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Experimental verification of a new mathematical model of ventilation of semi-closed circuit breathing apparatus

SUMMARY

In the paper course and results of experimental verification of mathematical models of ventilation of breathing space of semi-closed circuit rebreather (SCR), proposed in [1], are presented and discussed.

To simulate the respiratory process a unique laboratory stand was used. Evaluation of correctness of the assumed ventilation models was performed with the use of APW-6M underwater diving apparatus. Results of the experimental investigations were compared with those calculated by means of the theoretical models.

A higher modelling accuracy was observed in the case of application of two-bag SCR mathematical model than in the case when the earlier, simplified model was used.

INTRODUCTION

The aim of the presented work was to experimentally verify a mathematical model of semi-closed circuit breathing apparatus (SCR) ventilation of the constant breathing gas dosage, proposed in [1] and preliminarily tested by a manned research [4,6]. Two models are presented in detail in Appendix B and C of this paper for one and two-bag SCR. It would enable modelling of the processes appearing in the SCR. It would also save time and costs of design of the equipment and decompression technology. The existing lack of an adequate mathematical model of the diving systems makes it necessary to reconstruct several times the prototype after preliminary investigations. Moreover such model, tested and positively verified would enable evaluation of the currently used diving systems.

However manned experiments are hazardous and not enough precise [1, 2, 5]. Hence unmanned tests to simulate the respiratory process by mechanical representation of pulmonary ventilation and gas exchange by catalytic burning of acetone [5] during breathing would be more appropriate. The processes should be precisely and repeatedly represented in such simulator, although not any detail of the real respiratory cycle is required to be taken into account.

For this task a unique laboratory stand [5] was built to simulate the respiratory process.

THE APPLIED METHOD OF EXPERIMENTAL VERIFICATION

The evaluation of correctness of the proposed ventilation model was carried out under the following assumptions :

➢ approximate form of respiration process versus time is sinusoidal [1, 2, 4]
➢ the respiratory quotient $\varepsilon = 0.75$ (the volumetric ratio of emitted carbon dioxide amount and that of consumed oxygen)
➢ the simulation is realised with the use of the above mentioned, unique laboratory stand
➢ the APW-6M underwater diving apparatus [3] shown in Fig.1,2 is applied for the experimental verification.

The tested attachment which enables oxygen uptake and carbon dioxide emission, was connected to the respiratory and metabolic process simulator presented in [5].
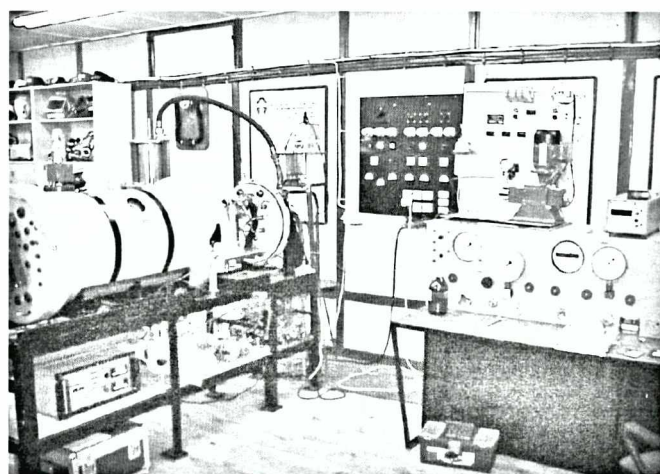

Fig.1. Experimental respiratory and metabolic hyperbaric process simulator stand
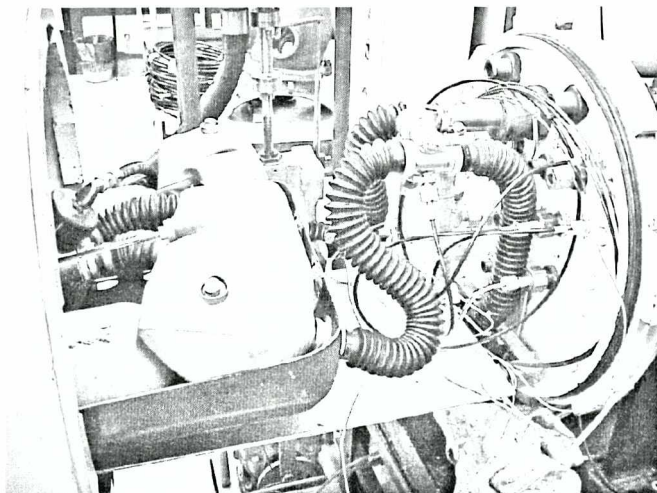
Fig.2. APW-6M SCR underwater diving apparatus connected to the simulator

Air was used as the breathing medium supplied to SCR. The SCR constant gas metering system maintained gas flow at the level of $\dot{V}_f = (8.61 \pm 0.06)$ dm^3·min^{-1} [3]. Experimental results concerning the breathing gas composition recorded by Multiwarn II gas analyser (see Appendix A) are presented in Tab.1.

Tab.1. Experimental results of oxygen contents versus time in the gas mixture inspired from APW-6M SCR

Time [s]	Oxygen contents [%vol] for run №								Mean of O₂ content [%vol]
	1	2	3	4	5	6	7	8	
0	20.9	20.9	20.9	20.9	20.9	20.9	20.9	20.9	20.9
10	20.9	20.8	20.9	20.9	20.8	20.8	20.0	19.8	20.6
20	19.6	19.8	20.0	20.3	19.4	19.6	18.6	18.5	19.5
30	18.3	18.8	18.9	19.0	17.9	18.6	17.7	17.6	18.3
40	17.5	18.0	18.1	18.1	17.0	18.0	16.8	16.7	17.5
50	16.8	17.2	17.2	17.3	16.1	17.3	16.1	16.0	16.7
60	16.2	16.5	16.6	16.6	15.4	16.6	15.4	15.4	16.1
70	15.7	16.0	16.0	16.0	14.8	16.1	14.9	15.0	15.6
80	15.1	15.5	15.5	15.5	14.3	15.7	14.4	14.6	15.1
90	14.7	15.1	15.1	15.0	13.9	15.3	14.0	14.4	14.7
100	14.4	14.8	14.7	14.6	13.5	15.0	13.7	13.9	14.3
110	14.0	14.4	14.3	14.3	13.3	14.8	13.4	13.7	14.0
120	13.8	14.2	14.1	14.0	12.9	14.6	13.1	13.5	13.8
130	13.3	13.9	13.8	13.8	12.7	14.4	12.9	13.3	13.5
140	13.2	13.6	13.6	13.5	12.6	14.1	12.7	12.7	13.3
150	13.1	13.3	13.5	13.4	12.4	14.0	12.5	12.6	13.1
160	12.9	13.1	13.3	13.2	12.3	13.9	12.4	12.5	13.0
170	12.7		13.2	13.1	12.2	13.8	12.3	12.4	12.8
180	12.6		13.1	12.9	12.1	13.7	12.2	12.3	12.7
190	12.5		13.0	12.8	12.1	13.6	12.1	12.2	12.6
200	12.3		12.9	12.6	12.0	13.5	12.0	12.1	12.5
210	12.4		12.8	12.5	11.9	13.4	12.0	12.0	12.4
220	12.3		12.7	12.4	11.9	13.4	11.9	11.9	12.4
230	12.2		12.7	12.3	11.9	13.3	11.8	11.9	12.4
240	12.2		12.6	12.2	11.8	13.2	11.8		12.3
250	12.1		12.5		11.8	13.1	11.8		12.2
260	12.1		12.5		11.8	12.9	11.7		12.2
270	11.9		12.4		11.7	12.7			12.2

Apart from oxygen content, CH_4, CO and CO_2 contents were measured. CH_4 and CO_2 contents were below measuring threshold, whereas CO content values reached 100 ppm.

PRESENTATION AND DISCUSSION OF THE RESULTS

To elaborate experimental results the same method was applied as that used in the preliminary model verification [4]. Such approach was necessary as the metering nozzle being placed in the reaction zone could change its characteristics, therefore acetone dosage (metering) could be different due to established conditions. Verification of the model was performed at assumption that the oxygen molar fraction in the fresh breathing medium $x_s = 0.122$ mol·mol^{-1}, is constant [3,4] :

$$\dot{\upsilon} = \frac{x_f - x_s}{1 - x_s} \dot{V}_f = \frac{0.209 - 0.122}{1 - 0.122} 8.61 = (0.85 \pm 0.07) \text{dm}^3 \cdot \text{min}^{-1} \tag{1}$$

Systematic error of oxygen consumption was determined by means of the total differential method :

$$\Delta\dot{\upsilon} = \left|\left(\frac{\partial\dot{\upsilon}}{\partial x_f}\right)\right| \cdot \Delta x_f + \left|\left(\frac{\partial\dot{\upsilon}}{\partial x_s}\right)\right| \cdot \Delta x(\infty) + \left|\left(\frac{\partial\dot{\upsilon}}{\partial\dot{V}_f}\right)\right| \cdot \Delta\dot{V}_f \cong 0.07 \, \text{dm}^3 \cdot \text{min}^{-1}$$

$$\left|\left(\frac{\partial\dot{\upsilon}}{\partial x_f}\right)\right| \cdot \Delta x_f = \frac{1}{1 - x_s} \dot{V}_f \cdot \Delta x_f =$$

$$= \frac{1}{1 - 0.122} 8.61 \cdot 0.001 = 0.0098 \, \text{dm}^3 \cdot \text{min}^{-1}$$

$$\left|\left(\frac{\partial\dot{\upsilon}}{\partial x(\infty)}\right)\right| \cdot \Delta x_s = \frac{1 - x_f}{1 - x_s} \dot{V}_f \cdot \Delta x_s =$$

$$= \frac{1 - 0.209}{1 - 0.122} 8.61 \cdot 0.007 = 0.0543 \, \text{dm}^3 \cdot \text{min}^{-1}$$

$$\left|\left(\frac{\partial\dot{\upsilon}}{\partial\dot{V}_f}\right)\right| \cdot \Delta\dot{V}_f = \frac{x_f - x_s}{1 - x_s} \Delta\dot{V}_f =$$

$$= \frac{0.209 - 0.122}{1 - 0.122} 0.06 = 0.0059 \, \text{dm}^3 \cdot \text{min}^{-1}$$

The obtained experimental relationship of the mean oxygen contents versus time (Tab.1) made it possible to compare, in Tab.2 and Fig.3, the measured data with the results calculated from the one bag SCR mathematical model [1] (see Appendix B). In Fig.3 the solid line represents the results obtained from the mathematical model (9⁰ Appendix B), and the points with marked measurement accuracy limits are the experimental results.

It is worth of notice that the curve is somewhat turned about some pivoting point, despite of satisfactory fitting of the theoretical curve and experimental results (the curve is contained within the calculated limits of the maximum systematic error) as well as correct representation of the theoretical curve by the measurement points. According to the error propagation theory, occurence of random errors should be manifested by a statistical scatter. However, occurence of a systematic statistical error was manifested by the observed mutual

Tab.2. *Comparison of oxygen contents measured in the inspiration bag of SCR with those calculated by means of the mathematical model [1]*

Time t	Measured C(t)	Calculated C(t)	Time t	Measured C(t)	Calculate C(t)
[s]	[%vol]	[%vol]	[s]	[%vol]	[%vol]
± 1	± 0.7	-	± 1	± 0.7	-
0	20.9	20.900	140	13.3	13.653
10	20.6	19.849	150	13.1	13.481
20	19.5	18.925	160	13.0	13.330
30	18.3	18.114	170	12.8	13.197
40	17.5	17.401	180	12.7	13.080
50	16.7	16.774	190	12.6	12.978
60	16.1	16.223	200	12.5	12.888
70	15.6	15.740	210	12.4	12.809
80	15.1	15.315	220	12.4	12.739
90	14.7	14.941	230	12.4	12.678
100	14.3	14.613	240	12.3	12.624
110	14.0	14.324	250	12.2	12.577
120	13.8	14.071	260	12.2	12.536
130	13.5	13.848	270	12.2	12.499

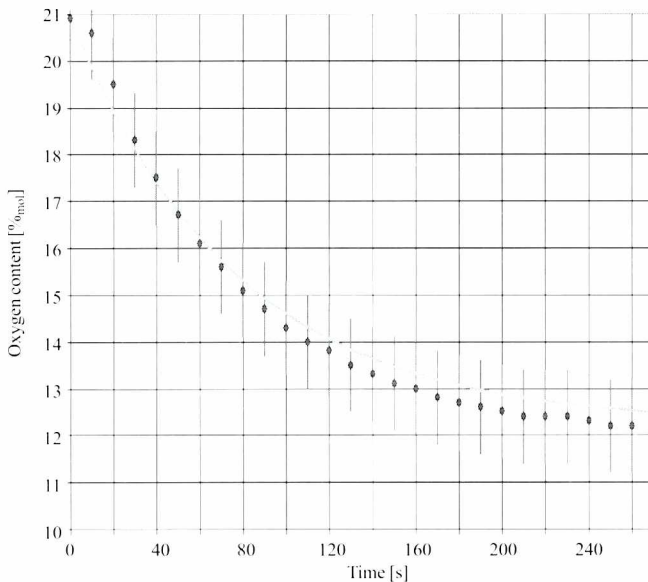C(t) – percentage content of oxygen [C(t) = x(t) 100%]

Fig.3. *Comparison of oxygen content measured in the inspiration bag of SCR with those calculated by means of the mathematical model [1]*

displacement of the experimental curve delineated by the measurement points and the theoretical one. Those effects have not been observed here.

However, a different kind of displacement of the experimental curve and the theoretical one can be observed here . This suggests some inaccuracy of the applied theoretical model. A model that was earlier developed for the two-bag SCR [1] (see also Appendix C) can be alternatively used. It seems that the earlier conclusions concerning negligible, from the practical point of view, differences between the one and two-bag SCR models [1], would be not very adequate as it is possible to measure these differences during experiments with the use of the respiratory and metabolic process simulator.

The two-bag SCR mathematical model
with respect to oxygen consumption is as follows (Appendix C) :

$$\dot{\upsilon} = \frac{\dfrac{p}{p_o} \cdot \dfrac{x_f - x_s}{x_f} \dot{V}_p + \dot{V}_f}{1 + \dfrac{p}{p_o} \cdot \dfrac{1 - x_s}{x_f} \cdot \dfrac{\dot{V}_p}{\dot{V}_f}} \qquad (2)$$

To verify the above mentioned idea
the following schedule of additional experiments was assumed :

- after gas mixture composition has been stabilised at the depth of 0 m H_2O, the experiment is switched to that at the depth of 10 m H_2O
- after gas mixture composition has been stabilised at the depth of 10 m H_2O, the experiment is switched to that at the depth of 20 m H_2O
- after gas mixture composition has been stabilised at the depth of 20 m H_2O, the experiment is switched back to that at the depth of 10 m H_2O
- after gas mixture composition has been stabilised at the depth of 10 m H_2O, the experiment is switched back to that at the depth of 0 m H_2O

Results of five so-arranged experiments are presented in Tab.3.

For the carried-out experiments, the maximum, calculated from (2), oxygen contents in the breathing mixture contained in the inspiration bag cannot be theoretically higher than 4%vol when changing the depths from 0 m H_2O to 20 m H_2O. Therefore the oxygen content differences should be well visible during the so-planned experiments. If the calculated oxygen consumption (Tab.3) is considered as measurement results of the same parameter and the investigated population of the results is of the normal distribution (according to Liliefor's, Shapiro's-Wilk's and K-S statistical tests) then it will be justified to conclude that the mean oxygen consumption amounts to (1.22 ± 0.02) dm³·min⁻¹.

Tab.3. *Variation of oxygen content in the inspiration from SCR gas mixture and resulting oxygen consumption, measured during the additional experiments*

Experiment №	Depth	Increasing pressure		Decreasing pressure	
		oxygen content	oxygen consumption	oxygen content	oxygen consumption
	[m H₂O]	[%vol]	[dm³·min⁻¹]	[%vol]	[dm³·min⁻¹]
	± 1	± 0.2	–	± 0.2	–
1	0	14.8	1.142	13.1	1.279
	10	12.0	1.126	–	–
	20	10.7	1.150		
2	0	13.5	1.247	12.8	1.302
	10	10.4	1.255	10.4	1.255
	20	9.4	1.255		
3	0	14.1	1.199	13.6	1.239
	10	10.5	1.247	10.7	1.231
	20	9.9	1.215		
4	0	13.4	1.255	12.9	1.295
	10	10.2	1.271	10.3	1.263
	20	9.6	1.239		
5	0	13.8	1.223	13.8	1.223
	10	11.4	1.175	11.3	1.183
	20	10.8	1.142		

CONCLUSIONS

The stable value of oxygen consumption, at about 4% oxygen content variability, observed, with high occuracy, in the experiments including 5 runs for 3 depths, suggests that the theoretical model developed for two-bag SCR, correctly represents behaviour of the real object, APW-6M SCR in this case. Therefore the two-bag SCR mathematical model (2) can be deemed more adequate than the earlier, simplified mathematical model (1) [4].

Appraised by Zbigniew Korczewski, Assoc.Prof.,D.Sc.

NOMENCLATURE

p	-	pressure at a given water depth [kPa]
p_o	-	atmospheric pressure [kPa]
ppm	-	content expressed by parts per million
SCR	-	semi-closed circuit breathing apparatus (rebreather)
t	-	time
$\dot{V}_f$	-	metering rate of the fresh breathing medium [$dm^3 \cdot min^{-1}$]
$\dot{V}_p$	-	pulmonary ventilation rate [$dm^3 \cdot min^{-1}$]
x	-	oxygen molar fraction [$mol \cdot mol^{-1}$]
x_f	-	oxygen molar fraction in the fresh breathing medium [$mol \cdot mol^{-1}$]
x_s	-	the stable value of oxygen molar fraction in the inspired breathing bag

$$x_s = \lim_{t \to \infty} x(t) \; [mol \cdot mol^{-1}]$$

Δ	-	absolute error
ε	-	respiratory quotient [$m^3 \cdot m^{-3}$]
$\dot{\upsilon}$	-	oxygen consumption rate [$dm^3 \cdot min^{-1}$]
%LEL	-	percentage of the lower explosive limit
$\%_{vol}$	-	percentage content by volume

BIBLIOGRAPHY

1. Kłos R.: *Mathematical models of ventilation of hyperbaric objects* (in Polish). Ergonomia No 18/1995
2. Kłos R.: *Breathing space ventilation in diving apparatuses of semi-closed circuit and constant breathing gas dosage* (in Polish). Ergonomia No 18/1995
3. Kłos R.: *Diving apparatuses with regeneration of breathing medium* (in Polish). KOOPgraf. Poznań, 2000
4. Kłos R.: *Mathematical modelling of the breathing space ventilation for semi-closed circuit diving apparatus*. Biocybernetics and Biomedical Engineering No 22/2002
5. Kłos R.: *Metabolic simulator supports diving apparatus researches*. Sea Technology No 12/2002
6. Reimers S.D., Hansen O.R.: *Environmental control for hyperbaric applications*. US Navy Experimental Diving Unit Report No 25/1972

Acknowledgement
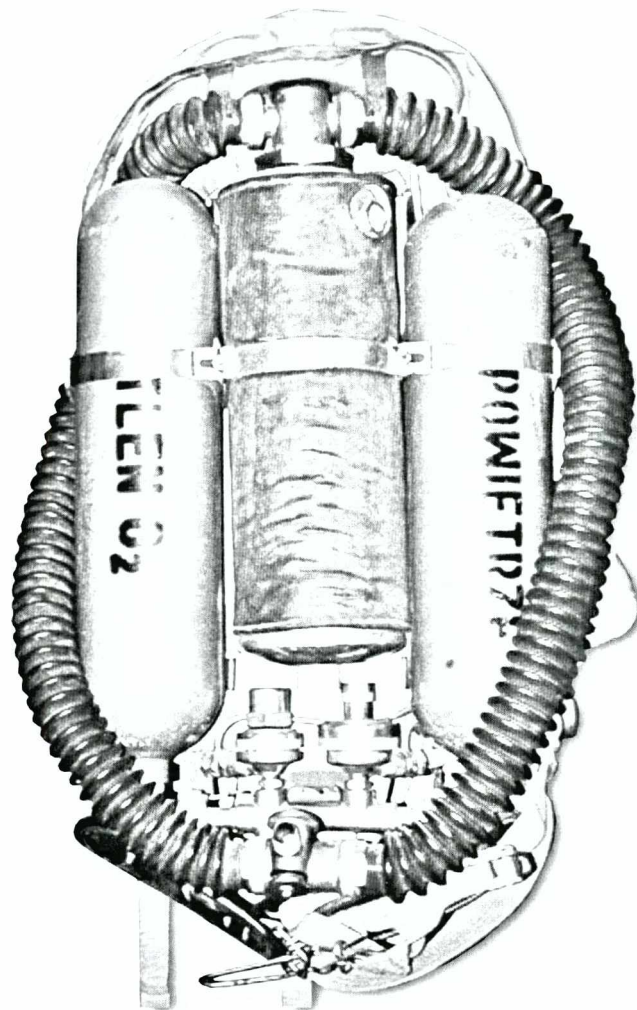
Fig.4. Polish historical construction of APW-3 one-bag SCR

Appendix A. Selected technical data of Multiwarn II gas analyser

Multiwarn II SEP 8314060 № ARPN-4194			
Environmental parameters :	-20 ÷ 40°C 700 ÷ 1300 hPa 10 ÷ 95 % relative humidity		
Working time :	over 8 hours		
Frequency of charging :	every 3 weeks (if it is not working)		
Dimensions :	55 mm x 110 mm x 65 mm		
Weight :	1 kg		

Sensor type		Sensor of CO_2	Sensor of CO	Sensor of H_2/CH_4	Sensor of O_2
		spectrophotometer IR	electrochemical sensor	pelistor	magneto-dynamic sensor
Principle of operation	Measuring range	$0 \div 25\ \%_{vol}$	$0 \div 200$ ppm	$0 \div 100$ % LEL	$0 \div 25\ \%_{vol}$
	Zero repeatability	$\leq \pm 0.01\ \%_{vol}$	$\leq \pm 2$ ppm	$\leq \pm 1\%$ LEL	$\leq \pm 0.2\ \%_{vol}$
	Sensitivity	$\leq \pm 5\ \%_{vol}$	$\leq \pm 1\%$ of the measured value	$\leq \pm 2.5\ \%$ of the measured value	$\leq \pm 1\%$ of the measured value
	Pressure effect	$\leq \pm 0.16\ \%$ of the measured value /hPa	$\leq \pm 0.01\ \%$ of the measured value /hPa	$\leq \pm 0.1\ \%$ of the measured value /hPa	$\leq \pm 0.01\ \%$ of the measured value /hPa
	Long - term drift	$\leq \pm 0.4\ \%$ of the measured value per month	$\leq \pm 1\ \%$ of the measured value per month	$\leq \pm 1\ \%$ of the measured value per month	$\leq \pm 1\ \%$ of the measured value per month
	Chemical compounds upsetting measurements	aldehydes, ketones, water	hydrocarbons and other organic compounds	carbon monoxide and other organic compounds	chlorine, ethane, nitrogen dioxide

Appendix B. Derivation of the relationship between oxygen molar fraction and time for the one-bag premix constant metering SCR by means of the integral calculus [1]

The scheme of oxygen balance in the breathing bag of the one-bag semi-closed circuit diving apparatus	
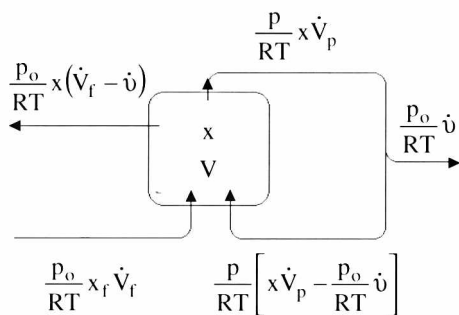
H – water depth [m H_2O]

p – pressure at a given water depth [kPa]

p_o – atmospheric pressure [kPa]

R – universal gas constant [$J \cdot K^{-1} \cdot mol^{-1}$]

t – time

T – absolute temperature [K]

V – breathing bag volume [dm^3]

$\dot{V}_f$ – metering of the fresh breathing medium [$dm^3 \cdot min^{-1}$]

$\dot{V}_p$ – pulmonary ventilation [$dm^3 \cdot min^{-1}$]

x – oxygen molar fraction in the inspiration bag [$mol \cdot mol^{-1}$]

x_o – initial oxygen molar fraction

x_f – oxygen molar fraction in the fresh breathing medium [$mol \cdot mol^{-1}$]

$\dot{\upsilon}$ – oxygen consumption [$dm^3 \cdot min^{-1}$]

Assumption	1°	$\dot{V}_f \neq f(H) \qquad \dot{\upsilon} \neq f(H) \qquad \dot{V}_p \neq f(H)$	
	2°	$\dot{V}_f = \dot{V}_f(p = p_o) \qquad \dot{\upsilon} = \dot{\upsilon}(p = p_o)$	
Thesis		$x = f(t)$	
Proof (evidence)	1°	$\dfrac{p}{RT} V \dfrac{\partial x}{\partial t} = \dfrac{p_o}{RT} \dot{V}_f x_f - \dfrac{p_o}{RT} \dot{\upsilon} - \dfrac{p_o}{RT} (\dot{V}_f - \dot{\upsilon}) x$	from the oxygen balance in the breathing bag
	2°	$\dfrac{\partial x}{\partial t} = \dfrac{p_o}{p} \cdot \dfrac{\dot{V}_f x_f - \dot{\upsilon}}{V} - \dfrac{p_o}{RT} \cdot \dfrac{\dot{V}_f - \dot{\upsilon}}{V} x$ fixed as : $\begin{cases} a = \dfrac{p_o}{p} \cdot \dfrac{\dot{V}_f x_f - \dot{\upsilon}}{V} = const \\ b = \dfrac{p_o}{p} \cdot \dfrac{\dot{V}_f - \dot{\upsilon}}{V} = const \end{cases}$	as for the stable conditions : $\dot{V}_f = const$ $\dot{V}_p = const$ $\dot{\upsilon} = const$
	3°	$\partial x = (a - bx)\partial t$	from 2°
	4°	$\dfrac{\partial x}{bx - a} + \partial t = 0$	from 3° by dividing by (bx-a)
	5°	$\displaystyle\int \dfrac{dx}{bx - a} + \int dt = C$ where : C – integral constant	from 4° and integral definition
	6°	$\dfrac{1}{b} \ln|bx - a| + t = C$ $\ln|bx - a| \equiv b(C - t) = C_1 - bt$ where : C_1 – the new constant	from 5°
	7°	$\exp(C_1 - bt) = C_2 \exp(-bt) \equiv bx - a$ where : C_2 – other new constant	from 6° and natural logarithm definition
	8°	If for boundary conditions t→0 ⇒ x→x_o then $C_2 = bx_o - a \Rightarrow (bx_o - a)\exp(-bt) = bx - a$	from 7°
	9°	$x(t) = \left(x_o - \dfrac{\dot{V}_f x_f - \dot{\upsilon}}{\dot{V}_f - \dot{\upsilon}} \right) \exp\left(-\dfrac{p_o}{p} \dfrac{\dot{V}_f - \dot{\upsilon}}{V} t \right) + \dfrac{\dot{V}_f x_f - \dot{\upsilon}}{\dot{V}_f - \dot{\upsilon}}$	from 2° i 8° **q.e.d.**

Appendix C. Derivation of the formula for stable value
of oxygen molar fraction for the two-bag premix constant metering SCR [3]

$$\frac{p_o}{RT}\dot{\upsilon}\,d\tau$$

$$\frac{p}{RT}x(i)\dot{V}_p d\tau \qquad \frac{p}{RT}\left[x(i)\dot{V}_p - \frac{p_o}{p}\dot{\upsilon}\right]d\tau$$

For : $\dot{V}_f \gg \dot{\upsilon}$

V_x $n_x(i)$ $x(i)$	V_y $n_y(i)$ $y(i)$

$$\frac{p_o}{RT}y(i)\left(\dot{V}_f - \dot{\upsilon}\right)d\tau$$

$$\frac{p_o}{RT}x_f\dot{V}_f d\tau \qquad \frac{p}{RT}y(i)\dot{V}_p d\tau$$

Indices :

- $d\tau$ – time interval
- i – current number of time intervals
- $x.y$ – oxygen molar fraction in the inspiration and expiration bag, respectively

Indices :
- x – of inspiration bag
- y – of expiration bag
- s – stable value
- f – of fresh medium
- p – of pulmonary ventilation

$$n_x(i+1) = n_x(i) + \frac{p_o}{RT}\dot{V}_f \cdot x_f \cdot d\tau + \frac{p}{RT}\dot{V}_p \cdot y(i) \cdot d\tau - \frac{p}{RT}\dot{V}_p \cdot x(i) \cdot d\tau \quad\Big|\quad N_x = \frac{p \cdot V_x}{RT}$$

$$\Delta x = \frac{1}{V_x}\left[\frac{p_o}{p}\dot{V}_f \cdot x_f + \dot{V}_p \cdot y(i) - \dot{V}_p \cdot x(i)\right] \cdot d\tau \quad\Big|\quad \Delta x = x(i+1) - x(i)$$

$$\text{Dla} \quad \Delta x = 0 \implies x(i) \to x_s \wedge y(i) \to y_s \qquad x_s = \lim_{t\to\infty} x(t) \qquad y_s = \lim_{t\to\infty} y(t)$$

$$\dot{V}_p[x_s - y_s] = \frac{p_o}{p}\dot{V}_f \cdot x_f \quad\to\quad x_s = \frac{p_o}{p}\cdot\frac{\dot{V}_f}{\dot{V}_p}x_f + y_s$$

$$n_y(i+1) = n_y(i) +$$

$$+\frac{p}{RT}\dot{V}_p \cdot x(i) \cdot d\tau - \frac{p_o}{RT}\dot{\upsilon} \cdot d\tau - \frac{p_o}{RT}\left(\dot{V}_f - \dot{\upsilon}\right)\cdot y(i) \cdot d\tau - \frac{p_o}{RT}\dot{V}_p y(i) \cdot d\tau \quad\Big|\quad N_y = \frac{p \cdot V_y}{RT}$$

$$\Delta y = \frac{1}{V_y}\left[\dot{V}_p \cdot x(i) - \frac{p_o}{p}\dot{\upsilon} - \frac{p_o}{p}\left(\dot{V}_f - \dot{\upsilon}\right)y(i) - \dot{V}_p \cdot y(i)\right]\cdot d\tau \quad\Big|\quad \Delta y = y(i+1) - y(i)$$

$$\text{Dla} \quad \Delta y = 0 \implies x(i) \to x_s \wedge y(i) \to y_s$$

$$\dot{V}_p[x_s - y_s] = \frac{p_o}{p}\left[\dot{\upsilon} + \left(\dot{V}_f - \dot{\upsilon}\right)y_s\right]$$

$$y_s = \frac{\dot{V}_f \cdot x_f - \dot{\upsilon}}{\dot{V}_f - \dot{\upsilon}}$$

$$x_s = \frac{p_o}{p}\cdot\frac{\dot{V}_f}{\dot{V}_p}x_f + \frac{\dot{V}_f \cdot x_f - \dot{\upsilon}}{\dot{V}_f - \dot{\upsilon}}$$

where :

- $x(i)$ – oxygen molar fraction in the inspiration bag at the moment $t = i\,d\tau$ [mol mol^{-1}]
- $y(i)$ – oxygen molar fraction in the expiration bag at the moment $t = i\,d\tau$ [mol mol^{-1}]
- $n_x(i)$ – number of oxygen moles in the inspiration bag at the moment $t = i\,d\tau$ [mol]
- $n_y(i)$ – number of oxygen moles in the expiration bag at the moment $t = i\,d\tau$ [mol]
- N_x – the total mole number in the inspiration bag
- N_y – the total mole number in the expiration bag
- V_x – the inspiration bag volume
- V_y – the expiration bag volume.

The remaining nomenclature as in Appendix B