# EXPERIMENTAL AND NUMERICAL STUDIES ON THE SHEAR STABILITY OF SHIP'S THIN PLATES 

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#### Abstract

The stability of thin plate plays an important role in the design and strength check of ship structure. In order to study the shear stability of ship's thin plates, in-plane shear buckling tests were carried out using a picture frame fixture and a 3D full-field strain measurement system. The critical buckling load, full-field displacement/strain information, and load-displacement curve were obtained. The finite element model with the frame fixture was established based on ABAQUS, with the eigenvalue buckling analysis and nonlinear buckling analysis being carried out to obtain the mechanical response information of the buckling and post-buckling of the ship's thin plate. The effectiveness and accuracy of the numerical simulation method are verified by comparing the numerical simulation with the experimental results. On this basis, the critical buckling load obtained by shear test, numerical simulation, and theoretical calculation is analyzed, and the function of the frame shear fixture and its influence on the critical buckling load are defined. The research in this paper provides a useful reference for the testing and simulation of in-plane shear stability of ship's thin plates.


Keywords: shear stability, ship's thin plate, full-field strain measurement, buckling, nonlinear

## INTRODUCTION

The stability of the plate has always played an important role in the design and strength check of the ship's structure. A large number of marine accidents show that the damage of ship structure is usually not caused by insufficient strength, but due to the loss of stability. When the overall longitudinal bending or torsion of the hull occurs, the hull plate may be subjected to axial compression load or shear load. When the axial compressive stress or shear stress reaches a certain value, the hull plate will be unstable [1-2].

The stability of thin-walled structures has always been a focus of scholars at home and abroad [3-456]. Previous studies
have focused on buckling and post-buckling of the stiffened plates. In literature [7], the trigonometric function was used to simulate rotational restraining rigidity of stiffener, and Ritz method was used to establish an analytical model for the local skin buckling problem of riveted stiffened panels under uniaxial compression. In literature [8], a rectangular plate model with inclined stiffener was used to simulate the pure shear stress state of the wedge-shaped webs, and its elastic buckling was numerically analyzed. In literature [9], eigenvalue buckling analysis and nonlinear buckling analysis were performed for marine composite stiffened plates based on a numerical method and finite element calculation. X. Shi (2005) used the commercial finite element software,

NASTRAN, to analyze the stability of composite panels[10]. Based on mechanical experiments and numerical simulation, Y. Peng, et al.(2020) studied the buckling and post-buckling behavior of the al-li alloy stiffened panel under shear load[11].

In the study of plate stability, Roberts and Azizian (1984) used the finite element method to analyze the elastic buckling of a square plate with holes under in-plane loads. In literature [12], the ABAQUS software was used to analyze the ultimate strength of the hull panel model under axial pressure. In literature [13], the elastic buckling and postbuckling behaviors of unstiffened panels under complex stress states were analyzed and compared with the empirical formula and American Bureau of Shipping specification. L. Sun (2014) applied near field dynamics to the analysis of the stability of metal plates under axial compression[14]. In literature [15], the Galerkin method was used to solve the elastoplastic buckling problem of rectangular plates under shear stress. In literature [16], the buckling problem of anisotropic rectangular plate under shear stress was solved by differential quadrature method. In literature [17], the differential equation of transverse displacement function in shear buckling of rectangular plate was established, and the determinant of coefficient matrix of homogeneous linear algebraic equation was simplified by using the point distribution method and antisymmetric characteristics of buckling deformation, and the analytical solution of critical buckling load of rectangular plate was obtained. Pham (2017) studied the shear buckling of a plate with holes by using the finite strip method and proposed an approximate formula for the shear buckling coefficient of a square plate with central circular holes and square holes[18].

Local rectangular thin plates under shear stress are widely used in engineering structures such as ships and buildings and buckling instability of local plates has become one of the main forms of engineering structural failure. In the past, there had been many studies dedicated to tensile, compressive, and bending load conditions. It is difficult to test the buckling instability of local rectangular plates under shear stress. The Galerkin method, differential quadrature method, analytical method, and finite element method were mainly used to solve the problem. The analytical solutions need programming operations. Finite element simulation [19-20] can avoid complicated programming for buckling analysis, but the calculation accuracy is limited to the design accuracy of the model and boundary conditions. Additionally, these methods need to be supported by mechanical experiments.

Therefore, this paper focuses on the buckling characteristics of a ship's thin plates under in-plane shear loading by means of mechanical experiments and numerical simulation. A picture frame fixture was designed and manufactured, and a diagonal tensile method was used to conduct the shear test on square thin plates. In the numerical simulation, eigenvalue analysis method and nonlinear analysis method were used to study the buckling behavior of ship's thin plates. The effectiveness of the numerical method was expected to be verified through


Fig. 1. Schematic of shearing condition of ship's thin plate

## TEST PLATE

The thin plate is one of the most common structural plate elements in the ship structure. The shear test plate is designed as shown in Fig. 2. Among them, the square with a side length of 310 mm is the shear area, and there are loading areas with a width of 40 mm around the shear area. There are multiple holes in the loading area to ensure uniform force. The thickness of the test plate is 1 mm , the material is ordinary marine steel, the elastic modulus is 210 GPa , and the Poisson's ratio is 0.3 . The stress-strain relationship of the test material was measured by tensile test, as shown in Fig. 3.


Fig. 2. Schematic of the test plate and its dimensions (all dimensions are in mm )

(a) Tensile test system

(b) Tensile test results

Fig. 3. True stress-strain curve from tensile test

## INSTRUMENTATION/EQUIPMENT DETAILS AND TEST PROCEDURE

The shear buckling test of the thin plate is a difficult structural test, which requires a reasonable fixture to transfer the shear load to the test plate. In this paper, a picture frame fixture using a diagonal tensile method was adopted as shown in Fig. 4. The four sides of the test plate are bolted to four pairs of shear plates, and the shear plates are hinged together at the top. When the testing machine is loaded, the tensile force P , provided by the testing machine, is decomposed into $\mathrm{T}_{\mathrm{xy}}$ along
the direction of the shear plate. The test piece is tensioned in the vertical diagonal ( $\Pi$ ) direction and compressed in the horizontal diagonal (I) direction, so that the test piece is subjected to the shear load. For the study of the buckling performance of the thin plate under shear load, the four sides of the plate are required to be simply supported and torsion limited, and the shear load is uniformly applied to the boundary of the plate. This is an ideal state of shear testing.


Fig. 4. Test setup of specimen and frame fixture
As shown in Fig. 5, the shear test was carried out on a WDW100-100C electronic universal test machine, with a maximum load of 100 kN and a continuous loading rate of $5 \mathrm{~mm} / \mathrm{min}$. In order to accurately measure the deformation of the plate surface, the XTDIC-CONST 3D full-field strain measurement and analysis system was applied to the test process. The surface of the test plate was sprayed with primer and speckle in advance, so that the change of displacement field and strain field on the surface of the test plate could be measured in real time. As shown in Fig. 5, the digital image correlation (DIC) measuring equipment is arranged at a specific position directly in front of the test plate.


Fig. 5. Shear buckling test of the ship's thin plate

## NUMERICAL SIMULATION

## BUCKLING ANALYSIS METHOD

## Eigenvalue Buckling

Linear buckling is also called eigenvalue buckling analysis. The buckling loads and buckling modes of the structure can be obtained by calculating the eigenvalues of the singular stiffness matrix. The corresponding buckling load can be determined by the following linear generalized eigenvalue equation [10,21]:

$$
\begin{equation*}
\left(\left[K_{0}\right]+\lambda\left[K_{\sigma}\right]\right)\{U\}=0 \tag{1}
\end{equation*}
$$

Where, $\left[K_{0}\right]$ is the linear stiffness matrix of the structure, [ $K_{\sigma}$ ] is the geometric stiffness matrix of the structure, $\lambda$ is the load scaling factor, and $\{U\}$ is the lateral displacement vector. It can be seen from Equation (1) that the linear stability problem of the structure is the eigenvalue problem, and the corresponding critical load and instability mode can be obtained by solving the eigenvalue and eigenvector.

## Nonlinear Buckling

The nonlinear buckling theory is to establish the equilibrium equation on the structure configuration which changes constantly during the loading process. For the imperfect plate with initial geometric defects or the laminate with tension-bending coupling effect, that is, under in-plane loading, the transverse displacement will appear from the beginning of loading and become a nonlinear bending problem. The governing equation is as follows:

$$
\begin{equation*}
K_{T} \Delta U=\Delta P \tag{2}
\end{equation*}
$$

Where, $K_{T}$, is the tangent stiffness matrix of the structure at a certain increment step, $\Delta P$ is the current external load increment of the structure, and $\Delta U$ is the current displacement increment of the structure. The above formula can also be written as follows:

$$
\begin{equation*}
\left(\left[K_{\mathrm{L}}\right]+\left[K_{\sigma}\right]+\left[K_{\mathrm{NL}}\right]\right) \Delta U=\Delta P \tag{3}
\end{equation*}
$$

Where, $K_{L}$ is the linear stiffness matrix, which is independent of node displacement. $K_{N L}$, is the initial displacement matrix, which represents the influence of element position change on element stiffness matrix.

According to nonlinear buckling theory, there can be several extreme points in the equilibrium path of a structure under load, and such extreme points are called nonlinear
buckling points. The buckling load of the structure is usually determined by the first extreme point. In the nonlinear analysis, the stability problem and strength problem of the structure are related to each other and can be studied from the change law of the whole process of load-displacement curve.

## FINITE ELEMENT MODEL

In order to effectively simulate the test conditions, the finite element model as shown in Fig. 6 was established based on ABAQUS. The plane continuous shell element S4R was used in test plate and fixtures. The S4R is a 4 -node first order reduction integral element, which uses linear interpolation methods, allows finite film strain and large rotation angle, considers the influence of shear deformation, and is suitable for geometrical and material nonlinear analysis. The constraint conditions between the four groups of shear plates and the loading area of the test plate were set as tie binding constraints to simulate bolted connection. The hinge connection units were used to simulate the effect of the hinge connection between the shear plate shaft hole and the pin.


Fig. 6. Finite element model

## RESULTS AND DISCUSSIONS

## LOAD-DISPLACEMENT CURVES

Fig. 7 indicates the load $(P)$-displacement $(D)$ curve of the ship's thin plate, where $D$ represents the displacement difference between the upper and lower diagonal points of the plate. As can be seen from the graph, the trend and results of the experiment and simulation are consistent. By comparing and analyzing the test and simulation results, it is found that as the load increases, there are obvious linear segments in the test and simulation curves, and the linear segment of the simulation curve has a longer duration and a greater slope. When the load increases to point $\mathrm{A}(P \approx$ 15.04 kN ), the structure produces buckling instability, the slope of load-displacement curve changes suddenly, and the structure stiffness begins to decrease. As the load
continues to increase, the load-displacement curve remains approximately linear until the load increases to point B ( $P \approx$ 26.02 kN ), at which time the maximum stress in the plate exceeds the yield limit of the material, and the structure begins to exhibit plastic deformation. With the increase of the plastic deformation area, the linear relationship of the loaddisplacement curve disappeared. When the load increases to point $\mathrm{C}(P \approx 35.68 \mathrm{kN})$, the slope of the curve changes greatly, and the structure stiffness decreases. After that, the slope of the curve changed relatively gently. Only when the load increases to point $\mathrm{D}(P \approx 62.30 \mathrm{kN})$, a small abrupt change occurs, and the structure stiffness decreases again. Based on the simulation curve, the key moments of the four slope sudden changes of A, B, C, and D can be extracted. In order to further verify the validity of the finite element results, the test results of the above four key moments under the same load are analyzed emphatically.


Fig. 7. Force versus displacement of the test sample

## FULL-FIELD DISPLACEMENT AT CRITICAL MOMENTS

Based on the DIC full-field strain measurement system, full-field displacement information at the four key moments of $A, B, C$, and $D$ during the shear test were extracted and compared with the numerical simulation results, as shown in Fig. 8. It is found that the displacement distribution of finite element simulation and experimental measurement results are basically the same. The normal displacements of the horizontal and vertical diagonals extracted from the experiment and simulation are shown in Fig. 8. The numerical simulation results of the four key moments $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D are basically consistent with the experimental values, and the curve change trend is consistent. On the vertical diagonal, there is a half wave symmetric along the horizontal diagonal of the plate, and the displacement of the center point of the wave is the largest and gradually decreases towards both ends. There are three half waves on the horizontal diagonal, the waveform is symmetrical along the vertical diagonal, there is a large wave at the center point, and there are two wavelets symmetrically on both sides. The amplitude of the three waves increases as the load increases. The results shown
in Fig. 9 are consistent with the displacement field shown in Fig. 7. Therefore, the validity and accuracy of the numerical simulation method are further verified.


Fig. 8. Full-field stress distributions of moment $A, B, C$, and $D$


Fig. 9. Distribution of diagonal normal displacement

## CRITICAL BUCKLING LOAD

Usually, the load at the first abrupt slope of the loaddisplacement curve is taken as the buckling instability load $(P c r)$ of the structure. The buckling time of the test plate can also be observed and judged by the strain bifurcation method, that is, the load at the bifurcation point of the strain-load curve is used as the critical buckling load of the structure. In order to obtain a more accurate buckling instability load, the 10 key points, shown in Fig. 10, are designed on the front and back sides of the plate. The vertical and horizontal diagonal lines of the plate are divided into eight equal parts. Considering the symmetry of the test plate, the equal points in the upper left quarter region are taken, and the corresponding strain-load curve was extracted. The points $0,3,4,5$, and 9 are located on the front of the plate, while the points $0^{\prime}, 3^{\prime}$, $4^{\prime}, 5$ ' and $9^{\prime}$ are on the back of the plate.


Fig. 10. The key points of the test plate

Fig. 11 shows the strain-load curves of the 10 key points. By comparing Fig. 11a and 11b, it is found that the positions of strain bifurcation of the key points on the vertical diagonal and horizontal diagonal are the same, and the corresponding loads are both 15.04 kN , which is the critical buckling load of the plate. The strain in the X direction along the vertical diagonal is much larger than that along the horizontal diagonal. On the vertical diagonal, the closer to the center of the plate, the smaller the strain value is. On the horizontal diagonal, the strain in the X direction is small and changes very little, but the closer to the center of the plate, the greater the strain value. The strain-load curve was partitioned based on time A, C, and D as shown in Fig. 7. At line A, the critical buckling load was consistent with the bifurcation point. At line C, the sudden change of the strain-load curve on the vertical diagonal is larger, which is consistent with the sudden change of the slope of the load-displacement curve at point C as shown in Fig. 7. At line D, the change of the strain-load curve on the horizontal diagonal is large, which is consistent with the sudden slope of the load-displacement curve at point D as shown in Fig. 7. Fig. 12 illustrates the full-field strain distribution corresponding to time C and D . The comparison shows that the finite element simulation is basically consistent with the test measurement results, which, once again, proves the effectiveness and accuracy of the numerical simulation.

(a) The key point of 0/0', 3/3', 4/4'


Fig. 12. Full-field strain distributions
The critical buckling loads of the thin plate obtained by different methods are summarized as shown in Table 1. Compared with the test results, it can be seen that the critical buckling load obtained by the eigenvalue analysis method is higher than the test one, with the error being 7.25\%. The buckling load calculated by the eigenvalue method is higher, partly because the nonlinearity of the structure and the defects of the structure are not considered. The nonlinear analysis method makes up for the deficiency of eigenvalue buckling analysis to a certain extent. Although the obtained buckling load is also higher than the experimental result, the error is relatively small, only $2.80 \%$. For the shear condition of rectangular thin plate, the critical buckling load can be calculated by the theoretical formula [22]. The theoretical value is smaller than the experimental result, and the corresponding error is large, about $11.35 \%$. The reason being that the theoretical formula is for the ideal shear condition with uniform shear force on the four sides, and the experiment
and simulation are not a real pure shear mode due to the existence of frame fixture. For this reason, it is necessary to consider the influence of the picture frame fixture on the experiment and simulation.

Table 1. Comparison of critical buckling loads

| Method | Critical Buckling Load <br> $(\mathrm{kN})$ | Error <br> $\%$ |
| :---: | :---: | :---: |
| Shear test $\left(P_{T}\right)$ | 14.63 | - |
| Finite element - eigenvalue <br> analysis $\left(P_{F E-L}\right)$ | 15.69 | 7.25 |
| Finite element - Nonlinear <br> analysis $\left(P_{F E}\right)$ | 15.04 | 2.80 |
| Theoretical calculation <br> $\left(P_{\text {theorr }}\right)$ | 12.97 | 11.35 |

## INFLUENCE OF THE PICTURE FRAME FIXTURE

In the study of the shear stability of thin plates, we usually think of obtaining the buckling and post-buckling performance of the plates when it is subjected to pure shear load. However, in the actual test, an auxiliary fixture must be used. The fixture will also deform during the stress process. Therefore, when the plate buckles, the load measured by the testing machine is not the true buckling load of the plate. The fixture is related. That is to say, in the shear test process, the picture frame fixture not only transmits the load, but also participates in the deformation of the structure, and shares part of the load, so the load recorded by the testing machine includes both the load that causes the plate to buckle, and the load that causes the deformation of the fixture. Therefore, the critical buckling load of the plate after using the fixture is greater than that under pure shear condition.

To this end, the influence factor $\lambda$ of picture frame fixture is proposed to characterize the influence of the fixture on the critical buckling load of the thin plate, and the relationship between numerical simulation and theoretical calculation or finite element simulation without fixture is established by $\lambda$. Suppose the following relationships exist:

$$
\begin{equation*}
\lambda=\left(P_{F E}-P_{\text {theory }}\right) / P_{F E} \tag{4}
\end{equation*}
$$

In the formula, PFE represents the critical buckling load obtained by the nonlinear finite element analysis method, and Ptheory represents the critical buckling load obtained by theoretical calculation. As shown in Fig. 13, the critical buckling loads of the plates with different thicknesses under the same working conditions are calculated. Comparing the simulation results with the theoretical values, it is found that the trend of the two is consistent, and there is a linear relationship between the critical buckling load and the cube of the plate thickness. It can be seen that the influence of the picture frame fixture on different thickness plates has a certain universality under the same working condition. However, by analyzing the curve between $\lambda$ and plate thickness (see

Fig. 14), it is found that $\lambda$ is not a constant value, it increases with the increase of the plate thickness. When the plate thickness is small, $\lambda$ is also small, indicating that the fixture has small influence on the critical buckling load. With the increase of plate thickness, the influence of the fixture on the critical buckling load increases, but when the plate thickness increases to a certain extent, the influence of the fixture on the critical buckling load gradually becomes gentle.

Based on the power index fitting method in MATLAB, the fitting formula of the influence factor $\lambda$ of the picture frame fixture is obtained as follows:

$$
\begin{equation*}
\lambda=0.05008 t^{0.9282}+0.1619 \tag{5}
\end{equation*}
$$

Where, $t$ is plate thickness. Therefore, the influence of the frame fixture on the critical buckling load of the thin plate under different plate thicknesses can be obtained.


Fig. 13. The relationship between $P_{c r}$ and $t_{3}$


Fig. 14. The relationship between $\lambda$ and $t_{3}$

## CONCLUSIONS

Based on mechanical experiment and numerical simulation, the in-plane shear stability of ship's thin plates was investigated, the buckling and post-buckling behavior, and the influence of the picture frame fixture on the shear instability of the rectangular thin plate were explored. The
mechanical response characteristics of load-displacement curves, load-strain curves, critical buckling loads, full-field deformation, and influence factor of the picture frame fixture were obtained. By comparing the numerical simulation with the mechanical tests, the following conclusions are drawn:

Based on ABAQUS, eigenvalue buckling analysis and nonlinear buckling analysis were carried out on the finite element model with the picture frame fixture. The critical buckling load, full-field displacement, and strain obtained by simulation are in good agreement with the results of full-field strain measurement system of DIC, which proves the effectiveness and accuracy of the numerical simulation method.

By comparing and analyzing the critical buckling load obtained by the three methods of shear test, numerical simulation, and theoretical calculation, it is concluded that the critical buckling load obtained by eigenvalue analysis method is higher than the test result, because the nonlinear and structural defects of the structure are not considered. The nonlinear buckling analysis method makes up for the shortcomings of linear buckling analysis, so the critical buckling load obtained is the closest to the test result. The error between the theoretical calculation result and the test value is the largest, because the theoretical formula is for the ideal shear condition with uniform shear force on the four sides, while the experimental and simulation results both consider the influence of the picture frame fixture.

The stress field provided by the picture frame fixture is quite different from that of pure shear. After the picture frame fixture is adopted, the buckling load of the structure will be greater than the buckling load of the thin plate under the ideal shear condition. Considering that the stability of thin plate is sensitive to boundary condition, the influence of the picture frame fixture must be considered in the numerical simulation of the in-plane shear stability test to ensure the validity of simulation result. In this paper, the influence factor $\lambda$ of the picture frame fixture is defined to represent the influence of this fixture on the critical buckling load of the thin plate, and the relationship between numerical simulation and theoretical calculation or finite element simulation without fixture is established through $\lambda$.

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## REFERENCES

1. W. Zhao, Z. Xie, X. Wang, X. Li, J. Hao, 'Buckling behavior of stiffened composite panels with variable thickness skin under compression', Mechanics of Advanced Materials and Structures, 2019. pp. 1-9, doi: 10.1080/15376494.2018.1495795.
2. Y. Chen, C. Yu, H. Gui, 'Research development of buckling and ultimate strength of hull plate and stiffened panel', Chinese Journal of Ship Research, 2017. pp. 54-62.
3. Q. Zhu, F. Wang, X. Wang, 'A correction method of loadend shortening curves for longitudinal multi-span beam column buckling', Shipbuilding of China, 2014. pp. 46-53.
4. O. Ozguc, 'Assessment of buckling behaviour on an fpso deck panel', Polish Maritime Research, 2020, doi: 10. 2478/ pomr-2020-0046.
5. P. Bielski, L. Samson, O. Wysocki, J. Czyzewicz, 'Simple computational methods in predicting limit load of highstrength cold-formed sections due to local buckling: A case study', Polish Maritime Research, 2018. pp. 73-82, doi: https:// doi.org /10.2478/pomr-2019-0064.
6. O. Ozgur, 'Estimation of buckling response of the deck panel in axial compression', Polish Maritime Research, 2018. pp. 98-105, https://doi.org/10.2478/pomr-2018-0136.
7. J. Chen, B. Kong, P. Chen, J. Yang, X. Gan, 'Local buckling analysis method of elastically restrained riveted stiffened panels under uniaxial compression', Journal of Nanjing University of Aeronautics \& Astronautics, 2020. pp. 989996, doi: 10.16356/j.1005-2615.2020.06.019.
8. Y. Jin, G. Tong, 'Elastic shear buckling of web plates in tapered I-girders', Engineering Mechanics, 2009. pp. 1-9.
9. X. Li, Z. Zhu, Y. Li, Z. Hu, 'Research on buckling and post buckling behavior of composite stiffened panel for ships', Shipbuilding of China, 2020. pp. 186-194.
10. X. Shi, 'Nonlinear buckling analysis of plates and shells with finite element method', Nanjing: Nanjing University of Aeronautics and Astronautics, 2005.
11. Y. Peng, Y. Ma, Y. Zhao, L. Zhu, 'Study on shear buckling performance of Al-Li Alloy Stiffened panel', Acta Aeronautica et Astronautica Sinica, 2020. pp. 408-417.
12. L. Feng, J. He, H. Shi, Q. Zhang, D. Li, 'Influence factors and sensitivity analysis of numerical calculation of hull panel ultimate strength', Ship Science and Technology, 2017. pp. 48-53.
13. J. Xia, E. Qi, 'Ultimate strength analysis of unstiffened plates under complex stress', Proceedings of the 20th Anniversary Academic Conference in Commemoration of Ship Mechanics, Zhoushan, Zhejiang, China, 2017, pp. 419-430.
14. L. Sun., 'Stability analysis method study of metallic plates based on peridynamics', Shanghai: Shanghai Jiao Tong University, 2014.
15. S. Renu, L. Roshan, 'Buckling and vibration of nonhomogeneous orthotropic rectangular plates with variable thickness using DQM', Advances in Intelligent Systems and Computing, 2014. pp. 295-304. doi: 10.1007/978-81-322-1602-5_33.
16. E. Jaberzadeh, M. Azhari, 'Elastic and inelastic local buckling of rectangular plates subjected to shear force using the Galerkin method', Applied Mathematical Modelling, 2009. pp. 1874-1885. doi: https://doi.org/10.1016/j. apm.2008.03.020.
17. D. Yang, Y. Huang, G. Li, 'Shear buckling analysis of anisotropic rectangular plates', Chinese Journal of Applied Mechanics, 2012. pp. 221-224.
18. C. H. Pham, 'Shear buckling of plates and thin-walled channel sections with holes', Journal of Constructional Steel Research, 2017. pp. 800-811. doi: https://doi.org/10.1016/j. jcsr.2016.10.013.
19. Y. Cui, F. Tu, Z. Xu, 'In-plane stress analysis of thin sheet with irregular shape', Machinery Design and Manufacture, 2012. pp. 218-220. doi: 10.19356/j.cnki.1001-3997.2012.08.081.
20. W. Fu, F. Yan, H. Wang, A. Lai, ‘The buckling numerical analysis of rectangular plates under shear force', Machinery Design and Manufacture , 2015. pp. 20-22,26. doi: 10.19356/j.cnki.1001-3997.2015.08.006.
21. N. Z. Chen, C. G. Soares, 'Buckling analysis of stiffened composite panels', III European Conference on Computational Mechanics. Springer Netherlands, 2006.
22. X. Liu, 'Ship structure and strength', Harbin: Harbin Engineering University Press, 2010.

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