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Influence of the number of tangential pressure ordinates of the diesel engine on the calculated values of the harmonic amplitude and its phase angle

SUMMARY

In this paper an attempt was undertaken to determine the lowest number of the tangential pressure ordinates, which should be used in harmonic analysis of 4-stroke combustion engine to obtain calculated values of amplitudes and phase angles sufficiently exact for practical applications.

INTRODUCTION

The driving system including an internal combustion engine is always excited to torsional vibrations by the torque periodically generated at all cranks of its crankshaft.

The torque exciting torsional vibrations is also caused by the inertia forces which occur in the crank mechanism. The last mentioned excitations as well as those from the side of an energy consumer, are not considered in this paper.

From the simple geometrical relations shown in Fig.1 it results that the torque M appearing in the crank of the radius r, at the arbitrary angular position α of the crankshaft can be determined by the formula (1) :

$$M = r\frac{\sin(\alpha+\beta)}{\cos\beta}P_g \qquad (1)$$

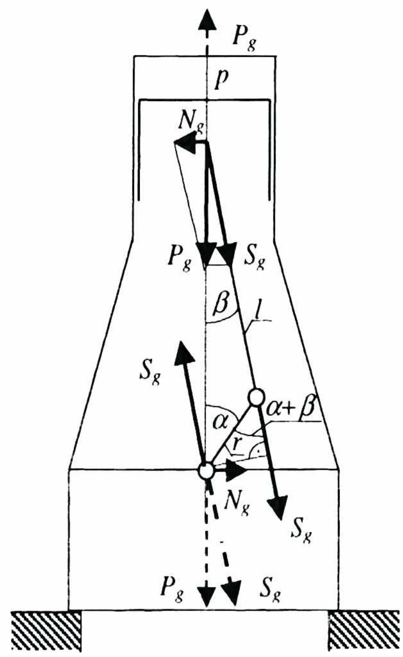

Fig. 1. Geometrical relationships of the crank mechanism

The formula can be transformed, by making the connecting rod angular position β dependent on the crank angle α, and by denoting the ratio of the crank radius r to the connecting rod length l as λ, to the following form :

$$M = r\sin\alpha\left(1+\frac{\lambda\cos\alpha}{\sqrt{1-\lambda^2\sin^2\alpha}}\right)P_g \qquad (2)$$

The force P_g is shortly called the „gas" force. In the trunk piston engines it results from the gases in the working space, having the pressure p exerted onto the piston with rings, as well as the air applied to the opposite side of the piston at the pressure p_k.

The area on which the gas and air acts is equal to the cylinder cross-section area F_{cyl}. In the crosshead engines the piston rod and pressure of the supercharging air in the piston under space as well as that contained in the crankcase should be additionally taken into account.

The tangential force can be obtained by relating the torque M to the radius r. This force related to the cross-section area of the piston represents the so called „tangential" pressure y_g expressed by the formula (3) :

$$y_g = \left(\frac{P_g}{r \cdot F_{cyl}}\right) \sin\alpha \left(1 + \frac{\lambda \cos\alpha}{\sqrt{1-\lambda^2 \sin^2\alpha}}\right) \tag{3}$$

The air pressure in the crankcase and the supercharging air pressure is usually assumed time independent. However the working space pressure p changes within a broad range during the thermodynamic cycle. Information on the course of the pressure changes can be obtained by means of indication of the particular cylinders. The measurement widely described in the subject literature (not referred to in this paper) is deemed extremely difficult and can be realized with a tolerable error only when maintaining many conditions and applying appropriate measuring instruments [1 to 5].

SUBJECT OF THE WORK

Within the scope of this work the pressure-crank angle diagram was obtained by applying a calculation-measurement procedure [8]. The following measured parameters of the thermodynamic cycle were used :

- p_{ind} - mean indicated pressure
- p_{max} - maximum combustion pressure
- p_c - compression end pressure
- p_k - supercharging air pressure
- ε - geometrical compression ratio
- g_e - specific fuel oil consumption
- n - engine speed.

The thermodynamic cycle was so modeled as to obtain the calculated values of the mean indicated pressure and maximum combustion pressure corresponding with those measured. It was obtained by choosing proper parameters of the heat emission process described by the Wibe's equation [6], with simultaneous taking into consideration the energy balance. The heat exchange process between the working medium and working space walls was described by the Woschni's equations [7]. The initial version of the thermodynamic calculations was described in [8]. It was supplemented and developed many times also in the course of elaborating of this paper. All parameters of the thermodynamic cycle were calculated using the 1° step of the crank angle. The tangential pressure was calculated also with the same step by applying the formula (3).

The obtained set of pairs of values, i. e. the crank angle α and tangential pressure y_g was the basis for performing the numerical harmonic analysis. The Fourier's trigonometric function approximating a given set of values is of the following form [4,9,10] :

$$y_{gi} \stackrel{def}{=} y_{g0} + \sum_{h=1}^{k} y_{g\,2h/\tau} \sin\left(\frac{2}{\tau} h\alpha_i + \delta_{2h/\tau}\right) \tag{4}$$

where :

- h - number of the harmonic
- τ - engine cycle factor (for two-stroke engine : 2, for four-stroke engine : 4)
- $2h/\tau$ - order of the harmonic
- $y_{g\,2h/\tau}$ - tangential pressure amplitude of the h-th harmonic of the $2h/\tau$ order
- $\delta_{2h/\tau}$ - phase angle of the h-th harmonic of the $2h/\tau$ order.

According to the principle of Fourier analysis the period of a considered physical phenomenon is always equal to 2π.

Dividing the period of the excitation from the engine side into $m = 2k$ parts the crank angle α_i can be calculated using the following equation :

$$\alpha_i = \frac{2\pi}{m} \cdot i = \frac{2\pi}{2k} \cdot i = \frac{\pi}{k} \cdot i \tag{5}$$

The Fourier series coefficients calculated in compliance with the commonly known expressions [e.g.11] are of the following form :

➔ the free term which represents the mean tangential pressure value :

$$y_{g0} = \frac{1}{2k} \sum_{i=0}^{2k-1} y_{gi} \tag{6}$$

➔ the coefficients of the terms a_h and b_h of the series for h = 1 to k

$$a_h = \frac{1}{k} \sum_{i=0}^{2k-1} y_{gi} \cos\left(i\frac{h\pi}{k}\right) \tag{7}$$

$$b_h = \frac{1}{k} \sum_{i=0}^{2k-1} y_{gi} \sin\left(i\frac{h\pi}{k}\right) \tag{8}$$

➔ the tangential pressure amplitude of h-th harmonic of $2h/\tau$ order :

for h = 1 to k-1

$$y_{g\,2h/\tau} = \sqrt{a_h^2 + b_h^2} \tag{9}$$

for h = k

$$y_{g\,2h/\tau} = \frac{1}{2}\sqrt{a_k^2 + b_k^2} \tag{10}$$

➔ the phase angles of h-th harmonic of 2h/t order :

$$\delta_{2h/\tau} = \mathrm{arctg}\frac{a_h}{b_h} \tag{11}$$

In this paper a four-stroke, supercharged engine was considered of the power output 33 kW per cylinder at the engine speed frequency n = 32 rps. The maximum combustion pressure was about 14 000 kPa. Other working parameters of the engine were also known from the test-bed measurements taken by its producer.

CALCULATION RESULTS

Fig.2 shows the calculated cylinder pressure-crank angle diagram.

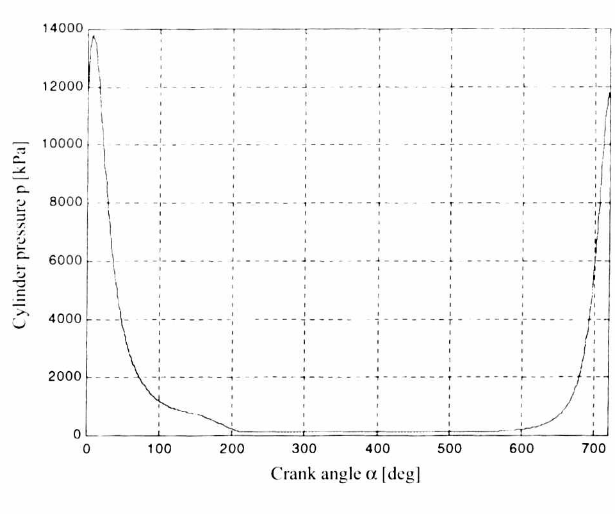

Fig.2. The calculated cylinder pressure - crank angle diagram

As the thermodynamic calculation step was equal to 1° degree of crank angle, the diagram graphically represents 720 pairs of the values : α_i, p_{gi}. Values of the tangential pressure y_{gi} were calculated by applying equation (3) for every i. The obtained set of 720 pairs of the values : α_i, y_{gi} was called the basic set. In Fig.3 the diagram of the calculated tangential pressure y_g is presented in function of the crank angle α.

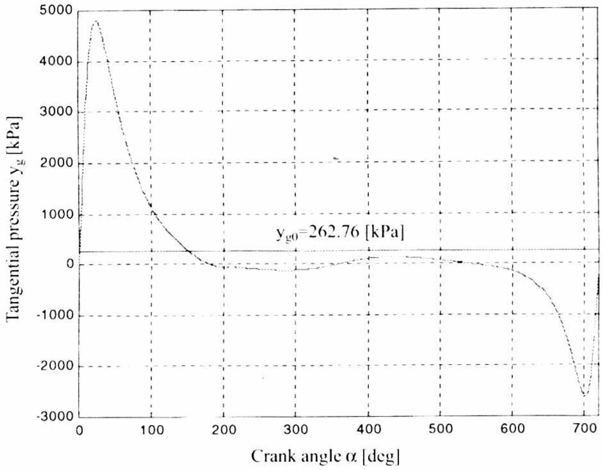

Fig.3. *The calculated tangential pressure y_g in function of the crank angle α*

From the basic set, also other sets with the calculation step equal to a multiple of the basic step can be formed. Hence the sets of the steps of 1, 2, 3, 4, 5, 10, 20 and 30° of crank angle were formed, i.e. these of the ordinate numbers equal to 720, 360, 240, 180, 144, 72, 36, and 24.

The so formed sets were subjected to the Fourier harmonic analysis. The free term y_{g0} as well as the tangential pressure amplitudes $y_{g\,2h/\tau}$ and phase angles $\delta_{2h/\tau}$ were calculated with the use of the equations (6) to (11). Also, by using additionaly (4), the ordinates denoted y_{fi} were calculated. The calculations were performed for the crank angle α_i varying from 0 to 720° by 1° degree of crank angle. The example results of the calculations (for the 24-ordinate set) are presented in Fig.4., where location of 24 points on the plane (α_i, y_{gi}) is denoted by asterisks , the curve denoted by '3' is the tangential pressure based on the basic set values, and the curve denoted by '2'is the relationship $y_f = f(\alpha)$.

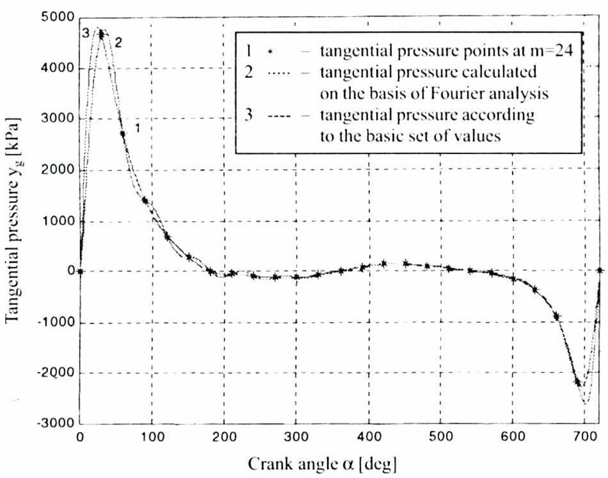

Fig.4. *The calculated tangential pressure in function of the cranks angle α*

As the elements of the 24-ordinate set, belong to the basic set, the curve $y_g = f(\alpha)$ passes through the 24 points and the curve $y_f = f(\alpha)$ does the same as well. However the intermediate values of the ordinates y_{fi} are different.

The mean square deviation was calculated to assess accuracy of the represenation of the tangential pressure with the use of the Fourier series (of amplitudes and phase angles calculated on the basis of m ordinates), as follows :

$$\Delta y_f = \left[\frac{1}{720}\sum_{i=0}^{719}\left(y_{gi} - y_{fi}\right)^2\right]^{\frac{1}{2}} \tag{12}$$

The so calculated value of the mean square deviation Δy_f is an absolute measure dealing with a given example, but directly rather illegible. Hence it was related to the mean tangential pressure y_{g0} of the basic set, shown in Fig.3. In result the mean square relative deviation is described by (13) :

$$\Delta = \frac{\Delta y_f}{\Delta y_{g0}} \cdot 100 \tag{13}$$

Results of the calculations performed for the considered four-stroke engine are presented in Tab.1.

Tab.1. *Accuracy of the representation of tangential pressure – the mean square relative deviation Δ*

Number of ordinates m	720	360	240	180	144	72	36	24
Δ [%]	1.605	1.720	0.110	0.222	0.400	2.172	19.785	81.730

From Tab.1. it results that the value of the mean square relative deviation Δ is the greatest for the smallest number of ordinates m, and it decreases along with increasing number of ordinates. The smallest value is obtained for m = 240. The authors did not succeeded in explanation why Δ values increased again for 360 and 720 ordinates.

Therefore the Fourier harmonic analysis on the basis of 240 ordinates was considered the most accurately representing the tangential pressure, and the values of the amplitudes and phase angles this way calculated were considered accurate.

Tab.2. *The percentage amplitude deviations SD calculated by means of the harmonic analysis for each of h harmonics at different numbers of ordinates m*

Harmonic		SD [%] for ordinate number m							
No.	Order	720	360	240	180	144	72	36	24
0	–	0.00	0.00	0.00	0.00	0.00	0.01	-0.35	-0.36
1	0.5	0.00	0.00	0.00	0.00	0.00	0.01	-0.20	-1.88
2	1.0	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	-1.34
3	1.5	0.00	0.00	0.00	0.00	0.00	0.01	0.04	-1.81
4	2.0	0.00	0.00	0.00	0.00	0.00	0.03	0.03	-2.20
5	2.5	0.00	0.00	0.00	0.00	0.00	0.03	0.03	-3.43
6	3.0	0.00	0.00	0.00	0.00	0.00	0.01	0.07	-5.40
7	3.5	0.00	0.00	0.00	0.00	0.00	0.05	0.05	-7.73
8	4.0	0.00	0.00	0.00	0.00	0.01	0.10	-0.01	-11.16
9	4.5	0.00	0.00	0.00	0.01	0.00	0.08	-0.01	-17.36
10	5.0	0.00	0.00	0.00	0.00	0.00	0.09	-0.26	-24.83
11	5.5	-0.01	-0.01	0.00	-0.01	0.01	0.20	-0.90	-28.70
12	6.0	0.00	0.00	0.00	0.00	0.01	0.27	-1.53	-60.08
13	6.5	0.01	0.01	0.00	-0.02	0.01	0.26	-2.24	–
14	7.0	0.00	0.00	0.00	0.01	0.03	0.43	-4.16	–
15	7.5	-0.02	-0.01	0.00	0.02	0.03	0.66	-6.95	–
16	8.0	0.00	-0.01	0.00	0.01	0.00	0.70	-8.62	–
17	8.5	0.01	0.01	0.00	0.04	0.02	0.96	-6.17	–
18	9.0	-0.01	0.00	0.00	0.00	0.12	1.68	-45.76	–
19	9.5	-0.01	-0.01	0.00	0.00	0.11	2.01	–	–
20	10.0	0.01	-0.01	0.00	0.07	-0.02	2.06	–	–
21	10.5	-0.04	-0.04	0.00	0.01	0.07	3.47	–	–
22	11.0	-0.06	-0.03	0.00	0.05	0.27	5.35	–	–
23	11.5	0.05	0.06	0.00	0.17	0.25	5.94	–	–
24	12.0	0.07	0.05	0.00	0.25	0.14	7.05	–	–
25	12.5	0.17	0.15	0.00	-0.20	0.30	10.55	–	–
26	13.0	-0.19	-0.19	0.00	-0.26	0.14	11.53	–	–
27	13.5	0.17	0.01	0.00	0.37	-0.31	10.13	–	–
28	14.0	0.32	0.32	0.00	0.47	0.06	13.53	–	–
29	14.5	-0.05	0.07	0.00	-0.54	0.54	12.78	–	–
30	15.0	-0.06	-0.07	0.00	-0.76	-0.72	-0.56	–	–
31	15.5	0.30	0.05	0.00	-0.04	-2.76	-9.22	–	–
32	16.0	0.21	0.11	0.00	-0.38	-2.12	-4.18	–	–
33	16.5	0.09	0.22	0.00	-1.14	-0.91	-13.63	–	–
34	17.0	0.57	0.61	0.00	-0.55	-1.58	-31.11	–	–
35	17.5	0.73	0.48	0.00	-0.14	-3.94	-16.29	–	–
36	18.0	-0.09	-0.18	0.00	-1.58	-3.01	-53.25	–	–

The earlier performed harmonic analyses for all the ordinate numbers *m* given in Tab.1. were used to investigate the influence of the number of tangential pressure ordinates on values of the amplitudes and phase angles.

For each of *h* harmonics the percentage deviations SD were calculated (defined as the difference of values of the amplitude at a given *m* and that at m = 240, related to the value of the amplitude calculated for m = 240).

Similarly, the percentage deviation SDE of the phase angle was determined.

Results of the calculations are presented in Tab.2 and 3. For formal reasons they are given with 1/100 % accuracy, although the authors are aware of the practically obtainable measurement accuracy of the pressure within the engine working space.

Tab.3. *The percentage phase angle deviations SDE calculated by means of the harmonic analysis for each of h harmonics at different numbers of ordinates m*

Harmonic		SDE [%] for ordinate number m							
No.	Order	720	360	240	180	144	72	36	24
0	–	–	–	–	–	–	–	–	–
1	0.5	0.00	0.00	0.00	0.00	0.00	-0.02	-0.19	-1.73
2	1.0	0.00	0.00	0.00	0.00	0.00	-0.03	-0.57	-5.51
3	1.5	0.00	0.00	0.00	-0.01	0.05	0.14	-4.05	-33.60
4	2.0	0.00	0.00	0.00	0.00	0.00	0.00	-0.05	-0.42
5	2.5	0.00	0.00	0.00	0.00	0.00	0.00	-0.06	-0.67
6	3.0	0.00	0.00	0.00	0.00	0.00	0.00	-0.08	-0.98
7	3.5	0.00	0.00	0.00	0.00	0.00	0.01	-0.12	-1.48
8	4.0	0.00	0.00	0.00	0.00	0.00	0.00	-0.17	-2.50
9	4.5	0.00	0.00	0.00	0.00	0.00	0.00	-0.26	-4.17
10	5.0	0.00	0.00	0.00	0.00	0.00	0.01	-0.43	-6.72
11	5.5	0.00	0.00	0.00	0.00	0.00	0.02	-0.65	-11.51
12	6.0	0.00	0.00	0.00	0.00	0.00	0.01	-0.95	-19.86
13	6.5	0.00	0.00	0.00	0.00	0.00	0.02	-1.55	–
14	7.0	0.00	0.00	0.00	0.00	0.00	0.04	-2.61	–
15	7.5	0.00	0.00	0.00	0.00	0.00	0.04	-4.21	–
16	8.0	0.00	0.00	0.00	0.01	0.00	0.05	-6.82	–
17	8.5	0.00	0.00	0.00	0.00	0.01	0.13	-11.26	–
18	9.0	0.00	0.00	0.00	0.00	0.01	0.16	-17.49	–
19	9.5	0.00	0.00	0.00	0.01	0.01	0.13	–	–
20	10.0	0.00	0.00	0.00	0.00	0.01	0.28	–	–
21	10.5	-0.01	0.00	0.00	0.00	0.04	0.55	–	–
22	11.0	0.00	0.01	0.00	0.02	0.05	0.67	–	–
23	11.5	0.01	0.00	0.00	0.04	0.03	0.84	–	–
24	12.0	-0.03	-0.03	0.00	-0.02	0.06	1.55	–	–
25	12.5	-0.04	-0.03	0.00	-0.01	0.10	2.32	–	–
26	13.0	0.01	0.01	0.00	0.11	0.12	3.27	–	–
27	13.5	0.02	0.03	0.00	0.16	0.24	5.30	–	–
28	14.0	-0.07	0.05	0.00	-0.01	0.48	7.95	–	–
29	14.5	-0.10	0.09	0.00	-0.01	0.33	9.43	–	–
30	15.0	-0.02	-0.05	0.00	0.21	0.13	12.02	–	–
31	15.5	-0.02	-0.02	0.00	0.21	0.33	17.75	–	–
32	16.0	-0.08	-0.02	0.00	0.00	0.73	20.17	–	–
33	16.5	0.00	0.02	0.00	0.11	0.54	20.29	–	–
34	17.0	0.05	0.01	0.00	0.29	0.10	25.98	–	–
35	17.5	0.14	-0.15	0.00	-0.03	0.29	33.28	–	–
36	18.0	0.14	0.10	0.00	-0.12	0.52	29.97	–	–

COMMENTS ON THE CALCULATION RESULTS, CONCLUSIONS

- The harmonic analysis of the four-stroke engine tangential pressure with the use of **24 ordinates** was performed solely for research purposes.
- At the number of ordinates **m = 36** the percentage deviations SD and SDE can be considered satisfactory up to those of 9-th harmonic.
- The ordinate number **m = 72** is recommended for practical use by the publication [3], a recognized torsional vibration handbook. In this case the percentage deviations SD and SDE are smaller than 1% up to those of 18-th harmonic.
- For the ordinate numbers **m > 72** the percentage deviations SD and SDE are smaller than 1% for all harmonics up to 36-th one, inclusive. Few exceptions concern the harmonics of such small amplitudes that, practically, they have no influence on final results of the calculations.
- If the errors of the calculated amplitudes and phase angles have to be maintained on the level permissible in engineering practice, it can be stated, that **the ordinate number *m* to be applied in the harmonic analysis, should be four times greater than the greatest number of the harmonic which has to be used for the calculations of torsional vibrations.**

Appraised by Jan Kruszewski, Prof.,D.Sc.,M.E.

NOMENCLATURE

a_h, b_h - Fourier series coefficients
F_{cyl} - cross-section of the cylinder liner area
g_e - brake specific fuel consumption
h - number of harmonic
i - number of segments into which the period of tangential pressure is divided
k - half of the number of parts into which the period of excitation is devided
l - length of the connecting rod
m - even number of parts into which the period of excitation is divided
M - torque on one crank of the engine crankshaft
n - engine speed
N_g - piston side-thrust force derived from the gas force
p - pressure of gases in the working space of the engine
p_c - compression pressure
p_{ind} - mean indicated pressure
p_k - supercharging air pressure
p_{max} - maximum combustion pressure
p_g - force shortly called „gas force"
r - crank radius
S_g - connecting rod force derived from the gas force
y_f - tangential pressure calculated by using the results of Fourier analysis
$y_f = f(\alpha)$ - calculated tangential pressure for any crank angle
y_g - tangential pressure
y_{g0} - free term of Fourier series equal to mean tangential pressure
$y_{g\,2h/\tau}$ - amplitude of tangential pressure of the h-th harmonic of the 2h/τ order
α - crank angle
β - connecting rod deflexion angle
$\delta_{2h/\tau}$ - phase angle of the h-th harmonic of the 2h/τ order
Δ_{yf} - mean square deviation
ε - compression ratio
$\lambda = r/l$ - crank radius - connecting rod length ratio
τ - engine cycle factor
SD - percentage deviation calculated as the difference of the h-th harmonic amplitude values obtained from harmonic analysis for m and 240 ordinates, related to that calculated for 240 ordinates
SDE - percentage deviation calculated as the difference of the h-th harmonic phase angle values obtained from harmonic analysis for m and 240 ordinates, related to that calculated for 240 ordinates
index i - position on the crank angle axis

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