

PREDICTION OF VORTEX-INDUCED VIBRATION RESPONSE OF DEEP SEA TOP-TENSIONED RISER IN SHEARED FLOW CONSIDERING PARAMETRIC EXCITATIONS

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ABSTRACT

It is widely accepted that vortex-induced vibration (VIV) is a major concern in the design of deep sea top-tensioned risers, especially when the riser is subjected to axial parametric excitations. An improved time domain prediction model was proposed in this paper. The prediction model was based on classical van der Pol wake oscillator models, and the impacts of the riser in-line vibration and vessel heave motion were considered. The finite element, Newmark- β and Newton-Raphson methods were adopted to solve the coupled nonlinear partial differential equations. The entire numerical solution process was realised by a self-developed program based on MATLAB. Comparisons between the numerical calculation and the published experimental test were conducted in this paper. The in-line and cross-flow VIV responses of a real size top-tensioned riser in linear sheared flow were analysed. The effects of the vessel heave amplitude and frequency on the riser VIV were also studied. The results show that the vibration displacements of the riser are larger than the case without vessel heave motion. The vibration modes and frequencies of the riser are also changed due to the vessel heave motion

Keywords: top-tensioned riser, vortex-induced vibration, wake oscillator model, time-varying axial tension force, sheared flow

INTRODUCTION

Since the vortex-induced vibrations (VIV) of deep sea top-tensioned risers can result in large amplitude responses in both the in-line (IL) and cross-flow (CF) directions, accurate prediction of the risers' VIV response has attracted wide attention among researchers. Meanwhile, a riser's axial tension force will change constantly with time due to the vessel heave motion. The time-varying axial tension force will cause transverse vibration of the riser, which is called parametric excitation vibration.

Wake oscillator models have been widely used to predict riser VIV responses in recent years. Nonlinear dynamic oscillators are adopted to predict the main features of vortex shedding in the wake region of the risers. Bishop and Hassan first put forward that the oscillator satisfied the van der Pol equation [1]. Hartlen and Currie first established the specific expression of the wake oscillator mode [2]. Then the wake

oscillator model was improved by Balasubramanian and Skop and Krenk and Nielsen [3-4]. Facchinetti et al. created three expressions for the wake oscillator model. They found that the acceleration coupling term was the best expression to predict the cylinder VIV response [5].

So far, many researchers have focused their attention on the study of the coupled in-line and cross-flow VIV response of different riser models [6-11]. However, the axial parametric excitations have not been considered in these studies. Spanos et al. and Lei et al. established frequency domain analysis models to predict the riser's transverse vibration considering the time-dependent axial tension force [12-13]. Considering the time-varying axial tension force, time domain analysis models to predict the riser transverse vibration have also been proposed in some works [14-18]. Some studies have investigated the effect of variable axial tension force on the vertical risers' cross-flow VIV when subjected to uniform and sheared flows [19-21].

From the above review, it can be seen that a number of works in the literature have analysed the effect of the time-varying axial tension force on the transverse vibration response and the VIV response which are induced by the sea current and/or waves. However, there are few studies on the coupled IL and CF VIV of deep sea top-tensioned risers in sheared flow considering time-varying axial tension force. In this paper, an improved three-dimensional time domain prediction model was proposed. The effects of the riser's in-line vibration and the vessel heave motion were considered in the prediction model. The coupled IL and CF VIV response of a real size top-tensioned riser was studied in this paper. The impact of the vessel heave amplitude and frequency on the riser's VIV response has also been discussed.

MODEL DESCRIPTION

STRUCTURE MODEL AND EQUATIONS OF MOTION

A two-end pinned Euler-Bernoulli beam model is used to simulate deep sea top-tensioned risers. A Cartesian coordinate system is defined, in which the x -axis is parallel to the direction of seawater flow and the z -axis is parallel to the axial direction of the un-deformed riser. Fig. 1 shows the schematic model of a deep sea top-tensioned riser.

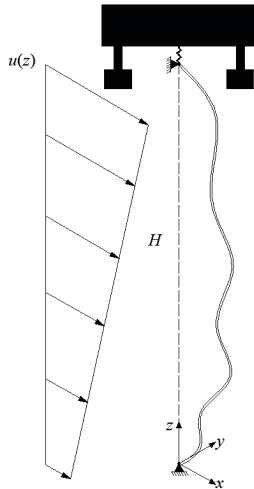


Fig. 1. Schematic model of deep sea top tensioned riser

The equations of the riser IL and CF vibrations can be written as [22-23]:

$$\begin{cases} EI \frac{\partial^4 x(z,t)}{\partial z^4} - T(z,t) \frac{\partial^2 x(z,t)}{\partial z^2} - w \frac{\partial x(z,t)}{\partial z} \\ + \bar{m} \frac{\partial^2 x(z,t)}{\partial t^2} + \bar{c} \frac{\partial x(z,t)}{\partial t} = f_x(z,t) \\ EI \frac{\partial^4 y(z,t)}{\partial z^4} - T(z,t) \frac{\partial^2 y(z,t)}{\partial z^2} - w \frac{\partial y(z,t)}{\partial z} \\ + \bar{m} \frac{\partial^2 y(z,t)}{\partial t^2} + \bar{c} \frac{\partial y(z,t)}{\partial t} = f_y(z,t) \end{cases} \quad (1)$$

In Eq. (1):

EI - the riser bending stiffness;

$T(z,t)$ - the effective axial tension force of the riser;

w - the wet weight of riser per unit length;

$\bar{m}$ - the equivalent mass of riser per unit length;

$\bar{c}$ - the damping coefficient;

$f_x(z,t)$, $f_y(z,t)$ - the IL and CF fluctuating hydrodynamic forces acting on the riser.

The vessel heave motion is modelled by a harmonic fluctuation in the present study. The time-varying effective axial tension force of the riser can be written as [18, 24-25]:

$$T(z,t) = f_{\text{top}} w H - \int_z^H w dz + K A \sin(\Omega t) \quad (2)$$

In Eq. (2):

f_{top} - the static top tension force coefficient;

H - the water depth;

K - the equivalent spring stiffness of the heave compensator ($K = wH/a_c$), a_c is the critical amplitude associated with the heave compensator (usually set as 10 m);

A , Ω - the heave motion amplitude and frequency of the vessel.

The wet weight and mass of riser per unit length are:

$$\begin{cases} w = (m_r + m_f - m_d)g ; \bar{m} = m_r + m_f + m_a \\ m_r = \pi \rho_r (D^2 - d^2) / 4 ; m_f = \pi \rho_f d^2 / 4 \\ m_d = \pi \rho_w D^2 / 4 ; m_a = C_a \pi \rho_w D^2 / 4 \end{cases} \quad (3)$$

In Eq. (3):

g - the gravity acceleration;

ρ_r, ρ_f, ρ_w - the density of the riser, internal fluid and sea water;

D, d - the riser's outer and inner diameters;

C_a - the added mass coefficient.

The damping coefficients are as follows [9]:

$$\begin{cases} \bar{c} = c_s + c_f \\ c_f = \gamma \Omega_f \rho_w D^2 \end{cases} \quad (4)$$

In Eq. (4):

c_s - the structure damping coefficient;

c_f - the fluid damping coefficient;

γ - the related coefficient ($\gamma = \bar{C}_D / 4\pi S_t$);

Ω_f - the vortex shedding angular frequency;

$\bar{C}_D$ - the mean drag force coefficient;

S_t - the Strouhal number.

Considering the effect of the riser's in-line vibration, the modified vortex shedding frequency can be expressed as follows:

$$\Omega_f = 2\pi S_t \frac{u(z) - \partial x(z,t) / \partial t}{D} \quad (5)$$

In Eq. (5):

$u(z)$ - the sea current velocity;

$\partial x(z,t) / \partial t$ - the riser's IL vibration velocity.

Considering the effect of the riser's in-line vibration, the fluctuating hydrodynamic forces can be expressed as follows:

$$\begin{cases} f_x(z,t) = 0.5\bar{C}_D\rho_w D u(z)^2 + 0.5C_D\rho_w D(u(z) - \partial x(z,t)/\partial t)^2 \\ \quad - 0.5C_L\rho_w D(u(z) - \partial x(z,t)/\partial t)^2 \partial y(z,t)/\partial t \\ f_y(z,t) = 0.5C_L\rho_w D(u(z) - \partial x(z,t)/\partial t)^2 \\ \quad + 0.5C_D\rho_w D(u(z) - \partial x(z,t)/\partial t)^2 \partial y(z,t)/\partial t \end{cases} \quad (6)$$

In Eq. (6):

C_D - the IL VIV drag force coefficient;

C_L - the CF VIV lift force coefficient;

$\partial y(z,t)/\partial t$ - the riser's CF vibration velocity.

WAKE OSCILLATOR MODEL AND EQUATIONS OF MOTION

According to the wake oscillator models [12, 16], C_D and C_L can be simulated by the time-varying fluid vortex variables p and q :

$$\begin{cases} \ddot{p} + 2\varepsilon_x\Omega_f(p^2 - 1)\dot{p} + 4\Omega_f^2 p = \frac{A_x}{D} \frac{\partial^2 x(z,t)}{\partial t^2} \\ \ddot{q} + \varepsilon_y\Omega_f(q^2 - 1)\dot{q} + \Omega_f^2 q = \frac{A_y}{D} \frac{\partial^2 y(z,t)}{\partial t^2} \end{cases} \quad (7)$$

In Eq. (7):

p, q - the IL and CF wake fluid variable parameter ($p = 2C_D/C_{D0}, q = 2C_L/C_{L0}$);

C_{D0}, C_{L0} - the IL VIV drag force coefficient and CF VIV lift force coefficient of the fixed riser;

$A_x, A_y, \varepsilon_x, \varepsilon_y$ - the non-dimensional coefficients.

BOUNDARY AND INITIAL CONDITIONS

The boundary conditions are expressed as follows:

$$\begin{cases} x(0,t) = y(0,t) = 0; \quad x(H,t) = y(H,t) = 0 \\ \frac{\partial^2 x(0,t)}{\partial z^2} = \frac{\partial^2 y(0,t)}{\partial z^2} = 0; \quad \frac{\partial^2 x(H,t)}{\partial z^2} = \frac{\partial^2 y(H,t)}{\partial z^2} = 0 \end{cases} \quad (8)$$

At the initial moment, the vibration displacement and velocity of the riser are assumed as zero. The values of the fluid variables p and q are set as 2.0, and their first derivatives with respect to time are set to zero.

MODEL PARAMETERS

According to [12, 13, 15, 16, 17], the values of the empirical coefficients are given in Table 1.

Tab. 1. Values of the empirical coefficients

Empirical coefficients	Values
S_t	0.17
$\bar{C}_{D0}$	1.2
C_{D0}	0.1
C_{L0}	0.3
A_x, A_y	12
$\varepsilon_x, \varepsilon_y$	0.3

NUMERICAL SOLUTION METHOD

The coupled form of Eq. (1) with Eq. (7) is nonlinear partial differential equations with variable coefficients. Some coefficients in the coupled equations change continuously along the spatial position, and some coefficients change continuously with time. The coupled equations are very complicated and the analytical solution cannot be obtained. So, numerical solution methods are used in the present study to solve the coupled equations.

The Galerkin-type finite element method is used to conduct the spatial discretisation of the coupled equations. Based on the parameters of the riser and seawater, as well as the boundary and initial conditions, the Hermite cubic interpolation functions are used to obtain the system's flexural stiffness matrix, geometric stiffness matrix, mass matrix, and damping matrix. Due to the uncertainty of the riser's damping coefficient, the Rayleigh damping theory is adopted in this paper to obtain the structural damping matrix. It is assumed that the vibration shape functions of wake oscillators are trigonometric functions. Then the wake oscillators are used to obtain the damping matrix of the fluid and the dynamic load matrix at the initial moment.

The Newmark- β method is used to solve the coupled equations in the time domain. In each time step, the Newton-Raphson iterative method is used to solve the nonlinear equations. The system's geometric stiffness matrix, mass matrix, damping matrix, dynamic load matrix and natural vibration frequency are all constantly updated at each time step. The entire numerical solution process is realised by a self-developed program based on MATLAB.

VALIDATION OF SIMULATION

In order to test the validity of the prediction models, comparisons with experimental results are carried out in this paper. The experiment was carried out at the Deep Sea Basin of the National Maritime Research Institute in Japan [26]. It should be pointed out that the water depth of the fluid flow in the experiment ranges from 0.4 m to 5 m, while the water depth of the fluid flow in the numerical calculation ranges from 0 to 5 m. The flow is stable and the flow velocity is set to 0.3 m/s in the water depth of 0 to 0.4 m in the numerical calculation. When the dynamic variation of the top tension is considered, two cases are calculated using the prediction model proposed in this paper. The added-tension/self-weight in the experiment is varied from 0.58 to 1.08 in a certain period. It should be pointed out that according to Eq. (2) and the equivalent principle, the top tension coefficient (f_{top}) is set to 1.83 and the KA is set to $0.25T_{top}$ in the numerical calculation.

Fig. 2 illustrates the experimental and numerical results of the model's cross-flow maximum VIV amplitude at different top tension variation periods. It can be seen that the maximum VIV amplitudes calculated in this paper are a little larger than the experimental results, especially in the upper part of the riser model. Comparing the experimental and numerical results, it can be seen that the vibration mode of the upper part

of the riser is comparatively consistent, but the vibration mode of the lower part of the riser is somewhat different. However, in general, the relative errors between the numerical and experimental results are acceptable in engineering applications. Overall, the prediction model proposed in this paper is valid in predicting the main features of the VIV response of large aspect (length/diameter) ratio top-tensioned risers.

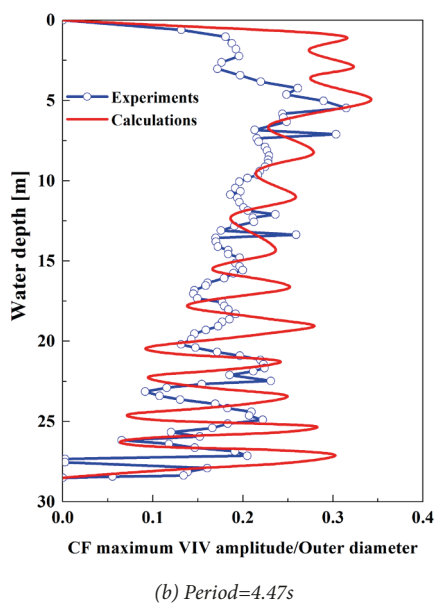
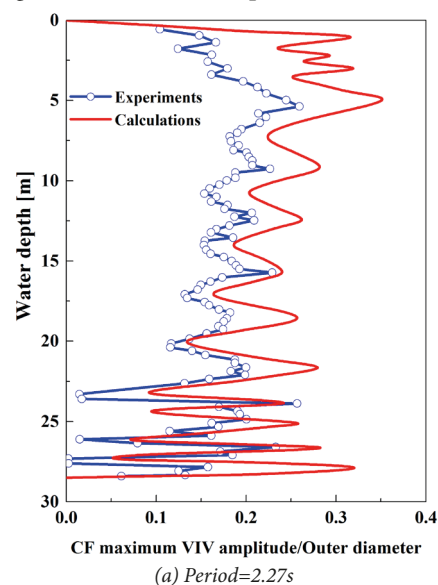


Fig. 2. VIV amplitude of the riser model

CASE STUDY

In the present study, the VIV response of a real size top-tensioned riser is studied. It is assumed that the current is a linear sheared flow. The main parameters of the riser and vessel are given in Table 2.

Tab. 2. Main parameters of the riser and vessel

Parameters	Values
Water depth (m)	2000
Outer diameter (m)	0.5334
Inner diameter (m)	0.4826
Elastic modulus (N/m ²)	2.1×10^{11}
Riser density (kg/m ³)	7850
Internal fluid density (kg/m ³)	1200
Seawater density (kg/m ³)	1030
Length/diameter ratio	3750
Static top tension coefficient	1.1~1.6
Sea surface current velocity (m/s)	0.5~1.5
Sea bottom current velocity (m/s)	0.2
Structural damping coefficient (%)	0.3
Vessel heave motion amplitude (m)	0.5~2.0
Vessel heave motion frequency (rad/s)	0.4~1.6

NATURAL FREQUENCIES OF THE RISER

Some key features of the riser's VIV response can be illustrated by inspecting the transverse vibration natural frequencies of the riser. Hence, the subspace iteration method is first applied to calculate the riser's natural frequencies. In this section, the time-varying dynamic component of axial tension is not considered. Fig. 3 shows the natural frequencies of the riser with the top tension coefficient of 1.6. As shown in Fig. 3, the natural frequencies of the riser are low and dense, which is a unique feature of the transverse vibration of deep sea top-tensioned risers.

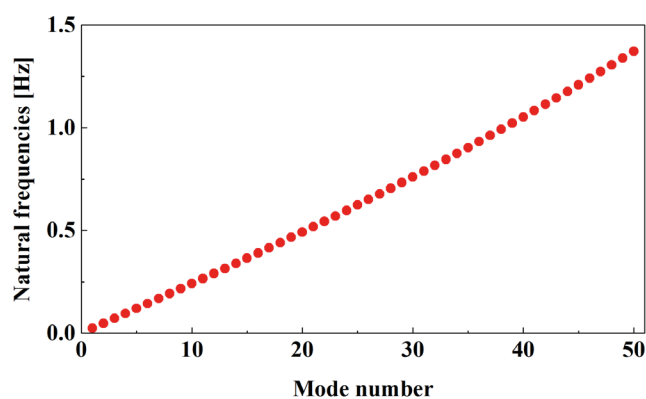


Fig. 3. Natural frequencies of the riser

DYNAMIC RESPONSE OF THE RISER

According to the element precision analysis, the riser is divided equally into 400 elements in the calculation. According to the maximum vortex shedding period and the calculation accuracy of the Newmark- β method, the interval time in the calculation is 0.01 s, and the total calculation time is 500 s. The top tension force coefficient is chosen to be 1.6 and the sea surface current velocity is chosen to be 1.5 m/s in the present study. In order to study the effect of the parametric excitation on the riser's VIV response, two typical cases are calculated and they are specified as follows. Case A: $A=2.0$ m, $\Omega=1.6$ rad/s; Case B: $A=0$ m, $\Omega=0$ rad/s. It is conformed such that the lock-in and parametric resonance in both cases does not occur.

Fig. 4 shows the root-mean square (RMS) of IL and CF displacements of the riser. It should be noted that the RMS of IL displacements of the riser were obtained after removing the mean displacement. It can be seen from Fig. 4 that whether or not the vessel heave motion is taken into account, the riser's VIV modes are all a mixture of travelling and standing waves. Meanwhile, the dominant modes of the riser in the in-line direction are larger than the cross-flow direction in both cases. In each case, the riser's VIV responses are all characterised by high order multi-mode vibrations in the non-locking region. As shown in Fig. 4, when considering the heave motion of the vessel, both the IL and CF vibration modes of the riser have changed remarkably. When the parametric excitation is considered, the fluctuation of the riser VIV is more intense. It can be seen that the VIV mode of the riser in linear sheared flow is very complicated when considering parametric excitations.

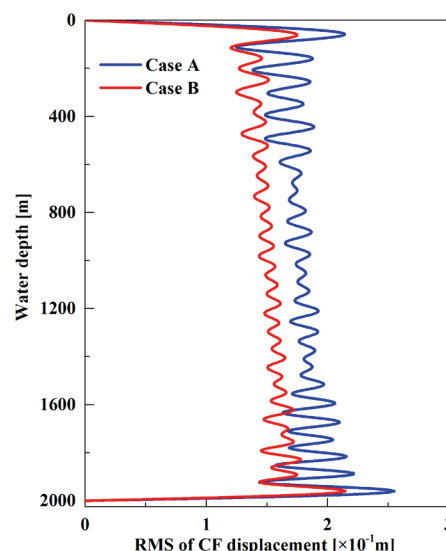
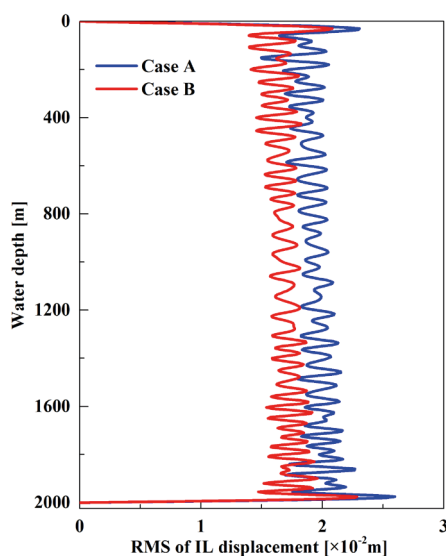


Fig. 4. RMS of IL and CF VIV displacements of the riser

According to the calculation results, when the heave motion of the vessel is not considered, the IL and CF maximum VIV amplitudes of the riser are 0.0754 m and 0.5184 m, respectively. In addition, when the heave motion of the vessel is considered, the IL and CF maximum VIV amplitudes of the riser are 0.0892 m and 0.5615 m, respectively. The IL and CF maximum VIV amplitudes of the riser are increased by 18.3% and 8.31%, respectively. It can be seen that the vessel heave motion has a greater effect on the riser IL VIV displacements. Overall, the heave motion of the vessel makes the riser VIV more severe.

Fig. 5 shows the amplitude spectrum of the riser vibration displacements at different water depths. It can be seen that, whether or not the vessel heave motion is considered, the bulks of the IL response are all between 0.021 Hz and 1.178 Hz, and the bulks of the CF response are all between 0.021 Hz and 0.594 Hz. According to Fig. 3, the 1st, 26th and 44th modes' natural frequencies are 0.024 Hz, 0.597 Hz and 1.177 Hz, respectively. It can be seen that the IL VIV participating modes reach the 44th, and the CF VIV participating modes reach the 26th. At the same time, the riser IL and CF VIV response are all shown to be a high order multi-mode response at each position. This is because even at the same position, the frequency of the vortex shedding is constantly changing due to the motion of the riser in the in-line direction, which causes the riser's vibration frequency to be constantly changing. As can be seen from Fig. 5, the highest peak frequencies at different positions of the riser are different, and the participating modes in the vibration are also different. This is because the sea current velocity and the axial tension force at different positions are all different, which makes the vortex shedding frequency different in different positions, thereby causing the above phenomenon. Meanwhile, the richness in frequency content indicates an irregular character of the VIV response of the riser. The IL response is much more irregular than the CF response, as can be seen from the width of the spectrum and the high number of participating modes. In addition, it can be seen from Fig. 5 that

when the heave motion of the vessel is considered, the IL and CF VIV dominant frequencies and participating frequencies at different positions of the riser are all changed. The heave motion of the vessel has different effects on the riser IL and CF VIV frequencies at different positions of the riser. It can be seen that the impact of the vessel heave motion on the IL and CF VIV response of the riser is very complicated.

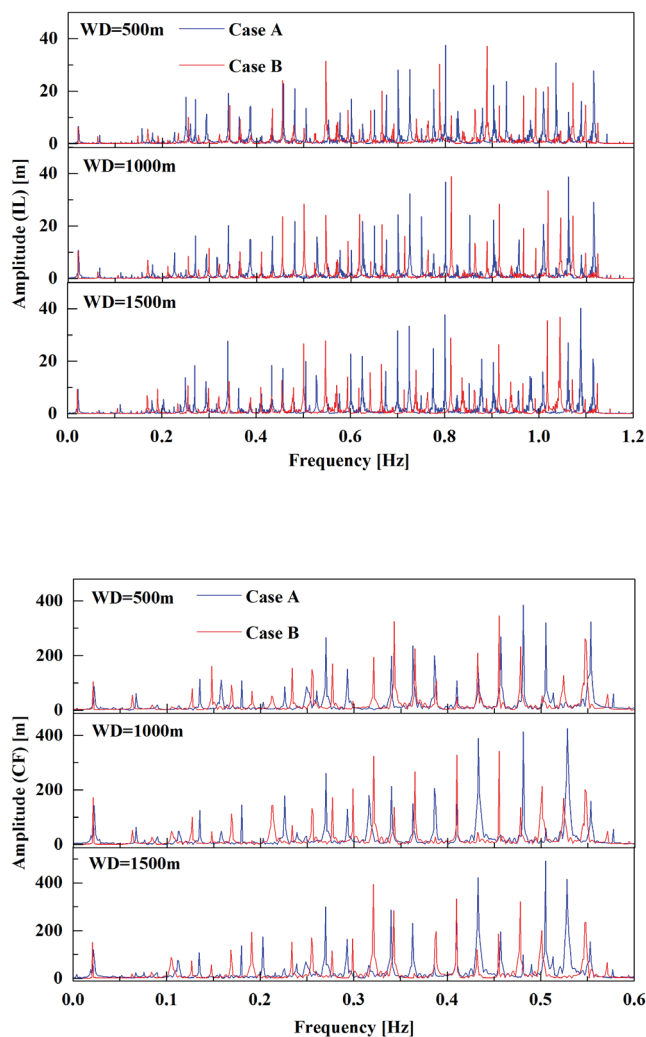


Fig. 5. Amplitude spectrum of the riser VIV displacements

EFFECT OF VESSEL HEAVE AMPLITUDE

According to the environmental condition in the South China Sea, the heave period of the vessel is commonly between 4 s and 15 s. So, the frequency of the axial parametric excitation falls into the range from 0.4 rad/s to 1.6 rad/s. The heave amplitude of the vessel is usually between 0.5 m and 2.0 m. In this section, four cases are studied to investigate how the vessel heave amplitude affects the riser's VIV response. The vessel heave frequency is chosen to be 1.6 rad/s. Fig. 6 shows

the riser IL and CF RMS displacements with different heave amplitudes.

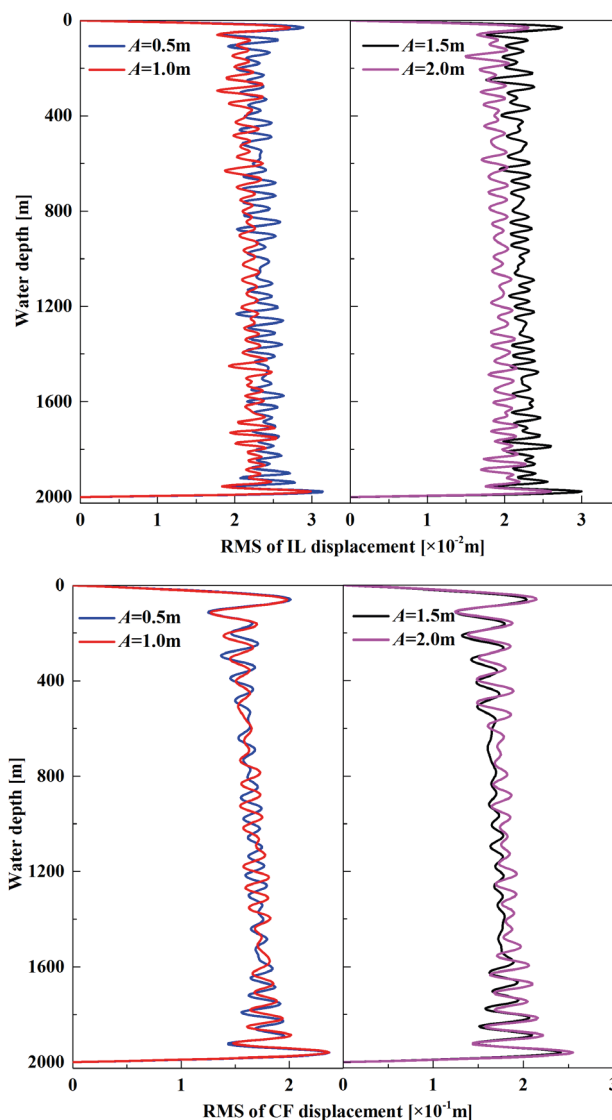


Fig. 6. RMS of VIV displacements of the riser with different heave amplitudes

As is shown in Fig. 6, when the vessel heave frequency is 1.6 rad/s, the effect of the variation of the vessel heave amplitude on the riser's VIV mode is complicated. The riser IL and CF VIV modes are different at different heave amplitudes. It can also be seen that the variation of the heave amplitude has a more significant effect on the riser's IL VIV mode than the CF. According to Eq. (2), the maximum axial tension force of the riser increases gradually with the increase of the heave amplitude, which causes the natural frequencies of the riser to change constantly. Therefore, at the same static top tension force and sea current velocity, the riser VIV mode changes gradually with the variation of the vessel heave amplitude.

According to the calculation results, the IL maximum VIV amplitudes of the riser in the four cases are 0.0744 m, 0.0763 m, 0.0805 m and 0.0892 m, respectively. Meanwhile, the CF maximum VIV amplitudes of the riser in the four cases

are 0.5517 m, 0.5539 m, 0.5573 m and 0.5615 m, respectively. It can be seen that with the increase of the heave amplitude, the IL and CF maximum VIV amplitudes of the riser increase continuously.

The displacements of the riser midpoint under different heave amplitudes are analysed by spectrum analysis, and the spectrum curves are shown in Fig. 7. As shown in Fig. 7, the dominant frequencies and the participating frequencies of the riser VIV change continuously when the heave amplitude changes. It can be concluded that the variation of the heave amplitude has a great influence on the riser VIV frequency.

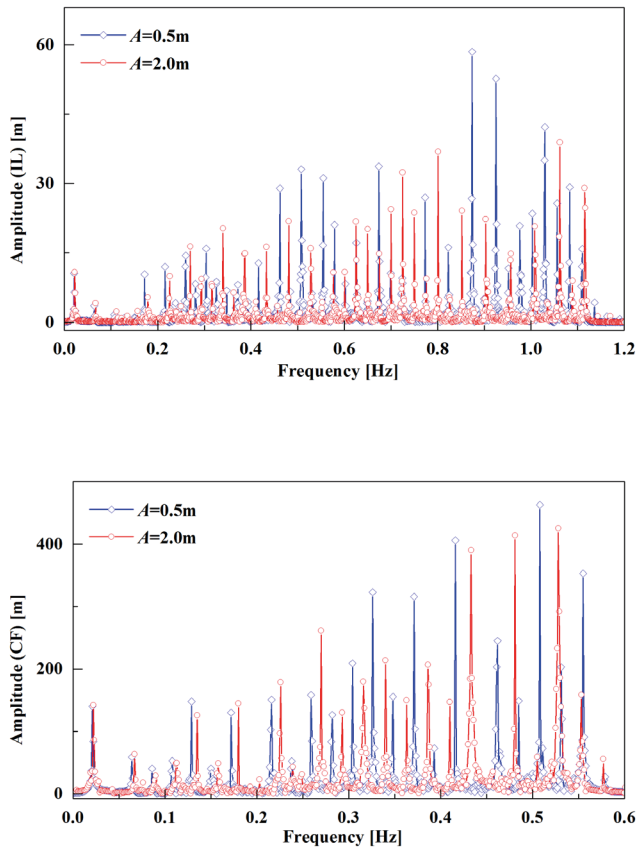


Fig. 7. Amplitude spectrum of the riser's VIV displacements with different heave amplitudes

EFFECT OF VESSEL HEAVE FREQUENCY

In this section, four cases are studied to investigate how the vessel heave frequency affects the riser VIV response. The heave amplitude of the vessel is chosen to be 2.0 m. Fig. 8 shows the riser's IL and CF RMS displacements with different heave frequencies. As shown in Fig. 8, when the heave frequency is small, the variation of the heave frequency has little effect on the riser's IL and CF VIV modes. However, when the heave frequency is large, the variation of the heave frequency has a relatively large influence on the riser's VIV mode, especially on its IL VIV mode.

According to the calculation results, the IL maximum VIV amplitudes of the riser in the four cases are 0.0813 m, 0.0812 m, 0.0826 m and 0.0892 m, respectively. Meanwhile, the CF maximum VIV amplitudes of the riser in the four cases are 0.5541 m, 0.5543 m, 0.5597 m and 0.5615 m, respectively. It can be seen that when the heave frequency changes from 0.4 rad/s to 0.8 rad/s, the IL and CF maximum VIV amplitudes of the riser basically remain unchanged. When the heave frequency is large, the IL and CF maximum VIV amplitudes of the riser increase gradually with the increase of the heave frequency.

The displacements of the riser midpoint under different heave amplitudes are analysed by spectrum analysis, and the spectrum curves are shown in Fig. 9. It can be seen from Fig. 9 that the variation of the vessel heave frequency does not change the components of the riser VIV frequency. However, the proportions of the VIV dominant frequency of the riser increase gradually with the increase of the vessel heave frequency. It can be concluded that the dominant frequency of the riser VIV is strengthened by the larger vessel heave frequency.

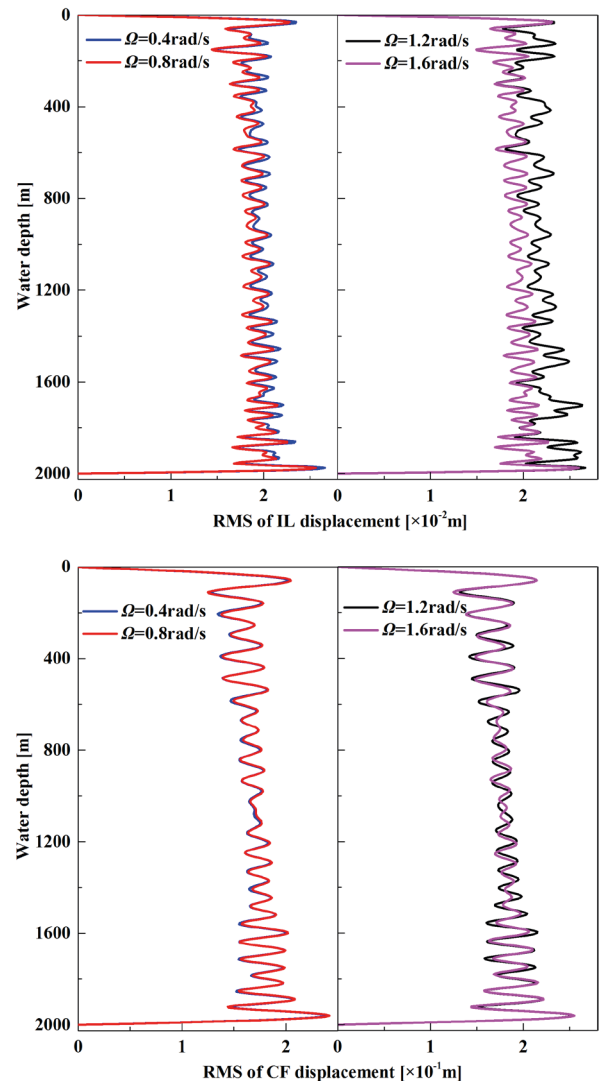


Fig. 8. RMS of VIV displacements of the riser with different heave frequencies

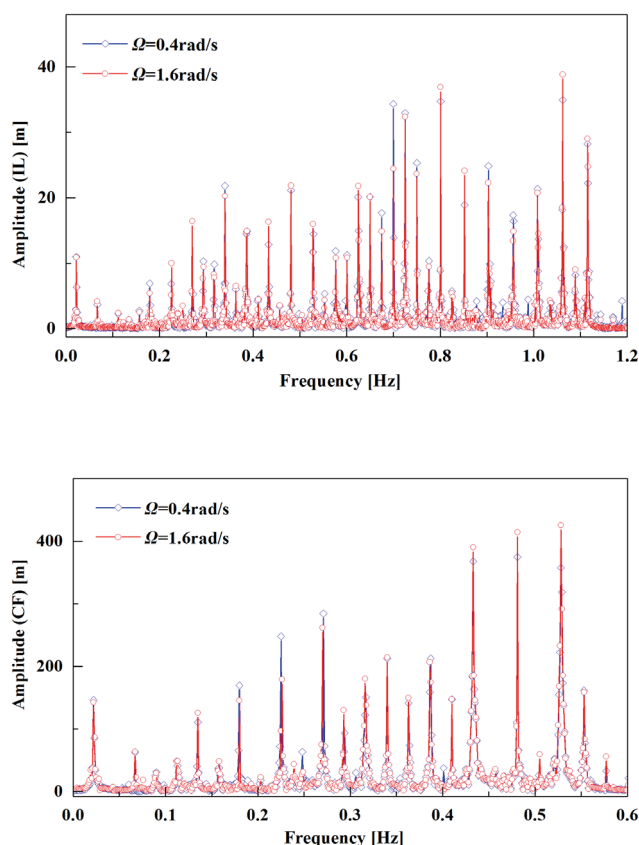


Fig. 9. Amplitude spectrum of the riser's VIV displacements with different heave frequencies

From the above analysis, it can be concluded that when the lock-in and parametric resonance of the riser does not occur, the effect of the variation of the vessel heave amplitude and frequency on the VIV response of the riser is complicated. The impact of the variation of the heave amplitude and frequency on the IL and CF VIV response of the riser is different. It should be pointed out that the above analysis is based on the same static top tension force and current velocity. However, the impact of the variation of the vessel heave amplitude and frequency on the riser's VIV response is different at different static top tension forces and current velocities. Hence, in practical operation, the relevant parameters should be adjusted in time to avoid serious VIV, which will lead to the damage of the riser.

CONCLUSIONS

An improved three-dimensional time domain prediction model of long flexible risers has been proposed in the present study to predict the IL and CF VIV response of top-tensioned risers. The impact of the riser's in-line vibration is considered in the prediction model. The periodic time-varying axial tension force caused by the vessel heave motion is also considered in the prediction model. Comparisons between the numerical calculation and experimental test results have shown that the

prediction model is reasonable to predict some main features of the riser's VIV response. It is assumed that the sea current is linear sheared flow, and the attention is focused on the study of the riser's VIV response when the lock-in and parametric resonance phenomenon does not occur. The time-domain simulation results show that the periodic time-varying axial tension force has a large effect on the IL and CF VIV response of the riser. At the same static top tension force and current velocity, when the periodic time-varying tension is considered, the VIV displacements of the riser increase and the VIV's dominant frequency and the participating frequencies of the riser have changed. Meanwhile, the riser's VIV mode has also changed when the periodic time-varying axial tension is considered. When the lock-in and parametric resonance phenomenon does not occur, the impact of the vessel heave amplitude and frequency on the riser's VIV response is complicated. The impacts of the vessel heave amplitude and frequency on the IL and CF VIV responses of the riser are different. So, it requires a detailed analysis of the impact at different operating conditions.

It is well known that the lock-in phenomenon easily occurs when the riser is subjected to VIV. The lock-in phenomenon is very complicated when the riser is in sheared flow. Meanwhile, parametric resonance of the riser is very likely to occur when the heave motion of the vessel is considered. To investigate the coupling between the VIV and parametric excitation will therefore be one of our research topics when the lock-in and parametric resonance occur simultaneously.

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REFERENCES

1. Bishop R. E. D., Hassan A. Y. (1964): The lift and drag forces on a circular cylinder oscillating in a flowing fluid. *Proceedings of the Royal Society of London, Series A: Mathematical and Physical Sciences*, Vol. 277(1368), 51-75.
2. Hartlen R. T., Currie I. G. (1970): Lift-oscillator model of vortex-induced vibration. *Journal of the Engineering Mechanics Division*, Vol. 96(5), 577-591.
3. Balasubramanian S., Skop R. A. (1996): A nonlinear oscillator model for vortex shedding from cylinders and cones in uniform and shear flows. *Journal of Fluids and Structures*, Vol. 10(3), 197-214.
4. Krenk S., Nielsen S. R. K. (1999): Energy balanced double oscillator model for vortex-induced vibrations. *Journal of Engineering Mechanics*, Vol. 125(3), 263-271.

5. Facchinetti M. L., Langre E. D., Biolley F. (2004): Coupling of structure and wake oscillators in vortex-induced vibrations. *Journal of Fluids and Structures*, Vol. 19(2), 123-140.
6. Furnes G. K., Sørensen K. (2007): Flow induced vibrations modeled by coupled non-linear oscillators. In: *Proceedings of the 17th International Offshore and Polar Engineering Conference*, Lisbon, Portugal, 2007.
7. Ge F., Long X., Wang L., Hong Y. S. (2009): Flow-induced vibrations of long circular cylinders modeled by coupled nonlinear oscillators. *Science in China Series G: Physics, Mechanics and Astronomy*, Vol. 52(7), 1086-1093.
8. Wu X. D., Ge F., Hong, Y. S. (2010): Effect of travelling wave on vortex-induced vibrations of submerged floating tunnel tethers. *Procedia Engineering*, Vol. 4(6), 153-160.
9. Srinil N., Zanganeh H. (2012): Modelling of coupled cross-flow /in-line vortex-induced vibrations using double Duffing and van der Pol oscillators. *Ocean Engineering*, Vol. 53(3), 83-97.
10. Gu J. J., An C., Levi C., Su J. (2012): Prediction of vortex-induced vibration of long flexible cylinders modeled by a coupled nonlinear oscillator: integral transform solution. *Journal of Hydrodynamics, Series B*, Vol. 24(6), 888-898.
11. Xue H. X., Wang K. P., Tang W. Y. (2015): A practical approach to predicting cross-flow and in-line VIV response for deepwater risers. *Applied Ocean Research*, Vol. 52, 92-101.
12. Spanos P. D., Tein W. Y., Ghanem R. (1990): Frequency domain analysis of marine risers with time dependent tension. *Applied Ocean Research*, Vol. 12(4), 200-210.
13. Lei S., Zhang W. S., Lin J. H., Yue Q. J., Kennedy D., Williams, F. W. (2014): Frequency domain response of a parametrically excited riser under random wave forces. *Journal of Sound and Vibration*, Vol. 333(2), 485-498.
14. Patel M., Park H. I. (1995): Combined axial and lateral responses of tensioned buoyant platform tethers. *Engineering Structures*, Vol. 17(10), 687-695.
15. Park H. I., Jung D. H. (2002): A finite element method for dynamic analysis of long slender marine structures under combined parametric and forcing excitations. *Ocean Engineering*, Vol. 29(11), 1313-1325.
16. Chatjigeorgiou I. K., Ioannis K., Mavrakos S. A. (2005): Nonlinear resonances of parametrically excited risers – numerical and analytic investigation for $\Omega=2\omega_1$. *Computers & Structures*, Vol. 83(8-9), 560-573.
17. Wang Y. B., Gao D. L., Fang J. (2015): Coupled dynamic analysis of deep water drilling riser under combined forcing and parametric excitation. *Journal of Natural Gas Science and Engineering*, Vol. 27(3), 1739-1747.
18. Lei S., Zheng X. Y., Kennedy D. (2017): Dynamic response of a deepwater riser subjected to combined axial and transverse excitation by the nonlinear coupled model. *International Journal of Non-Linear Mechanics*, Vol. 97, 68-77.
19. da Silveira L. M. Y., Martins C. D. A., Cunha L. D., Pesce C. P. (2007): An investigation on the effect of tension variation on VIV of risers. In: *Proceedings of 26th International Conference on Offshore Mechanics and Arctic Engineering*, California, USA, 2007.
20. Chen W. M., Li M., Guo S. X., Gan K. (2014): Dynamic analysis of coupling between floating top-end heave and riser's vortex-induced vibration by using finite element simulations. *Applied Ocean Research*, Vol. 48, 1-9.
21. Thorsen M. J., Sævik S. (2017): Vortex-induced vibrations of a vertical riser with time-varying tension. *Procedia Engineering*, Vol. 199, 1326-1331.
22. Burke B. G. (1974): An analysis of marine risers for deep water. *Journal of Petroleum Technology*, Vol. 26(4), 455-465.
23. Wang Y. B., Gao D. L., Fang J. (2014): Static analysis of deep-water marine riser subjected to both axial and lateral forces in its installation. *Journal of Natural Gas Science and Engineering*, Vol. 19(7), 84-90.
24. Chucheepsakul S., Monprapussorn T., Huang T. (2003): Large strain formulations of extensible flexible marine pipes transporting fluid. *Journal of Fluids and Structures*, Vol. 17(2), 185-224.
25. Kuiper G. L., Brugmans J., Metrikine A. V. (2008): Destabilization of deep-water risers by a heaving platform. *Journal of Sound and Vibration*, Vol. 310, 541-557.
26. Fujiwara T., Uto S., Kanada S. (2011): An experimental study of the effects that change the vibration mode of riser VIV. In: *Proceedings of 30th International Conference on Ocean, Offshore and Arctic Engineering*, Rotterdam, Netherlands, 2011.

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