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Integrated safety factors in design for infinite and finite fatigue lives of steel elements at multiaxial static and periodic loadings

SUMMARY

The paper deals with strength calculations of steel elements under combined static and periodic loadings. Two design criteria, (i) and (ii), are presented which ensure that not only the multiaxial stress does not exceed the ultimate (or yield) strength of the material but also that the combination of multiaxial mean stress and multiaxial periodic stress does not lead to fatigue failure in design (i) for an infinite fatigue life and (ii) for a finite fatigue life. It is assumed that the stress components are physically independent of each other and that for each of them the corresponding σ-N curve is given. The load modes and material anisotropy are taken into account.

INTRODUCTION

If an engineering component is exposed to static and dynamic loadings both the time-independent and time-varying components of the stress tensor must be taken into account in strength calculations. For example, when a structural element subject to combined in-phase bending and torsion is considered, the resulting stress has the following components :

$$\sigma_x(t) = \bar{\sigma}_x + a_x \sin \omega t$$
$$\sigma_{xy}(t) = \bar{\sigma}_{xy} + a_{xy} \sin \omega t \qquad (1)$$

where :

$\bar{\sigma}_x, \bar{\sigma}_{xy}$ - mean values
a_x, a_{xy} - amplitudes of the stress components
ω - circular frequency.

For calculation purposes in design for infinite fatigue life it is convenient to distinguish :

- partial safety factors, f_s and f_d, related to time-independent and time-varying parts of the stress components, respectively
- fatigue safety factor, δ, relating to fatigue performance under static-dynamic loading conditions
- integrated safety factor, f, which combines f_s and f_d.

According to [1÷3] at the stress (1) these factors are :

$$f_s = \left[\left(\frac{\bar{\sigma}_x}{S_x}\right)^2 + \left(\frac{\bar{\sigma}_{xy}}{S_{xy}}\right)^2\right]^{-1/2} \qquad (2)$$

$$f_d = \left[\left(\frac{\beta_x a_x}{\varepsilon_x F_x}\right)^2 + \left(\frac{\beta_{xy} a_{xy}}{\varepsilon_{xy} F_{xy}}\right)^2\right]^{-1/2} \qquad (3)$$

$$\delta = \left[\frac{1}{F_x^2}\left(\frac{\beta_x a_x}{\varepsilon_x} + \psi_x \bar{\sigma}_x\right)^2 + \frac{1}{F_{xy}^2}\left(\frac{\beta_{xy} a_{xy}}{\varepsilon_{xy}} + \psi_{xy} \bar{\sigma}_{xy}\right)^2\right]^{-1/2} \qquad (4)$$

$$f = f_d\left(1 - f_s^{-1}\right) \quad \text{or} \quad f = f_d\left(1 - f_s^{-2}\right) \qquad (5)$$

where :

S_i $(i = x, xy)$ - ultimate or yield strength at i-th static load
F_i - fatigue limit at i-th zero mean loading
β_i - notch sensitivity index at i-th loading
ε_i - size factor at i-th loading
ψ_i - asymmetry sensitivity index at i-th loading.

Satisfaction of the condition :

$$f \geq 1 \qquad (6)$$

ensures that not only the stress does not exceed the ultimate or yield strength of the material but also the combination of mean stress and time-varying stress components does not lead to fatigue failure.

Similarly, for ductile materials in the three-dimensional state of stress with in-phase components of mean values $\bar{\sigma}_i$ and amplitudes a_i :

$$\sigma_i(t)=\bar{\sigma}_i+a_i\sin\omega t \qquad i=x,y,z,xy,yz,zx \tag{7}$$

the use can be made of (5) and of the modified factors based on the reduced stress [2,4] :

$$f_d=\left(\sum_i b_i^2-b_xb_y-b_yb_z-b_zb_x\right)^{-1/2} \tag{8}$$

$$f_s=\left(\sum_i c_i^2-c_xc_y-c_yc_z-c_zc_x\right)^{-1/2} \tag{9}$$

where :

$$b_i=\frac{\beta_i a_i}{\varepsilon_i F_i} \qquad c_i=\frac{\bar{\sigma}_i}{S_i}$$

In the general case of periodic stress its components :

$$\sigma_i(t)=\sigma_i(t+T_0) \qquad i=x,y,\dots,zx \tag{10}$$

can be expanded into Fourier series :

$$\sigma_i(t)=\bar{\sigma}_i+\sum_{p=1}^{\infty}a_i^{(p)}\sin\left(p\omega_0 t+\alpha_i^{(p)}\right) \tag{11}$$

and modelled with in-phase components [4] :

$$\sigma_i^{(eq)}(t)=\bar{\sigma}_i+a_i^{(eq)}\sin\omega_{eq}t \tag{12}$$

where :

$$\omega_{eq}=k\omega_0 \qquad k=Round(\kappa) \qquad \omega_0=2\pi/T_0$$

$$\kappa=\left[\frac{\sum_i\sum_{p=1}^{\infty}\frac{\eta_i}{E_i^2}\left(pa_i^{(p)}\right)^2}{\sum_i\sum_{p=1}^{\infty}\frac{\eta_i}{E_i^2}\left(a_i^{(p)}\right)^2}\right]^{1/2} \tag{13}$$

$$a_i^{(eq)}=\left\{\frac{8}{k^2T_0}\int_0^{T_0}\left[\sum_{p=1}^{\infty}a_i^{(p)}\sin\left(p\omega_0t+\alpha_i^{(p)}\right)\right]^2\times\right.$$

$$\left.\times\left[\sum_{p=1}^{\infty}pa_i^{(p)}\cos\left(p\omega_0t+\alpha_i^{(p)}\right)\right]^2dt\right\}^{1/4}$$

- $a_i^{(p)},\alpha_i^{(p)}$ - amplitude and phase angle of p-th sinusoidal term in Fourier expansion of i-th stress component, respectively
- E_i - Young modulus or shear modulus, associated with i-th stress component
- k - natural number obtained by rounding the number κ
- T_0 - common period of the stress components
- η_i - internal damping coefficient in Kelvin-Voigt's model of the material, associated with i-th stress component.

The mean values $\bar{\sigma}_i$ in (11) and (12) are the same, and $a_i^{(eq)}$ are the amplitudes of the stress components equivalent in terms of fatigue life to the sinusoidal terms in (11). Consequently, (5) can be used also in the case if in (8) the amplitudes a_i are replaced by $a_i^{(eq)}$.

Of course, (3) ÷ (6) and (8) cannot be utilized in design for a finite fatigue life. Therefore in that case the $\sigma-N$ curves (Wöhler curves) are taken into account. The stress components are assumed physically independent of each other and the corresponding $\sigma-N$ curve is given for each of them.

INTEGRATED SAFETY FACTOR IN DESIGN FOR FINITE FATIGUE LIFE

Let us consider the stress :

$$\sigma(t)=\bar{\sigma}+a\sin\omega t \tag{14}$$

produced by an axial force. In design for a finite fatigue life, the following relations are frequently used [5] :

$$a=\sigma_N\left(1-\frac{\bar{\sigma}}{S_u}\right) \quad \text{(acc. to Goodman)} \tag{15}$$

$$a=\sigma_N\left(1-\frac{\bar{\sigma}}{S_v}\right) \quad \text{(acc. to Soderberg)} \tag{16}$$

$$a=\sigma_N\left[1-\left(\frac{\bar{\sigma}}{S_u}\right)^2\right] \quad \text{(acc. to Gerber)} \tag{17}$$

where :

- σ_N - amplitude of the zero mean normal stress at a given number, N, of cycles to fatigue failure
- a - amplitude of the stress (14) which will lead to that fatigue life
- S_u - ultimate tensile strength
- S_v - tensile yield strength.

In the high-cycle regime, the most widespread relationship between σ_N and N is :

$$N\sigma_N^m=K \tag{18}$$

for

$$F<\sigma_N\le L \tag{19}$$

where :

- F - fatigue limit at fully reversed tension-compression
- K - fatigue strength coefficient
- L - the maximum stress amplitude satisfying (18), above which low-cycle fatigue may occur [6]
- m - fatigue strength exponent.

Denoting :

$$N_a=Ka^{-m}$$

- N_r - number of stress cycles required to achieve a given design life
- $n_a=N/N_r$ - integrated safety factor in design for a finite fatigue life
- $n_{ad}=N_a/N_r$, $f_{as}=S_{u(v)}/\bar{\sigma}$ - partial safety factors

one obtains from (15), (16) and (18) :

$$n_a=n_{ad}\left(1-f_{as}^{-1}\right)^m \tag{20}$$

and from (17) and (18) :

$$n_a = n_{ad}\left(1 - f_{as}^{-2}\right)^m \qquad (21)$$

In the case of multiaxial stress with the in-phase components (7) the conventional strength theories (e.g., Huber-Mises theory) can be used [7]. Therefore in this paper it is suggested to model complex stress patterns under static and periodic loading by the equivalent stress with the in-phase components (12). Then the Huber-Mises theory can be adopted which leads to the reduced stress [4] :

$$\sigma_{red}(t) = \overline{\sigma}_{red} + a_{red}\sin\omega t \qquad (22)$$

where $\overline{\sigma}_{red}$ and a_{red} are its mean value and amplitude, respectively.

Since the reduced stress is referred to as the equivalent tensile stress, the integrated safety factor, n, can be expressed similarly to (20) as :

$$n = n_d\left(1 - f_s^{-1}\right)^m \qquad (23)$$

where :

m - the fatigue strength exponent in the $\sigma - N$ curve equation at symmetrical tension-compression and :

$$n_d = \frac{N_{red}}{N_r} \qquad f_s = \frac{S_{u(v)}}{\overline{\sigma}_{red}}$$

$N_{red} = Ka_{red}^{-m}$ - number of cycles to fatigue failure at the reduced zero mean stress.

In order to account for various load modes and material anisotropy the expressions for n_d and f_s must be modified which results in the formula (9) for f_s and the following one for n_d [4] :

$$n_d = \frac{1}{N_r}\left(\sum_i u_i^2 - u_x u_y - u_y u_z - u_z u_x\right)^{-1/2} \qquad (24)$$

valid for :

$$f < 1 \le l \qquad (25)$$

where :

$$f = f_d\left(1 - f_s^{-1}\right) \qquad l = l_d\left(1 - f_s^{-1}\right)$$

$$l_d = \left(\sum_i v_i^2 - v_x v_y - v_y v_z - v_z v_x\right)^{-1/2} \qquad (26)$$

$$u_i = \frac{a_i^{m_i}}{K_i} \qquad v_i = \frac{\beta_i a_i}{\varepsilon_i L_i} \qquad i = x, y, \ldots zx$$

f_d, f_s – quantities given by (8) and (9), respectively

K_i, m_i – parameters in the $\sigma - N$ curve equation at i-th zero mean stress

L_i – maximum stress amplitude satisfying this equation.

In order to adopt (23) at multiaxial static and periodic stress the amplitudes $a_i^{(eq)}$ of the equivalent stress components must be calculated and inserted into (8), (24) and (26) in place of a_i. Then n_d will obviously represent the ratio of the number of cycles to failure at the equivalent zero mean stress to the number of equivalent stress cycles required to achieve a given design life.

When the inequalities (25) are not fulfilled the presented calculation procedure cannot be applied. Should it happen that $l < 1$ the low-cycle fatigue is possible. As stated earlier at $f \ge 1$ infinite fatigue life may be expected.

Surface finish, details of the geometry etc. are accounted for by proper choice of the σ - N design curve [1,5] for each stress component.

EXAMPLE

Calculate the integrated safety factor and fatigue life of an engineering component subject to combined bending and torsion if at a given point the stress components are :

$$\left(\overline{\sigma}_i \text{ and } a_i^{(p)}[\text{MPa}]\right):$$

$$\sigma_x(t) = -25.2 + 65.2\sin 1.5t + 11.7\sin 4.5t + 11.7\sin 7.5t$$

$$\sigma_{xy}(t) = 22.4 + 39.6\sin 6t + 19.7\sin 12t$$

The design data are :

$$E_x = E = 2.1\cdot 10^6\,\text{MPa} \qquad E_{xy} = G = 8.077\cdot 10^4\,\text{MPa}$$

$$S_x = 310\,\text{MPa} \qquad S_{xy} = 160\,\text{MPa}$$

$$F_x = 180\,\text{MPa} \qquad F_{xy} = 110\,\text{MPa}$$

$$L_x = 300\,\text{MPa} \qquad L_{xy} = 150\,\text{MPa} \qquad m_x = m_{xy} = m = 3$$

$$K_x = 10^6\cdot F_x^3 \qquad K_{xy} = 10^6\cdot F_{xy}^3$$

$$\beta_x = 1.5 \qquad \beta_{xy} = 1.3 \qquad \varepsilon_x = \varepsilon_{xy} = 0.8 \qquad \eta_x = \eta_{xy}$$

Solution

Equations (13) give :

$$a_x^{(eq)} = 37.7\,\text{MPa} \qquad a_{xy}^{(eq)} = 55.2\,\text{MPa}$$

$$\kappa = 4.43 \qquad k = 4 \qquad \omega_{eq} = 6\,\text{s}^{-1}$$

By applying (26) one gets :

$$f = \left[\left(\frac{\beta_x a_x^{(eq)}}{\varepsilon_x F_x}\right)^2 + \left(\frac{\beta_{xy} a_{xy}^{(eq)}}{\varepsilon_{xy} F_{xy}}\right)^2\right]^{-1/2} \times$$

$$\times\left\{1 - \left[\left(\frac{\overline{\sigma}_x}{S_x}\right)^2 + \left(\frac{\overline{\sigma}_{xy}}{S_{xy}}\right)^2\right]^{1/2}\right\}$$

$$l=\left[\left(\frac{\beta_x a_x^{(eq)}}{\varepsilon_x L_x}\right)^2+\left(\frac{\beta_{xy} a_{xy}^{(eq)}}{\varepsilon_{xy} L_{xy}}\right)^2\right]^{-1/2}\times$$

$$\times\left\{1-\left[\left(\frac{\bar{\sigma}_x}{S_x}\right)^2+\left(\frac{\bar{\sigma}_{xy}}{S_{xy}}\right)^2\right]^{1/2}\right\}$$

Hence $f = 0.93 < 1$, $l = 1.3 > 1$ which satisfies the condition (25).

For the equivalent stress one obtains from (23) and (24) :

$$n=\frac{1}{N_r}\left\{\left[\frac{\left(a_x^{(eq)}\right)^{m_x}}{K_x}\right]^2+\left[\frac{\left(a_{xy}^{(eq)}\right)^{m_{xy}}}{K_{xy}}\right]^2\right\}^{-1/2}\times$$

$$\times\left\{1-\left[\left(\frac{\bar{\sigma}_x}{S_x}\right)^2+\left(\frac{\bar{\sigma}_{xy}}{S_{xy}}\right)^2\right]^{1/2}\right\}^m$$

which yields :

$$n=4.6\cdot 10^6\cdot\frac{1}{N_r}$$

It means that the fatigue life is :

$$T=4.6\cdot 10^6\,\frac{2\pi}{\omega_{eq}}=4817\cdot 10^3\,\mathrm{s}$$

CONCLUSIONS

At multiaxial static-dynamic stresses the accurate fatigue assessment of structural elements may be complicated especially if the stress components are non-proportional to each other and stress cycles have to be counted [8]. In the presented method the cycle counting is avoided by virtue of the transformation of the actual stress components (10) into the equivalent ones (12). The resulting formulae for the integrated safety factor n are also based on the experimentally verified models (15) and (16) ; similarly the model (17) can be applied. However, none of the relations (15) ÷ (17) are generally valid as they are grounded on the experimental data obtained under different conditions [5].

The amplitudes $a_i^{(eq)}$ of the equivalent stress components are defined by (13) as positive quantities. So, the relevant signs „–" in (8), (24) and (26) must be replaced with „+" in order to account for the stress in the outer fibres on both sides of the cross-section with respect to the neutral axis of an engineering component subject to bending moment(s) .

For the materials which show no true fatigue limits and those at corrosive environmental conditions, equations (3) ÷ (6) and (8) cannot be used ; and the equations in design for a finite fatigue life must be modified (e.g., by applying the $\sigma - N$ design curves without marked fatigue limits or with reduced fatigue limits [1,5]).

NOMENCLATURE

- a – amplitude of the uniaxial normal stress
- a_i – amplitude of i-th component of in-phase stress ($i = x,y,z,xy,yz,zx$)
- $a_i^{(eq)}$ – amplitude of i-th component of the equivalent stress
- $a_i^{(p)}$ – amplitude of p-th sinusoidal term in Fourier expansion of i-th component of periodic stress
- a_{red} – amplitude of the reduced stress
- E_i – Young modulus or shear modulus associated with i-th stress component
- f – integrated safety factor in design for infinite fatigue life
- f_d – partial safety factor related to time-varying parts of the stress components
- f_s, f_{as} – partial safety factors related to mean stress
- F – fatigue limit at fully reversed tension-compression
- F_i – fatigue limit at i-th zero mean stress
- k – natural number obtained by rounding the number κ
- K – fatigue strength coefficient at fully reversed tension-compression
- K_i – fatigue strength coefficient at i-th zero mean stress
- l, l_d – quantities appearing in (26)
- L – the maximum stress amplitude satisfying equation of the $\sigma - N$ curve at fully reversed tension-compression, above which low-cycle fatigue may occur
- L_i – the maximum amplitude satisfying equation of the $\sigma - N$ curve at i-th zero mean stress , above which low-cycle fatigue may occur
- m – fatigue strength exponent at fully reversed tension-compression
- m_i – fatigue strength exponent at i-th zero mean stress
- n, n_a – integrated safety factors in design for a finite fatigue life
- n_d, n_{ad} – partial safety factors related to time-varying parts of the stress components
- N, N_a – numbers of stress cycles to fatigue failure
- N_r – number of stress cycles required to achieve a given design life
- N_{red} – number of cycles to failure at the reduced zero mean stress
- S_i – ultimate or yield strength at i-th static load
- S_u – ultimate tensile strength
- S_y – tensile yield strength
- $S_{u(y)}$ – S_u or S_y arbitrarily
- t – time
- T – fatigue life
- T_0 – stress period
- $\alpha_i^{(p)}$ – phase angle of p-th sinusoidal term in Fourier expansion of i-th component of periodic stress
- β_i – notch sensitivity index at i-th stress
- ε_i – size factor at i-th stress
- η_i – internal damping coefficient in Kelvin-Voigt's model of the material, associated with i-th stress component
- κ – number apearing in (13)
- σ – uniaxial normal stress
- σ_i – i-th stress component
- $\bar{\sigma}$ – mean value of the uniaxial normal stress
- σ_N – amplitude of the zero mean normal stress at a given number, N, of cycles to fatigue failure
- $\bar{\sigma}_i$ – mean value of i-th stress component
- σ_{red} – reduced stress
- $\bar{\sigma}_{red}$ – mean value of the reduced stress
- ψ_i – asymmetry sensitivity index at i-th stress
- ω – circular frequency
- ω_0 – fundamental circular frequency of the periodic stress
- ω_{eq} – circular frequency of the equivalent stress

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