

# ESTIMATION OF HYDRODYNAMIC COEFFICIENTS FROM RESULTS OF REAL SHIP SEA TRIALS

Daewon Kim

University of Rostock, Rostock, Germany

## ABSTRACT

*This paper suggests an estimation method for ship's hydrodynamic coefficients, which is based on the system identification method that calculates an optimum value in a mathematical way. For the purpose of modelling existing ships, this study collects real ship sea trial data as benchmarks. Prior to the optimization, a sensitivity analysis is carried out for easy and effective optimization. The simulation results using optimized coefficients agree well with corresponding benchmarks. Following this, with various trim and draught conditions, this study suggests new estimation formulas that concern all trim and draught conditions. Simulation results applying the estimation formulas are satisfactory in regard to a corresponding benchmark, compared to a result obtained by using an existing regression formula.*

**Keywords:** ship manoeuvrability, system identification method, hydrodynamic coefficients, sea trial, mathematical optimization

## INTRODUCTION

Modelling is an important way to predict ship's manoeuvrability, and the need for it is continuously growing with the development of marine information technology. The forces and moments acting especially on a submerged part of the hull can be described by hydrodynamic coefficients. International Towing Tank Conference (ITTC) summarized various methods for estimating hydrodynamic coefficients for a ship's manoeuvrability [6].

Fig. 1 and 2 show an overview of all methods and their accuracy with respect to effort/cost characteristics [6]. In the design stage, the captive model test and the Computational Fluid Dynamics (CFD) are common because there is no real ship yet and those are the most reliable sources of hydrodynamic coefficients in the environment [12, 15]. For existing ships, the full-scale trial is the most reliable, but it is not preferred due to its relatively high cost [5, 18]. Thus, a combination of empirical methods and tuning is widely used to model a ship.

This paper suggests an estimation method based on the system identification method that calculates an optimum

value in a mathematical way. This method estimates the hydrodynamic coefficients in a ship's mathematical model by a mathematical optimization algorithm that runs a manoeuvre simulation and compares it with benchmark data at every iteration. Finally, it provides optimized hydrodynamic coefficients.

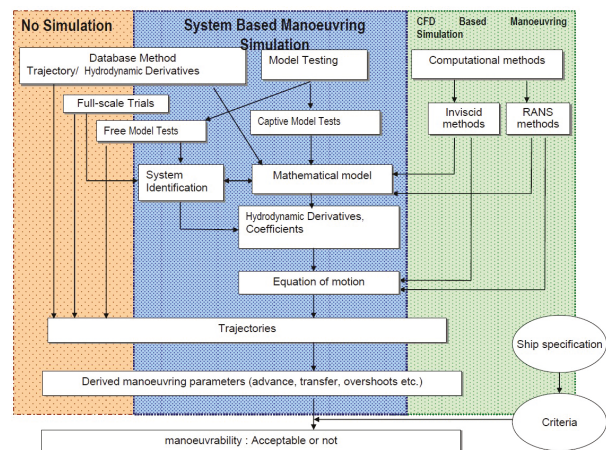


Fig. 1. Overview for prediction methods

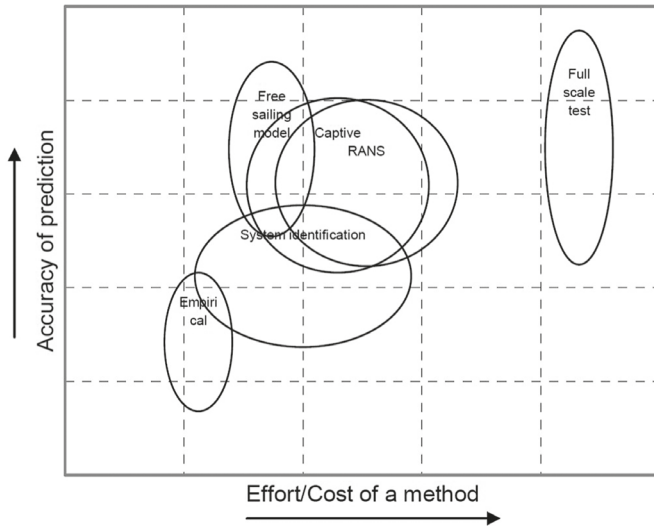


Fig. 2. Accuracy of prediction methods with respect to effort/cost ratio

The system identification method has been studied with various ideas. Abkowitz was the first to apply the Extended Kalman Filter (EKF) to full-scale sea trial data. Rhee and Zhang conducted system-based (SB) free running tests with the EKF algorithm [1, 3, 13, 19]. Tran introduced Sequential Quadratic Programming (SQP) and Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithms for an optimization process [14, 16]. Kim optimized hydrodynamic coefficients with an interior point algorithm using simulation results as benchmark data [4].

Change of trim and draught has a significant impact on ship's manoeuvrability. This leads to a change in corresponding ship's conditions, such as displacement, location of the centre-of-pressure for the sway force, rudder inflow angle and so on. Much of the previous studies on changing manoeuvrability with various loading conditions have focused on the corresponding changes of displacement, stern shape and rudder area. Kijima and Kose studied influence and importance of trim and draught conditions on a ship's manoeuvrability. In order to estimate ship's manoeuvrability in different trim and draught conditions, they performed captive model tests with multiple ships in four trim and draught conditions: fully loaded, half loaded, ballast with even keel and ballast aft trim conditions [7, 8, 9]. The prediction results based on the estimation agreed well with the measured results of free running model tests. Yasukawa et al. investigated influence of the load condition on the effect of rudder force [17].

Based on previous studies, this paper estimates hydrodynamic coefficients in a mathematical way. A set of results from real ship sea trials in various trim and draught conditions is the benchmark data in an optimization process, and a regression formula is suggested to estimate hydrodynamic coefficients for all trim and draught conditions.

## MODELLING AND BENCHMARK DATA

### MATHEMATICAL MODEL

Three degrees of freedom (3DOF) ship- and earth-fixed coordinate systems are applied to the modelling for the optimization process. Both coordinate planes are placed on the undisturbed free surface. In the mathematical model, a ship is considered a massive and rigid body. Forces and moment acting on the hull are described as follows.

$$\begin{aligned} X &= m(\dot{u} - vr - x_g r^2) \\ Y &= m(\dot{v} + ur - x_g \dot{r}) \\ N &= I_z \dot{r} + m x_g (\dot{v} + ur) \end{aligned} \quad (1)$$

To each force and moment the modular structure composed by hull, propeller, rudder and other external forces or moments, is applied as follows:

$$\begin{aligned} X &= X_H + X_P + X_R \\ Y &= Y_H + Y_P + Y_R \\ N &= N_H + N_P + N_R \end{aligned} \quad (2)$$

The hydrodynamic forces and moments acting on the ship's hull are comprised of several velocity and acceleration elements. The basic empirical regression formulas for the initial values of the optimization process are taken from Norrbin and Clarke [2, 11]. Each hydrodynamic coefficient can be expressed by a function of ship's main dimensions, such as length, beam, draught, its trim and displacement. The nonlinear components  $Y_{non}$  and  $N_{non}$  are influenced by the position of the ship's turning point.

$$\begin{aligned} X'_H &= X'_{up} \dot{u} + X'_{vr} vr + X'_{uu} |u| \\ &+ X'_{u^4} u^3 |u| + X'_{uvv} uv^2 |v| \end{aligned} \quad (3)$$

$$\begin{aligned} Y'_H &= Y'_{up} \dot{v} + Y'_{rp} \dot{r} + Y'_{ur} ur + Y'_{uv} |u| |v| + Y'_{non} \\ N'_H &= N'_{rp} \dot{r} + N'_{vp} \dot{v} + N'_{ur} |u| r + N'_{uv} uv + N'_{non} \\ \{Y'_{uv}, Y'_{ur}, N'_{uv}, N'_{ur}, Y'_{non}, N'_{non}\} &= \\ &f(L, B, T, \Delta) \end{aligned} \quad (4)$$

### BENCHMARK DATA

Benchmark data for the optimization are recorded on a 4,500 TEU class container carrier. Five zigzag manoeuvres with three trim and draught conditions were performed for this study. Tab. 1 shows details of the vessel, and Tab. 2 shows manoeuvre conditions of each trial.



Tab. 1. Particulars of benchmark vessel

Type of vessel	4,500 TEU class container carrier
Length overall [m]	294.12
Length between perpendiculars [m]	283.20
Beam [m]	32.20
Design draught [m]	12.00
Scantling draught [m]	13.00
Maximum speed [knots]	23.70

## OPTIMIZATION OF HYDRODYNAMIC COEFFICIENTS

### SENSITIVITY ANALYSIS

A sensitivity analysis of hydrodynamic coefficients is essential for easy and effective optimization. Procedures of the sensitivity analysis are as follows.

- 1) Divide the coefficients into two groups according to the corresponding manoeuvre: straight motion with constant speed and zigzag manoeuvre.
- 2) Run simulations with regard to certain variations of a specific coefficient.
- 3) Calculate derivative of data set for the coefficient and conduct min-max normalization for all hydrodynamic coefficients to find sensitivity in the group.

Tab. 2. Conditions of sea trials

	Data1	Data2	Data3	Data4	Data5
Manoeuvre	ZZ10	ZZ10	ZZ10	ZZ10	ZZ20
Latitude	32.8N	32.0N	10.7N	9.7N	9.7N
Longitude	119.9W	117.3W	67.2W	79.6W	79.6W
Heading [°]	110	110	260	250	250
RPM	843	620	676	422	422
Draught fore [m]	12.75	12.75	10.00	9.10	9.10
Draught mid [m]	12.55	12.55	10.00	-	-
Draught aft [m]	13.00	13.00	10.00	9.60	9.60
Wind direction [°]	270	310	20	50	50
Wind speed [knots]	12	15	5	15	15
Current direction [°]	160.47	251.56	169.50	23.62	23.62
Current speed [knots]	1.37	0.88	0.28	1.25	1.55
Water depth [m]	>1000	>1000	>1000	>1000	>1000

Stepwise optimization is carried out based on results of the sensitivity analysis, as shown in Fig. 3 and 4.

The first step optimizes two coefficients for the force acting along X-axis, using straight motion with constant speed. The second step optimizes four linear coefficients for the force acting along Y-axis and the moment acting around Z-axis, using zigzag manoeuvres.

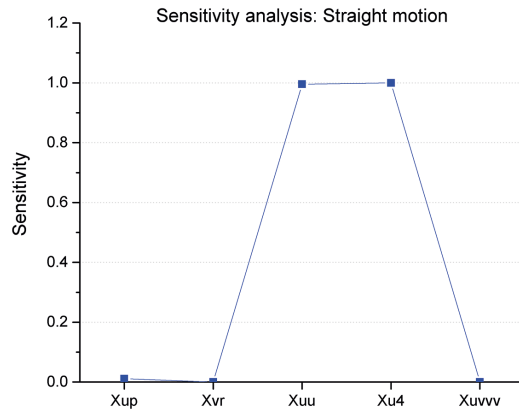


Fig. 3. Sensitivity analysis for straight motion

### MATHEMATICAL OPTIMIZATION

A mathematical optimization is a process to minimize or maximize an objective function which is subject to constraints imposed on variables [10]. A basic idea of the mathematical optimization is as follows.

$$\min f(x), \text{ subject to } x \in R^n \tag{5}$$

$$c_i(x) = 0, i \in E$$

$$c_i(x) \geq 0, i \in I$$

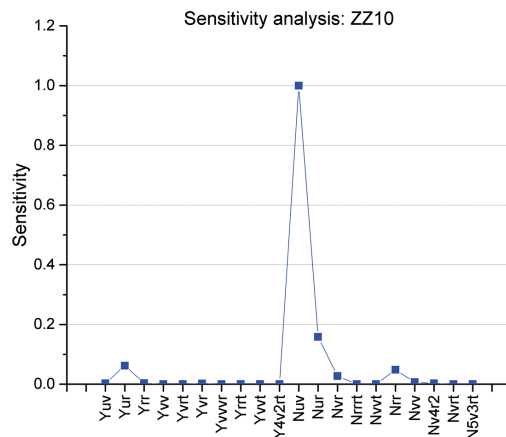


Fig. 4. Sensitivity analysis for zigzag manoeuvre

where,

- $x$  is/are variable(s) to be optimized;
- $f$  is an objective function that returns minimum or maximum scalar; and
- $c_i$ s are constraints, both equality and inequality conditions, in which variable(s)  $x$  must follow the whole optimization process.

Tab. 3 presents an example of the optimization conditions. According to the result of sensitivity analysis, stepwise optimization is applied to the process. Initial values that are essential for the mathematical optimization are calculated by the empirical formulas of Clarke and Norrbín. Objective functions for both steps calculate the sum of X- and Y-coordinate differences between the benchmark data and an optimization result at each iteration.

Optimizations are carried out only for the Data 2 to 4, which have different trim and draught conditions. Remaining measurement data are used for the first validation of optimization results.

Tab. 3. Optimization conditions for Data 3

	Step 1		Step 2	
Solver	fmincon			
Algorithm	interior-point			
Initial values	Xuu	-0.0373	Yuv	-1.3811
	Xu4	-0.4534	Yur	0.3820
			Nuv	-0.4401
			Nur	-0.2348
Lower bounds	Xuu	-0.3700	Yuv	-13.811
	Xu4	-4.5000	Yur	0.0001
			Nuv	-4.4019
			Nur	-2.3480
Upper bounds	Xuu	-0.0001	Yuv	-0.0001
	Xu4	-0.0001	Yur	3.8201
			Nuv	-0.0001
			Nur	-0.0001
Objective function	Track difference			
	Straight motion		Zigzag 10 degrees	
Constraints	None		None	

## VALIDATION OF OPTIMIZED COEFFICIENTS

### OPTIMIZATION RESULTS

Tab. 4 presents Clarke estimation coefficients and optimization results, and Tab. 5 compares manoeuvre characteristics with corresponding benchmark data. The results show that a simulation result obtained by using finally optimized coefficients is relatively more close to the corresponding benchmark data than a result by using Clarke estimation coefficients.

Tab. 4. Optimization results for hydrodynamic coefficients

		Xuu	Xu4	Yuv	Yur	Nuv	Nur
Data 2	C	-0.0280	-0.3405	-1.5857	0.4281	-0.5625	-0.2675
	S1	-0.0250	-0.2865				
	S2			-1.9472	0.3426	-1.2354	-0.2783
Data 3	C	-0.0373	-0.4534	-1.3811	0.3820	-0.4401	-0.2348
	S1	-0.0515	-0.5873				
	S2			-2.2214	0.4827	-3.4181	-0.6116
Data 4	C	-0.0407	-0.4948	-1.3947	0.3934	-0.3965	-0.2339
	S1	-0.0665	-0.4536				
	S2			-2.2611	0.3919	-0.9541	-0.2335
Remarks	C: Clarke estimation S1: Step 1 (Straight motion) S2: Step 2 (Zigzag manoeuvre)						

Tab. 5. Optimization results for manoeuvre characteristics

		Way/Lpp	Init.	Yaw	Ovst1	Ovst2
Data 2	Clarke	3.33	82	266	1.87	2.73
	Step 1	3.52	79	319	1.99	2.55
	Step 2	3.52	69	378	5.31	9.66
	Bench	3.53	58	370	6.70	11.80
Data 3	Clarke	5.58	57	272	1.87	2.51
	Step 1	5.22	64	286	1.68	2.42
	Step 2	5.22	38	265	3.63	7.13
	Bench	5.22	47	279	4.80	7.40
Data 4	Clarke	3.89	89	414	1.71	1.79
	Step 1	3.50	93	438	1.53	1.72
	Step 2	3.51	87	423	2.98	3.90
	Bench	3.52	78	398	3.20	4.60
Remarks	Bench: Benchmark data (Measured data) Way/Lpp: Distance from start point/Lpp Init.: Initial turning time (s) Yaw: Yaw checking time (s) Ovst1: First overshoot angle (°) Ovst2: Second overshoot angle (°)					

Fig. 5 and 6 show comparisons for track coordinates and heading angle of the measurement Data 3, respectively.

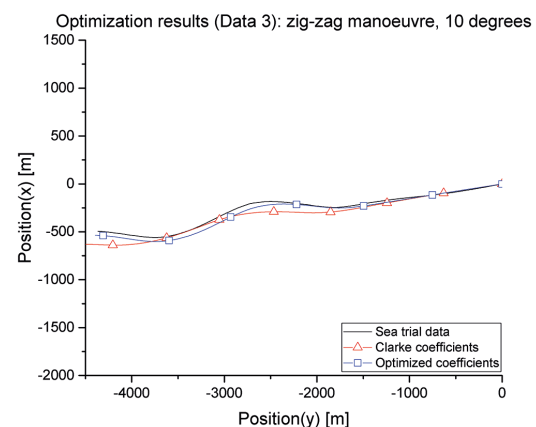


Fig. 5. Trajectory comparison for Data 2

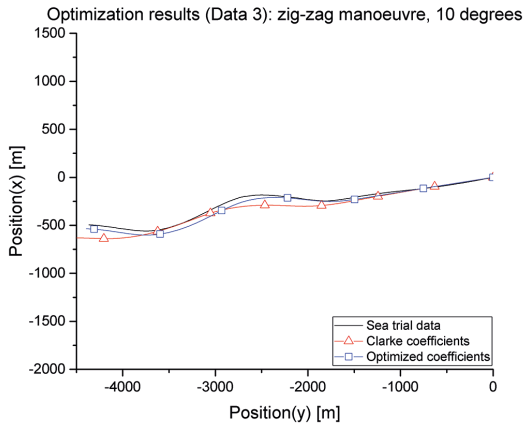


Fig. 6. Heading comparison for Data 2

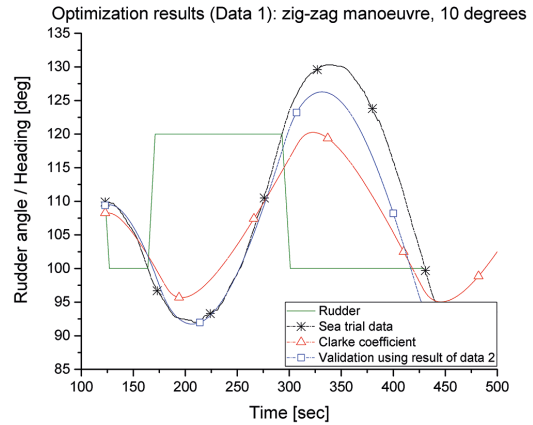


Fig. 8. Heading comparison for Data 1

### VALIDATION WITH OTHER BENCHMARKS

Benchmark Data 1 and 5 are compared with simulation results that use optimized coefficients of Data 2 and 4, respectively. Tab. 6 compares the benchmark data and the simulation results.

Tab. 6. Validation with additional benchmarks

		Way/Lpp	Init.	Yaw	Ovst1	Ovst2
Data 1	Clarke	4.7	76	267	2.34	2.26
	Val.	4.8	51	278	6.27	8.28
	Bench	4.42	46	293	6.00	12.30
Data 5	Clarke	2.15	93	441	3.21	3.25
	Val.	1.93	84	436	5.59	6.26
	Bench	1.96	81	405	5.60	6.10
Remarks		Val.: Validation of hydrodynamic coefficients using optimization results of Data 2 and Data 4, respectively Bench: Benchmark data (Measured data)				

Fig. 7 to 10 compare benchmark data and corresponding simulation results. In the case of Data 1, it is supposed that difference in a second overshoot angle and an item Way/Lpp is caused by difference in trajectories of the benchmark and the simulation result.

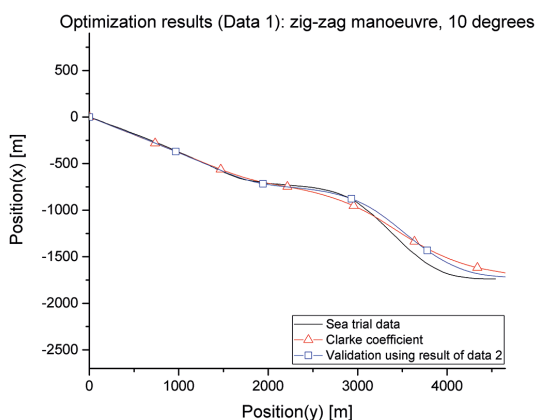


Fig. 7. Trajectory comparison for Data 1

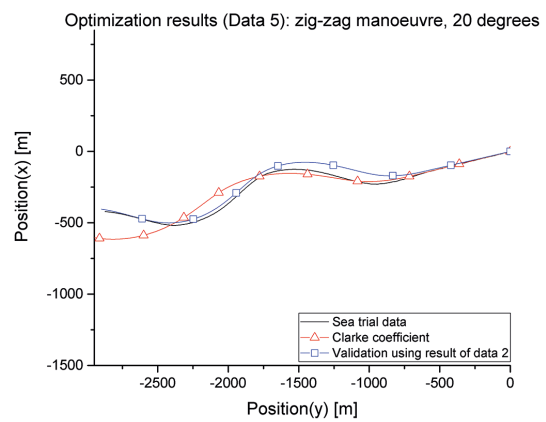


Fig. 9. Trajectory comparison for Data 5

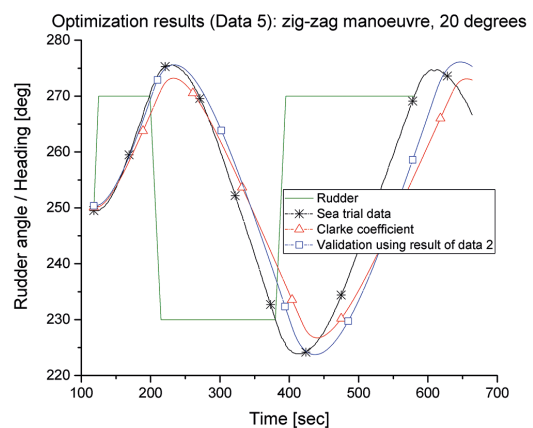


Fig. 10. Heading comparison for Data 5

### SUGGESTION ON NEW ESTIMATION FORMULAS

Based on the results of the optimization and the Clarke estimation, a new estimation formula is suggested regarding all trim and draught conditions. A set of optimized coefficients under certain trim and draught condition is set to a default condition, and differences of trim and draught are calculated to other optimized coefficients. Details of the estimation formula are as follows.

$$\begin{aligned}
& \text{Coeff.new} \\
& = OT.Coeff.standard.trim\&draught \\
& + (Corr.draught * CE.Diff.draught \\
& + Corr.trim * CE.Diff.trim)
\end{aligned}$$

(6)

simulation result using coefficient with the new estimation method is relatively closer to the benchmark data than that obtained by using Clarke estimation.

$$\begin{aligned}
& CE.Diff.draught \\
& = CE.Coeff.standard.trim\&draught \\
& - CE.Coeff.target.draught\&standard.trim
\end{aligned}$$

$$\begin{aligned}
& CE.Diff.trim \\
& = CE.Coeff.standard.trim\&draught \\
& - CE.Coeff.target.trim\&standard.draught
\end{aligned}$$

where,

- *Coeff.new* is an estimated coefficient;
- *OT.Coeff.standard.trim&draught* is a coefficient that has been optimized (Own Tuned) under the standard trim and draught condition;
- *Corr.xxx* is a correlation value that is calculated by all of the optimized coefficients;
- *CE.Diff.xxx* is the difference between standard trim and draught Clarke estimation coefficient and target trim and draught Clarke estimation coefficient; and
- *CE.Coeff.xxx* is a Clarke estimation coefficient.

A standard condition for this study is set to be : 10m draught and even keel. Tab. 7 shows results of Clarke estimation and the new estimation method.

Tab. 7. Estimation results: hydrodynamic coefficients

Tm=11.85m t=-0.1m	Clarke	Estimated
	76	267
Xuu	-0.0304	-0.0260
Xu4	-0.3699	-0.5263
Yuv	-1.4991	-2.3778
Yur	0.4056	0.5005
Nuv	-0.5144	-3.0489
Nur	-0.2527	-0.5670

To validate the estimated result, Tab. 8 compares characteristics of a zigzag manoeuvre with 20 deg rudder angle inclination. A benchmark is taken from the official manoeuvring booklet of the vessel. Since there is no trajectory data in the booklet, only the numeric values and a heading angle graph are applied to the validation. As Fig. 11 shows, the

Tab. 8. Validation of estimation formulas

Tm=11.85m t=-0.1m	Way/Lpp	Init.	Yaw	Ovst1	Ovst2
Clarke	28.5491	56	285	3.4	6.8
Val.	28.5608	48	275	7.7	11.9
Bench	-	53	278	8.8	13.7
Remarks	Val.: Validation of hydrodynamic coefficients using estimation formulas Bench: Benchmark data (Measured data)				

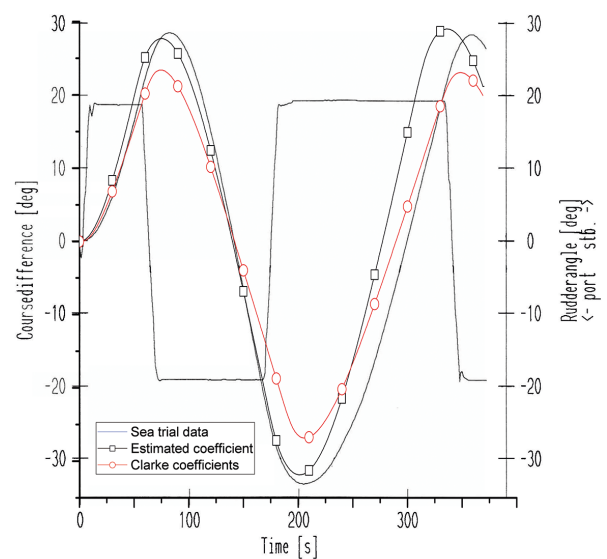


Fig. 11. Heading comparison with the use of manoeuvring booklet (sea trial data)

## CONCLUSION

This paper presents a study on estimating hydrodynamic coefficients by using real ship sea trial data, especially for all trim and draught conditions. A mathematical optimization, a part of system identification, is used to calculate optimum hydrodynamic coefficients. Benchmarks are taken from the sea trial of a container carrier. Results of this study can be summarized as follows.

Simulation results that use optimized hydrodynamic coefficients are closer to the corresponding benchmarks than those obtained from Clarke estimation.

The benchmarks Data 1 and 5 which are not used to optimize hydrodynamic coefficients, and their corresponding simulation results using optimized coefficients are similar to each other, compared with simulation results using Clarke estimation coefficients.

Hydrodynamic coefficients, estimated by using the suggested formulas, are satisfactory for simulation running, compared to a corresponding benchmark from a manoeuvring booklet ( sea trial data).



This summary show that the proposed estimation formulas can complement the existing Clarke formulas. However, five measurement data are not enough to attribute a high reliability to the proposed estimation method. In addition, various manoeuvres such as turning and other emergency manoeuvres should be included in the optimization process. These issues should be considered in future studies.

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**CONTACT WITH THE AUTHOR**

**Daewon Kim**

*e-mail: kim.daewon@hotmail.com*

University of Rostock  
Albert-Einstein-Strasse 2  
18059 Rostock  
**GERMANY**