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LIMITING DISTRIBUTION OF THE THREE-STATE SEMI-MARKOV MODEL OF TECHNICAL STATE TRANSITIONS OF SHIP POWER PLANT MACHINES AND ITS APPLICABILITY IN OPERATIONAL DECISION-MAKING

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ABSTRACT

The article presents the three-state semi-Markov model of the process {W(t): t ≥ 0} of state transitions of a ship power plant machine, with the following interpretation of these states: s_1 – state of full serviceability, s_2 – state of partial serviceability, and s_3 – state of unserviceability. These states are precisely defined for the ship main engine (ME). A hypothesis is proposed which explains the possibility of application of this model to examine models of real state transitions of ship power plant machines. Empirical data concerning ME were used for calculating limiting probabilities for the process {W(t): t ≥ 0}. The applicability of these probabilities in decision making with the assistance of the Bayesian statistical theory is demonstrated. The probabilities were calculated using a procedure included in the computational software MATHEMATICA, taking into consideration the fact that the random variables representing state transition times of the process {W(t): t ≥ 0} have gamma distributions. The usefulness of the Bayesian statistical theory in operational decision-making concerning ship power plants is shown using a decision dendrite which maps ME states and consequences of particular decisions, thus making it possible to choose between the following two decisions: d_1 – first perform a relevant preventive service of the engine to restore its state and then perform the commissioned task within the time limit determined by the customer, and d_2 – omit the preventive service and start performing the commissioned task.

Keywords: decision, probability, ship power plant machine, semi-Markov process, ship internal combustion engine

INTRODUCTION

The operation of ship power plant machines on sea-going vessels is the phase of their existence which, compared to the remaining phases (design and production), consumes most energy and materials. That is why of utmost importance in this phase is solving their operational problems in the way which will lead to most effective performance of these machines. An essential problem of this type is developing relevant models and methods to control the process of appearance of consecutive technical states in their operation [4, 5, 11, 18].

The above process control can be considered as:

- Operator's action on selected parameters of the constructional structure of a power plant machine (including adjustment) to initiate and maintain its faultless operation [4, 16, 22, 24],
- Making operational decisions to ensure rational (optimal, if possible) execution of the process of appearance of consecutive technical states [1, 4, 9, 11, 24].

For ship power plant machines, the issue of control of parameters of their constructional structure with the assistance of technical diagnostics is now well established. This control makes it possible to shape energy states of these machines in the way ensuring their best durability and reliability. However, it still does not provide opportunities for rational control of the appearance of consecutive technical states of high importance in their operational phase.

Such control requires making relevant operational decisions. In turn, the literature review shows that this decision-making requires establishing, at least, a three-state set S of technical states s_i (i = 1, 2, 3) of ship power plant machines, with the following interpretation of these states:

s_1 – state of full serviceability, s_2 – state of partial serviceability, s_3 – state of unserviceability. The experience gained by the author from past research indicates that the decision-making procedures can be developed based either on the theory of semi-Markov processes, or on the statistical theory of decision-making [1, 4, 11, 14, 17]. In both cases, there is a need to develop a semi-Markov model of transitions of technical states of a given ship power plant machine in which this machine may stay. These models are essential for defining relations required to assess the probability of existence of states s_i ($i = 1, 2, 3$). Justification for the applicability of semi-Markov processes as models of technical state transitions of ship power plant machines is presented in [3, 4, 5, 24]. The hypotheses formulated in those articles are the proof that the following hypothesis can also be considered true: *the state s_{i+1} of an arbitrary ship power plant machine and the time T_{i+1} of its duration depend significantly on state s_i(i = 1, 2) which existed directly before state s_{i+1} and not on earlier states and time durations, as after the time duration of each state, full recovery of functional properties of this machine takes place which restores its full serviceability (s_1), the beginning of which depends of user's capabilities*. Hence, when the state s_1 has place, the appearance of state s_2 depends on the time duration of state s_1 and not on whether the state s_3 existed before state s_1 or not. Then, the appearance of state s_3 depends on the time duration of the directly preceding state s_2, and not on the earlier presence or absence of state s_1. This makes a basis for developing a three-state model of state transitions $s_i \in S$ ($i = 1, , 3$) in the form of a semi-Markov process, discrete in states and continuous in time.

The application of the semi-Markov process as the model of transitions of the abovementioned technical states of an arbitrary ship power plant machine makes it possible to easily determine, inter alia, the limiting distribution of the appearance of states s_i(i = 1, 2, 3) of this machine after the elapse of time intervals T_i(i = 1, 2, 3). This distribution includes the probabilities: $P_1 = P(s_1)$, $P_2 = P(s_2)$ and $P_3 = P(s_3)$ with their interpretations as: P_1 – probability of appearance of state s_1, P_2 – probability of appearance of state s_2 , and P_3 – probability of appearance of state s_3. These probabilities are used in operational decision-making processes taking into account an optimisation criterion, which can be, for instance, the expected value of the consequence of making a decision belonging to the set of possible decisions permissible in given circumstances [1, 4, 9, 10, 11].

The states s_i(i = 1, 2, 3) should have precise interpretations, individual for each type of machine, but the procedure of developing semi-Markov models for all machines is the same. It requires taking into account the operational status of the machine.

OPERATIONAL STATUS OF SHIP POWER PLANT MACHINE AND PROBLEM FORMULATION

During the operation of a ship power plant machine (main or auxiliary internal combustion engine, steam or water boiler, pump, compressor, electric motor, current generator, etc.), its technical state worsens as a result of wear. Consequently, its action (D) is characterised by lower ability to transmit energy and convert it to heat (Q) and work (L). This can be easily demonstrated by looking, for instance, at the action of the ship propulsion system with single engine and single propeller, the scheme of which is shown in Fig. 1. The action of this system consist in energy conversion to heat and work, and transmitting this energy from the main engine (*ME*) to the screw propeller (*SP*) [16, 22, 25]. As a result of excessive wear, the available engine action (D_M) can be smaller than the required action (D_W) to perform the transport task by the ship. For the above propulsion system, this case can be expressed as [5, 6]:

$$D_M \equiv D_{L_e} = \int_0^t L_e(\tau)\mathrm{d}\tau = 2\pi\int_0^t n(\tau)M_o(\tau)\tau\mathrm{d}\tau < D_W \quad (1)$$

where, in general:

$$n \cdot M_o = N_e \quad (2)$$

and:

$D_M \equiv D_{Le}$ – available ME action which allows effective work L_e to be done

D_W – required *ME* action for performing the task by the ship,

L_e – effective work done by *ME*,

n – rotational speed of *ME*,

M_o – average torque of *ME*,

N_e – power output of *ME*,

t – time of *ME* action.

The *ME* power output N_e is needed to do the work L_e (Fig. 1) [5, 22, 25]. This power output is an important index of *ME* operation, as it contains the information on how fast the work L_e can be done.

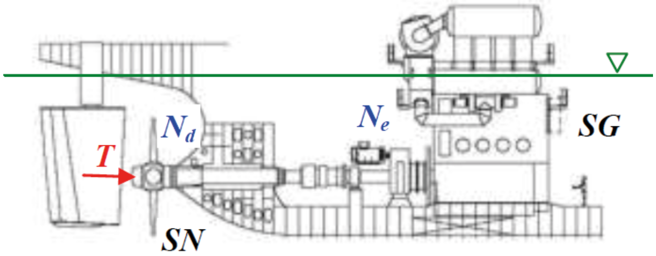

Fig. 1. Constructional scheme of the propulsion system of a ship with two-stroke high-power low-speed engine and large-diameter screw propeller: T – thrust force, SP – screw propeller, ME – main engine, N_d – power transmitted to SP, N_e – ME power output, ∇ – outboard seawater level indicator.

The increasing wear of *ME*, in particular of its tribological piston/cylinder liner system, causes the decrease of power output N_e and, consequently, of power N_d transmitted to the screw propeller *SP*. All this leads to the decrease of the thrust force *T* (Fig. 1), and ship's speed as the final result. In an extreme case, the wear of the abovementioned tribological system may cause broad damages of *ME* pistons and cylinder liners which will make further *ME* operation impossible. Such piston damage is shown in Fig. 2, while Fig. 3 presents excessive damage of *ME* cylinder liner.

Fig. 2. View of damaged side surface of a piston due to its seizure in cylinder liner and breaking of sealing rings. [8, 23, 25]

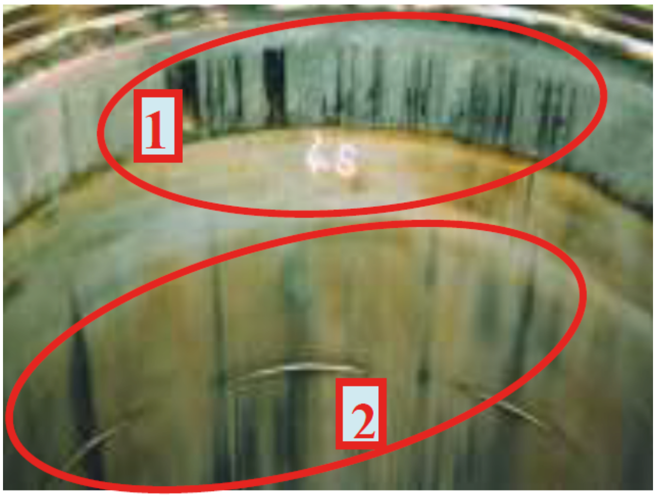

Fig. 3. View of damaged inner sliding surface of a cylinder liner due to excessive wear caused by: 1 – friction corrosion, 2 – pitting corrosion [6, 21, 25]

ME damages similar to those shown in Fig. 2 and Fig. 3 make that the technical state of this engine is to be considered as state of unserviceability (s_3). This state can also be a source of other *ME* damages. On the other hand, the *ME* user should aim at maintaining the state of full serviceability (s_1). Therefore, according to the suggestion presented in Introduction, the set *S* of states:

$$S = \{ s_1, s_2, s_3 \}. \quad (3)$$

should be unambiguously defined. This can be done based on engine characteristics shown in Fig. 4.

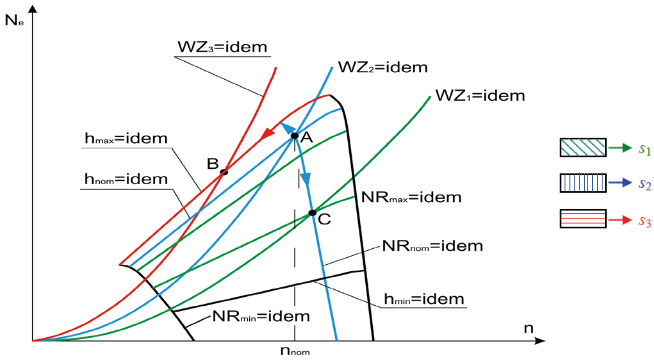

Fig. 4. SG characteristics illustrating engine load changes according to the regulator characteristic NR_{nom} = idem: N_e – ME power output, n – ME rotational speed, WZ – ambient conditions ($WZ_3 > WZ_2 > WZ_1$), NR_{min} – minimal regulator setting, NR_{nom} – nominal (rated) regulator setting, NR_{max} – maximal regulator setting, h_{max} – maximal setting of injection pumps, h_{nom} – nominal (rated) setting of injection pumps, h_{min} – minimal setting of injection pumps, s_1 – state of full serviceability, s_2 – state of partial serviceability, s_3 – state of unserviceability.

ME characteristics determine unambiguously possible ranges of engine loading with power output (N_e) and rotational speed (*n*). In the case shown in Fig. 4, the *ME* load is limited by screw propeller characteristics WZ_3 = idem and WZ_1 = idem, regulator characteristics NR_{min} = idem and NR_{max} = idem, and the external characteristic h_{max} = idem along the segment from point *B* to the beginning of characteristic NR_{max} = idem. The screw propeller characteristic WZ_3 = idem corresponds to the toughest ambient conditions (*WZ*) of ship navigation, while the characteristic WZ_1 = idem – to most favourable conditions. The main engine operation according to the external characteristic h_{max} = idem provides an opportunity to load the engine with maximal power output (N_{emax}). *ME* can be loaded in this way only when in the state of full serviceability (s_1). Progressing *ME* wear which takes place during its lifetime makes applying such a load impossible. A situation may occur that the engine can be loaded only according to the characteristic h_{nom} = idem, or even to the characteristic N_e – *n* situated lower in the diagram (Fig. 4). According to this characteristic, the engine can only be loaded with rated power (N_{nom}). The main engine which can be loaded according to the characteristic h_{nom} = idem is not any longer in the state of full serviceability, but only in the state of partial serviceability (s_2). When the engine can be loaded only according to characteristics situated below the curve h_{nom} = idem, its state should be considered as the state of unserviceability (s_3), which results from the need to observe principles of rational engine operation. In general, the current technical state of *ME* can be classified to one of three, at least, classes of states s_i (i = 1, 2, 3), belonging to the set *S* (3) of states, with the following descriptive interpretations:

– state of full serviceability s_1, i.e. the technical state which enables safe engine operation according to the external

characteristic h_{max} = idem, in arbitrary ambient conditions (WZ) and in the load range for which it was intended in the design and production phases (load field marked with green lines in Fig. 4),
– state of partial serviceability s_2, i.e. the technical state which enables engine operation only according to the external characteristic h_{nom} = idem in limited ambient conditions (WZ) and in the load range lower than that for which it was intended in the design and production phases (load field marked with blue lines in Fig. 4),
– state of unserviceability s_3, i.e. the technical state which does not make engine operation possible according to the external characteristic h_{nom} = idem, i.e. in limited ambient conditions (WZ) and in the load range much lower than that for which it was intended in the design and production phases (load field marked with red lines in Fig. 4), or even makes it totally impossible due to, for instance, damage of pistons, rings, or cylinder liners, similar to those shown in Fig. 2 and Fig. 3.

In the main engine operation phase, the probabilities of occurrence of states $s_i \in S(i = 1, 2, 3)$ should be predicted. This can be done taking into account an arbitrary time t of engine operation (1), or a very long time interval t of its operation, theoretically $t \to \infty$.

Similar considerations can be done for other ship power plant machines, and for each machine, a relevant set of states $s_i \in S(i = 1, 2, 3)$ can be predicted. For a steam boiler, these states will result from its heating output. When the boiler is in the state of full serviceability (s_1), it can be loaded to produce the maximal heating output to meet ship's needs. With time, the thermal capacity of the boiler decreases due to a growing layer of limescale on its heated walls. As a consequence, its state will worsen initially to partial serviceability s_2 , and then to unserviceability s_3. This also refers to water boilers. For other machines: pumps, compressors, various heat exchangers, such as water and lubricating oil coolers or fuel and lubricating oil heaters, etc., the states s_i $(i = 1, 2, 3)$ can be determined in a similar way. Like for ME, in the operation phase of these machines, the probability of occurrence of states $s_i \in S(i = 1, 2, 3)$ at arbitrary time t of machine operation, or in extremely long time interval, theoretically $t \to \infty$, should be assessed.

However, these considerations are only possible once the models of transitions of the states belonging to the set S of states have been developed. The review of the available literature suggests that these models can take a form of semi-Markov processes which have a discrete set of states and are continuous in time t. For ME, the set of these states (3) has been unambiguously defined by giving interpretations of states $s_i(i = 1, 2, 3)$.

Further in the article, a model will be presented which refers not only to main engines (ME) composing the ship propulsion systems (Fig. 1), but also to other ship power plant machines.

For an arbitrary ship power plant machine, like for ME, it is possible to develop the semi-Markov model of transitions of states $s_i(i = 1, 2, 3)$ in the form of a semi-Markov process, as for these states, when relevant technical diagnostics is applied, the time duration of state s_i $(i = 1, 2, 3)$ existing at time and the state $s_j(j = 1, 2, 3)$ which can appear at time do not depend stochastically on earlier states and their time durations [3, 4, 11, 13, 14, 17, 24].

SEMI-MARKOV PROCESS OF TECHNICAL STATE TRANSITIONS OF SHIP POWER PLANT MACHINE

The technical state of an arbitrary ship power plant machine, not only ME, is determined by a set of technical features of its constructional structure which enables functioning of this machine as intended in the design and production phases. At arbitrary time t of machine operation, this state depends not only on this time, but also on the technical state of the machine at its start-up time $t_0 < t$, as well as on changes of loads applied in time interval $[t_0, t]$ and the course of control of various processes during this time interval. This control has a fundamental impact on technical state transitions of each ship power plant machine [4, 7, 8, 9, 16, 18, 20].

Technical state transition of a ship power plant machine which takes place at time t due to its wear is a stochastic process, continuous in states and time, which means that the realisations of possible types of technical states of this machine comprise an infinite set.

Identifying all technical states of an arbitrary ship power plant machine is neither possible nor justified for both technical and economic reasons. Consequently, there is a need to define a small number of classes (subsets) of its technical states. Assuming the machine's ability to perform intended tasks as the criterion for the above classification, the following classes (subsets) of technical states, further simply referred to as states, can be named [3, 4, 9, 10, 24]:
– state of full serviceability s_1, which enables the machine to operate in all ambient conditions and in the entire load range for which it was intended in the design and production phases,
– stan of partial serviceability s_2, which enables the machine to operate in limited ambient conditions and in the load range smaller than that for which it was intended in the design and production phases,
– state of unserviceability s_3, which does not allow the machine to operate as intended, for instance due to its damage, performing necessary preventing actions, etc.

These states should be precisely defined individually for each ship power plant machine.

The technical states of ME, denoted as $s_i \in S(i = 1, 2, 3)$, were defined in the previous Section, see Fig. 4. Possible transitions of these states can be arranged in a graph [3, 4, 5, 10]. The graph of transitions of states s_i $(i = 1, 2, 3)$ which is valid for an arbitrary ship power plant machine, including ME, is shown in Fig. 5.

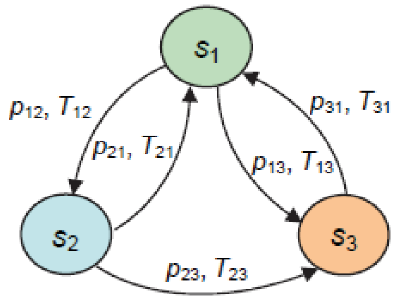

Fig. 5. Graph of technical state transitions of an arbitrary ship power plant machine: s_1 – state of full serviceability, s_2 – state of partial serviceability, s_3 – state of unserviceability, p_{ik} – probability of occurrence of state transition from s_i to s_k, T_i – time of unconditional existence of state s_i regardless of which state s_k appears next, T_{ik} – time duration of state s_i provided that then next state is s_k; i, k = 1, 2, 3; i ≠ k

This graph illustrates the possibility of rational appearance of the abovementioned states, starting from state s_1 which lasts from time t_0 to time t_1. These states appear consecutively at times t_1, t_2, t_3, …, t_n, where $n = 1, 2, 3, …$(Fig. 6). State transitions of each ship power plant machine are characterised by probabilities p_{ik} of state transition from s_i to s_k($i, k = 1, 2, 3$; $i \neq$ k) and random variables, which are time intervals T_i($i = 1, 2, 3$) of unconditional existence of state s_i regardless of which state s_k appears next, and time intervals T_{ik}($i, k = 1, 2, 3; i \neq k$) of existence of state s_i provided that the next state is s_k.

A machine staying in the state of full serviceability (s_1) can change the state to partial serviceability (s_2) after time T_{12}. This state transition may occur with probability p_{12}. In case the crew during the voyage, or after returning to the port, restores the technical state of the machine, its state changes again to s_1. This state transition will occur with probability p_{21} after time interval T_{21}. During the voyage, the crew does not always have the opportunity to perform such a repair. Then, after time T_{23}, the technical state of the machine may change to unserviceability (s_3), and this state transition will occur with probability p_{23}. When it refers to a machine having an essential impact on ship safety, the main engine (*ME*) for instance, this may lead to a dangerous situation, and even catastrophic when in storm, as the ship may sink in those conditions. The machine staying in state s_3 should undergo full overhaul, which will take place with probability p_{31} after time T_{31}. In rational operation, partial repair of this machine to restore it to state s_2 is not permitted, as this situation may again lead to the appearance of state s_3 during the ship voyage, with the resulting threat of occurrence of dangerous, or even catastrophic threats. That is why the graph of state transitions in Fig. 5 does not include a curve illustrating state transition from s_3 to s_2. This means that the probability of this state transition is $p_{32} = 0$.

We can assume that when neither state s_2 nor state s_3 has place, then the machine stays in state s_1.

Particular machine states $s_i \in S(i = 1, 2, 3)$ can be identified using a relevant diagnostic system (*SD*) which consist of two subsystems: diagnosing subsystem (*SDG*), and the power plant machine being the diagnosed subsystem (*SDN*). The operating applicability of the diagnostic system (*SD*) depends on the quality of the applied subsystem *SDG* and adaptation of the power plant machine, i.e. subsystem *SDN*, to its state identification.

It results from the above considerations that states $s_i \in S(i = 1, 2, 3)$, which consecutively take place during the operation of the power plant machine, compose a set of transitions of these states $\{W(t): t \geq 0\}$. An exemplary execution of this process is shown in Fig. 6.

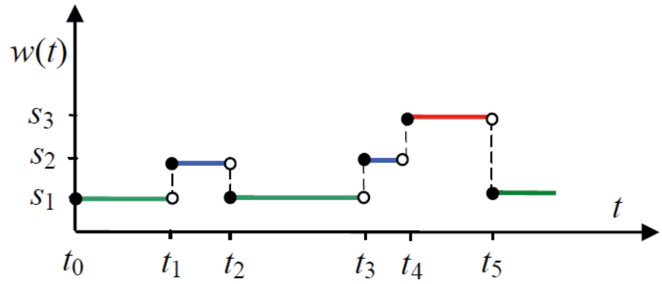

Fig. 6. Execution of process {W(t): t ≥ 0} for an arbitrary ship power plant machine: {W(t): t ≥ 0} – technical state transition process, t – operating time, s1 – state of full serviceability, s2 – state of partial serviceability, s3 – state of unserviceability

In mathematical approach, the analysed process of technical state transitions of a ship power plant machine is the function which maps states $s_i \in S$ ($i = 1, 2, 3$) of this machine on the operating time.

Hence, the set $S = \{s_1, s_2, s_3\}$ of technical states can be considered as a set of values of the stochastic process $\{W(t): t \geq 0\}$ with realisations constant in intervals and continuous on the right (Fig. 6).

The initial distribution of this process (Fig. 7) is given by the formula [4, 9, 11]:

$$P_i = P\{W(0) = s_i\} = \begin{cases} 1 & dla \quad i = 1 \\ 0 & dla \quad i = 2,\ 3 \end{cases} \tag{4}$$

According to the graph in Fig. 5, the functional matrix $Q(t)$ is:

$$Q(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & Q_{23}(t) \\ Q_{31}(t) & 0 & 0 \end{bmatrix} \tag{5}$$

where [11]:

$$Q_{ij}(t) = P\{W(\tau_{n+1}) = s_j, \tau_{n+1} - \tau_n \leq t \mid W(\tau_n) = s_i\}; \quad i,\ j = 1,\ 2,\ 3; \quad i \neq j$$

where:

p_{ij} – probability of going a process $W(t)$: $t \geq 0\}$ from state s_i to state s_j,

F_{ij} – cumulative distribution function random variable T_{ij} ($i, j = 1, 2, 3; i \neq j$)

This case of a matrix (5) can occur if correctness all maintenance activities will be checked after their carrying out by running the ship power plant machine.

Based on this matrix, we can determine transition probabilities that are significant in operation of the ship power plant machines. They are defined as the conditional probability:

$$P_{ij} = P\{W(t) = s_j \mid W(0) = s_i\}, \; i, j = 1, 2, 3; \quad i \neq j \tag{6}$$

which, in turn, fulfill Fellera equations [11, 24].

As the time passes by, theoretically to t → ∞, the conditional probabilities $P_{ij}(t)$ given by formula (7) and the probabilities $P_j(t)$ of states sj, j = 1, 2, 3

$$P_j(t) = P\{W(t) = s_j\}, \tag{7}$$

stabilise and tend towards constant values. Consequently, these probabilities can be replaced by limiting probabilities [11, 13, 19]

$$P_{ij} = \lim_{t\to\infty} P_{ij}(t), \qquad P_j = \lim_{t\to\infty} P_j(t),$$

These values can be determined based on matrix (5) and making use of the corresponding matrix of transition probabilities of the Markov chain embedded in the process $\{W(t): t \geq 0\}$. The matrix of the Markov chain embedded in the considered process *SM* has the form:

$$\mathbf{P} = \begin{bmatrix} 0 & p_{12} & p_{13} \\ p_{21} & 0 & p_{23} \\ p_{31} & 0 & 0 \end{bmatrix} \tag{8}$$

We can determine these probabilities based on the matrix (5) using the corresponding probability matrix of going a process $\{W(t): t \geq 0\}$ of Markov chain. This chain that is embedded to a semi-Markov process will take a form:

$$\mathbf{P} = \begin{bmatrix} 0 & p_{12} & p_{13} \\ p_{21} & 0 & p_{23} \\ p_{31} & 0 & 0 \end{bmatrix} \tag{8}$$

To specify the boundary distribution of the Markoy chain embedded in the process $\{W(t): t \geq 0\}$ we should to solve the following equations:

$$\left.\begin{aligned} &[\pi_1, \pi_2, \pi_3] \cdot \mathbf{P} = [\pi_1, \pi_2, \pi_3] \\ &\pi_1 + \pi_2 + \pi_3 = 1 \end{aligned}\right\} \tag{9}$$

The process $\{W(t): t \geq 0\}$ has boundary distribution $P_j = \lim_{t\to\infty} P\{W(t) = s_j\}, \; j = 1, 2, 3$, because it is unpredicted [11, 24] and the mentioned random variables T_j(j = 1, 2, 3) have finite positive expected values $E(T_j)$. The limiting probabilities of the process can be calculated from formulas [11]

$$P_{ij} = \lim_{t\to\infty} P_{ij}(t) = P_j = \lim_{t\to\infty} P_j(t) = \frac{\pi_j \mathrm{E}(T_j)}{\sum \pi_j \mathrm{E}(T_j)}, \quad S = \{1, 2, 3\} \tag{10}$$

where:

$$E(T_j) = \sum_{i=1}^{3} p_{ij} E(T_{ij}),$$

and:

$$\pi_j = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} P\{W(\tau_k) = s_j / W(0) = s_i\}; \; s_i, s_j \in S, \; j = 1, 2, 3,$$

is the boundary distribution of Markov chain $\{W(\tau_n): n = 0, 1, 2, \dots\}$ embedded to a process $\{W(t): t \geq 0\}$.

Appearance of consecutive states s_1, s_2, s_3 of the ship power plant machines is the result of the impact of the cumulative loads on them. This means that random variables T_{ij} have approximately gamma distributions with the appropriate parameters [1, 2, 24].

The data published by companies producing ship engines, as well the empirical studies, [21, 22, 23, 24] have shown that some high-power main engines (*ME*) have the expected values m_{ij} of times of state transition from s_i to s_j equal to:

$$m_{12} = 20\, tys.[h], \quad m_{13} = 10\,0\, tys.[h] \quad k_{12} = \tfrac{1}{2\sqrt{2}} \approx 0{,}354 \quad k_{13} = 1$$
$$m_{21} = 0{,}12\, tys.[h], \quad m_{23} = 4\,0\, tys.[h] \quad k_{12} = \tfrac{1}{2\sqrt{2}} \approx 0{,}354 \quad k_{23} = 1,$$
$$m_{31} = 0{,}40\, tys.[h] \quad k_{31} = \tfrac{1}{2\sqrt{2}} \approx 0{,}354$$

while the probabilities of these transitions (p_{ij}), being the elements of matrices (5) and (8), are equal to:

$$p_{12} = 0.8 \quad p_{13} = 0.2$$
$$p_{21} = 0.9 \quad p_{23} = 0.1$$
$$p_{31} = 1$$

The limiting probabilities can be calculated using a procedure included in the computational software MATHEMATICA. The values of these probabilities calculated in the above way for the obtained estimates are:

$$\left.\begin{aligned} P_1 &= P(s_1) = 0{,}9138, \\ P_2 &= P(s_2) = 0{,}0834, \\ P_3 &= P(s_3) = 0{,}0028 \end{aligned}\right\} \tag{11}$$

The limiting probabilities (10) with values (11) can be used in real operational decision-making with the assistance of the Bayesian statistical theory. This theory makes it possible to develop a decision-making model which, along with the abovementioned probabilities, takes into account the expected value of consequence of a particular decision as a criterion.

STATISTICAL MODEL OF DECISION-MAKING

The information on the values of probabilities P_1, P_2 and P_3 (23a) and the consequences of particular decisions makes it possible to apply the statistical decision-making theory when choosing, for instance, between the following two decisions [1, 4, 9]:

- decision d_1 – first perform a relevant preventive service of the engine to restore its state, as a precondition for performing the commissioned task, and then start performing the task within the time limit determined by the customer,
- decision d_2 – omit the preventive service and start performing the commissioned task.

Making a choice between these two decisions can be facilitated by the use of the statistical decision-making theory, which says that the expected value of a consequence (gain or loss) of each choice can be a criterion for making the most favourable choice from among the available options. However, this requires proper assessment of consequences of particular decisions and presenting these consequences as relative numbers. In particular, the gain resulting from making a given decision will be expressed as a positive number, while loss – as negative [1, 4].

The Bayesian statistical theory says that in such a decision-making situation, for instance for the considered process $\{W(t): t \geq 0\}$ of technical main engine state transitions, a decision dendrite can be used. The dendrite representative for the above situation is shown in Fig. 7.

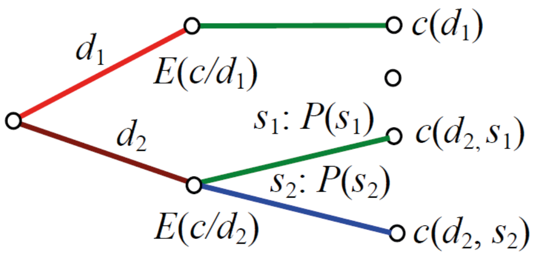

Fig. 7. Decision dendrite for making decision d_1 or d_2: d_1 – decision that preventive service of engine should be performed first, and then the commissioned task, d_2 – decision that the commissioned task should be performed without prior preventive engine service, $P(s_1)$ – probability that the engine is in state s_1, $P(s_2)$ – probability that the engine is in state s_2, $c(d_1, s_1)$ – consequence of decision d_1 when the engine is in state s_1, $c(d_2, s_1)$ – consequence of decision d_2 when the engine is in state s_1, $c(d_2, s_2)$ – consequence of decision d_2 when the engine is in state s_2

The decision dendrite in Fig. 7 shows that the expected value can be calculated from the relations [1, 4]:

$$\left.\begin{aligned} E(c|d_1) &= c(d_1, s_1) \\ E(c|d_2) &= P(s_1)c(d_2, s_1) + P(s_2)c(d_2, s_2) \end{aligned}\right\} \qquad (12)$$

taking into account the following relation (11):

$$P(s_1) + P(s_2) = 0{,}9927$$

The principle of decision-making is as follows: when $E(c|d_1) > E(c|d_2)$, decision d_1 is to be made, and when $E(c|d_1) < E(c|d_2)$, decision d_2 is considered correct.

In such a simple decision-making situation, the probability P^* of correct operation of an arbitrary machine (an internal combustion engine *SG* in this case) can be found for which, for formal reasons, it does not matter which decision will be made. For this probability the following relations are obtained:

$$P^* \rightarrow E(c/d_1) = E(c/d_2) \rightarrow d_1 \cup d_2, \qquad (13)$$

Hence, this probability can be calculated from the equation (Fig. 7):

$$c(d_1) = P^* c(d_2, s_1) + (0{,}9927 - P^*)\, c(d_2, s_1) \qquad (14)$$

since
$E(c/d_1) = c(d_1)$ and $P_1 + P_2 = 0{,}9927$ (11).

Determining the values of consequences $c(d_1)$, $c(d_2, s_1)$ and $c(d_2, s_1)$ is not easy. However, to demonstrate that these values are of higher practical importance that the values of probabilities $P(s_1)$ and $P(s_2)$, it is sufficient to assume that: $c(d_1) = 0{,}5jp$, $c(d_2, s_1) = 1jp$ while $c(d_2, s_2) = -0{,}5jp$ (*jp* – monetary unit). The assumed values illustrate the fact that performing an unnecessary preventive service as a result of decision $c(d_1)$ leads to profit decrease from $1jp$ to $0{,}5jp$, i.e. $c(d_1) = 0{,}5jp$. At the same time, resigning from this service when the engine is in state s_1 leads to profit amounting to $c(d_{2,} s_1) = 1jp$. On the other hand, when the service is not performed for the engine in state s_2, it will result in not completing the task and the loss amounting to $c(d_2, s_2) = -0{,}5jp$.

After some transformations, the formula (14) can be written as

$$P^* = \frac{c(d_1) - 0{,}9927c(d_2, s_2)}{c(d_2, s_1) - 0{,}9927c(d_2, s_2)} \qquad (15)$$

The rule of decision-making can be formulated as:

$$\left.\begin{aligned} P < P^* &\rightarrow d_1, \text{ because then } E(c/d_1) > E(c/d_2) \\ P > P^* &\rightarrow d_2, \text{ because then } E(c/d_1) < E(c/d_2) \end{aligned}\right\} \qquad (16)$$

The calculated values of probabilities P_1, P_2 and P_3, along with the selected values of consequences $c(d_1)$, $c(d_2, s_1)$ and $c(d_2, s_1)$, make it possible to calculate, from formula (15), that $P^* = 0{,}6658$. Since the inequality $P_1 > P^*$ takes place, the decision-making rule (16) says that decision d_2 should be made. The decision-making situation in Fig. 7 shows that the accuracy of assessment of probability *P* is not particularly important. It results from this situation that decision d_2 – *"omit preventive service and start performing the commissioned task"* can be made when the values of probabilities range within $P_1 = P(s_1) = 0,6658 \div 1,0$. Hence, when applying the presented decision-making model, the calculated probability $P_1 = 0,7832$ leads to making the above operational decision.

REMARKS AND CONCLUSIONS

The article describes a relatively simple model of technical state transitions of an arbitrary ship power plant machine, with the main engine (*ME*) and the three-state set of technical states: s_1, s_2 and s_3 as an example. The model has been developed based on the theory of semi-Markov processes and therefore has the form of a semi-Markov process, discrete in states and continuous in time. This model made it possible to calculate values of limiting probabilities of the considered process, as a result of application of gamma distributions with different values of shape parameter r and scale parameter λ referring to the times of machine operation and repair as random variables. Adopting these distributions was justified by cumulative action of loads on the machine which resulted in its gradual wear, and by the fact that the empirical distribution of repair times is a random variable which can take negative values and cannot be described by normal distribution.

Semi-Markov processes as models of real processes of technical state transitions of ship power plant machines are a convenient tool in practical studies. This convenience results from the fact that their application as models of real processes allows making use of professional computer software for solving different equation systems describing real processes.

The article demonstrates the applicability of limiting probabilities (P_1, P_2 and P_3) of distribution of the stochastic process $\{W(t): t \geq 0\}$ when making operational decisions with the assistance of the Bayesian stochastic theory. It is shown that this theory is easy to apply once the abovementioned probabilities and consequences of particular decisions are known. This ease results from the existence of one well-defined criterion for selecting the most favourable decision from among the available options. This criterion is the expected value of consequence (gain or loss) of making one or another decision.

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