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DYNAMIC CHARACTERISTIC STUDY OF RISER WITH COMPLEX PRE-STRESS DISTRIBUTION

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ABSTRACT

In this study, the dynamic characteristic problem of riser structure with complex pre-stress distribution is investigated. At first, the differential equation of the riser structure with complex pre-stress distribution is derived. The analytical expression of the free vibration of a riser structure with complex pre-stress distribution is discussed by using the orthogonal property of the trigonometric series. A top-tensioned riser (TTR) for example, the influences of the amplitude and direction of complex pre-stress on natural frequency and mode shape characteristics are compared. This study provides a new method for addressing the riser structure response problem with complex loading.

Keywords: complex pre-stress, top-tensioned riser (TTR), natural frequency, mode analysis, free vibration

INTRODUCTION

Owing to the growing demand for crude oil and gas, offshore equipment has been developed for deep-water exploitation. A riser structure, as the key equipment that links the platform and wellhead at the sea base, has become a popular issue in engineering design. In addition to the influence of gravity, riser structures are subjected to wave - and current-induced loading and high-pressure oil and gas. Thus, several dynamic responses, such as parametric and vortex-induced vibrations (VIV), occur in riser structures. The parametric vibration which is caused by the heave of the floating platform might destabilize the straight equilibrium of a riser structure. An accurate prediction of dynamic characteristics is vital to the design, installation and operation of the riser structure. Researchers have conducted numerous studies that involve theoretical analysis, numerical calculation and experimental investigation to investigate the fundamental mechanism of free vibration for the riser structure. The nonlinear resonance that arises from parametric excitation problems has been discussed, and closed-form solutions for the riser have been obtained based on first and second modes through extensive mathematical manipulations [3,4]. The parametric vibration that is caused by the wave-induced motions of the floating platform is practically important because this vibration can destroy a riser structure [11]. A finite element method has also been used to analyse the influence of water depths, environmental conditions and vessel motions under combined parametric and forcing excitations [10].

Structural natural frequency is an important dynamic property of a riser structure. However, the natural frequencies of a riser structure possess a low modal, and these frequencies are near one another given the structural stiffness problem of risers. A closed-form solution for the natural frequencies and associated mode shapes of axial loading has been deduced by Timoshenko beam theory [6,15]. Based on Spark's theory, the influence on bending stiffness is defined as tension force, and natural frequency and mode are deduced through a segmentation method [12].

Pre-stress (initial stress) often exists in complex structures and may be caused by welding residual stress, structural manufacturing defects, material thermal effects, static external loading, and so on. A riser is a large welded structure, and the welding residual stress in the structure will not be completely eliminated with the operation of risers. Pre-stress can resist or aid in structural deformation and alter the static and dynamic characteristics of a structure. Pre-stress significantly influences local and global stiffness matrices. The effects of uniform pre-stress distribution on natural frequencies and dynamic responses have been investigated [7,8]. The natural frequencies of a structure may increase or decrease if the pre-stress is considered in terms of uniform Euler-Bernoulli beams under linearly varying fully tensile stress [1,2,11]. The effects of welding residual stress on the added virtual mass and the quality factor of the diaphragm have been presented [14]. In addition, the differential equation of the vibration of a cylindrical shell with welding residual stress has been derived, and a theoretical solution has been presented [9]. The analytic expression of the influence of complex pre-stress force on a riser structure is derived in a previous study, and the VIV response is compared [5]. The influence of complex pre-stress on the mechanical response of risers, especially on their structural dynamic characteristics, requires further study. However, only a few studies have considered the effects of complex pre-stress on the dynamic response of a riser structure.

The main objective of the present study is to investigate the influence of complex pre-stress (welding residual stress) on the natural frequency and modal shape for the dynamic characteristic of a top-tensioned riser (TTR). The developed analytical method to analyze the dynamic behaviour of the riser structures with or without, local area or overall and even non-uniform pre-stress distributions. The outline of the present paper is as follows. A model of complex pre-stress force theory is described in Section 1. The differential equation of the pre-stressed beam is presented in Section 2. A free vibration analytical solution of a riser structure with complex pre-stress is introduced in Section 3. The governing equation of the parametric vibration for the riser structure with complex pre-stress distribution is expressed in Section 4. A numerical analysis is implemented in Section 5. The conclusions are presented in Section 6.

PRE-STRESS MODEL OF BEAM

Pre-stress (initial stress) typically exists in continuum structures, the influence of complex pre-stress on the static and dynamic characteristics of riser structures, especially on their structural dynamic characteristics, is worthy of study. In the present study, only welding residual stress and axial tension are discussed. These types of pre-stress can be defined as complex pre-stress that is unaffected by external dynamic excitation force. The complex pre-stress that satisfies the linear superposition principle can be expressed as follows:

$$\sigma_r = \sigma_T + \sigma_R \tag{1}$$

where σ_r is the complex pre-stress in the structure, σ_T is the pre-stress caused by axial riser tension force and σ_R is the welding residual stress. In the riser structures, complex pre-stress σ_r is a non-uniform distribution stress and the function of time and space. In the present study, only the axial direction of pre-stress is considered; the complex pre-stress can be written as $\sigma_r = \sigma_{r,z}$.

DIFFERENTIAL EQUATION OF THE PRE-STRESS BEAM

The riser structure can be idealized as a beam structure. The motion equations of beam structure with pre-stress distribution must consider the pre-stress distribution.

EULER–BERNOULLI BEAM MODEL

In Fig. 1, the length of the beam structure is L. A rectangular Cartesian coordinate system x, y and z are used to describe the loading and structure deformations. The original points of the structural member are referred to as a Cartesian coordinate system (x, y and z), where the z-axis is parallel to the longitudinal axis of the beam, whereas x and y are the principal axes of the cross-section of the structure. The displacement of the beam satisfies the 'plane cross-sectional assumption'.

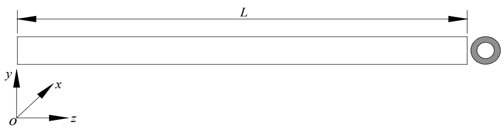

Fig.1. Beam geometry model

A differential element with length dz is selected, as illustrated in Fig. 2. The internal member forces during the beam structure vibration include two parts. The first part consisted of the moment and axial forces that are caused by dynamic external loading. The second part comprised of the moment and axial forces that are caused by the coupling of complex pre-stress force and dynamic vibration displacement.

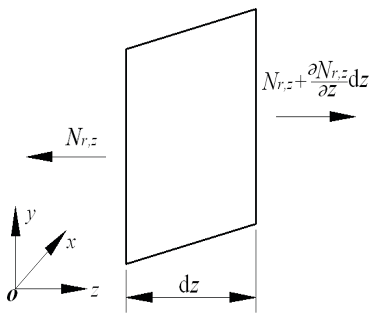

Fig.2. Section force that is caused by complex pre-stress

In the beam structure, the pre-stress $\sigma_{r,z}$ is distributed along the z-direction; this pre-stress is a function of variable z.

The axial internal forces $\mathbf{N}_{r,z}$ at the end of the differential element can be written as follows:

$$\mathbf{N}_{r,z} = \sigma_{r,z} S \tag{2}$$

where S is the cross-sectional area of the beam. Based on the beam bending theory, the strain ε_z in the differential element can be stated as follows:

$$\varepsilon_z = -y \frac{\mathrm{d}^2 w(z,t)}{\mathrm{d}z^2} \tag{3}$$

where $w(z,t)$ is the displacement that is vertical to the symmetry plane. The stress in the beam structure can be expressed as $\sigma_z = E\varepsilon_z = -Ey\frac{\mathrm{d}^2 w}{\mathrm{d}z^2}$. The moment in the differential element can be written as follows:

$$M_z = EI \frac{\mathrm{d}^2 w}{\mathrm{d}z^2} \tag{4}$$

where M_z is the moment which is caused by stress σ_z, I is the moment of inertia of the beam and E is Young's modulus of the beam material.

COUPLING FUNCTION OF PRE-STRESS FORCE AND VIBRATION DISPLACEMENT

If the beam structure is vibrated by the dynamic external loading, then the coupling function between complex pre-stress and dynamic vibration displacement can be obtained. In addition, the amplitude and distribution of the complex pre-stress are constant during structural vibration. The structural deformation l_y of the cross-sectional neutral plane with unit length can be expressed as follows:

$$l_y = 1 + \varepsilon_z \tag{5}$$

Based on the 'plane cross-sectional assumption', the volume of the unit length of the beam structure can be stated as follows:

$$V = \int_{-\frac{y}{2}}^{\frac{y}{2}} l_y \mathrm{d}y = S \tag{6}$$

In Eq. (6), the volume of the differential element of the beam cross-section is constant. The internal forces $\mathbf{N}_{r,z}$ will be static while the beam vibrates. Owing to the structural displacement w, a torsional angle for the y-axis emerges, and the value of the angle is $\partial w / \partial z$.

In the differential element, internal forces $\mathbf{N}_{r,z}$ are parallel to the z-direction, the components in the other directions are zero and internal forces $\mathbf{N}_{r,z}$ includes a component in the y-direction. This situation is written as $\Delta\mathbf{N}_{y,z}$ and can be expressed as follows:

$$\Delta\mathbf{N}_{y,z} = \boldsymbol{\sigma}_{r,z} S \frac{\partial w}{\partial z} \tag{7}$$

In Eq. (7), the component force $\Delta\mathbf{N}_{y,z}$ depends on the complex pre-stress and dynamic vibration displacement. Then, a new equilibrium equation is obtained for the differential element of the beam structure. This equilibrium equation is defined as a coupling force of complex pre-stress and dynamic vibration displacement, as depicted in Fig. 3.

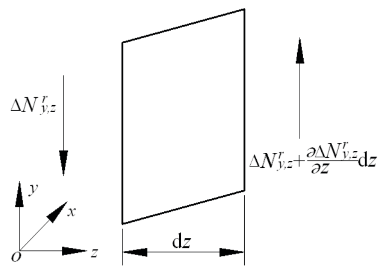

Fig.3. Coupling force of the differential element

EQUILIBRIUM EQUATION

The equilibrium equation for internal force and moment are established to address the vibration function of the differential element. Based on the small deformation of beam bending theory, the force in the z-direction satisfies the equilibrium equation. Then, the equilibrium equation in the y-direction which combines force N_z and coupling force $\Delta\mathbf{N}_{y,z}$ can be stated as follows:

$$\frac{\partial \mathbf{Q}_z}{\partial z}\mathrm{d}z + \frac{\partial \Delta\mathbf{N}_{y,z}}{\partial z}\mathrm{d}z = m\frac{\partial^2 w}{\partial t^2}\mathrm{d}z \tag{8}$$

where m is the mass of the unit length of the beam structure. The moment in the y-direction satisfies the equilibrium equation because no coupling moment is caused by the complex pre-stress. The moment in the z-direction can be stated as

$$\mathbf{Q}_z \mathrm{d}z + \frac{\partial M_z}{\partial z}\mathrm{d}z = 0 \tag{9}$$

The differential equation is gained by substituting Eq. (9) into Eq. (8), as follows:

$$\frac{\partial M_z}{\partial z^2} - \frac{\partial \Delta\mathbf{N}_{y,z}}{\partial z} = -\rho S \frac{\partial^2 w}{\partial t^2} \tag{10}$$

where ρ is the equivalent density of the beam, and S is the cross-sectional area of the beam.

MOTION DIFFERENTIAL FORMULATION OF BEAM

The free vibration differential formulation of the beam structure with complex pre-stress can be expressed by substituting Eqs. (4) and (7) into Eq. (10), as follows:

$$EI\frac{\partial^4 w}{\partial z^4}-\frac{\partial}{\partial z}(\sigma_{r,z}S\frac{\partial w}{\partial z})=-\rho S\frac{\partial^2 w}{\partial t^2} \quad (11)$$

In Eq. (11), the elemental stiffness matrix comprises two parts, that is, the elasticity and geometric stiffness matrices; the former can be written as $EI\frac{\partial^4 w}{\partial z^4}$, while the latter can be defined written as $C(w,\sigma_{r,z},t)=-\frac{\partial}{\partial z}(\sigma_{r,z}S\frac{\partial w}{\partial z})$. This quasi-linear fourth-order partial differential equation governs the beam to a general dynamic and distributed external excitation. If the complex pre-stress in the beam structure is zero, then $C(w,\acute{o}_{r,z},t)=0$. This scenario represents a free vibration differential formulation of a simple beam structure. The vibration equation will be solved with given boundary conditions.

FREE VIBRATION OF THE RISER WITH COMPLEX PRE-STRESS DISTRIBUTION

The structural modes and modal problem of the riser with a complex pre-stress distribution are discussed.

DEFINITION OF BOUNDARY CONDITIONS

The physical boundary condition at both ends of the riser structure can be modelled as a simple support. The displacement at each end is zero, and the boundary conditions can be written as follows:

$$w(z,t)\big|_{z=0}=0\ ,\ \frac{\partial^2 w(z,t)}{\partial z^2}\bigg|_{z=0}=0\ ,\ w(z,t)\big|_{z=L}=0\ ,\ \frac{\partial^2 w(z,t)}{\partial z^2}\bigg|_{z=L}=0 \quad (12)$$

Modal decomposition is based on the assumption that the riser mode may be expressed as a sum of eigenmodes or eigenfunctions at any point in time. The solution of Eq. (11) can be obtained with a form of power series expansion. Galerkin's procedure is used to obtain the solution of Eq. (11). The solution of linearized Eq. (11) under the simple support boundary can be expressed as follows:

$$w(z,t)=\phi(z)\mathrm{e}^{\mathrm{i}\omega t}=\sum_{\mathrm{n}=1}^{N} B_{\mathrm{n}}\sin(n\beta z)\mathrm{e}^{\mathrm{i}\omega t} \quad (13)$$

where z is the axial coordinate, L is the length of the riser, t is the time parameter, $w_{\mathrm{n}}(t)$ is the modal weight function and $\phi_{\mathrm{n}}(z)$ is the mode shape function, where $n=1,2,3,\cdots$. The mode shapes can be defined as sinusoid functions as follows: $\phi_{\mathrm{n}}=\sin(n\beta z)$, where $\beta=\pi/L$, and ω is the angular frequency.

The free vibration differential formulation of the riser with complex pre-stress can be obtained by introducing orthogonal series $\sin(\xi\beta z)$ and substituting Eq. (13) into Eq. (11). $\int_0^L \sin(m\pi z/L)\cdot\sin(n\pi z/L)=0$ exists by using the orthogonal property of the trigonometric series because $m\neq n$ integrates the function from $z=0$ to $z=L$. Then, Eq. (11) can be written as follows:

$$EI(n\beta)^4 B_{\mathrm{n}}-\frac{2}{L}\int_0^L\frac{\partial}{\partial z}(\sigma_{r,z}S\frac{\partial w}{\partial z})\sin(\xi\beta z)\mathrm{d}z=\rho S\omega^2 B \quad (14)$$

where $R=-\frac{2}{L}\int_0^L\frac{\partial}{\partial z}(\sigma_{r,z}S\frac{\partial w}{\partial z})\sin(\xi\beta z)\mathrm{d}z$ which is defined as the integration item of the complex pre-stress and dynamic vibration displacement.

SOLUTION OF THE BEAM STRUCTURE

The free vibration of the riser structure with a complex pre-stress distribution is analysed, and the analytical solution for Eq. (14) is discussed as follows:

(1) If the complex pre-stress satisfies the equation $\sigma_{r,z}=0$, then Eq. (14) can be expressed as a vibration function of a simple beam. The solution equation can be written as $EI(\mathrm{n}\beta)^4=\rho S\omega^2$.

(2) If the complex pre-stress is in a uniform distribution form, then the pre-stress can be written as $\sigma_{r,z}S=T_0$, where A is the cross-sectional area. The solution equation can be written as $EI(\mathrm{n}\beta)^4+T_0(\mathrm{n}\beta)^2=\rho S\omega^2$ by substituting the complex pre-stress into Eq. (14). If parameter $\sigma_{r,z}$ is the tensile stress, then the natural frequency of the beam structure increases. If parameter $\sigma_{r,z}$ is a compressive stress, then the natural frequency of the beam structure decreases.

(3) If the complex pre-stress is a function of variable z, then the complex pre-stress is a one-dimensional complex pre-stress distribution problem. The complex pre-stress distribution can be fitted by a trigonometric function and can be expressed as follows:

$$\sigma_{r,z}=\sigma_{r,z0}\cos(g\beta z) \quad (15)$$

where $\sigma_{r,z0}$ is the amplitude of the complex pre-stress, and g is an integral number that is not less than 1. Function R can be obtained by separating the variables and substituting the complex pre-stress function into Eq. (14), as follows:

$$R=-\frac{2\sigma_{r,z0}S}{L}\int_0^L\frac{\partial}{\partial z}[\cos(g\beta z)\frac{\partial w}{\partial z}]\sin(\xi\beta z)\mathrm{d}zR$$
$$=-\frac{2\sigma_{r,z0}S}{L}\sum_{n=1}^{N}B_n(n\beta)\int_0^L\frac{\partial[\cos(n\beta z)\cos(g\beta z)]}{\partial z}\sin(\xi\beta z)\mathrm{d}z \quad (16)$$

Following the trigonometric function, function R can be defined as follows:

$$R=-\frac{\sigma_{r,z0}S}{L}\sum_{n=1}^{N}B_n(n\beta)\int_0^L\frac{\partial\{\cos[(n+g)\beta z]+\cos[(n-g)\beta z]\}}{\partial z}\cdot\sin(\xi\beta z)\mathrm{d}z$$
$$=\frac{\sigma_{r,z0}S}{L}\{\sum_{n=1}^{N}B_n(n\beta^2)(n+g)\int_0^L\sin[(n+g)\beta z]\cdot\sin(\xi\beta z)\mathrm{d}z$$
$$+\sum_{n=1}^{N}B_n(n\beta^2)(n-g)\int_0^L\sin[(n-g)\beta z]\cdot\sin(\xi\beta z)\mathrm{d}z\} \quad (17)$$

Based on the orthogonal characteristics of the trigonometric function, the solution of Eq. (17) can be written as follows:

$$\int_0^L\sin[(n+g)\beta z]\cdot\sin(\xi\beta z)\mathrm{d}z=\begin{cases}L/2 & n+g=\xi\\ 0 & n+g\neq\xi\end{cases} \quad (18.1)$$

$$\int_0^L\sin[(n-g)\beta z]\cdot\sin(\xi\beta z)\mathrm{d}z=\begin{cases}L/2 & n-g=\xi\\ -L/2 & n-g=-\xi\\ 0 & |n-g|\neq\xi\end{cases} \quad (18.2)$$

By substituting Eq. (21) into Eq. (17), the function R can be expressed as follows:

$$R=\begin{cases}\frac{\sigma_{r,z0}S}{2}\sum_{g=1}^{N}[B_{n-g}(n\beta^2)(n-g)+B_{n+g}(n\beta^2)(n+g)] & n>g\\ \frac{\sigma_{r,z0}S}{2}\sum_{g=1}^{N}[-B_{|n-g|}(n\beta^2)(n-g)+B_{n+g}(n\beta^2)(n+g)] & n<g\end{cases} \quad (19)$$

Based on Eq. (19), N number functions exist and can be written in a matrix form as follows:

$$(\Lambda+R_g)X=0 \quad (20)$$

where $X=[w_1(z,t),w_2(z,t),\cdots,w_N(z,t)]^T$, and $\mathbf{\Lambda}=\mathrm{diag}\{EI(n\beta)^4-\rho S\omega^2\}$ is a diagonal matrix. The element parameter in the diagonal matrix can be written as $EI(n\beta)^4-\rho S\omega^2$. R_g is a sparse matrix, and the value of function R is as follows:

If $p=n$ and $q=|n-g|$, then the following equation can be deduced as

$$R_{pq}=\begin{cases}\frac{\sigma_{r,z0}S}{2}(n\beta^2)(n-g), & n>g\\ -\frac{\sigma_{r,z0}S}{2}(n\beta^2)(n-g), & n<g\end{cases} \quad (21.1)$$

If $p=n$ and $q=n+g$, then the following equation can be deduced as

$$R_{pq}=\frac{\sigma_{r,z0}S}{2}(n\beta^2)(n+g) \quad (21.2)$$

(4) If the distribution of the complex pre-stress has high complexity, then the complex pre-stress is a function of variable z and a one-dimensional complex pre-stress distribution problem. The distribution of the complex pre-stress can be fitted by a trigonometric function and expressed as follows:

$$\sigma_{r,z}=\sum_{g=1}^{G}\sigma_{r,g}\cos(g\beta z) \quad (22)$$

where $\sigma_{r,g}$ $(g=1,2,\cdots,J-1,J)$ is the amplitude of the complex pre-stress force.

N number functions exist by substituting the complex pre-stress function into Eq. (6) and can be written in a matrix form as follows:

$$(\Lambda+R)X=0 \quad (23)$$

where Λ is a diagonal matrix. An increase in the series leads to a change in complex pre-stress influence matrix R because the complex pre-stress expression is highly complex. Matrix R is not a sparse matrix. In addition, the series of complex pre-stress follows the linear superposition principle and can be expressed as follows:

$$\mathbf{R}=\sum_{g=1}^{J}\mathbf{R}_g \quad (24)$$

MODAL ANALYSIS

Following the complex pre-stress function, the dynamic response function of the beam structure with complex pre-stress can be obtained because the structural complex pre-stress can be expressed as a trigonometric function. The equations are linear in a beam structure with or without a complex pre-stress distribution, and the determinant factor to the characteristic equation is zero and can be expressed as follows:

$$|\Lambda+R|=0 \quad (25)$$

In Eq. (25), if no complex pre-stress force distribution ($R=0$) exists, then this equation can be written as a classical elastic beam free vibration equation. If a complex pre-stress distribution ($R\neq0$) exists, then the matrix $\Lambda+R$ is not diagonal. Therefore, the characteristic equation and structural modes of the free vibration beam with complex pre-stress must be modified.

FREE VIBRATION OF THE RISER STRUCTURE

SYSTEM MODELLING

The TTR in a complex ocean environment is discussed to reveal the parametric vibration problem by considering the heaving of the platform and the motion of the tension ring. The riser structure is subjected to a current. The riser structure could be considered a long, continuous, tubular member that is straight and vertical. The boundary conditions at the two ends are known. The heaving of a floating platform which induces axial tension fluctuation in the riser structure is considered. The top of the riser is connected to the main body of the platform through a compensator. The heave compensator can be simplified as an equivalent spring with stiffness *K*.

Several hypotheses for the riser structure are defined as follows. (1) The material and mechanical properties are uniform along the overall of the riser structure. (2) The tension variation along the riser length varies linearly with depth. (3) Only the cross-flow vibration is considered, whereas the in-line vibration was excluded. (4) The effect of the shear strain is minimal and can be neglected. In addition, the pipe wall behaves elastically; that is, no internal damping was considered.

In this study, a rectangular Cartesian coordinate system is defined to establish the deformation of the riser structure, as demonstrated in Fig. 4.

In Fig.4, the sea surface is set as the origin of the coordinate system, the *x*-axis is parallel to the flow velocity and the *z*-axis is measured from the top of the riser.

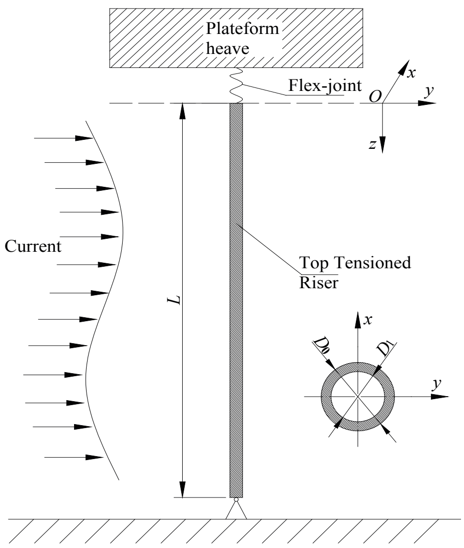

Fig. 4. Mechanical model and reference frame of the riser

Owing to the neglected effects of rotational inertia, the motion equations in the two principal vertical planes are identical and can be derived independently for each plane because of the symmetry of the riser cross-section. The riser structure can move only in the plane of the figure (i.e., in the *z*-direction). The lateral deflection ($w(z,t)$) is considered minimal in which every cross-section remains plane perpendicular to the axis, and the riser structure can be modelled as a beam structure.

FREE VIBRATION EQUATION OF THE RISER

Owing to the assumption of the riser structure, the governing equation of the lateral deflection $w(z,t)$ of the riser structure by vibration can be written in the form of the following partial differential equation:

$$EI\frac{\partial^4 w(z,t)}{\partial z^4}-\frac{\partial}{\partial z}[T(z,t)\frac{\partial w(z,t)}{\partial z}]+ \\ (m_\mathrm{r}+m_\mathrm{f}+m_\mathrm{a})\frac{\partial^2 w(z,t)}{\partial t^2}+c_S\frac{\partial w(z,t)}{\partial t}=f(z,t) \tag{26}$$

where the first term is the bending stiffness of the riser structure; the second term is the axial riser tensioning force; the third term is the inertia of the riser structure and includes the riser structure, internal fluid and fluid addition masses. *EI* is the bending stiffness of the riser structure, $T(z,t)$ is the effective axial tension of the riser, $w(z,t)$ is the displacement that is vertical to the riser structure axis, *z* is the axial position and *t* is the time parameter.

In the partial differential equation, $m_\mathrm{r}=\frac{1}{4}\rho_\mathrm{s}\pi(D_0^2-D_1^2)$ is the mass per unit length of the riser, $m_\mathrm{f}=\frac{1}{4}\rho_\mathrm{f}\pi D_1^2$ is the mass per unit length of the internal fluid and $m_\mathrm{a}=\frac{1}{4}C_\mathrm{a}\rho_\mathrm{w}\pi D_0^2$ is the mass per unit length of the added mass; in addition, the influence of the motion of internal fluid in the riser structure is omitted. D_0 is the outer diameter of the riser, D_1 is the inner diameter of the riser, ρ_s is the riser structural density, ρ_f is the water density, ρ_w is the internal fluid density and C_a is the added mass coefficient. c_S is the damping parameter of the riser structure and is defined as 0 in the present study.

PARAMETRIC FORCE

The effect of tension in the riser includes static and dynamic components. The static component of the tension results from the pretension imposed by the heave compensator and submerged weight. The dynamic component is caused by the heaving of the platform. This tension component depends only on time given the assumption of inextensibility. If the platform vibrates harmonically, the effective tension can be expressed as follows:

$$T(z,t) = T_0 - W_a z + Ka\sin(\tau t) \quad \textbf{(27)}$$

where $T_0 = Sg(\rho_s - \rho_w) f_{top} L$ is the static tension on the top of the riser, and f_{top} is the top tension coefficient. W_a is the submerged weight of the riser per unit length which is equal to $m_r + m_f + m_a$. *K* is the stiffness of the heave compensator. Parameters τ and *a* are the amplitude and frequency of the heave of the platform, respectively. In the structural vibration analysis, if the effective axial tension is defined, then the structural modes and structural modal of the riser can be obtained, and the analytical solution of the dynamic response can be obtained.

Based on Eq. (1), the complex pre-stress force in the riser structure is $\sigma_{r,z} = \frac{[T_0 - W_a z + Ka\sin(\tau t)]}{S}$, where T_0 is the static axial tension force, $U_m z$ is the influence of gravity, $K\mu$ is the amplitude of dynamic axial tension force and τ is the frequency of platform heaving. The partial differential equation of a deepwater riser structure can be obtained by substituting Eq. (27) into Eq. (14) and can be written as follows:

$$EI\frac{\partial^4 w(z,t)}{\partial z^4} - \frac{\partial}{\partial z}[\sigma_{r,z} S \frac{\partial w(z,t)}{\partial z}] + (m_r + m_f + m_a)\frac{\partial^2 w(z,t)}{\partial t^2} + c_S \frac{\partial w(z,t)}{\partial t} = f(z,t) \quad \textbf{(28)}$$

Eq. (28) is a modified function of the riser structure that considers the complex pre-stress force distribution. The traditional differential equation of the deepwater riser structure is compared, and the parameter of pre-stress $\frac{\partial}{\partial z}[\sigma_{r,z} S \frac{\partial w(z,t)}{\partial z}]$ is obtained. The description of the stress conditions of the riser structure is comprehensive. If the axial tension and welding residual stress are defined, then the structural modes and structural modal of the riser and the solution of the dynamic response can be determined.

NUMERICAL RESULT AND DISCUSSION

MODEL DESCRIPTION

The design parameters of the riser structure for the numerical analysis of the parametric vibration problem are listed in Table 1. The effects of complex pre-stress on the natural frequency and mode shape of the riser are compared. The design parameter is similar to that in Reference [13].

Tab. 1. Design parameters of the model system

Parameter	Symbol	Values	Unit
Length	*L*	1000	m
Outer Diameter	D_0	0.30	m
Thickness	*t*	0.025	m
Young's Modulus	*E*	2.1E11	Pa
Material Density	ρ_s	7850	kg/m^3
Density of Water	ρ_w	1025	kg/m^3
Density of Oil	ρ_f	800	kg/m^3
Equivalent Coefficient	λ	10	m
Top Tension Coefficient	f_{top}	1.3	-
Addition Mass Coefficient	C_a	1.0	-
Stiffness of Compensator	*K*	320000	N/m
Heave Amplitude	μ	5	m
Heave Frequency	τ		s

MODELLING OF WELDING RESIDUAL STRESS

Two types of welding residual stress distribution are compared to analyse the influence of welding residual stress on the dynamic characteristics of the riser. The two types of stress are mixed tensile and compressive residual stresses.

The three types of welding residual stress distribution are as follows. Distribution model I is a tensile stress. Distribution model II is a tensile stress with a maximum amplitude that is smaller than that of Distribution model I. Distribution model III is defined as mixed tensile and compressive residual stresses.

The welding residual stress in the riser follows for a non-uniform distribution. Based on the common welding technology for risers, the peak value of stress is distributed periodically by a certain distance. In the present study, every 2 m is defined as peak stress, and the welding residual stress is zero at the end of the riser structure. The welding residual stress in the riser can be fitted by a trigonometric series. Five peaks with the characteristics of the periodic distribution of welding residual stress are illustrated in Fig. 5 to describe the welding residual stress distribution.

Fig. 5(a) depicts Distribution model I of the welding residual stress; a positive value denotes the tensile stress.

Fig. 5(b) exhibits Distribution model II of the welding residual stress, and a positive value denotes the tensile stress. The peak value of the welding residual stress in Fig. 5(b) is less than that in Fig. 5(a).

Fig. 5(c) presents Distribution model III of the welding residual stress. A negative value denotes the compressive stress, and a positive value denotes the tensile stress. The absolute value is similar to that in the Distribution model I compared with the values displayed in Figs. 5(a) and 5(b).

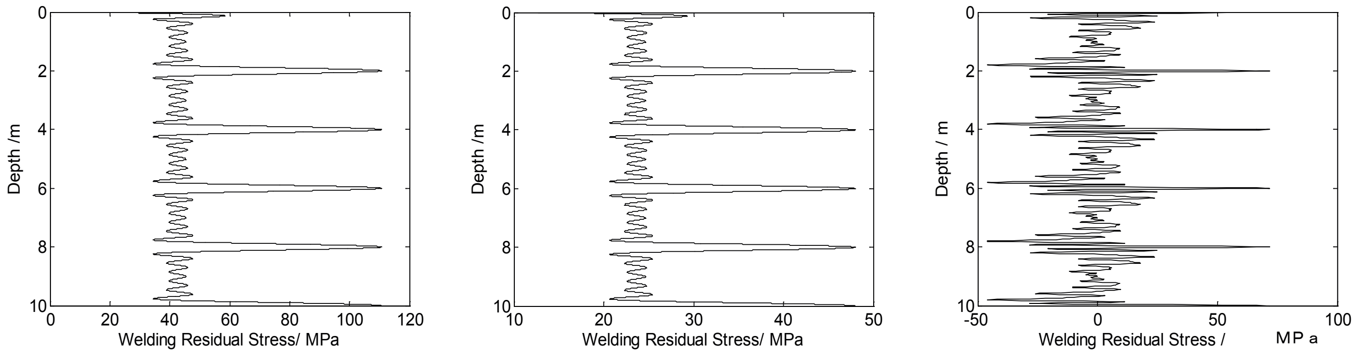

Fig. 5. Distribution model of the welding residual stress

EFFECT OF WELDING RESIDUAL STRESS ON NATURAL VIBRATION FREQUENCY

Based on the function of the welding residual stress, the complex pre-stress in the riser can be written as $\sigma_{r,z} = \frac{[T_0 - U_{\mathrm{m}} z + K\mu \sin(\pi t)]}{A} + \sigma_R$. The vibration characteristics of the riser can then be obtained.

(1) Constant axial loads

Constant axial loading is also considered, in which the sole weight is neglected. The complex pre-stress in the riser structure can be written as $\sigma_{r,z} = T_0 / A$. The mode shapes of the riser structure can be written as $EI\frac{d\phi^4(z)}{dz^4} - T_0\frac{d\phi^2(z)}{dz^2} - (m_{\mathrm{r}} + m_{\mathrm{f}} + m_{\mathrm{a}})\omega^2\phi(z) = 0$. The mode function of the vertical tube under fixed axial force can be calculated by using the method of separation of variables.

(2) Varying axial loads

Vertical pipe weight and internal tension are considered. The complex pre-stress in the riser structure can be written as $\sigma_{r,z} = (T_0 - W_{\mathrm{a}} z)/A$. The mode function of the vertical tube under fixed axial force can be calculated by using the method of the separation of variables, and the modes are not standard sine functions.

(3) Varying axial loading and welding residual stress

In this case, the welding residual stress, vertical pipe weight and internal tension are considered. The values of the welding residual stress are obtained from the measurement data of the riser structure or through a numerical simulation analysis. The cosine function is used to fit the welding residual stress parameter. The welding residual stress in the riser can be written as $\sigma_{r,z} = \sum_{g=1}^{18} \sigma_{r,zg} \cos(g\beta z)$, and the distribution of welding residual stress is exhibited in Figs. 5(a), 5(b) and 5(c). The complex pre-stress in the riser can be written as $\sigma_{r,z} = \frac{T_0 - U_m z}{A} + \sum_{g=1}^{18} \sigma_{r,zg} \cos(g\beta z)$. By substituting the complex pre-stress into Eq. (3), the differential formulation can be written as follows:

$$EI\frac{d\phi^4(z)}{dz^4} - [T_0 - U_m z + A\sum_{g=1}^{18} \sigma_{r,zg} \cos(g\beta z)]\frac{d\phi^2(z)}{dz^2} + W_a\frac{d\phi(z)}{dz} - (m_{\mathrm{r}} + m_{\mathrm{f}} + m_{\mathrm{a}})\omega^2\phi(z) = 0$$

Table 2 summarizes the natural frequencies of the riser structure with variation from the first to the tenth order. Table 2 indicates that the complex pre-stress exerts a significant effect on the natural frequency of the riser. If the welding residual stress is positive, then the natural frequency of the riser will increase. If the welding residual stress is negative, the natural frequency of the riser will decrease. In addition, the influence of complex pre-stress on natural frequency becomes increasingly significant with the increase in order. These results show that the peak value of the welding residual stress distribution is inevident in natural frequency because the range of the riser in the peak area is relatively narrow. Table 2 indicates that analysing the influence of the welding residual stress is necessary for the riser design.

Tab.2. Comparison of the natural frequencies of the riser

Order	Constant axial force	Axial force and sole weight	Welding residual stress mode I	Welding residual stress mode II	Welding residual stress mode III
1	0.257	0.190	0.273	0.240	0.176
2	0.514	0.381	0.545	0.478	0.360
3	0.771	0.572	0.818	0.718	0.546
4	1.029	0.764	1.092	0.959	0.733
5	1.287	0.957	1.367	1.200	0.923
6	1.547	1.153	1.642	1.442	1.115
7	1.807	1.350	1.918	1.686	1.309
8	2.069	1.550	2.196	1.931	1.507
9	2.332	1.752	2.476	2.178	1.707
10	2.596	1.956	2.757	2.427	1.910

MODES OF THE RISER STRUCTURE

The first-order to sixth-order modes of the riser structure are illustrated in Fig. 6. The modes are not standard sine functions.

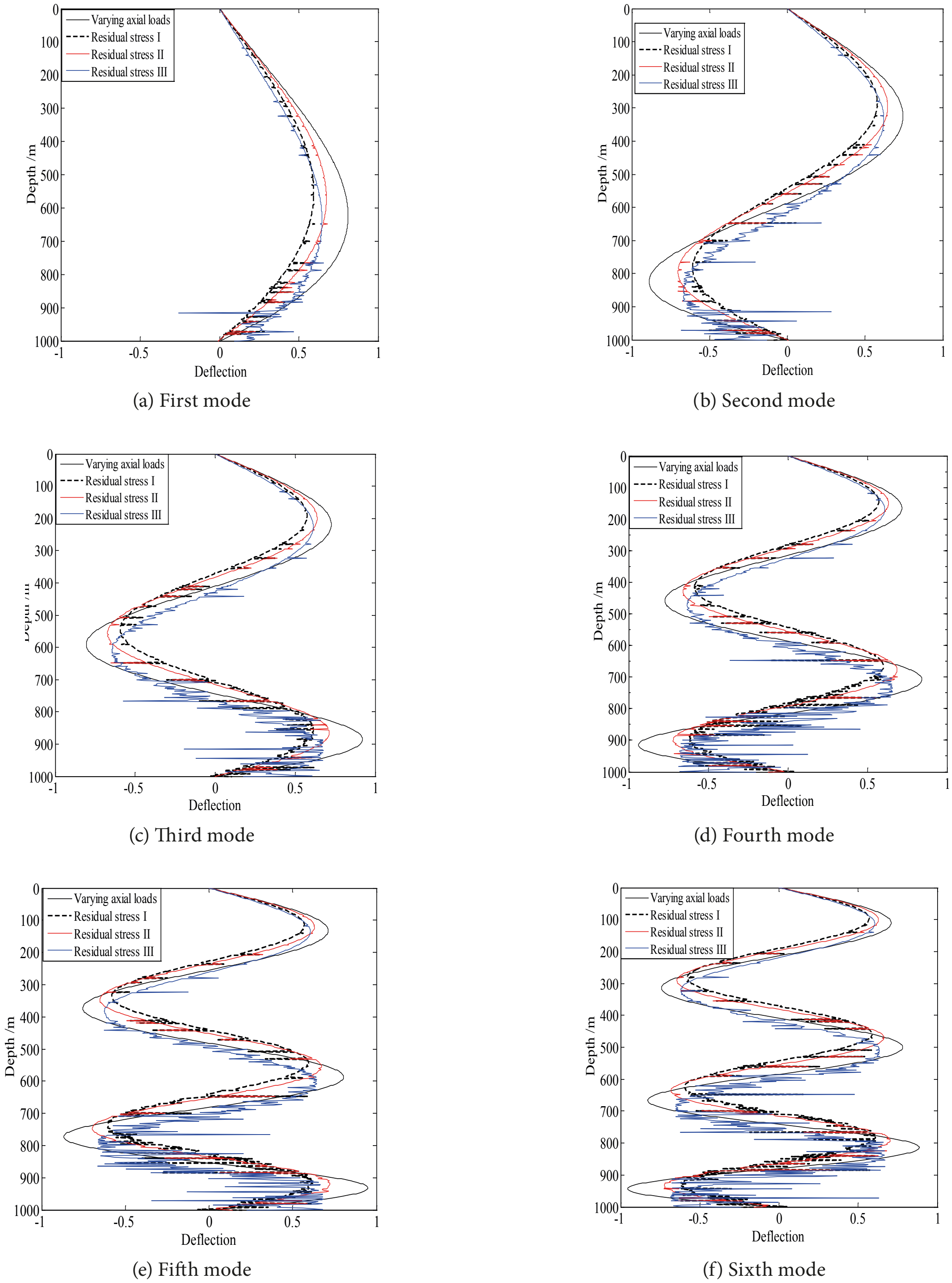

Fig.6 First six mode shapes in different loading cases

In Fig. 6, the maximum amplitude of the mode is moved to the bottom when the sole weight is considered. Welding residual stress significantly affects the mode shapes of the riser structure. The mode shapes become increasingly complex when the welding residual stress is considered. The influence of the welding residual stress on the riser structure vibration mode is evident. The mode shape of the riser becomes a rough curve with mutation when the welding residual stress exists. The mutation direction depends on the direction of the welding residual stress because the welding residual stress had changed the local stiffness of the riser.

CONCLUSIONS

In this study, a new approach to analysing the dynamic characteristics of the riser structure is proposed to investigate the influence of the welding residual stress on the natural frequency and modal shape. A corresponding differential equation is established. The numerical results show that complex pre-stress force significantly influences parametric vibration. The distribution of complex pre-stress causes the resonance point of the riser to migrate, and the migration direction corresponds to the complex pre-stress direction. The mode shape is not a smooth curve, and the distortion direction depends on the amplitude of complex pre-stress. The approach that is proposed in this study can be applied to address the uniform or non-uniform distribution stress force problem, thereby extending the research area of the application for pre-stress problems.

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