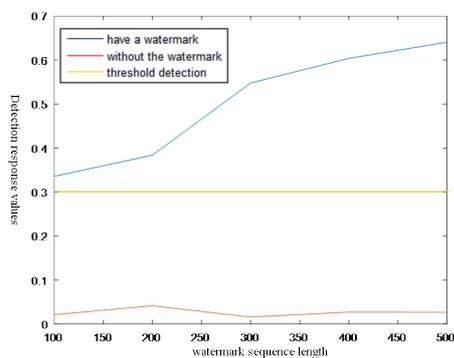
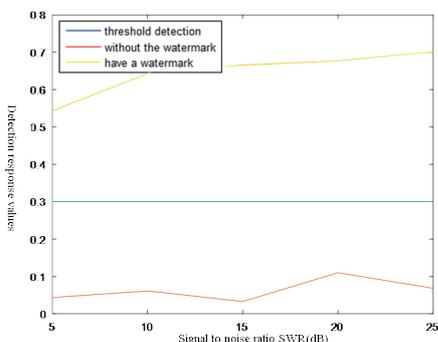


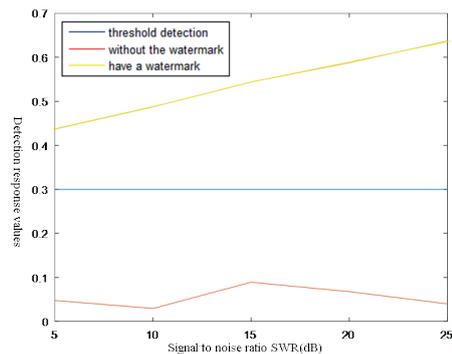
Fig.4 Detector response of different watermark length

As shown in the above figure, the results are in accordance with the theoretical results of (10), and the watermark embedding length is 400. If the length of the watermark sequence is too large, it will have an impact on the signal itself. Figure 4 and Figure 3 have the same principle, Figure 4 shows that when the transform order is fixed, the embedded strength  $K$  is constant, with the increase of the embedded sequence, the watermark SWR is decreasing, and at the same time it is consistent with Figure 3, Figure 4 is mainly used to select the appropriate sequence length.

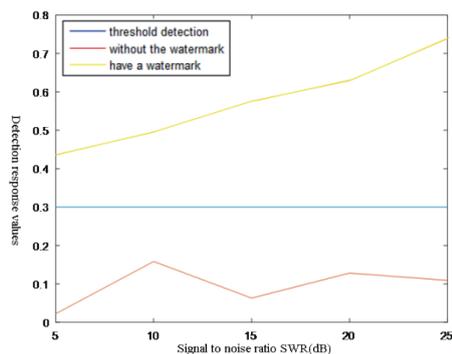
### SELECTION OF P ORDER AND ANALYSIS OF WATERMARK ROBUSTNESS



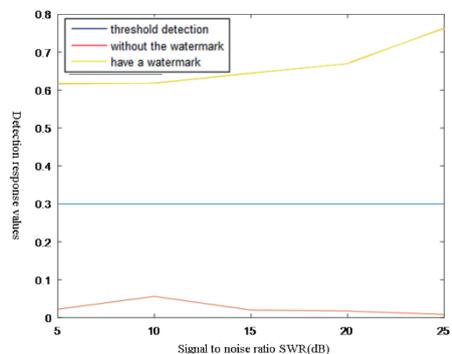
Conversion order  $P = 0.2$  (a)



Conversion order  $P = 0.4$  (b)



Conversion order  $P = 0.6$  (c)



Conversion order  $P = 0.8$  (d)

Fig.5 Response of signal to noise ratio detectors with different transform orders

In the above figures (a) to (d), the signal noise ratio varies from 5 dB to 25 dB, and the conversion order  $p$  is 0.2, 0.4, 0.6, 0.8. Figure 5, adding noise are different variance and mean of 0 Gaussian noise, can be seen from the figure with the increase of signal noise ratio, the detection response value is getting bigger and bigger, when the signal noise ratio is small, if the selected order is also relatively small, the detection response is close to the threshold as shown in Fig. 5 (a). According to the transform order, when the transform order is chosen to be 0.8, the detection response value produces a large difference, which is easily distinguishable from the watermark detection value, and the false alarm probability. The watermark in different transform orders to add Gaussian

noise, with the increase of Gaussian noise variance, the watermark detection response value is close to the detection threshold, but there is no false alarm. According to Figure 5 (a) to Figure 5 (d), even if the SNR value is 5dB, the detection response is still greater than the threshold which we set, so the probability of false alarm is very low, and the threshold selection of appropriate circumstances can strictly control the occurrence of false alarm situation. So it shows the superiority in the detection, which can be obtained in the given SNR case, in order to achieve better detection efficiency to select the appropriate threshold and the transformation order can be achieved, the detection threshold chosen by this method is high, It can be seen that the identification method can obtain high detection accuracy in the watermark with higher watermark robustness.

Through the above simulation analysis shows that when the conversion order and watermark related parameters are not known, it is basically impossible to detect whether it contains watermarks to identify the signal, strong robustness makes it difficult to destroy already embedded watermarks by setting different parameters. Therefore, the security of this method in sonar signal authentication is guaranteed, and the robustness is further improved.

## CONCLUSION

This article using the fractional Fourier transform (FRFT) as a tool, utilizing all the features that gradually transform the signal form the time domain to the frequency domain as the transform order increases, and uses the good time-frequency domain characteristics of the fractional Fourier transform to design a sonar identification method based on FRFT. The watermark is embedded into the FRFT domain of the signal, the watermark embedding of different transform orders and complex random sequences improves the security and robustness of the algorithm. Through the robustness of the algorithm, the watermark capacity, and the selection of the transformation order are analyzed. The experimental results show that the sonar signal identification based on FRFT digital watermarking algorithm has better robustness and detection efficiency than other algorithms.

## ACKNOWLEDGEMENTS

The authors acknowledge the National Natural Science Foundation of China (Grant: 11574120,U1636117), the Natural Science Foundation of Jiangsu province of China (Grant: BK20161359), the Open Project Program of the Key Laboratory of Underwater Acoustic Signal Processing, Ministry of Education, China (UASP1503), Six Talent Peaks project of Jiangsu Province and Foundation of Key Lab of Intelligent Ocean Information Perception and Transmission Technology.

## REFERENCES

1. JinCong. Digital Watermarking Theory and Technology. [M]. Beijing Tsinghua University press, 2008
2. Mobasseri B G, Lynch R S, Carter G C. Information Embedding in Sonar for Authentication and Identification[J]. IEEE Aerospace Conference Proceedings, 2008:1-5.
3. [3] Mobasseri B G, Lynch R S. Sonar authentication performance evaluation under realistically simulated undersea channels[J]. Proceedings of SPIE - The International Society for Optical Engineering, 2010, 7678:76780R-76780R-10.
4. Mobasseri B G, Lynch R S, Chakilam N. Watermarking sonar waveforms using knowledge of channel coherence[C]// Oceans. IEEE, 2010:1-8.
5. PING Xian-jun Tao-ran ZHOU Si-yong. A new fast algorithm of fractional Fourier transform [J]. Electronic journal, 2001,29 (3): 406-408
6. Wang Juan-feng. Research on digital watermarking algorithm based on fractional Fourier domain image features.[D] Zhengzhou University, 2007
7. Bultheel A, Sulbaran H E M. Computation of the fractional Fourier transform[J]. Applied & Computational Harmonic Analysis, 2004,16(3):182-202.
8. Cheng Xue. Digital watermarking technology based on fractional Fourier transform [D]. Nanjing University of Science and Technology, 2009
9. Pei S C, Ding J J. Fractional Fourier transform, wigner distribution, and filter design for stationary and nonstationary random processes[J]. Signal Processing IEEE Transactions on, 2010, 58(8):4079-4092.
10. Zhu Jian-dong, Zhao Yong-jun, Tang-Jiang. Detection and estimation of linear frequency modulated continuous wave signal in periodic fractional Fourier transform [J]. Journal of electronic and information science, 2013, 35 (8): 1827-1833.

## CONTACT WITH THE AUTHORS

**Wang Biao, Ph.D.**

*e-mail: wangbiao@just.edu.cn*

Faculty of School of Electronic and Information  
Jiangsu University of Science and Technology  
Zhenjiang, Jiangsu, 212003  
CHINA