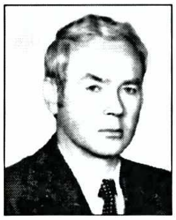

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On the use of theory of energy transformation systems in fatigue calculations of steel elements

SUMMARY

The paper deals with simulation of fatigue performance of steel elements under uniaxial periodic loading in the high-cycle regime. For this purpose a sinusoidal stress model has been defined which is equivalent in terms of fatigue life to the actual stress given in the form of Fourier series. The equivalence conditions follow from the theory of energy transformation systems and are related to internally and externally dissipated energy by the average shear strain energy and Kelvin-Voigt's model of the material. Thereby the cycle counting and use of a fatigue damage accumulation rule is avoided.

INTRODUCTION

Fatigue has been the subject of research investigations for over a hundred years, and despite the progress made, failures continue to occur. This situation is due in part to the complex nature of the fatigue process and the stress-material-environmental interactions involved therein. From a review of the literature on multiaxial fatigue it appears that no single universal criterion exists for different materials and loading conditions. Presently, special attention is being paid to energy criteria [1]. As far as the high-cycle fatigue is concerned, the criteria related to elastic energy and formulated by means of static energy hypotheses, e.g. the Huber-Mises-Hencky shear strain energy hypothesis, are considered satisfactory. Among them , a criterion based on the average strain energy [2] should be mentioned because it showed relatively small errors in comparison with experimental results. However, this criterion does not include the phase shifts between normal and shear stress components.

On the other hand, every system in operation has one energy input with the power P_t , and two outputs: the desired power P_u and the externally dissipated power P_{de}. Along with the internally accumulated power P_{da} the following balance equation can be written :

$$P_t = P_u + P_{da} + P_{de} \tag{1}$$

According to the theory of energy transformation systems [3,4], the dissipation capacity of every system, as well as any volume of material in a system, is finite. Thus, the internal accumulation of dissipated energy, E_{da}, is finite too, that is :

$$E_{da} = \int_0^{\Theta} P_{da}(\Theta, P_{de})d\Theta \le E_{db} \tag{2}$$

where Θ is the operation time of the system, and E_{db} is a limit value (breakdown value) of the dissipation capacity of the system (volume). Due to finite dissipation capacity of the system (volume), its life ends with the breakdown time $\Theta = \Theta_b$. However, the latter depends also on the externally dissipated power P_{de}, see (2). Hence the following conclusion can be drawn :

A stress model adopted in fatigue calculations and an actual stress can be regarded as equivalent in terms of fatigue life when during a sufficiently long period of time the internally and externally dissipated energies per unit volume in these two stress states are equal, respectively.

Such an approach to stress modelling was used in [5,6] under assumption that the components of stress tensor are physically and statistically independent of each other. In the present paper the average shear strain energy is taken into account and an alternative stress model is considered in the case of uniaxial periodic stress.

FATIGUE CRITERIA FOR STEEL ELEMENTS UNDER UNIAXIAL PERIODIC STRESS

Let the zero mean stress $\tilde{\sigma}(t)$ of the period T_0 be produced by an axial force and given in the form of Fourier series :

$$\tilde{\sigma}(t) = \sum_{p=1}^{\infty} \sigma_p \sin(p\omega_0 t + \alpha_p) \tag{3}$$

where σ_p and α_p are the amplitude and phase angle of p-th term, and :

$$\omega_0 = \frac{2\pi}{T_0} \tag{4}$$

is the fundamental circular frequency of the stress. Such stress can be viewed as falling :

either into the safe region of the basic variable space if :

$$\left[\tilde{\sigma}(t)\right]_{max} \leq F \tag{5}$$

where F is the fatigue limit under fully reversed tension-compression,

or into the failure region if :

$$\left[\tilde{\sigma}(t)\right]_{max} > F \tag{6}$$

In what follows it is assumed that the high-cycle fatigue failure region is not exceeded, i.e., that :

$$\left[\tilde{\sigma}(t)\right]_{max} \leq L \tag{7}$$

where L is the maximum stress amplitude satisfying equation of the σ - N curve (Wöhler curve) :

$$N\sigma^m = K \tag{8}$$

In (8), N is the number of stress cycles to failure, σ is the stress amplitude under sinusoidal tension-compression, and K and m are the material dependent constants.

As stated earlier, the average shear strain energy will be here taken into account. In the considered stress state, the shear strain energy per unit volume is :

$$\Phi = \frac{1+\nu}{3E}\tilde{\sigma}^2 \tag{9}$$

where : E – Young modulus
ν – Poisson's ratio.

In order to allow for the internal energy dissipation the Kelvin--Voigt's model of the material :

$$\tilde{\sigma} = E\tilde{\varepsilon} + \eta\dot{\tilde{\varepsilon}} \tag{10}$$

can be utilized. Here η is the coefficient of internal viscous damping of the material, and

$$\tilde{\varepsilon}(t) = \sum_{p=1}^{\infty}\varepsilon_p \sin\left(p\omega_0 t + \beta_p\right) \tag{11}$$

is the strain corresponding to the stress (3). Now the main goal is to determine the amplitudes σ_{eq}, ε_{eq} and frequency ω_{eq} of the reduced stress and strain :

$$\tilde{\sigma}_{eq}(t) = \sigma_{eq}\sin\left(\omega_{eq}t + \varphi\right) \tag{12}$$

$$\tilde{\varepsilon}_{eq}(t) = \varepsilon_{eq}\sin\left(\omega_{eq}t + \psi\right) \tag{13}$$

equivalent to the actual stress and strain in terms of fatigue lifetime. For this purpose the expression for the shear strain energy per unit volume in the equivalent stress state, is used :

$$\Phi_{eq} = \frac{1+\nu}{3E}\tilde{\sigma}_{eq}^2 \tag{14}$$

and the corresponding Kelvin-Voigt's model :

$$\tilde{\sigma}_{eq} = E\tilde{\varepsilon}_{eq} + \eta\dot{\tilde{\varepsilon}}_{eq} \tag{15}$$

Thus, the integral average values of Φ and Φ_{eq} over the period T_0 become :

$$\frac{1}{T_0}\int_0^{T_0}\Phi(t)dt = \frac{1+\nu}{3ET_0}(A+B) \tag{16}$$

$$\frac{1}{T_0}\int_0^{T_0}\Phi_{eq}(t)dt = \frac{1+\nu}{3ET_0}\left(A_{eq}+B_{eq}\right) \tag{17}$$

where :

$$A = E^2\int_0^{T_0}\tilde{\varepsilon}^2 dt \qquad B = 2E\eta\int_0^{T_0}\tilde{\varepsilon}\dot{\tilde{\varepsilon}}dt + \eta^2\int_0^{T_0}\dot{\tilde{\varepsilon}}^2 dt$$

$$A_{eq} = E^2\int_0^{T_0}\tilde{\varepsilon}_{eq}^2 dt \qquad B_{eq} = 2E\eta\int_0^{T_0}\tilde{\varepsilon}_{eq}\dot{\tilde{\varepsilon}}_{eq}dt + \eta^2\int_0^{T_0}\dot{\tilde{\varepsilon}}_{eq}^2 dt \tag{18}$$

The modelling method presented in this paper is based on the assumption that the function A is proportional to the externally dissipated energy and that the function B is proportional to the internally dissipated energy. Consequently, the equivalence conditions can be read as :

$$A = A_{eq} \tag{19}$$

$$B = B_{eq} \tag{20}$$

To make use of these conditions, it is convenient to express the circular frequency of the equivalent stress / strain as :

$$\omega_{eq} = k\omega_0 \tag{21}$$

where k is a natural number to be determined.

With accounting for (11), (13), (18) and (21), the conditions (19) and (20) give :

$$\varepsilon_{eq}^2 = \sum_{p=1}^{\infty}\varepsilon_p^2 \tag{22}$$

$$k^2\varepsilon_{eq}^2 = \sum_{p=1}^{\infty}\left(p\varepsilon_p\right)^2 \tag{23}$$

Since the quotient :

$$\kappa = \left[\frac{\sum_{p=1}^{\infty}\left(p\varepsilon_p\right)^2}{\sum_{p=1}^{\infty}\varepsilon_p^2}\right]^{1/2} \tag{24}$$

is a real number, k can be approximated as :

$$k = \mathrm{Round}\ (\kappa) \tag{25}$$

According to (13) and (15), the amplitude of the equivalent stress is :

$$\sigma_{eq} = \varepsilon_{eq}\left[E^2 + (\eta\omega_{eq})^2\right]^{1/2} \tag{26}$$

For structural steels the values of η are relatively small so that in practice the following relationships are justified :

$$\sigma_{eq} = E\varepsilon_{eq} \qquad \sigma_p = E\varepsilon_p \tag{27}$$

and (22) and (24) can be expressed in the form :

$$\sigma_{eq} = \left(\sum_{p=1}^{\infty} \sigma_p^2 \right)^{1/2} \tag{28}$$

$$\kappa = \left[\frac{\sum_{p=1}^{\infty} (p\sigma_p)^2}{\sum_{p=1}^{\infty} \sigma_p^2} \right]^{1/2} \tag{29}$$

By taking advantage of the equivalent stress, the criterion in design for an infinite fatigue life can be fomulated as :

$$\sigma_{eq} < F \tag{30}$$

By making use of (8), the criterion in design for a finite fatigue life becomes :

$$\frac{\omega_{eq} \sigma_{eq}^m T_d}{2\pi K} < 1 \tag{31}$$

where T_d is the required design life.

EXAMPLE

Task : Compare fatigue lives, Θ_1 and Θ_2 , at the stresses :

$$\tilde{\sigma}_1(t) = \sigma \sin \omega_0 t \qquad F < \sigma < L$$

$$\tilde{\sigma}_2(t) = \sigma \left[H(t) - 2H\left(t - \frac{\pi}{\omega_0} \right) + 2H\left(t - 2\frac{\pi}{\omega_0} \right) - \ldots \right]$$

shown also in Fig.1, where H is the Heaviside's step function.

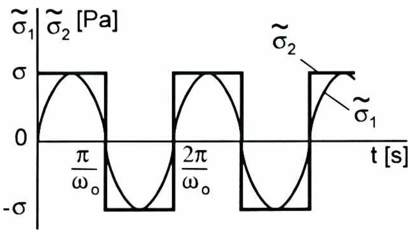

***Fig. 1.** Time histories of the stresses $\tilde{\sigma}_1$ and $\tilde{\sigma}_2$*

Solution : Fourier expansion of the function $\tilde{\sigma}_2(t)$ yields :

$$\tilde{\sigma}_2(t) = \frac{4\sigma}{\pi} \left(\sin \omega_0 t + \frac{1}{3} \sin 3\omega_0 t + \right.$$

$$\left. + \frac{1}{5} \sin 5\omega_0 t + \frac{1}{7} \sin 7\omega_0 t + \ldots \right)$$

With these four terms retained, (21), (25), (28) and (29) give :

$$\sigma_{eq} = 1.378\sigma \quad \kappa = 1.848 \quad k = 2 \quad \omega_{eq} = 2\omega_0$$

Thus, according to (8) and (31) :

$$\Theta_1 = \frac{2\pi K}{\omega_0 \sigma^m} \qquad \Theta_2 = \frac{2\pi K}{\omega_{eq} \sigma_{eq}^m} \qquad \frac{\Theta_1}{\Theta_2} = k\left(\frac{\sigma_{eq}}{\sigma} \right)^m = 2 \times 1.378^m$$

It means that in such comparison not only stress patterns, but also the fatigue strength exponent m plays an important role.

CONCLUSIONS

In this paper the equivalence conditions are presented which enable a uniaxial periodic stress to be replaced by the sinusoidal stress equivalent, in terms of fatigue life of the material. The fatigue criteria (30) and (31) are based on the assumption that the so defined equivalent stress may be used for simulation of fatigue performance of steel elements subject to uniaxial periodic loading in the high-cycle regime. The assumption follows from the theory of energy transformation systems which is fundamental for all considerations connected with the lifetime of mechanical systems. Hereby the cycle counting and use of a fatigue damage accumulation rule is avoided. Similar approach to a multiaxial periodic stress is considered in [7].

Moreover, despite the infinite Fourier series being used for describing a given periodic stress, only a finite number of its sinusoidal terms can be taken into account in calculation of the equivalent stress. The question remains which terms in Fourier expansion of the given stress can be neglected. However, similar problem appears when the response of structural element is known in the form of a power spectral density curve which represents contributions of an infinite number of vibration modes of the structure and the time history is to be simulated [8].

Appraised by Marek Sperski, Assoc.Prof.,D.Sc.

NOMENCLATURE

E – Young modulus
E_{da} – internally dissipated energy
E_{db} – limit value of the internally dissipated energy
F – fatigue limit under fully reversed tension-compression
H – Heaviside's step function
k – natural number obtained by rounding the number κ
K – fatigue strength coefficient at fully reversed tension-compression
L – maximum stress amplitude satisfying equation of the σ-N curve at fully reversed tension-compression
m – fatigue strength exponent at fully reversed tension-compression
N – number of stress cycles to fatigue failure
P_{da} – internally dissipated power; P_{dc} – externally dissipated power
P_t – input power; P_u – desired output power
t – time; T_d – required design life
T_0 – stress period
α_p – phase angle of p-th term in Fourier expansion of periodic stress
β_p – phase angle of p-th term in Fourier expansion of periodic strain
$\tilde{\varepsilon}$ – periodic strain
ε_p – amplitude of p-th term in Fourier expansion of periodic strain
$\tilde{\varepsilon}_{eq}$ – equivalent strain
ε_{eq} – amplitude of the equivalent strain
η – coefficient of internal viscous damping of the material
Θ – operation time of a system; Θ_b – breakdown time of a system
Θ_1, Θ_2 – fatigue lives; κ – number defined by (24)
ν – Poisson's ratio; σ – amplitude of the sinusoidal stress
$\tilde{\sigma}$ – periodic stress
σ_p – amplitude of p-th term in Fourier expansion of periodic stress
$\tilde{\sigma}_{eq}$ – equivalent stress; σ_{eq} – amplitude of the equivalent stress
φ – phase angle of the equivalent stress
Φ – shear strain energy per unit volume
Φ_{eq} – shear strain energy per unit volume in the equivalent stress state
ψ – phase angle of the equivalent strain
ω_{eq} – circular frequency of the equivalent stress/strain
ω_0 – fundamental circular frequency of periodic stress

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