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Design of the robust PID course-keeping control system for ships

SUMMARY

The paper presents the design of PID ship course-keeping autopilot of high steering performance and which is robust against ship dynamic motion variations. The basis for the design was a simple linear model of ship dynamic motion, but model's variations due to velocity were also considered.

The PID controller coefficients were selected by using the performance indices: the settling time and ITAE integral performance index. The designed PID controller was tested by means of computer simulations. For that aim a nonlinear, directionally unstable ship model was used.

INTRODUCTION AND MATHEMATICAL DESCRIPTION

Ship motion dynamics is strongly non-linear and difficult for identification. Moreover ship control system is expected to work properly under all possible sea states and changes of ship speed and loading. In terms of the control theory this means that the model of the process is time-variant. The goal of the paper is to design a ship course-keeping autopilot which has high steering performance and is robust against dynamic variations.

The basis for the design is Nomoto's 1st order model, a simple and often-used model of ship planar motion dynamics, namely :

$$T \cdot \dot{r} + r = k\delta \;\Rightarrow\; P(s) = \frac{r(s)}{\delta(s)} = \frac{k}{1+sT} \tag{1}$$

where :

- T, k - dynamic model parameters
- r - course turning rate ($=d\psi/dt$)
- ψ - actual ship course angle
- t - time
- $\dot{r}$ - turning rate time derivative ($=dr/dt$)
- δ - rudder deflection angle
- $P(s)$ - transfer function describing ship dynamic motion
- s - Laplace variable.

The parameters T, k are strongly coupled to the ship velocity V [3] :

$$k = k_m \frac{V}{V_m} \qquad T = T_m \frac{V_m}{V} \tag{2}$$

where :

- k_m, T_m - parameters identified for the velocity V_M
- V - actual ship velocity
- V_m - the ship velocity for which the ship dynamic motion model was identified.

A scheme of the proposed control system is presented in Fig.1.

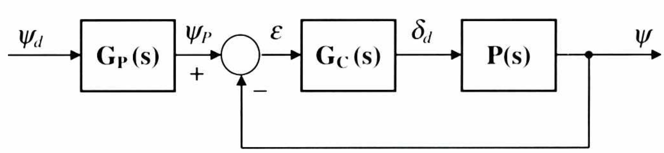

Fig.1. Scheme of ship course-keeping control system
Notation : P(s) - mathematical model of ship motion dynamics; $G_P(s)$, $G_C(s)$ - two controllers to be designed; ψ_d - desired ship course angle; ψ - actual ship course angle; δ_d - desired ruder actuator; ψ_p - Gp controller output signal ; ε - feedback loop error signal

A PID controller, very popular in ship autopilots, is considered as $G_C(s)$. The popularity of PID (Proportional Integral Derivative) controllers results from their functional simplicity and robust performance in a wide range of operating conditions. Therefore the transfer function of $G_C(s)$ is :

$$G_C(s) = H_1 + \frac{H_2}{s} + H_3 s = \frac{H_3 s^2 + H_1 s + H_2}{s} \tag{3}$$

where : H_1, H_2, H_3 - coefficients of the proportional, integral and differential part of the controller, respectively.

The coefficients are selected by using the following performance indices : the settling time t_r and ITAE (Integral Time Absolute Error) performance index. The ITAE index is defined as the integral of product of time t and absolute error |e(t)| as follows :

$$\eta_{ITAE} = \int_0^T t \cdot |e(t)| \cdot dt \tag{4}$$

where : $e(t) = \psi_d - \psi$

By assuming that the closed-loop transfer function has the following form :

$$T(s) = \frac{L(s)}{M(s)} = \frac{b_0}{s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0} \tag{5}$$

the ITAE index optimum coefficients b_0, b_1, ..., b_n which minimize the index for the unit step transient response, can be calculated [1]. For example, for 3rd order transfer function T(s) the denominator M(s) should be :

$$M(s) = s^3 + 1.75\omega_o s^2 + \omega_o^2 s + \omega_o^3 \tag{6}$$

where : ω_o – natural frequency.

For linear systems the natural frequency can be approximately determined by using the 2% settling time t_r (i.e. that required for the system to settle within 2% of the final output signal) and the damping factor ξ :

$$\omega_o \approx \frac{4}{t_r \xi} \tag{7}$$

In the designed control system the damping factor ξ is unknown, but for the ITAE optimum system its value is near $\xi = 0.8$. Therefore the approximate expression for ω_o is :

$$\omega_o \approx \frac{5}{t_r} \tag{8}$$

The closed-loop transfer function of the control system (Fig.1) is :

$$T(s) = G_P(s)\frac{G_C(s) \cdot P(s)}{1 + G_C(s) \cdot P(s)} = G_P(s)\frac{\frac{k}{T}\left(H_3 s^2 + H_1 s + H_2\right)}{s^3 + s^2\frac{1 + kH_3}{T} + s\frac{kH_1}{T} + \frac{kH_2}{T}} \tag{9}$$

The PID controller coefficients H_1, H_2, H_3 are selected to create the ITAE optimum transfer function denominator (6). The PID coefficients obtained by using (8) are as follows :

$$H_1 = 53.75T / (t_r^2 k)$$
$$H_2 = 125T / (t_r^3 k) \tag{10}$$
$$H_3 = (8.75T/t_r - 1) / k$$

The pre-filter $G_P(s)$ is so determined that the closed-loop T(s) does not have any zeros, as required by equation (5) :

$$G_P(s) = \frac{125T}{(8.75T - t_r)t_r^2 s^2 + 53.75Tt_r s + 125T} \tag{11}$$

When the formulas (2) are applied into the equations (10) and (11) the control system model is made dependent on ship speed variations.

SHIP MOTION MODEL FOR SIMULATION

For computer simulations the Norrbin's non-linear model of directionally unstable ship motion was used :

$$T_m \cdot \dot{r} + a_3 \cdot r^3 + a_2 \cdot r^2 + a_1 \cdot r + a_0 = k_m \delta \tag{12}$$

where the non-linear components :

$$a_3 \cdot r^3 + a_2 \cdot r^2 + a_1 \cdot r + a_0 = C(r) \tag{13}$$

describe the ship course-stability curve.

For the directionally unstable vessels : $a_1 = -1$; and for the stable ones : $a_1 = 1$.

The block diagram of the model is presented in Fig.2.

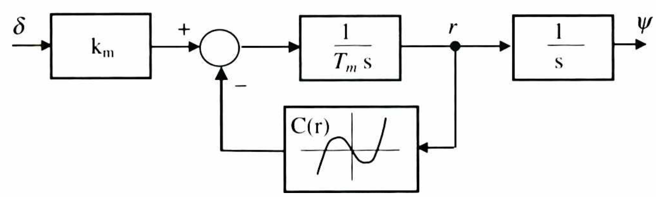

Fig.2. Ship dynamic motion model for simulation

The model (12) was identified for the 1:24 scale model of 176 000 DWT tanker [4]. Main parameters of the tanker and its scale model (used in the Ship Handling Research and Training Centre in Iława, Poland) are presented in Tab.1. The identified model parameters are shown in Tab.2.

Tab.1. Main particulars of the considered tanker and its scale model

Main particulars	Tanker	1:24 scale model
Length overall	330.65 [m]	13.78 [m]
Length between perpendiculars	324.00 [m]	13.50 [m]
Beam	47.00 [m]	2.38 [m]
Draft – loaded condition	20.60 [m]	0.86 [m]
Displacement – loaded condition	323 660 [t]	22.83 [t]
Draft – ballast condition	12 [m]	0.5 [m]
Displacement – ballast condition	176 000 [t]	12.46 [t]
Speed	15.2 [kn]	3.1 [kn]

Tab.2. Parameters of the directionally unstable motion model of the considered tanker

a_3	a_2	a_1	a_0	k_m	T_m
1.2322	0.0665	-1	0.07536	0.1256	48.5

RUDDER MODEL

In most ships the rudder deflection angle is restricted within the range from –35° to 35°. The turning angle rate is also limited. Moreover the real rudder usually has the dead zone which causes the rudder blade insensitive to small changes of the desired rudder angle. Hence behaviour of the real steering gear is simulated by means of a model shown in Fig.3 [2].

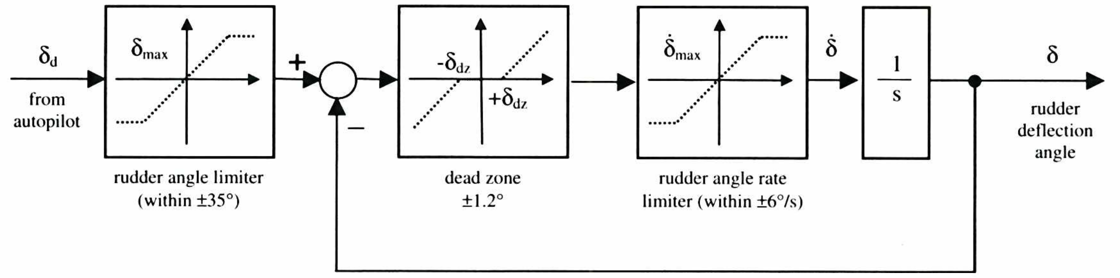

Fig.3. Scheme of steering gear model used for simulation

SIMULATION RESULTS

Fig.4 and 5 shows simulation results for the nominal model (with the same k_m and T_m values as those used in the design) and for the modified model, respectively. In Tab.3 and Fig.6 influence of the parameters k_m and T_m on the ITAE performance index, is presented.

The ITAE index value strongly increases when the ship inertial time T_m increases and k_m decreases. Therefore values of the parameters T, k of the basic mathematical model (1) should be set near the upper limit of possible changes of T_m, and near the lower limit of expected changes of k_m, respectively.

Tab.3. Dependence of the ITAE performance index on T_m and k_m ship motion model parameters

k_m	T_m	ITAE
0.1256	30	3014.43
	40	3169.96
	48.5	3233.85
	60	3743.09
	70	4583.65

T_m	k_m	ITAE
48.5	0.0700	6318.0
	0.0900	4241.0
	0.1256	3233.9
	0.2000	3006.9

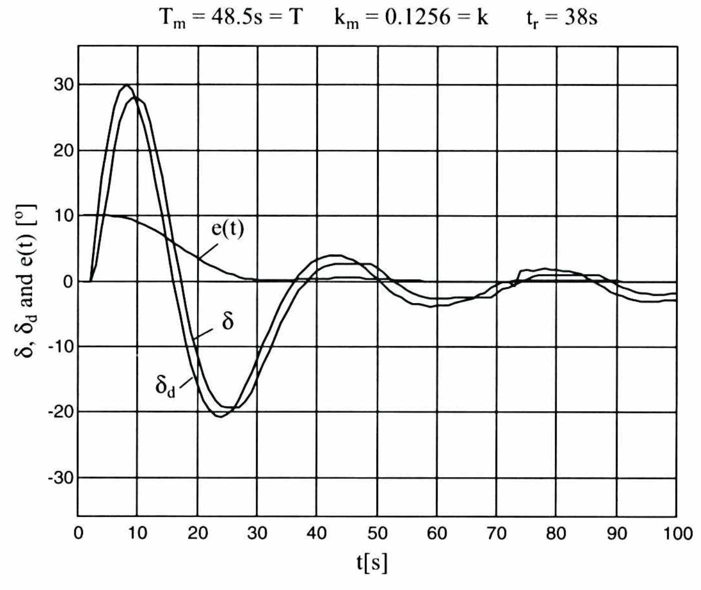

Fig.4. Rudder deflection angles : desired δ_d and real δ, and course angle error e(t) versus time t for the nominal model (k_m = k; T_m = T); t_r = 38s

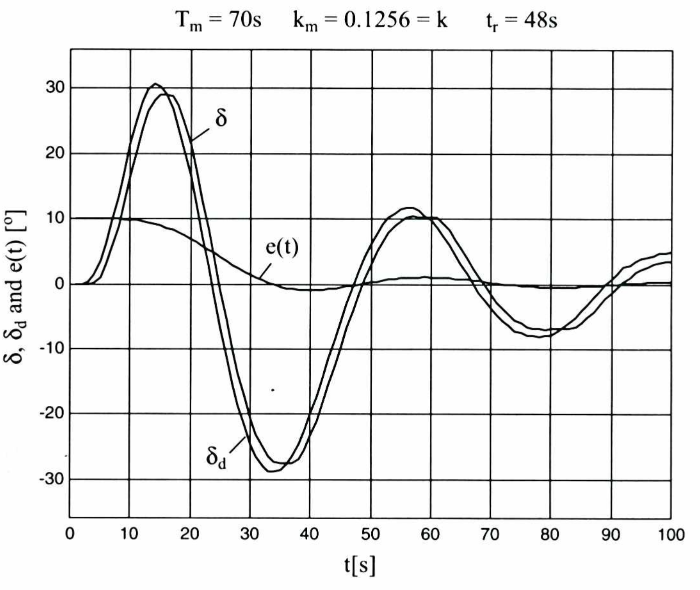

Fig.5. Rudder deflection angles : desired δ_d and real δ, and course angle error e(t) versus time t for the modified model (T_m > T); t_r = 48s

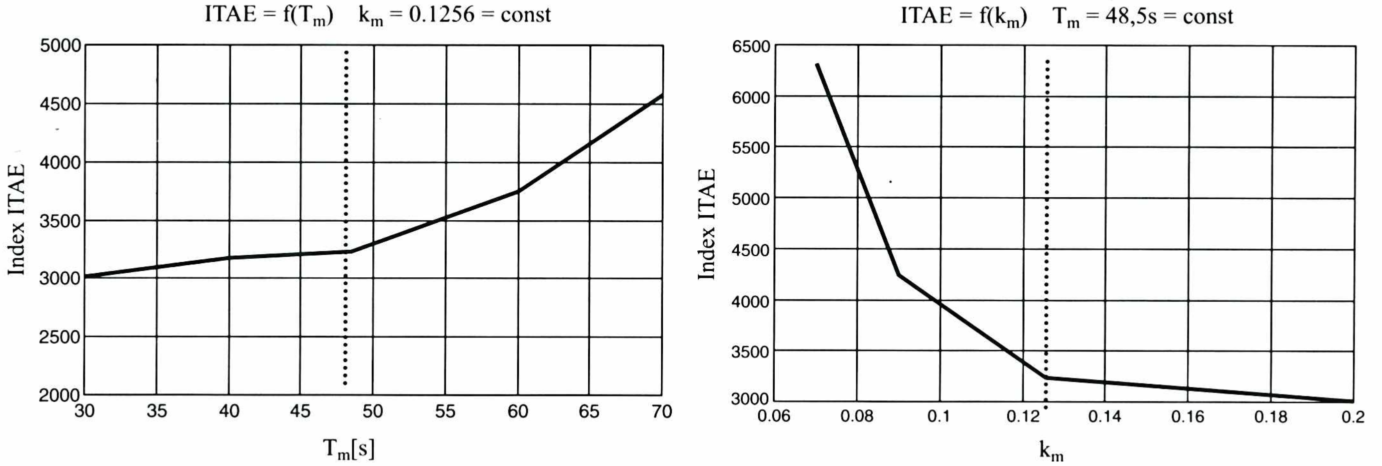

Fig.6. Dependence of the ITAE integral performance index on T_m and k_m ship motion model parameters

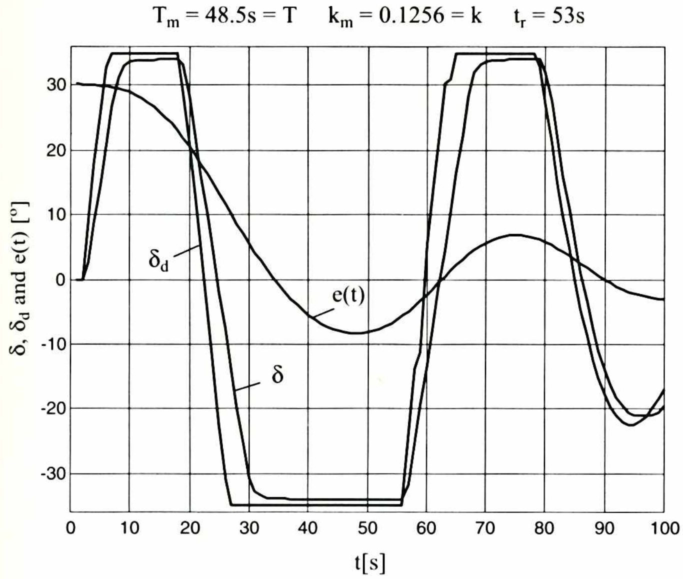

Fig.7. *Rudder deflection angles : desired δ_d and real δ, and course angle error e(t) versus time t for the nominal model ($k_m = k$; $T_m = T$) at the big overshoot when the rudder stalling (saturation) occurs (30° turning); t_r = 53s*

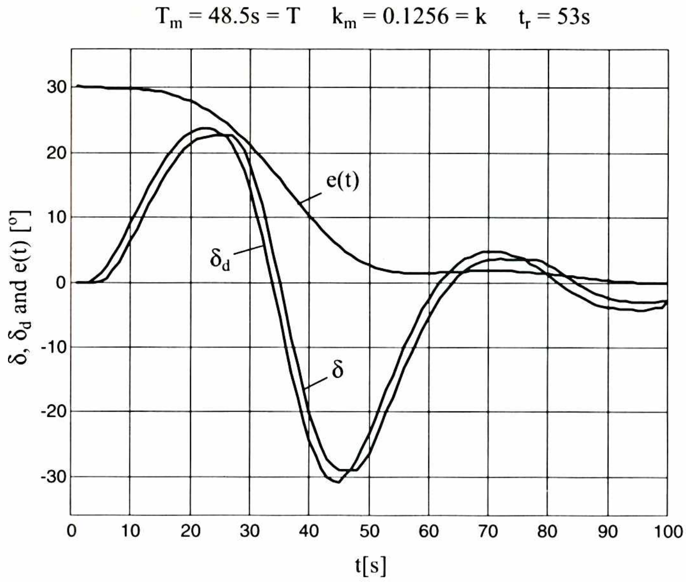

Fig.8. *Rudder deflection angles : desired δ_d and real δ, and course angle error e(t) versus time t for the nominal model ($k_m = k$; $T_m = T$) with the course turning rate limiter set at 1.2°/s (30° turning); t_r = 53s*

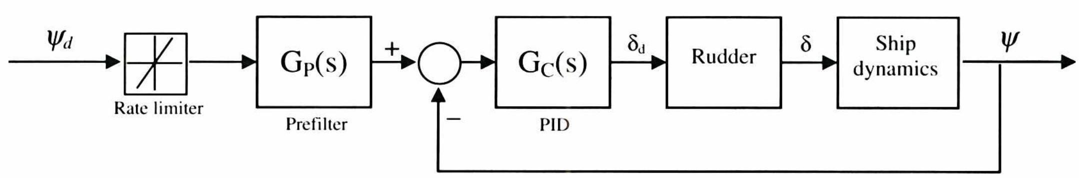

Fig.9. *Scheme of final version of ship course-keeping control system*

The control system does not work properly when rudder stalling (saturation) occurs during, for example, big desired turning (Fig.7). The control system can be protected against big overshoot and long regulation time in two ways :

1st by making the settling time t_r in the final control relationships (10) and (11) dependent on the turning angle, e.g. the bigger turning angle the longer t_r time.

2nd by limiting the desired course turning rate $d\psi/dt$ to the value which does not cause the rudder stalling (saturation).

The second method was applied. The trial simulation of the system with the course turning rate limiter set at 1.2°/s is presented in Fig.8. The final version of the ship course-keeping control system is presented in Fig.9.

Appraised by Józef Lisowski, Prof.,D.Sc.

NOMENCLATURE

a_0, a_1, a_2, a_3 - parameters of a nonlinear model of ship dynamic motion (of ship course-stability curve)
C(r) - ship course-stability curve
$e(t) = \psi_d - \psi$ - course angle error
$G_P(s)$, $G_C(s)$ - two controllers to be designed
H_1, H_2, H_3 - proportional, integral and differential part of the PID controller, respectively
k, T - parameters of Nomoto model of ship dynamic motion
k_m, T_m - parameters of Nomoto model of ship dynamic motion, identified for the velocity V_m
P(s) - ship dynamic motion model
$r = d\psi / dt$ - course turning rate
$\dot{r} = dr / dt$ - turning rate time derivative
s - Laplace variable
t - time
t_r - 2% settling time (time required for the system to settle within 2% of the final output signal)
V - ship velocity
V_m - the ship velocity for which the ship dynamic motion model was identified
δ - rudder deflection angle
δ_d - desired rudder actuator
δ_{dz} - rudder angle dead zone
δ_{max} - maximum rudder deflection angle
$\dot{\delta}_{max}$ - maximum rudder turning rate
η_{ITAE} - integral performance index
ξ - damping factor
ψ - actual ship course angle
ψ_d - desired ship course angle
ω_o - natural frequency

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