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Approximation method of determining the analytical model of naval gas turbine axial compressor

SUMMARY

The paper presents a way of determining the analytical model of an axial compressor on the basis of the least squares and polynomial, multidimensional regression methods.

The approach was applied to the axial compressors of DR77 engine and proved adequate. A satisfying consistence of the full-scale and model characteristics was obtained, hence usefulness of the method to modelling such objects was confirmed.

INTRODUCTION

Operation parameters of the gas turbine depend on co-operation conditions and characteristics of its subsystems [1]. Large number of them and measurement difficulties make real object investigations very expensive. Therefore computer simulation methods are wider and wider used to carrying out computational identification of gas turbine energy processes in different operation conditions.

MATHEMATICAL MODEL OF THE AXIAL COMPRESSOR

Problems of forming the simulation models of naval gas turbine axial compressors are connected with the range of the assumed approximations which determine accuracy of the mathematical models [2]. Simulation investigations require converting the compressor characteristic usually presented in a generalized graphical form (Fig.1) into an analytical one applicable to numerical calculations :

$$\dot{m}_{zr} = f(\pi^*, n_{zr}, \alpha_{kw}) \tag{1}$$

$$\eta^*_{is} = f(\dot{m}_{zr}, n_{zr}, \alpha_{kw}) \tag{2}$$

where :

- $\dot{m}_{zr}$ - reduced air flow rate
- π^* - compression
- n_{zr} - reduced rotational speed
- α_{kw} - setting angle of the intake controllable guide vane
- η^*_{is} - isentropic efficiency

Obtaining the analytical form of the functions (1) and (2) which model the real compressor characteristic, at a possible low approximation error, is connected with some difficulties of the following sources :

- form of the functions is complex
- an ambiguity appears as in the low rotational speed range the isodroms of the characteristic are of a moderate slope corresponding to $\pi^* \approx idem$, and in the high rotational speed range they have steep segments corresponding to $\dot{m}_{zr} \approx idem$ (Fig.1).

In result of this observation the least squares and multidimensional polynomial regression method was chosen as the probably most effective way of analytical description of the axial compressor characteristic [3].

It results from the preliminary investigations that the method makes it possible to determine an arbitrary operational point at the compressor characteristic with the deviations of its model values from real ones maintained within measurement error limits. Such general compressor model was searched with the use of the following set of equation which approximates its universal characteristic :

$$\dot{m}_{zr} = a_0 + a_1\pi^* + a_2(\pi^*)^3 + a_3 n_{zr} + a_4\pi^* n_{zr} + a_5(\pi^*)^2 n_{zr} + a_6\pi^*(n_{zr})^2 + a_7(\pi^* n_{zr})^2 \tag{3}$$

$$\eta^*_{is} = b_0 + b_1\dot{m}_{zr} + b_2(\dot{m}_{zr})^3 + b_3 n_{zr} + b_4\dot{m}_{zr} n_{zr} + b_5(\dot{m}_{zr})^2 n_{zr} + b_6\dot{m}_{zr}(n_{zr})^2 + b_7(\dot{m}_{zr} n_{zr})^2 \tag{4}$$

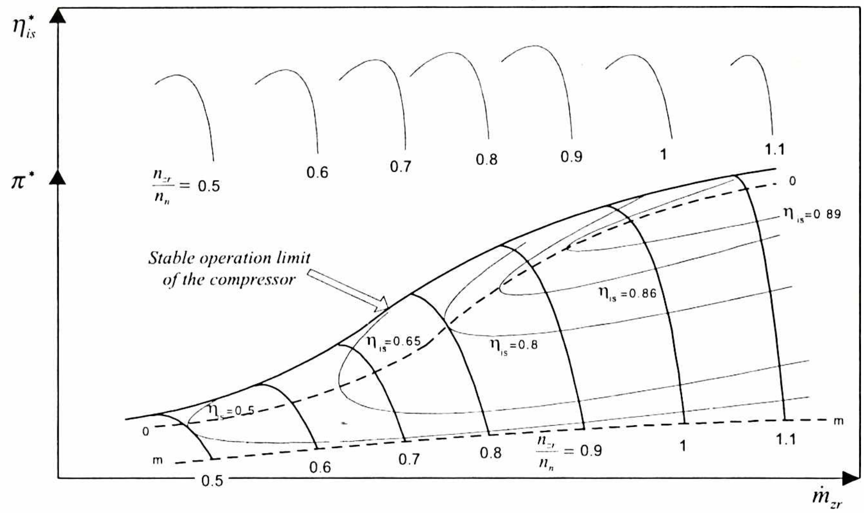

Fig.1. An example of the universal characteristic of the axial compressor

$$\eta^*_{is} = f\left(\dot{m}_{zr}, \frac{n_{zr}}{n_n}\right)$$

$$\pi^* = f\left(\dot{m}_{zr}, \frac{n_{zr}}{n_n}, \eta_{is}\right)$$

$\left.\begin{matrix} o-o \\ m-m \end{matrix}\right\}$ *boundaries of working range of compressor characteristic*

Values of the regression coefficients a_i and b_i are determined on the basis of the Gauss-Markov theorem by searching for minimum values of the following functionals :

$$J_{\dot{m}_{zr}}(a_0,...,a_7) = \sum_{k=1}^{p}[(\dot{m}_{zr})_k - (\overline{\dot{m}}_{zr})_k]^2 \qquad (5)$$

$$J_{\eta^*_{is}}(b_0,...,b_7) = \sum_{k=1}^{p}[(\eta^*_{is})_k - (\overline{\eta}^*_{is})_k]^2 \qquad (6)$$

The functionals are the sums of squares of the model deviations from reality. The regression equations are supplemented by the boundary conditions which determine the operational space of the characteristic :

$$\pi^*_1 = c_1\dot{m}_{zr1} + d_1 \qquad (7)$$

$$\pi^*_2 = c_2\dot{m}_{zr2} + d_2 \qquad (8)$$

where :

c_1, c_2, d_1, d_2 - coefficients of the equations limiting the operational space of the characteristic.

CALCULATION RESULTS

To check the accuracy of the method in question an attempt was made to describe mathematically operational characteristics of the compressors of DR77 propulsion gas turbine within the range of the rotational speed $(n_{zr})_{HPC}$ from 4500 to 9500 rpm.

Adequacy assessment of the compressor model was performed by determining the multidimensional correlation coefficients :

$$r_{\dot{m}_{zr}} = \frac{\sum_{k=1}^{p}[(\dot{m}_{zr})_k - \dot{m}_{zr}][(\overline{\dot{m}}_{zr})_k - \overline{\dot{m}}_{zr}]}{\sqrt{\sum_{k=1}^{p}[(\dot{m}_{zr})_k - \dot{m}_{zr}]^2 \sum_{k=1}^{p}[(\overline{\dot{m}}_{zr})_k - \overline{\dot{m}}_{zr}]^2}} \qquad (9)$$

$$r_{\eta^*_{is}} = \frac{\sum_{k=1}^{p}[(\eta^*_{is})_k - \eta^*_{is}][(\overline{\eta}^*_{is})_k - \overline{\eta}^*_{is}]}{\sqrt{\sum_{k=1}^{p}[(\eta^*_{is})_k - \eta^*_{is}]^2[(\overline{\eta}^*_{is})_k - \overline{\eta}^*_{is}]^2}} \qquad (10)$$

where :

$\dot{m}_{zr} = \frac{1}{p}\sum_{k=1}^{p}(\dot{m}_{zr})_k$, $\eta^*_{is} = \frac{1}{p}\sum_{k=1}^{p}(\eta^*_{is})_k$ - the arithmetic mean value of the model output

$\overline{\dot{m}}_{zr} = \frac{1}{p}\sum_{k=1}^{p}(\overline{\dot{m}}_{zr})_k$, $\overline{\eta}^*_{is} = \frac{1}{p}\sum_{k=1}^{p}(\overline{\eta}^*_{is})_k$ - the arithmetic mean value of the measurement output

In Fig.2,3,4 and 5 the model characteristics of the DR77 gas turbine compressors of low pressure (LPC) and high pressure (HPC) at the angular position of the intake controllable guide vane a_{kw} = - 10° are presented. To demonstrate effectiveness of the applied method the measurement values (points) obtained by the turbine manufacturer from experimental investigations were marked on the calculated characteristics.

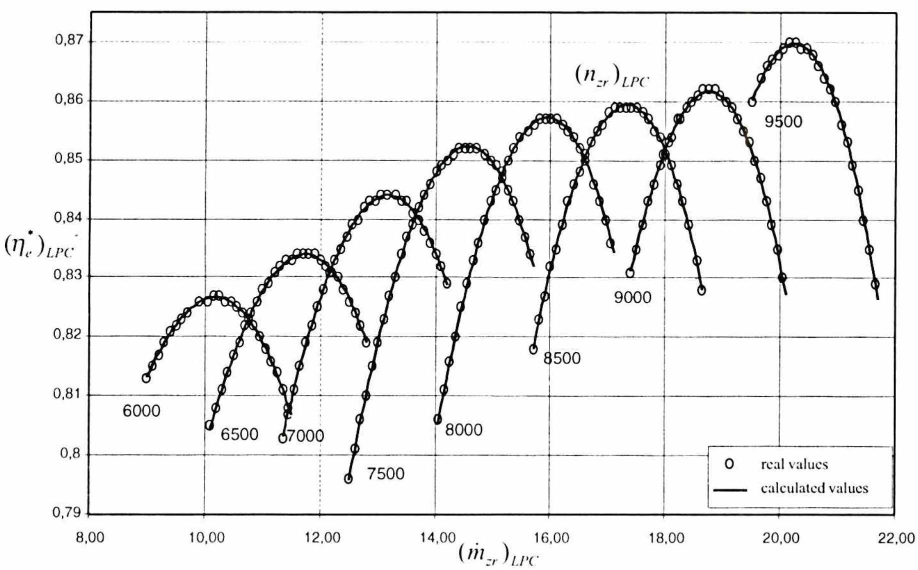

Fig.2. DR77 gas turbine low pressure compressor characteristic

$$(\eta^*_e)_{LPC} = f[(\dot{m}_{zr})_{LPC}, (n_{zr})_{LPC}]$$

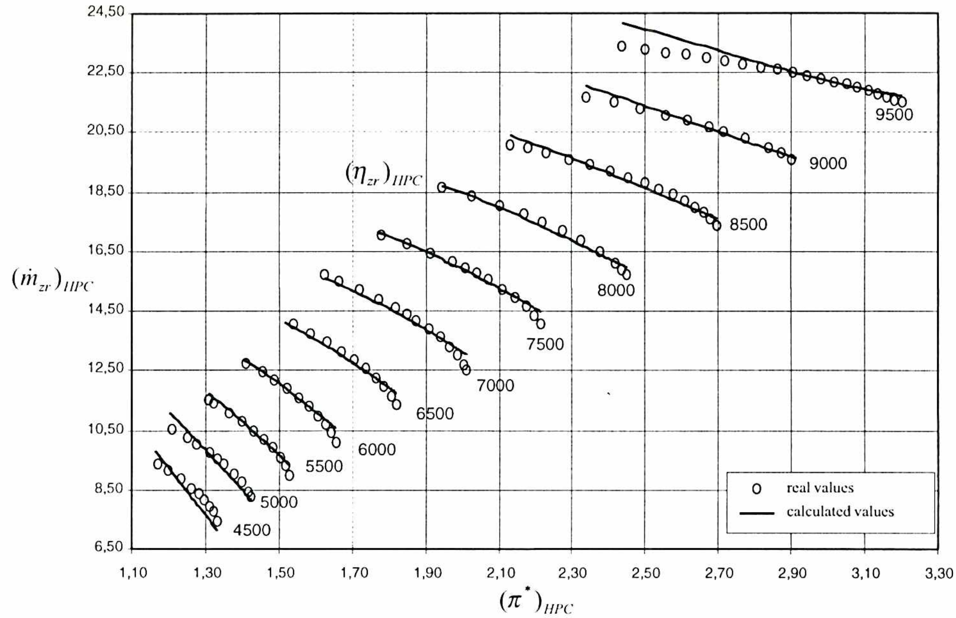

Fig.3. DR77 gas turbine high pressure compressor characteristic

$$(\dot{m}_{zr})_{HPC} = f[(\pi^*)_{HPC}, (n_{zr})_{HPC}]$$

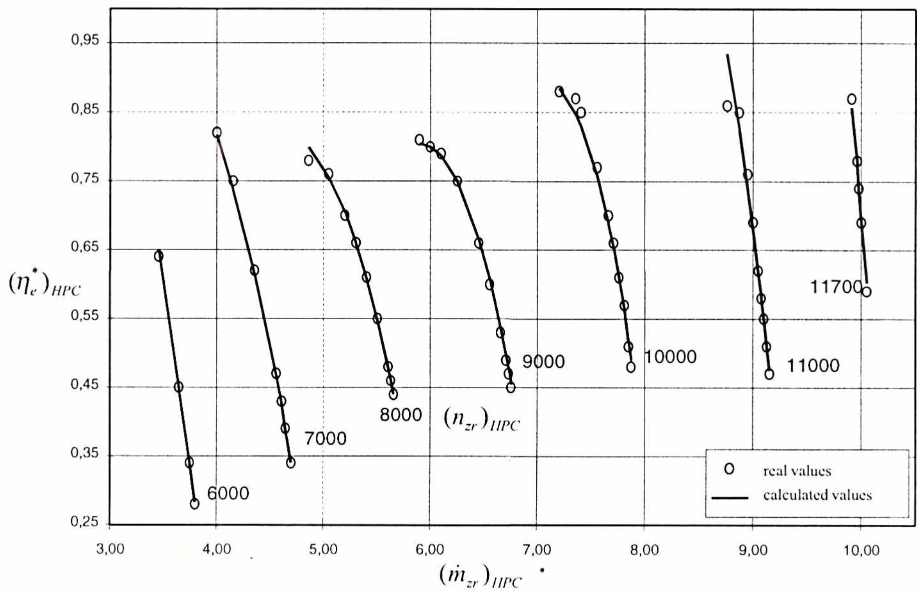

Fig.4. DR77 gas turbine high pressure compressor characteristic

$$(\eta_e^*)_{HPC} = f[(\dot{m}_{zr})_{HPC}, (n_{zr})_{HPC}]$$

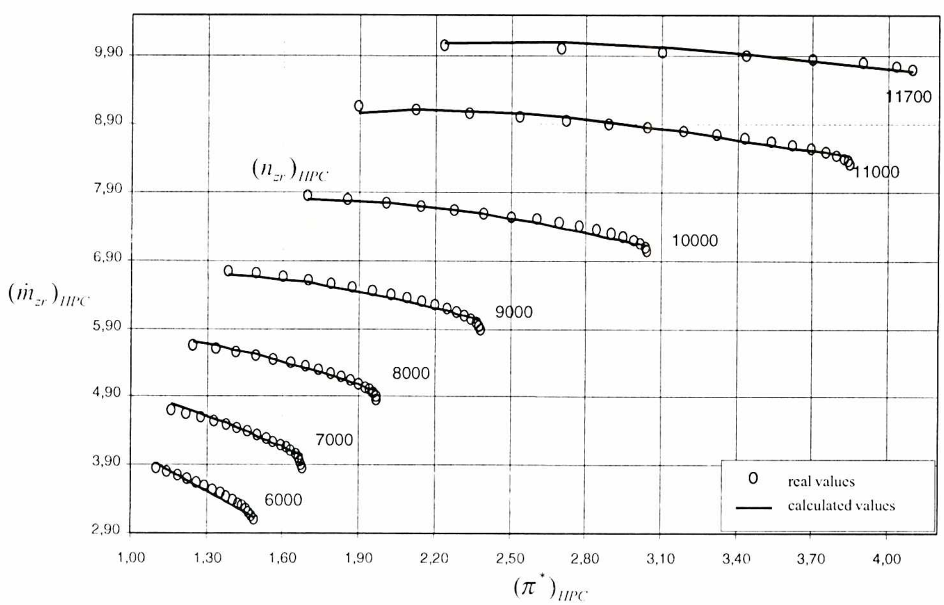

Fig.5. DR77 gas turbine high pressure compressor characteristic

$$(\dot{m}_{zr})_{HPC} = f[(\pi^*)_{HPC}, (n_{zr})_{HPC}]$$

Tab.1 and 2 contain values of the statistical parameters which demonstrate consistence of the calculation and measurement results.

Tab.1. Adequacy degree of the axial compressor model : $\dot{m}_{zr} = f(\pi^*, n_{zr})$ *against its real characteristics*

Gas turbine type	Compressor type	Correlation coefficient **r**
DR77	LPC (→ Low Pressure)	0.999051
	HPC (→ High Pressure)	0.999582

Tab.2. Adequacy degree of the axial compressor model : $\eta_e^* = f(\dot{m}_{zr}, n_{zr})$ *against its real characteristics*

Gas turbine type	Compressor type	Correlation coefficient **r**
DR77	LPC (→ Low Pressure)	0.999912
	HPC (→ High Pressure)	0.996344

In Tab. 3 and 4 values of the regression coefficients of the equations (3) and (4) approximating the compressor characteristics are presented.

Tab.3. Regression coefficient values of DR77 gas turbine compressor model : $\dot{m}_{zr} = f(\pi^*, n_{zr})$

Regression coefficient	**Low pressure compressor (LPC)**	**High pressure compressor (HPC)**
a_0	5,312967E+01	5,153248E+00
a_1	-6,106490E+01	-7,058496E+00
a_2	2,621641E+00	1,170619E-01
a_3	-4,506188E-01	3,002407E-03
a_4	9,654960E-01	6,963144E-02
a_5	-1,948155E-01	-8,550140E-03
a_6	-1,527891E-03	-6,122171E-05
a_7	3,84533E-04	1,050332E-05

Tab.4. Regression coefficient values of DR77 gas turbine compressor model : $\eta_e^* = f(\dot{m}_{zr}, n_{zr})$

Regression coefficient	**Low pressure compressor (LPC)**	**High pressure compressor (HPC)**
b_0	2,477361E+00	5,56834E+00
b_1	-3,655833E-01	-5,350542E+00
b_2	-7,351419E-04	-8,726627E-02
b_3	-2,147000E-02	-5,024347E-03
b_4	7,516718E-02	6,872460E-02
b_5	6,370980E-05	5,546236E-03
b_6	-4,013629E-05	-4,154724E-04
b_7	7,214879E-07	1,385779E-05

COMPUTER ANIMATION

The mathematical model in question became the basis for elaborating a computer animation program with the use of BORLAND DELPHI language. The program makes it possible to visualize graphically the way of determining an arbitrary operation point of the axial compressor characteristics. Also, it signals any exceedance of stable operation limits.

A monitor display of the program in action is demonstrated in Fig.6.

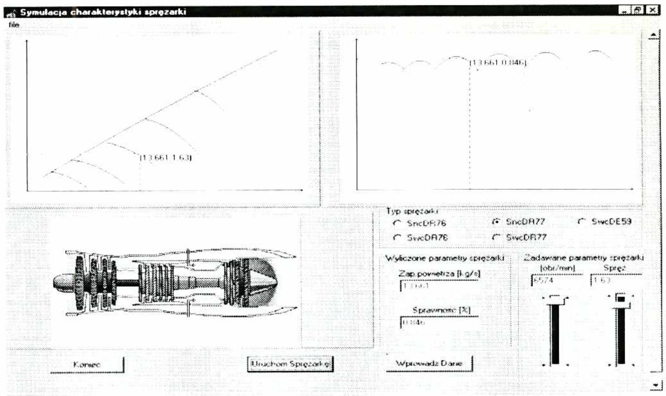

Fig.6. A monitor display obtained from the graphical animation program in action

CONCLUSIONS

- The presented method of modelling the axial compressor characteristics is very simple and of the accuracy sufficient for simulation purposes.
- Its accuracy was checked by modelling the low-pressure and high pressure compressors of DR gas turbine, operating in series.
- In the case of application of a two-variable function the sufficient accuracy was already obtained at choosing the modelling functions in the form of third order polynomials.

NOMENCLATURE

$\overline{\dot{m}}_{zr}, \dot{m}_{zr}$	-	reduced air flow rate
n_n	-	rated speed
n_{zr}	-	reduced rotational speed
p	-	number of measurements
r	-	multidimensional correlation coefficient
α_{kw}	-	setting angle of the intake controllable guide vane
η_e	-	overall efficiency
$\overline{\eta}_{is}^*, \eta_{is}^*$	-	isentropic efficiency
π^*	-	compression
HPC	-	high pressure compressor
LPC	-	low pressure compressor

Notation : „-" real values (as measured)

Appraised by Adam Charchalis, Prof.,D.Sc.,M.E.

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