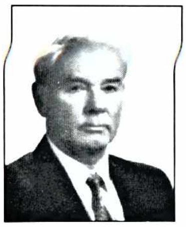

JANUSZ KOLENDA, Prof., D.Sc.
Gdańsk University of Technology
and Polish Naval University, Gdynia

Fatigue criterial parameters of variable amplitude stress

SUMMARY

In this paper new fatigue criterial parameters of variable amplitude stress are presented. To obtain them it is assumed that the energy expenditure in the fatigue process is proportional to the elastic strain energy. If the increment of damage which is caused by n_j stress cycles of constant amplitude, a_j, is related to the elastic strain energy transferred to unit volume in N_j cycles which cause failure at the stress amplitude a_j, then the Palmgren-Miner rule is obtained. If, however, this increment is related to the elastic strain energy transferred to unit volume in n_j cycles which cause failure at the sufficiently high stress amplitude, A_j, another formula for the total fatigue damage and additional criterial parameters are obtained.

INTRODUCTION

One of the current topics in fatigue research is determination of a suitable damage parameter which would allow to predict with sufficient accuracy the fatigue strength of the material under cyclic loads of various levels and patterns. The fatigue process associated with nucleation and subsequent propagation of fatigue cracks is generally controlled by the local stress/strain fields [1]. Fatigue damage increases with applied load cycles in a cumulative manner which may lead to fracture. The cumulative fatigue damage is an old [2,3] but not yet solved problem [4]. Starting with the linear damage cumulation rule suggested by Palmgren [2] and expressed by Miner [3], a number of relationships have been derived, in particular those relating the high-cycle and low-cycle fatigue life to the elastic and plastic strain energy [4].

In this paper the elastic strain energy is taken into account and the high-cycle fatigue in which failure occurs in excess of 10^3 load cycles, is considered. In this regime the deformations are small and can be characterized by elastic behaviour of the material [5]. High strain amplitudes experienced in the low-cycle regime can include a significant plastic strain, and accumulated damage may therefore be history-dependent. It can be noted that the original paper [3] dealt with the tests of nominally elastic behaviour under load control.

However, the Palmgren-Miner rule may in many applications be biased quite often in an unpredictable manner, leading to large uncertainties in the fatigue life calculations [6]. It is why considerable attention is still focused on the high-cycle fatigue criteria.

BACKGROUND

A commonly accepted technique in the high-cycle fatigue life estimation is the use of the Palmgren-Miner rule [6 , 7]. According to the original concept :

- the fatigue process is cumulative
- the fatigue effect is proportional to the work of the active loads
- the increment of damage which is caused by n stress cycles of constant amplitude, a, can be estimated as n / N where N is the number of stress cycles which would cause failure at the same amplitude a.

As to the variable amplitude loads, the concept holds that the total damage can be estimated as the sum of the damage increments, each corresponding to the specified stress level. This can be symbolically written as :

$$D = \sum_j \frac{n_j}{N_j} = 1 \tag{1}$$

where :

D – cumulative fatigue damage under variable amplitude stress of cycle ratios n_j / N_j (j = 1, 2, ...)
n_j – the number of cycles at the j-th stress level
N_j – the number of cycles that is expected to cause failure at the j-th stress level, usually obtained from the S-N curve equation (Wöhler curve) [6,7] :

$$N\sigma^m = K \tag{2}$$

where :

σ – stress amplitude
K – fatigue strength coefficient
m – fatigue strength exponent
N – number of cycles to cause failure.

Failure is expected – as in constant amplitude testing – to occur when D = 1. The method implicitly assumes that D is independent of the order in which the cycles of different levels are applied.

As derived in the original analysis [3], (1) is an alternative way of stating that a material will fail when the expended net work reaches the critical value, Ω :

$$\Omega = \sum_j \frac{w_j}{W_j} = 1 \tag{3}$$

where :

w_j – the energy expenditure per cycle
W_j – the failure energy.

Their values can be determined, e.g., by using the area of hysteresis loops.

As follows from the experiments, the total fatigue damage may significantly differ from unity (the values lower than 0.1 and higher than 10 were reported [7]). Nevertheless the total fatigue damage (1) still remains the most frequently used parameter for predicting the high-cycle fatigue life.

The aim of this paper is to present other stress parameters which might be useful in design considerations.

CRITERIAL PARAMETERS

It has been shown that the energy expenditure per cycle and unit volume is independent of the state of hardening or softening of the material [8]. On the other hand, (2) implies that this quantity is proportional to the term σ^m which allows to assume that the energy expenditure per unit volume in n_j stress cycles of the amplitude a_j is dependent on the elastic strain energy transferred to unit volume in n_j cycles of the same amplitude. Consequently, it can be stated that :

- the fatigue process is cumulative
- the fatigue effect is proportional to the elastic strain energy
- the increment of damage which is caused by n_j stress cycles of the amplitude a_j can be estimated as :

$$\frac{\phi_{n_j}(a_j)}{\phi_{N_j}(a_j)}$$

where :

$\phi_{n_j}(a_j)$ – the maximum strain energy of distortion transferred to unit volume in n_j cycles
$\phi_{N_j}(a_j)$ – the maximum strain energy of distortion transferred to unit volume in N_j cycles
N_j – the number of cycles that would cause fatigue failure at the stress amplitude a_j , as in (1).

Thus, the total fatigue damage is given by :

$$D = \sum_j \frac{\phi_{n_j}(a_j)}{\phi_{N_j}(a_j)} = 1 \tag{4}$$

The maximum strain energy of distortion (shear strain energy) transferred to unit volume is :

in a single stress cycle of the amplitude a_j :

$$\phi_1(a_j) = \frac{1+\nu}{3E} a_j^2 \tag{5}$$

in n_j such cycles :

$$\phi_{n_j}(a_j) = \frac{1+\nu}{3E} n_j a_j^2 \tag{6}$$

in N_j such cycles :

$$\phi_{N_j}(a_j) = \frac{1+\nu}{3E} N_j a_j^2 \tag{7}$$

where :

E – Young modulus
ν – Poisson's ratio.

Apparently, by substituting (6) and (7) into (4) the equation (1) is obtained. However, there is such stress of the amplitude $A_j > a_j$ that even its n_j cycles would cause fatigue failure. In other words, the maximum strain energy of distortion, transferred to unit volume until fatigue failure, can be also calculated for n_j cycles as :

$$\phi_{n_j}(A_j) = \frac{1+\nu}{3E} n_j A_j^2 \tag{8}$$

where :

A_j – the stress amplitude that would lead to fatigue failure in n_j cycles, i.e. :

$$A_j = \left(\frac{K}{n_j}\right)^{1/m} \tag{9}$$

The quantity A_j will be called the *j-th* fatigue-critical stress amplitude.

Since the S-N curve equation and the strain energy of distortion are non-linear functions of stress amplitude, the values of $\phi_{N_j}(a_j)$ and $\phi_{n_j}(A_j)$ do not coincide.

Consequently, the following assumptions can be formulated :

- the fatigue process is cumulative
- the fatigue effect is proportional to the strain energy of distortion
- the increment of damage which is caused by n_j stress cycles of the amplitude a_j can be estimated as :

$$\frac{\phi_{n_j}(a_j)}{\phi_{n_j}(A_j)}$$

where :

$\phi_{n_j}(a_j)$ – the maximum strain energy of distortion transferred to unit volume in n_j stress cycles of the amplitude a_j
$\phi_{n_j}(A_j)$ – the maximum strain energy of distortion transferred to unit volume in n_j stress cycles of the amplitude A_j.

Now the formula for the total fatigue damage, Δ, reads :

$$\Delta = \sum_j \frac{\phi_{n_j}(a_j)}{\phi_{n_j}(A_j)} = 1 \tag{10}$$

By using (6) and (8), the equation (10) can be rewritten as :

$$\Delta = \sum_j \left(\frac{a_j}{A_j}\right)^2 = 1 \tag{11}$$

(11) is valid in the application range of (2), that is :

$$Z < a_j \le A_j \le L \quad (12)$$

$$\frac{K}{L^m} \le n_j < \frac{K}{Z^m} \quad (13)$$

where :

L – the maximum stress amplitude satisfying (2), above which the low-cycle fatigue may occur
Z – the fatigue limit under fully reversed stress.

The mean stress effect can be taken into account with the aid of the modified Goodman's or Soderberg's equation [6].

EXAMPLE APPLICATION

A metallic element is to be subjected to $n_1 = 0.4\, N_1$ stress cycles of the amplitude a_1 and $n_2 = 0.6\, N_2$ stress cycles of the amplitude a_2 so that the total fatigue damage (1) would equal unity :

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} = 0.4 + 0.6 = 1.0$$

If the number n_1 of stress cycles of the amplitude a_1 remains unchanged, what number n_2^* of stress cycles of the amplitude a_2 could be accepted at the fatigue strength exponent m = 3 if (11) were used ?

Solution

From (2) the following is obtained :

$$N_j a_j^m = K \qquad n_j A_j^m = K \quad (14)$$

so that :

$$\frac{a_j}{A_j} = \left(\frac{n_j}{N_j}\right)^{1/m} \quad (15)$$

Hence (11) gives :

$$\Delta = \sum_j \left(\frac{n_j}{N_j}\right)^{2/m} = 1 \quad (16)$$

for the cycle numbers, n_j, comprised within the interval (13).

In the considered case :

$$\Delta = 0.4^{2/3} + 0.6^{2/3} = 0.54 + 0.71 = 1.25 > 1$$

Consequently, in order to obtain the value $\Delta = 1$ the number n_2 should be reduced to :

$$n_2^* = (1 - 0.54)^{3/2} N_2 = 0.31\, N_2$$

CONCLUSIONS

- From the presented example it follows that the formula (16) – as well as its version (11) – is too conservative, especially for higher fatigue strength exponents *m* and lower cycle ratios n_j / N_j.
- Moreover, (16) is inapplicable to the cycle numbers $n_j < K/L^m$.
- Hence the conclusion can be drawn that the Palmgren-Miner rule is superior to (11) and (16).
- However, bearing in mind that the fatigue lifetime prediction is subjected to uncertainties, the above defined fatigue-critical stress amplitudes A_j as well as the quantities a_j / A_j and $(a_j / A_j)^2$ may be useful in design considerations as additional criterial parameters.

Appraised by Marek Sperski, Assoc.Prof.,D.Sc.

NOMENCLATURE

a – stress amplitude
a_j – stress amplitude at *j-th* stress level
A_j – *j-th* fatigue-critical stress amplitude (that leads to failure in n_j cycles)
D – total fatigue damage acc. to (1),
E – Young modulus
K – fatigue strength coefficient in (2)
L – maximum stress amplitude satisfying (2), above which low-cycle fatigue may occur
m – fatigue strength exponent in (2),
n – number of stress cycles
n_j – number of cycles at the stress amplitude a_j, number of cycles to cause failure at the stress amplitude A_j
N – number of stress cycles to cause failure
N_j – number of cycles to cause failure at the stress amplitude a_j
w_j – energy expenditure per cycle
W_j – failure energy
Z – fatigue limit under fully reversed stress
Δ – total fatigue damage acc. to (11)
ν – poisson's ratio
σ – stress amplitude
$\phi_{n_j}(a_j)$ – maximum strain energy of distortion transferred to unit volume in n_j stress cycles of the amplitude a_j
$\phi_{n_j}(A_j)$ – maximum strain energy of distortion transferred to unit volume in n_j stress cycles of the amplitude A_j
$\phi_{N_j}(a_j)$ – maximum strain energy of distortion transferred to unit volume in N_j stress cycles of the amplitude a_j
Ω – total fatigue damage acc. to (3).

BIBLIOGRAPHY

1. Kujawski D.: *Fatigue failure criterion based on strain energy density.* Mechanika Teoretyczna i Stosowana, No 1, 1989
2. Palmgren A.: *Die Lebensdauer von Kugellagern.* Verfahrenstechnik, No 68, 1924
3. Miner M.A.: *Cumulative damage in fatigue.* Journal of Applied Mechanics, No 67, 1945
4. Fatemi A., Yang L.: *Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials.* Int. Journal of Fatigue, No 1, 1998
5. Blake A. (Ed.) : *Handbook of mechanics, materials and structures.* J. Wiley & Sons. New York, 1985
6. Almar-Naess A. (Ed.) : *Fatigue handbook. Offshore steel structures.* Tapir Publishers. Trondheim, 1985
7. Kocańda S., Szala J.: *Fundamentals of fatigue calculations* (in Polish). PWN. Warszawa, 1997
8. Skelton R.P.: *Energy criterion for low cycle fatigue failure.* Mater. Sci. Tech., No 7, 1991

Conference

HYDRONAV 2003

On 22 - 23 October 2003 Ship Design and Research Centre, Gdańsk, organized 15th yearly conference on :

Hydrodynamics in Ship Design, Safety and Operation

Its participants had an opportunity to be acquainted with 28 papers devoted to the following problems :

- Ship resistance, propulsion and propellers
- Ship manoeuvrability
- Ship motions and seakeeping
- Sea loads on ship structures
- Safety of ships
- High speed crafts
- Special problems in ship hydrodynamics and design

The presented papers were prepared by representatives of scientific research centres and ship classification institutions : 6 - from Poland, 9 - from Russia, Great Britain, Denmark, Iran, Germany, Ukraine, USA, India and Switzerland.