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ROBUST TRAJECTORY TRACKING CONTROL OF UNDERACTUATED SURFACE VEHICLES WITH PRESCRIBED PERFORMANCE

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ABSTRACT

In this paper, a robust sliding mode tracking controller with prescribed performance is developed for an underactuated surface vehicle (USV) with time-varying external disturbances. Firstly, to guarantee the transient and steady-state performance of the closed-loop system, the error transformation technique is presented. Further, the design of the prescribed performance function implements predefined tracking performance constraints, which eliminate the requirement for prior knowledge about the initial errors. Then, a Lyapunov stability synthesis shows that all closed-loop signals remain bounded and the tracking errors remain strictly within the predefined bounds. Finally, simulations and a comparative study are performed to illustrate the robustness and effectiveness of the proposed robust sliding mode control scheme.

Keywords: robust sliding mode control,prescribed performance,guaranteed transient performance,trajectory tracking,underactuated surface vehicle

INTRODUCTION

Tracking control of underactuated surface vehicles (USVs) has been widely used in military and civil fields such as coastal patrols, mine countermeasures, oceanographic sampling, military reconnaissance, and so on [1], [2]. In the case of tracking problems, trajectory tracking is more difficult than path following, since the control laws require the USV to be driven to reach and track a time-varying trajectory on time. The challenge comes mainly from the nonlinearity and underactuation of USV dynamical systems. At the same time, USVs inevitably suffer from uncertainties, complex hydrodynamics, and time-varying external disturbances such as wind, waves, and ocean currents [3]. Therefore, an actual mathematical model of a USV is hard to obtain precisely using current modeling methods. The other challenge, however, is that the number of USV control inputs is less than the degrees of freedom, which means that its system causes non-integral constraints [4].

On the one hand, it is clear that a USV is an underactuated system with nonholonomic constraints that cannot be stabilized by continuous time-invariant feedback [5]. Many nonlinear control works have been conducted to solve the tracking problem of underactuated vehicles, such as Lyapanov-based techniques [6], robust adaptive control [7], the backstepping technique [8], sliding mode control [9], neural networks [10], and disturbance-observer-based control [11]. Considering external perturbations, a full state feedback control algorithm based on a backstepping scheme was presented to track a straight line in [8]. Reference [12] proposes a point-to-point navigation technique combined with an adaptive and backstepping method to improve the robustness under the model uncertainties. However, the repeated differential of virtual control laws in the backstepping method increases the complexity of calculations. Moreover, due to the use of the adaptive technique and neutral networks, some parameters need to be adjusted online, which makes the computing process and parameter selection harder. Nevertheless, a sliding mode control structure is simple and provides excellent tracking performance even when uncertainty and bounded disturbance are acting on the underactuated vehicle.

On the other hand, it is necessary to consider the prescribed transient and steady-state control performance in practical controller design. More recently, the prescribed performance control (PPC) method was introduced in [13]. Furthermore, PPC was used in the tracking control for several classes of nonlinear systems [14] and robots [15] and for autonomous underwater vehicles [16] and surface vehicles [17]. However, logarithmic error mapping functions used in previous studies have led to the potential singularity problem of the designed control law. For tracking control of an underactuated vehicle with ensured transient performance [18], a robust adaptive controller under environmental disturbances was put forward, but it only solved the path-following problem. In [19], a robust fault-tolerant controller was proposed to ensure that the tracking errors of a USV were within predefined performance bounds. Reference [20] investigated the distributed containment control of networked surface vessels, which achieved the prescribed transient and steady-state performance. PPC is common in fully actuated nonlinear systems but has not been innovatively extended to the control of underactuated vehicles [21].

Motivated by the above observations, to guarantee the predefined performance of USVs under time-varying external disturbances, a robust sliding mode technology is used to design a control scheme. The main contributions of this article include the following: (1) unlike the existing literature, the proposed controller can guarantee that tracking errors will not exceed a prescribed performance bound; (2) the proposed method eliminates the requirement for prior knowledge about the initial errors.

PROBLEM FORMULATION

MATHEMATICAL MODELS

Before presenting the design procedure, mathematical models of the USV are described. Figure 1 shows the three-degrees-of-freedom motions of the USV with body-fixed and earth-fixed frames. It should be noted that the kinematics and dynamics of the USV with time-varying external disturbances in the horizontal plane (surge, sway, and yaw) are as follows [22]:

$$\begin{cases} \dot{x} = u\cos\psi - v\sin\psi \\ \dot{y} = u\sin\psi + v\cos\psi \\ \dot{\psi} = r \\ \dot{u} = M_1\left(a_{23}vr - D_u u + \tau_u + \tau_{wu}(t)\right) \\ \dot{v} = M_2\left(a_{13}ur - D_v v + \tau_{wv}(t)\right) \\ \dot{r} = M_3\left(a_{12}uv - D_r r + \tau_r + \tau_{wr}(t)\right) \end{cases} \quad (5)$$

where $\eta = [x \quad y \quad \psi]^T, \psi \in [-\pi, \pi]$ is the north and east positions and heading of the USV in the earth-fixed frame and $\upsilon = [u \quad v \quad r]^T$ represents the surge and sway velocities and yaw rate in the body-fixed frame. Then, the only two control inputs are the surge force τ_u and the yaw moment τ_r, and the USV model has underactuated characteristics. Then, $\tau_{wj}(t), j = u, v, r$ are the vectors of time-varying external disturbances acting on the USV. Also, $M_1 = 1/(m - X_{\dot{u}})$, $M_2 = 1/(m - Y_{\dot{v}})$, $M_3 = 1/(I_z - N_{\dot{r}})$, $D_u = -X_u - X_{|u|u}|u|$, $D_v = -Y_v - Y_{|v|v}|v|$, $D_r = -N_r - N_{|v|r}|v|$, $a_{12} = Y_{\dot{v}} - X_{\dot{u}}$, $a_{13} = X_{\dot{u}} - m$, and $a_{23} = m - Y_{\dot{v}}$. I_z is the moment of inertia. $X_j, j = u, v, r$ are the hydrodynamic added mass terms. D_j comprises the linear and nonlinear hydrodynamic damping. m is the mass of the USV.

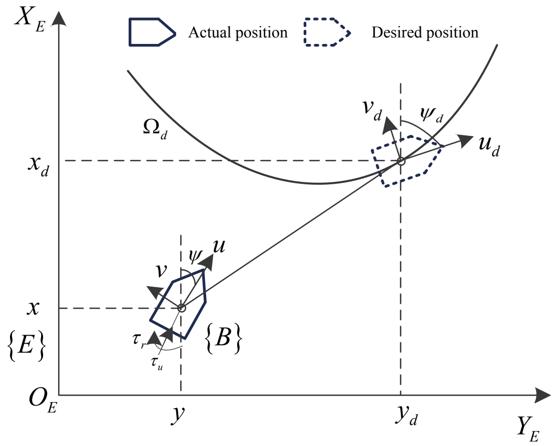

Fig. 1. Definition of reference coordinate frame of the USV trajectory tracking

Control objective: The control goal of the USV system is to design a robust sliding mode tracking controller with prescribed performance under time-varying external disturbances, ensuring that the system output can track the arbitrary smooth reference trajectory Ω_d at the corresponding time (refer to Figure 1). All signals in the closed-loop system remain bounded.

Assumption 1: The external environment disturbances $\tau_{wj}(t), j = u, v, r$ are bounded by unknown constants, that is, $|\tau_{wj}(t)| \le \tau_{wj\max}$.

Assumption 2: The desired trajectory $\Omega_d(t): U \to \mathbb{R}^2, t \in U \subset [0,\infty)$ and its first order derivative are bounded.

Assumption 3: All the states are measurable and available for feedback.

PRESCRIBED PERFORMANCE

As shown in Figure 2, to guarantee the prescribed transient performance and ensure that the steady-state tracking error is bounded, the following definition is introduced first.

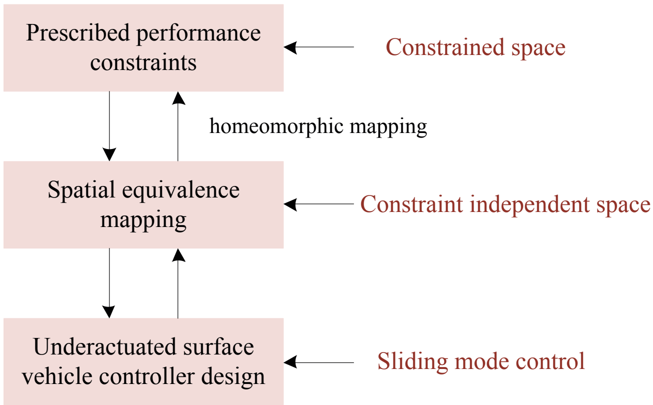

Fig. 2. Structure of prescribed performance method

Definition 1 [23]. A smooth bounded function $\rho(t)$: $\mathbb{R}^+ \to \mathbb{R}^+$ can be called a performance function; if $\rho(t)$ is decreasing, $|e(0)| < |\rho(0)|$ and $\lim_{t\to\infty} \rho(t) = \rho_\infty > 0$.

The prescribed performance can be achieved if the following conditions hold:

$$-\delta\rho(t) < e(t) < \rho(t), e(0) \geq 0 \tag{2}$$

$$-\rho(t) < e(t) < \delta\rho(t), e(0) \leq 0 \tag{3}$$

where $e(t)$ is the system state error, $\delta \in [0,1]$ is the designed parameter, and $t \in [0,\infty)$. Because of the convergence performance in exponential form, the prescribed performance function in this paper is defined as:

$$\rho(t) = (\rho_0 - \rho_\infty)e^{-lt} + \rho_\infty \tag{4}$$

where $\rho_0 > \rho_\infty$, $l > 0$.

To achieve the control performance (2) and (3), the error transformation method is employed to eliminate the requirement for prior knowledge about the initial errors, and the error transformation function is constructed as:

$$e(t) = \rho(t)S(\sigma) \text{ or } \sigma(t) = S^{-1}\left(\frac{e(t)}{\rho(t)}\right) \tag{5}$$

where σ is the transformation error, and the transformation function $S(\sigma)$ is strictly increasing and satisfies:

$$\begin{cases} \lim_{\sigma\to-\infty} S(\sigma) = -\delta, e(0) \geq 0 \\ \lim_{\sigma\to-\infty} S(\sigma) = -1, e(0) \leq 0 \end{cases} \begin{cases} \lim_{\sigma\to\infty} S(\sigma) = 1, e(0) \geq 0 \\ \lim_{\sigma\to\infty} S(\sigma) = \delta, e(0) \leq 0 \end{cases} \tag{6}$$

$$S(\sigma) = \frac{e^\sigma - \delta e^{-\sigma}}{e^\sigma + e^{-\sigma}}, e(t) \geq 0 \quad S(\sigma) = \frac{\delta e^\sigma - e^{-\sigma}}{e^\sigma + e^{-\sigma}}, e(t) \leq 0 \tag{7}$$

From the definition of $S(\sigma)$ and , we can get:

$$\sigma(t) = F(S(\sigma)) = \frac{1}{2}\ln\frac{\delta + S(\sigma)}{1 - S(\sigma)} \tag{8}$$

where $F(\cdot)$ is the homeomorphic mapping function, and the derivative of $\sigma(t)$ is:

$$\dot{\sigma}(t) = \frac{\dot{e}(t) - \dot{\rho}(t)S(\sigma)}{\rho(t)\left(\frac{\partial S}{\partial \sigma}\right)} \tag{9}$$

Remark 1. From (5) and (8), it can be seen that the bounded transformation error $\sigma(t)$ can guarantee that the prescribed performance constraint is met. Meanwhile, the two systems are the homeomorphic mapping.

DESIGN OF PRESCRIBED PERFORMANCE TRACKING CONTROL

In this section, a robust sliding mode tracking controller based on prescribed performance (robust sliding mode with prescribed performance – RSMPP) will be designed for the system (1) with time-varying external disturbances. The whole control architecture can be seen in Figure 3. The detailed design procedure is as follows:

Step 1. Tracking error transformation

Define the position and velocity error variables as:

$$\begin{bmatrix} e_x \\ e_y \end{bmatrix} = \begin{bmatrix} x - x_d \\ y - y_d \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} u_e \\ v_e \end{bmatrix} = \begin{bmatrix} u - \alpha_u \\ v - \alpha_v \end{bmatrix} \tag{11}$$

where x_d, y_d and α_u, α_v are the desired position and velocity, respectively.

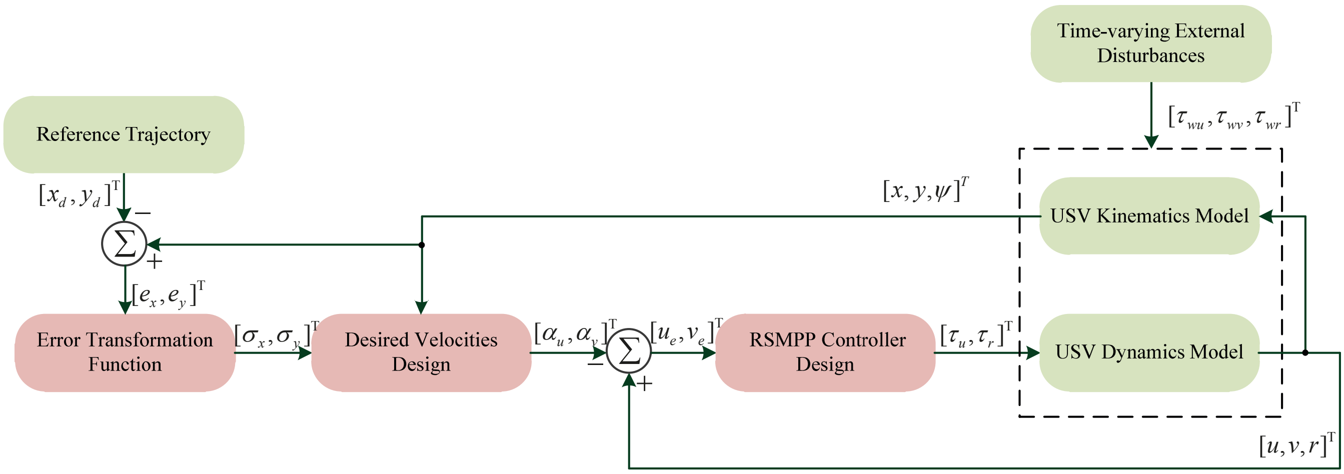

Fig. 3. The whole control architecture

Considering (1) and (10), the time derivative of the variable e_x, e_y can be calculated as:

$$\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \end{bmatrix} \tag{12}$$

Here, the design purpose is to make the position vector track the desired position vector with prescribed performance. As mentioned above in Section 2.2, the error transformation function is designed as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\ln\left(\delta + \frac{e_x}{\rho}\right) - \frac{1}{2}\ln\left(1 - \frac{e_x}{\rho}\right) \\ \frac{1}{2}\ln\left(\delta + \frac{e_y}{\rho}\right) - \frac{1}{2}\ln\left(1 - \frac{e_y}{\rho}\right) \end{bmatrix} \tag{13}$$

Then, we have:

$$\begin{bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\left(\dot{e}_x - e_x\frac{\dot{\rho}}{\rho}\right)\left(\frac{1}{\rho\delta + e_x} - \frac{1}{e_x - \rho}\right) \\ \frac{1}{2}\left(\dot{e}_y - e_y\frac{\dot{\rho}}{\rho}\right)\left(\frac{1}{\rho\delta + e_y} - \frac{1}{e_y - \rho}\right) \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \end{bmatrix} = \begin{bmatrix} \xi_x\left(\dot{e}_x - e_x\frac{\dot{\rho}}{\rho}\right) \\ \xi_y\left(\dot{e}_y - e_y\frac{\dot{\rho}}{\rho}\right) \end{bmatrix} = \begin{bmatrix} \xi_x & 0 \\ 0 & \xi_y \end{bmatrix} \begin{bmatrix} \dot{e}_x - e_x\frac{\dot{\rho}}{\rho} \\ \dot{e}_y - e_y\frac{\dot{\rho}}{\rho} \end{bmatrix} \tag{15}$$

where $= \frac{1}{\ }\left(\frac{1}{\rho\delta + e_x} - \frac{1}{e_x - \rho}\right), \xi_y = \frac{1}{2}\left(\frac{1}{\rho\delta + e_y} - \frac{1}{e_y - \rho}\right)$.

Substituting (12) into (15) yields:

$$\begin{bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \end{bmatrix} = \begin{bmatrix} \xi_x & 0 \\ 0 & \xi_y \end{bmatrix} \left(\begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} \dot{x}_d + e_x\frac{\dot{\rho}}{\rho} \\ \dot{y}_d + e_y\frac{\dot{\rho}}{\rho} \end{bmatrix} \right) \tag{16}$$

Step 2. Design of the desired velocities

To avoid design flaws in Remark 2 [24], the desired surge and sway velocities, that is, the virtual control law, are chosen as:

$$\begin{bmatrix} \alpha_u \\ \alpha_v \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \dot{x}_d + \frac{e_x\dot{\rho}}{\rho} - \frac{k_x\sigma_x}{\xi_x} \\ \dot{y}_d + \frac{e_y\dot{\rho}}{\rho} - \frac{k_y\sigma_y}{\xi_y} \end{bmatrix} \tag{17}$$

where the choice of the controller gains $k_x > 0, k_y > 0$ can be tuned and determines how fast the tracking errors converge to zero so that the performance of the USV is robust against uncertainties.

Based on (1) and (17), the derivative of the variable α_u, α_v is constructed by:

$$\begin{bmatrix} \dot{\alpha}_u \\ \dot{\alpha}_v \end{bmatrix} = r\begin{bmatrix} \alpha_v \\ -\alpha_u \end{bmatrix} + \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \ddot{x}_d + \frac{\dot{\rho}\dot{e}_x + \ddot{\rho}e_x}{\rho} - \frac{e_x\dot{\rho}^2}{\rho^2} - k_x\frac{\dot{\sigma}_x\xi_x - \sigma_x\dot{\xi}_x}{\xi_x^2} \\ \ddot{y}_d + \frac{\dot{\rho}\dot{e}_y + \ddot{\rho}e_y}{\rho} - \frac{e_y\dot{\rho}^2}{\rho^2} - k_y\frac{\dot{\sigma}_y\xi_y - \sigma_y\dot{\xi}_y}{\xi_y^2} \end{bmatrix} \tag{18}$$

Remark 2. Avoiding large tracking errors causes the virtual control law to exceed the maximum velocity of the USV and thus affects the control effect. So the virtual control law selects (17), not the following:

$$\begin{bmatrix} \alpha_u \\ \alpha_v \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \dot{x}_d - k_x\sigma_x \\ \dot{y}_d - k_y\sigma_y \end{bmatrix} \tag{19}$$

Step 3. Prescribed performance tracking controller design

Combined with (11), the sliding surfaces are chosen such that:

$$L_1 = u_e + \lambda_1 \int_0^t u_e(\tau)d\tau \tag{20}$$

$$L_2 = \dot{v}_e + \lambda_2 \int_0^t v_e(\tau)d\tau + \lambda_3 v_e \tag{21}$$

where $\lambda_1, \lambda_2, \lambda_3 > 0$. Using (1), the time derivatives of these sliding surfaces are such that:

$$\dot{L}_1 = M_1\left(a_{23}vr - D_u u + \tau_u + \tau_{wu}(t)\right) - \dot{\alpha}_u + \lambda_1 u_e \tag{22}$$

$$\dot{L}_2 = M_2\left[a_{13}\dot{u}r - D_v\dot{v} + \dot{\tau}_{wv}(t) + a_{13}uM_3\left(a_{12}uv - D_r r + \tau_r + \tau_{wr}(t)\right)\right] - \ddot{\alpha}_v + \lambda_2 v_e + \lambda_3\dot{v}_e \tag{23}$$

To ensure the finite-time convergence of L_1 and L_2 to zero, the design is as follows.

$$\dot{L}_1 = -K_1 \operatorname{sign}(L_1) \tag{24}$$

$$\dot{L}_2 = -K_2 \operatorname{sign}(L_2) \tag{25}$$

where $K_1, K_2 > 0$.

Therefore, the robust sliding mode control law based on prescribed performance is:

$$\tau_u = -a_{23}vr + D_u u - \eta_1 + \frac{1}{M_1}\left(\dot{\alpha}_u - \lambda_1 u_e - K_1 \operatorname{sign}(L_1)\right) \tag{26}$$

$$\tau_r = -a_{12}uv + D_r r - \eta_3 - \frac{1}{b}K_2 \operatorname{sign}(L_2) + \frac{1}{b}\left[-M_2\left(a_{13}\dot{u}r - D_v\dot{v} + \eta_2\right) + \Gamma - \lambda_2 v_e - \lambda_3\dot{v}_e\right] \tag{27}$$

where $\eta_2 \geq \tau_{wv\max}$, $\eta_3 \geq \tau_{wr\max}$, $b \neq 0$, and $b = M_3\left(M_2 a_{13}u + \alpha_u\right)$. Then according to (18), we can get $\ddot{\alpha}_v = \Gamma - \alpha_u\dot{r}, \Gamma = -\dot{\alpha}_u r + f$, and

$$f = \left[\ddot{y}_d + \frac{\dot{\rho}\dot{e}_y + \ddot{\rho}e_y}{\rho} - \frac{e_y\dot{\rho}^2}{\rho^2} - k_y\frac{\dot{\sigma}_y\xi_y - \sigma_y\dot{\xi}_y}{\xi_y^2}\right]\cos\psi - \left[\ddot{x}_d + \frac{\dot{\rho}\dot{e}_x + \ddot{\rho}e_x}{\rho} - \frac{e_x\dot{\rho}^2}{\rho^2} - k_x\frac{\dot{\sigma}_x\xi_x - \sigma_x\dot{\xi}_x}{\xi_x^2}\right]\sin\psi.$$

Theorem 1. Consider the USV model under time-varying external disturbances, which are given by , and the error transformation function (13). Under Assumptions 1 to 3, the design parameters $\lambda_1, \lambda_2, \lambda_3, K_1, K_2, k_x, k_y$ *and* δ are carefully adjusted. Then the proposed controller (26) and (27) with the virtual control law (17) guarantees the stability of the USV closed-loop system. All states remain bounded and it can be ensured that the tracking errors remain within the given prescribed performance bounds for $\forall t \geq 0$.

Proof: Firstly, the complete Lyapunov function is assigned:

$$V_1 = \frac{1}{2}L_1^2 + \frac{1}{2}L_2^2 \tag{28}$$

In virtue of (22) and (23), the derivative of V_1 satisfies:

$$\begin{aligned} \dot{V}_1 &= L_1\dot{L}_1 + L_2\dot{L}_2 \\ &= L_1\left[M_1\left(a_{23}vr - D_u u + \tau_u + \tau_{wu}(t)\right) - \dot{\alpha}_u + \lambda_1 u_e\right] \\ &\quad + L_2\left\{ \begin{array}{l} M_2\begin{bmatrix} a_{13}\dot{u}r - D_v\dot{v} + \dot{\tau}_{wv}(t) \\ +a_{13}uM_3\left(a_{12}uv - D_r r + \tau_r + \tau_{wr}(t)\right) \end{bmatrix} \\ -\ddot{\alpha}_v + \lambda_2 v_e + \lambda_3\dot{v}_e \end{array} \right\} \end{aligned} \tag{29}$$

Integrating (26) and (27), one has:

$$\begin{aligned} \dot{V}_1 &= L_1\dot{L}_1 + L_2\dot{L}_2 \\ &= L_1\left[M_1\left(a_{23}vr - D_u u + \tau_u + \tau_{wu}(t)\right) - \dot{\alpha}_u + \lambda_1 u_e\right] + L_2\left\{ \begin{array}{l} M_2\begin{bmatrix} a_{13}\dot{u}r - D_v\dot{v} + \dot{\tau}_{wv}(t) \\ +a_{13}uM_3\left(a_{12}uv - D_r r + \tau_r + \tau_{wr}(t)\right) \end{bmatrix} \\ -\ddot{\alpha}_v + \lambda_2 v_e + \lambda_3\dot{v}_e \end{array} \right\} \\ &= -K_1L_1\operatorname{sign}(L_1) + L_2M_2\left[a_{13}\dot{u}r - D_v\dot{v} + \dot{\tau}_{wv}(t)\right] + L_2\left[-\Gamma + b\left(a_{12}uv - D_r r + \tau_r + \tau_{wr}(t)\right) + \lambda_2 v_e + \lambda_3\dot{v}_e\right] \\ &= -K_1L_1\operatorname{sign}(L_1) - K_2L_2\operatorname{sign}(L_2) + \left(\tau_{wu}(t) - \eta_1\right) + \left(\dot{\tau}_{wv}(t) - \eta_2\right) + \left(\tau_{wr}(t) - \eta_3\right) \\ &\leq -K_1|L_1| - K_2|L_2| \end{aligned} \tag{30}$$

where $\tau_{wu}(t) - \eta_1 \leq 0$, $\dot{\tau}_{wv}(t) - \eta_2 \leq 0$, and $\tau_{wr}(t) - \eta_3 \leq 0$. Γ and b are defined in (27). It is obvious from (30) that $\dot{V}_1 < 0$ for $(L_1, L_2) \neq (0,0)$ since $K_1 > 0$and $K_2 > 0$. This indicates that sliding surfaces in (20) and (21) can reach zero; that is, the tracking errors satisfy $(u_e, v_e) \to (0,0)$.

Then, the following Lyapunov function candidate is chosen:

$$V_2 = \frac{1}{2}{\sigma_x}^2 + \frac{1}{2}{\sigma_y}^2 \tag{31}$$

Substituting (17) into (16), the time derivative of V_2 can be written as follows:

$$\dot{V}_2 = \sigma_x\dot{\sigma}_x + \sigma_y\dot{\sigma}_y = -k_x{\sigma_x}^2 - k_y{\sigma_y}^2 \tag{32}$$

Since $k_x > 0$ and $k_y > 0$, it is clear from (32) that $\dot{V}_2 < 0$ for $(\sigma_x, \sigma_y) \neq (0,0)$. So, it can be found that both e_x and e_y converge asymptotically to zero.

Furthermore, consider the following Lyapunov function candidate:

$$V_3 = \frac{1}{2} r^2 \tag{33}$$

The time derivative of V_3 is given by:

$$\dot{V}_3 = M_3 r \left[a_{12} uv - D_r r + \tau_r + \tau_{wr}(t) \right] \tag{34}$$

Knowing that $M_3 > 0$, the condition $\dot{V}_3 < 0$ is satisfied when:

$$a_{12} uv + \tau_r + \tau_{wr}(t) < D_r r \quad if \;\; r > 0 \tag{35}$$

$$a_{12} uv + \tau_r + \tau_{wr}(t) > D_r r \quad if \;\; r < 0 \tag{36}$$

which implies the following:

$$\left| D_r r \right| > \left| a_{12} uv + \tau_r + \tau_{wr}(t) \right| \tag{37}$$

Therefore, $\dot{V}_3 < 0$ if the inequality (37) is satisfied. Moreover, $\dot{V}_3 < 0$ implies that V_3 is a decreasing function, which means that $\left| r \right|$ is decreasing as well from .

Consequently, the control laws proposed in (26) and (27) ensure the convergence of position and velocity errors while the yaw motion remains bounded. In accordance with the above depiction, all states of the USV closed-loop control system remain bounded.

SIMULATION RESULTS AND ANALYSIS

In this section, simulation studies are performed to illustrate the effectiveness and robustness of the proposed control scheme. A USV from the Norwegian University of Science and Technology [22] is adopted and the parameters used for it are given in Table 1. The reference trajectory is selected as $x_d = 100\sin\left(0.1t\right)$ and $y_d = 100\cos\left(0.1t\right)$. The initial position and velocity of the USV are $\eta = \left[30 \quad 110 \quad -\pi/9\right]^T$ and $\upsilon = \left[2 \quad 0 \quad 0\right]^T$. The time-varying external disturbances are given as $\tau_{wu} = 10\left[\sin\left(0.1t\right) + \cos\left(0.05t\right)\right]$, $\tau_{wv} = 0.01\left[0.01\sin\left(0.1t\right) + 0.01\cos\left(0.05t\right)\right]$, and $\tau_{wr} = 0.1\left[0.2\sin\left(0.1t\right) + 0.1\cos\left(0.05t\right)\right]$, and the units are $\left(N\right)$ and $\left(N \cdot \mathrm{m}\right)$, respectively. From (20) and (21), we can find that λ_1 and λ_2 are integral factors. If the values chosen are too small, the system may be unstable. However, small integral factors can eliminate steady-state error and improve the accuracy of the system. And λ_3 is a proportional coefficient, which can speed up the reaction speed of the system and reduce the steady-state error. The coefficient selected should be neither too large nor too small: a large coefficient may make the system unstable and a small coefficient can reduce the reaction speed of the system. Therefore, λ_1, λ_2, and λ_3 should be selected carefully. Then, it can be found from the theoretical analysis of **Theorem 1** that the remaining control parameters K_1, K_2, k_x *and* k_y should be designed to be positive. As with the performance function parameters, it is obvious that the parameter ρ_0 denotes the bound of the overshoot, ρ_∞ represents the maximum allowable steady-state tracking error, and l influences the convergence rate of the tracking error. Based on the above analysis and the debugging results, the control parameters are chosen as λ_1 = 3, λ_2 = 0.9, λ_3 = 0.2, K_1 = 5, K_2 = 0.7, k_x = 0.4, k_y = 0.04, and δ = 0.4 and the performance function is chosen as $\left(t\right) = \left(80 - 0.06\right)e^{\,0.2} + 0.06$. To show the advantages of the proposed algorithms, two conditions are compared and evaluated: with and without the predefined performance bounds.

Table 1. Parameters of the USV

Parameter	Value	Unit
m	23.8	kg
L	1.255	m
B	0.29	m
I_z	1.760	$\mathrm{kg \cdot m^2}$
$X_{\dot{u}}$	-2	kg
$Y_{\dot{v}}$	-10	kg
$Y_{\dot{r}}$	0	kg
$N_{\dot{v}}$	0	kg
$N_{\dot{r}}$	-1	$\mathrm{kg \cdot m^2}$
X_u	-0.72253	kg / s
Y_v	-0.88965	kg / s
N_r	-1.900	$\mathrm{kg \cdot m^2 / s}$
$X_{\lvert u \rvert u}$	-1.32742	kg / s
$Y_{\lvert v \rvert v}$	-36.47287	kg / s
$N_{\lvert v \rvert r}$	0.080	kg / s

Remark 3: Without loss of generality, simulations are adopted with the same initial position, reference trajectory, and control parameters. The blue solid line represents the proposed robust sliding mode control method based on prescribed performance. The dashed line in green expresses the proposed method without prescribed performance (PP) and the black dashed line represents the proposed method without time-varying external disturbances (TED). The simulation time is set as 100 s. Then, the simulation results are as shown below.

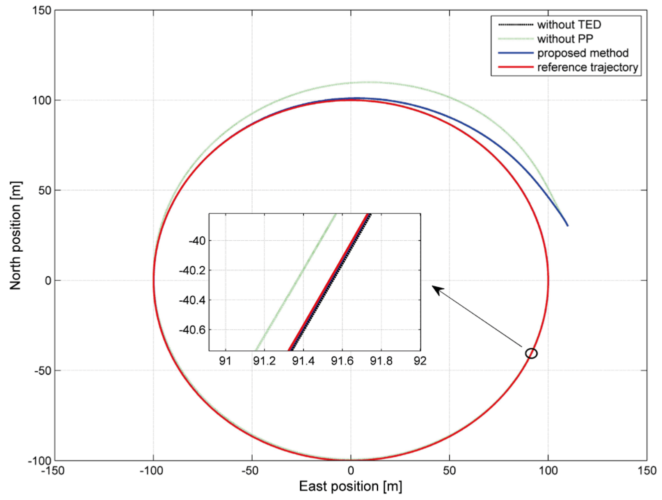

Fig. 4. The position curve of trajectory tracking for the USV

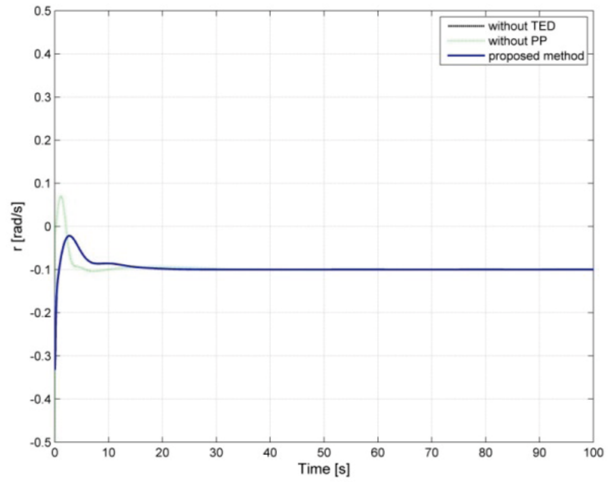

Fig. 7. The yaw rate of trajectory tracking for the USV

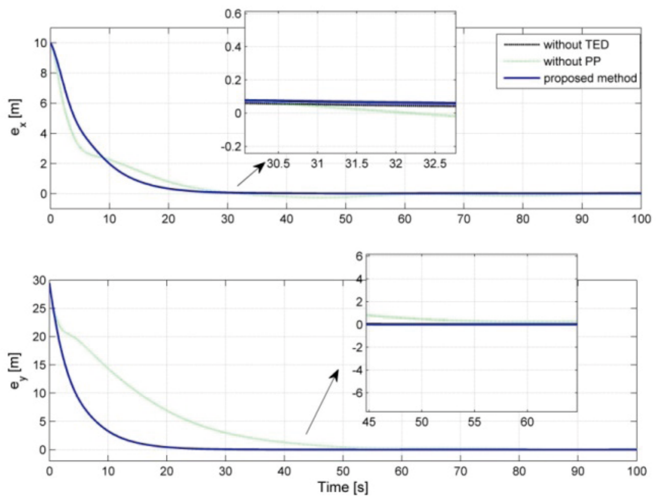

Fig. 5. Time response of the tracking errors for the USV

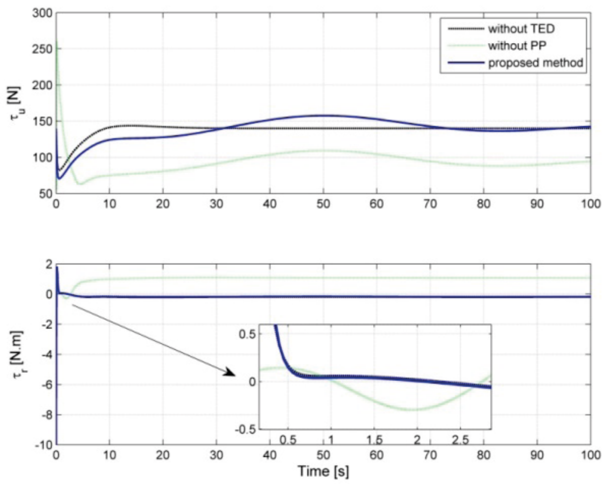

Fig. 8. Time response of the control input curve for USV

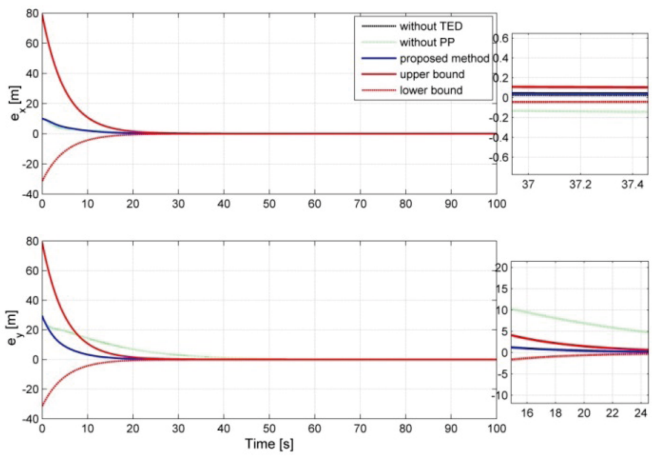

Fig. 6. Trajectory tracking errors and performance bound

Figure 4 gives the change of positions and heading. Whether or not the impact of time-varying external disturbances is taken into account, the controller performs the tracking mission well. In comparison, the proposed controller with prescribed performance converges faster. It is shown that the proposed controller has strong robustness to time-varying external disturbances. The tracking errors are shown in Figure 5, where the convergence time of the method without prescribed performance is more than 50 s. Besides, it will lead to undesirable transient-state performance when the large overshoot exceeds the boundary. As can be seen from Figure 6, the constraints for the case without the prescribed performance method where the tracking error is greater than 0.1 m, clearly exceed the performance boundary. On the contrary, the proposed method can quickly converge to the bounded sets and will always be maintained between the prescribed upper bound and lower bound with small overshoots. The prescribed performance function is designed to ensure the performance of the tracking control without any prior knowledge about the initial errors. The yaw rate of trajectory tracking for the USV is shown in Figure 7. Compared with the proposed controller, the method without prescribed performance requires greater control forces, which results in larger overshoots. The control inputs $\tau = \left[\tau_u \quad 0 \quad \tau_r\right]^{\mathrm{T}}$ are depicted in Figure 8, where only surge force and yaw moments exist. It is shown that the control force and moment can guarantee the stability of the closed-loop system under time-varying external disturbances. It can be concluded that the designed controller ensures that the tracking errors are within the predefined bounds and produce better transient and steady-state performance compared to the one without prescribed performance.

CONCLUSION

In the presence of time-varying external disturbances, the problem of trajectory tracking control for a USV with prescribed performance has been solved in this paper. Based on error transformation technique, a robust prescribed performance tracking control law is designed in combination with sliding mode technology. Meanwhile, the prescribed performance bound approach, which is combined with Lyapunov theory, successfully ensures that the tracking errors of the USV remain within the required performance constraints. The results of the simulation and comparison show that the tracking errors of the proposed controller remain strictly within the predefined bounds, while the one without prescribed performance cannot guarantee this.

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