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A modification of distortion-energy strength theory at static-periodic loading conditions

SUMMARY

The aim of this paper is to present the analytical procedure capable of dealing with complex stress patterns in the high-cycle fatigue regime formed simultaneously by static and periodic loads. It consists in modelling the actual multiaxial stress by means of the equivalent in-phase stress, transforming the equivalent stress into the reduced stress, and in using the reduced stress to fatigue assessment of the structural element by means of the formulae accepted in the case of simple sinusoidal stress. The reduced stress is determined with the aid of the average-distortion-energy strength hypothesis based on the distortion-energy strength theory. Thereby the mean stress is taken into account and the cycle counting is avoided.

INTRODUCTION

In general, structural elements are simultaneously exposed to static and cyclic loads. In addition to the applied cyclic stress, the steady stress resulting from static load and the mean stress (the average of the maximum and minimum of the cyclic stress) can have a significant effect on the fatigue life of a structural element [1]. The tensile steady and/or mean stresses are detrimental to fatigue performance, whereas compressive ones are beneficial. It is important to recognize that the total strength of the structural element is altered if residual stresses (caused by cold forming, heat treatment, welding, etc.) exist. No distinction is made between mean, steady and residual stresses in this paper, since residual stresses have similar influence on the fatigue behaviour of materials as that of mechanically imposed static stresses of the same magnitude [2].

In design for an infinite fatigue life under multiaxial stress, the mean stress effect on the fatigue process can be taken into account either by means of the fatigue failure criterion which includes mean-dependent constants [3] or by using equivalent, completely reversed stress components [4].

Loading history must be examined and stress cycles counted when analyzing the effect of mean stress in the stress (strain) - fatigue life equation. A multiaxial, non-proportional cycle counting method and a fatigue damage calculation procedure that incorporate the mean stress effect described in [5], are based on the critical plane concept and plastic deformation response in the low-cycle regime. However the mean stress is more important for the high-cycle fatigue [5]. In this regime the deformations are small and can be characterized by material's elastic behaviour [1] which makes the methodology suggested in [5] inapplicable.

In the following, an analytical procedure capable of dealing with complex stress patterns in the high-cycle fatigue regime is considered for static-periodic loading and materials compliant with the Kelvin-Voigt's model. The proposed calculation scheme consists in :

- modelling the actual multiaxial stress by means of the equivalent in-phase stress [6]
- transforming the equivalent stress into the reduced stress, and
- using the reduced stress to fatigue assessment by means of the formulae accepted in the case of simple sinusoidal stress [2,4].

The reduced stress is determined here with the aid of the average-distortion-energy strength hypothesis. Thereby the mean stress is taken into account and cycle counting is avoided.

EQUIVALENT IN-PHASE STRESS

A three-dimensional stress with Cartesian components is considered :

$$\tilde{\sigma}_i(t)=\tilde{\sigma}_i(t+\tau_0) \qquad i = x, y, z, xy, yz, zx \tag{1}$$

where τ_0 is the common period of the stress components. These components are assumed physically independent of each other and given in the form of Fourier series as follows :

$$\tilde{\sigma}_i(t)=\sigma_{mi}+\sum_{p=1}^{\infty} a_{ip}\sin\left(p\omega_0 t+\alpha_{ip}\right) \tag{2}$$

where :

- σ_{mi} - mean value of i-th stress component
- α_{ip}, a_{ip} - phase angle and amplitude of p-th sinusoidal term, respectively
- $\omega_0 = 2\pi/\tau_0$ - fundamental circular frequency

As follows from [6] such stress can be modelled on the basis of the Cempel's theory [7] by the equivalent stress with in-phase components :

$$\sigma_i^{(eq)}(t)=\sigma_{mi}+a_i^{(eq)}\sin\omega_{eq}t \qquad i=x,\,y,\,\ldots,\,zx \tag{3}$$

According to the theory, two stress states are equivalent in terms of fatigue lifetime if the energies dissipated internally and externally in both the states are equal. The magnitudes σ_{mi} in (2) and (3) are the same since the mean stress does not influence the energy dissipation in the case of Kelvin-Voigt's model, and the equivalent stress amplitudes $a_i^{(eq)}$ and circular frequency ω_{eq} can be determined by means of the following formulas [6] :

$$a_i^{(eq)}=\left\{\frac{8}{k^2\tau_0}\int_0^{\tau_0}\left[\sum_{p=1}^{\infty}a_{ip}\sin\left(p\omega_0 t+\alpha_{ip}\right)\right]^2\cdot\left[\sum_{p=1}^{\infty}pa_{ip}\cos\left(p\omega_0 t+\alpha_{ip}\right)\right]^2dt\right\}^{1/4} \tag{4}$$

$$\omega_{eq}=k\omega_o \qquad k=\text{Round}\,(\kappa) \tag{5}$$

$$\kappa=\left[\frac{\sum_i\sum_{p=1}^{\infty}\eta_i\left(pa_{ip}/E_i\right)^2}{\sum_i\sum_{p=1}^{\infty}\eta_i\left(a_{ip}/E_i\right)^2}\right]^{1/2} \tag{6}$$

where :

E_i - Young or shear modulus

η_i - internal viscous damping coefficient in the Kelvin-Voigt's model of the material, both corresponding to i-th stress component.

AVERAGE-DISTORTION-ENERGY STRENGTH HYPOTHESIS

The distortion-energy theory is widely accepted for the strength assessment of ductile materials exposed to multiaxial static load. According to the theory the reduced stress is equivalent in terms of static effort of the material to a given multiaxial stress if the strain energies of distortion per unit volume at these stresses are equal. In dynamic problems no single theory has been able to correlate the test data for a variety of loading conditions and materials [1] and all existing multiaxial fatigue criteria can demonstrate large scatter [8]. The question arises as to whether the distortion-energy strength theory could form the basis for a more appropriate fatigue criterion to account for static-dynamic loading conditions. The question cannot be answered within the scope of the current paper. Therefore the main objective is to present the hypothesis which is a modification of the distortion-energy strength theory and correlates with the experimental results reported in [9,10]. The proposed formulation of the hypothesis is as follows:

The reduced stress is equivalent in terms of static-dynamic effort of a material to a given multiaxial stress if :

(A) the time-independent parts of the strain energies of distortion per unit volume at both these stresses are equal

(B) the integral averages of the instantaneous values of the strain energies of distortion per unit volume at both these stresses are equal

(C) the reduced stress and the components of the multiaxial stress have the same frequency.

The condition (A) follows directly from the distortion-energy strength theory [11]. The condition (B) lays special emphasis on the mean values of the instantaneous strain energy of distortion. The condition (C) reflects the correlation between the fatigue performance of structural materials under in-phase loading and the fatigue predictions based on the distortion-energy strength theory [3]. The calculation (A) refers to mean stresses, the condition (B) is expressible as an integral relation between the strain energies of distortion at the reduced and multiaxial stresses under static-dynamic loading conditions, and the condition (C) is concerned with the nature of the stress oscillations.

The presented hypothesis will be called the average-distortion-energy strength hypothesis. Its limitations result from those of the distortion-energy strength theory. In particular the hypothesis is confined to such steels whose static interaction equation between normal and shear stress components is of a quadratic form.

REDUCED STRESS AT THE EQUIVALENT IN-PHASE STRESS

With respect to (3) and condition (C) the reduced stress can be expressed in the following form :

$$\tilde{\sigma}_{red}(t)=\sigma_m+a_{red}\sin\omega_{eq}t \tag{7}$$

where :

σ_m - mean value of the reduced stress

a_{red} - reduced stress amplitude

The formula for the instantaneous strain energy of distortion per unit volume at the reduced stress is [11] :

$$\tilde{\phi}_{red}=\frac{1+\nu}{3E}\tilde{\sigma}_{red}^2 \tag{8}$$

and that at the equivalent stress :

$$\tilde{\phi}_{eq}=\frac{1+\nu}{3E}\Big\{\left(\tilde{\sigma}_x^{(eq)}\right)^2+\left(\tilde{\sigma}_y^{(eq)}\right)^2+\left(\tilde{\sigma}_z^{(eq)}\right)^2+ -\tilde{\sigma}_x^{(eq)}\cdot\tilde{\sigma}_y^{(eq)}-\tilde{\sigma}_y^{(eq)}\cdot\tilde{\sigma}_z^{(eq)}-\tilde{\sigma}_z^{(eq)}\cdot\tilde{\sigma}_x^{(eq)}+ +3\left[\left(\tilde{\sigma}_{xy}^{(eq)}\right)^2+\left(\tilde{\sigma}_{yz}^{(eq)}\right)^2+\left(\tilde{\sigma}_{zx}^{(eq)}\right)^2\right]\Big\} \tag{9}$$

or, if the structural element is exposed to bending moment(s)

$$\tilde{\phi}_{eq}=\frac{1+\nu}{3E}\Big\{\left(\tilde{\sigma}_x^{(eq)}\right)^2+\left(\tilde{\sigma}_y^{(eq)}\right)^2+\left(\tilde{\sigma}_z^{(eq)}\right)^2+ +\tilde{\sigma}_x^{(eq)}\cdot\tilde{\sigma}_y^{(eq)}+\tilde{\sigma}_y^{(eq)}\cdot\tilde{\sigma}_z^{(eq)}+\tilde{\sigma}_z^{(eq)}\cdot\tilde{\sigma}_x^{(eq)}+ +3\left[\left(\tilde{\sigma}_{xy}^{(eq)}\right)^2+\left(\tilde{\sigma}_{yz}^{(eq)}\right)^2+\left(\tilde{\sigma}_{zx}^{(eq)}\right)^2\right]\Big\} \tag{10}$$

where :

E - Young's modulus

ν - Poisson's ratio

The condition (A) is fulfilled if :

$$\sigma_m=\left[\sigma_{mx}^2+\sigma_{my}^2+\sigma_{mz}^2-\sigma_{mx}\sigma_{my}-\sigma_{my}\sigma_{mz}-\sigma_{mz}\sigma_{mx}+ +3\left(\sigma_{mxy}^2+\sigma_{myz}^2+\sigma_{mzx}^2\right)\right]^{1/2} \tag{11}$$

In the considered case the condition (B) is of the form :

$$\frac{1}{\tau_0}\int_0^{\tau_0}\tilde{\phi}_{red}\,dt = \frac{1}{\tau_0}\int_0^{\tau_0}\tilde{\phi}_{eq}\,dt \qquad (12)$$

The equations (3) and (7) through (12) yield :

$$\begin{aligned} a_{red} = \Big\{ & \left(a_x^{(eq)}\right)^2 + \left(a_y^{(eq)}\right)^2 + \left(a_z^{(eq)}\right)^2 + \\ & \mp a_x^{(eq)}a_y^{(eq)} \mp a_y^{(eq)}a_z^{(eq)} \mp a_z^{(eq)}a_x^{(eq)} + \\ & +3\left[\left(a_{xy}^{(eq)}\right)^2 + \left(a_{yz}^{(eq)}\right)^2 + \left(a_{zx}^{(eq)}\right)^2\right]\Big\}^{1/2} \end{aligned} \qquad (13)$$

The reduced stress in question is defined by (7), (11) and (13).

REDUCED STRESS AT THE OUT-OF-PHASE STRESS

In the similar way as above a multiaxial out-of-phase stress with synchronous components :

$$\tilde{\sigma}_i(t) = \sigma_{mi} + a_i \sin(\omega t + \alpha_i) \qquad (14)$$

$$i = x, y, \ldots, zx$$

can be treated, where :

- a_i and α_i - amplitudes and phase angles of the stress components, respectively
- ω - circular frequency.

In this case (7), (9), (10) and (12) can be replaced by the following expressions :

$$\tilde{\sigma}_{red}(t) = \sigma_m + a_{red}\sin\omega t \qquad (15)$$

$$\begin{aligned} \tilde{\phi} = \frac{1+\nu}{3E}\Big[& \tilde{\sigma}_x^2 + \tilde{\sigma}_y^2 + \tilde{\sigma}_z^2 \mp \tilde{\sigma}_x\tilde{\sigma}_y \mp \tilde{\sigma}_y\tilde{\sigma}_z \mp \tilde{\sigma}_z\tilde{\sigma}_x + \\ & +3\left(\tilde{\sigma}_{xy}^2 + \tilde{\sigma}_{yz}^2 + \tilde{\sigma}_{zx}^2\right)\Big] \end{aligned} \qquad (16)$$

$$\frac{1}{\tau}\int_0^{\tau}\tilde{\phi}_{red}\,dt = \frac{1}{\tau}\int_0^{\tau}\tilde{\phi}\,dt$$

$$\tau = 2\pi/\omega \qquad (17)$$

The equation (11) is obtained again in result of applying the condition (A) to $\tilde{\phi}_{red}$ and $\tilde{\phi}$, and the integration indicated in (17) leads to :

$$\begin{aligned} a_{red} = \Big[& a_x^2 + a_y^2 + a_z^2 \mp a_x a_y\left(\sin\alpha_x \sin\alpha_y + \right. \\ & \left. +\cos\alpha_x \cos\alpha_y\right) \mp a_y a_z\left(\sin\alpha_y \sin\alpha_z + \right. \\ & \left. +\cos\alpha_y \cos\alpha_z\right) \mp a_z a_x\left(\sin\alpha_z \sin\alpha_x + \right. \\ & \left. +\cos\alpha_z \cos\alpha_x\right) + 3\left(a_{xy}^2 + a_{yz}^2 + a_{zx}^2\right)\Big]^{1/2} \end{aligned} \qquad (18)$$

CONCLUDING REMARKS

The presented average-distortion-energy strength hypothesis concerns the situation where a structural element is exposed not only to the steady stresses but also to simultaneously applied cyclic stresses. The hypothesis makes it possible to determine the reduced stress which can be readily employed in fatigue calculation.

For load systems made up from two synchronous, sinusoidal, out-of-phase normal stresses :

$$\begin{aligned} \tilde{\sigma}_1(t) &= \sigma_{m1} + a_1\sin\omega t \\ \tilde{\sigma}_2(t) &= \sigma_{m2} + a_2\sin(\omega t + \alpha_2) \end{aligned} \qquad (19)$$

where σ_{m1} and σ_{m2} - their mean values, a_1 and a_2 - their amplitudes, and α_2 - phase shift, the reduced stress becomes :

$$\begin{aligned} \tilde{\sigma}_{red}(t) = & \left(\sigma_{m1}^2 + \sigma_{m2}^2 - \sigma_{m1}\sigma_{m2}\right)^{1/2} + \\ & + \left(a_1^2 + a_2^2 - a_1 a_2 \cos\alpha_2\right)^{1/2}\sin\omega t \end{aligned} \qquad (20)$$

The amplitude of the stress (20) is enlarged and, consequently, the fatigue strength is clearly reduced by the phase shift : the fatigue strength decreases when α_2 is increasing, and its minimum value occurs at $\alpha_2=\pi$, which correlates with the test data [9]. The monotonous increase of the reduced stress amplitude with increasing α_2 is not in strict accordance with the experimental results by Mielke [12] where the optimum phase shift $\alpha_2 = \pi/3$ is found.

Another type of out-of-phase stress of interest in fatigue problems is that due to bending and torsion :

$$\begin{aligned} \tilde{\sigma}_x(t) &= \sigma_{mx} + a_x\sin\omega t \\ \tilde{\sigma}_{xy}(t) &= \sigma_{mxy} + a_{xy}\sin(\omega t + \alpha_{xy}) \end{aligned} \qquad (21)$$

According to (18) the phase shift in out-of-phase bending and torsion does not affect the fatigue strength which is in agreement with the outcome of experiments on hard metals [10]. However mild and brittle metals under such load conditions exhibit a different behaviour characterized by marked influence of the phase shift [9,10].

It means that the average-distortion-energy strength hypothesis cannot be applied to some kinds of structural materials under out-of-phase loading conditions.

NOMENCLATURE

- a_i - amplitude of i-th stress component
- $a_i^{(eq)}$ - amplitude of i-th component of the eqvivalent stress
- a_{ip} - amplitude of p-th sinusoidal term in Fourier expansion of i-th stress component
- a_{red} - amplitude of i-th reduced stress
- a_1, a_2 - amplitudes of the principal stresses
- E - Young modulus
- E_i - modulus of elasticity associated with i-th stress component ($i=x,y,z,xy,yz,zx$)
- k - natural number obtained by rounding the number κ
- t - time
- τ_0 - stress period
- α_i - phase angle of i-th stress component
- α_{ip} - phase angle of p-th sinusoidal term in Fourier expansion of i-th stress component
- α_2 - phase shift between the principal stresses
- η_i - internal viscous damping coefficient of the material, associated with i-th stress component
- κ - number given by Eq. (6)
- ν - Poisson's ratio
- $\tilde{\sigma}_i$ - i-th stress component
- $\tilde{\sigma}_i^{(eq)}$ - i-th component of the equivalent stress
- $\tilde{\sigma}_{red}$ - reduced stress
- σ_m - mean value of the reduced stress
- σ_{mi} - mean value of i-th stress component
- $\tilde{\sigma}_1$, $\tilde{\sigma}_2$ - principal stresses
- σ_{m1}, σ_{m2} - mean values of the principal stresses

$\tilde{\Phi}, \tilde{\Phi}_{eq}, \tilde{\Phi}_{red}$ - strain energies of distortion per unit volume at the out-of-phase stress, at the equivalent stress and at the reduced stress, respectively

ω - circular frequency

ω_{eq} - equivalent circular frequency

ω_0 - fundamental circular frequency.

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Conferences

Safety at sea

On 22 February 1998 a conference on safety at sea was organized by Polish Navy in Gdynia. During the conference, held under the official heading : „I Conference on Safety at Sea and Oversea Flights", 8 papers were presented, two of them devoted to Polish internal, technical and organizational problems of co-operation of various search and rescue services, the remaining papers of general character were focussed on the following topics :

- Specific aspects of executing the rescue flights over sea independently and in co-operation with naval vessels and other floating objects
- New rescue equipment of the Polish Navy ship, PIAST
- Medical rescue aspects during sea disasters
- Influence of active fire protection on ship safety at sea
- Development trends in the area of ship systems intended for providing ship operation safety
- Magnetic field measurements related to safety of ship operation at sea and helicopter flights.

Moreover two practical rescue operations were demonstrated to the conference participants, namely :

- Performance demonstration of Mi-14PS and W-3 ANAKONDA rescue helicopters
- Demonstration of a rescue episode by applying a computer aided decision making program.

Much interest was paid to the conference topics which was manifested in vivid discussion and support given to the proposal of continuation of the theme at the next conferences devoted to safety at sea.

Current reports

TECHNICAL UNIVERSITY OF SZCZECIN
MECHANICAL FACULTY

RESEARCH ON HEAT AND MASS EXCHANGE IN HYPERBARIC ENVIRONMENT

For many years the Thermodynamics Division of Thermal Technique Department, Technical University of Szczecin, has carried out research work on heat and mass exchange in hyperbaric environment. Results of the solved problems are the basis for elaborating the ventilation and air-conditioning models of stationary diving complexes. The experimental investigations are aimed at determining the influence of pressure, temperature, humidity and content of the gas mixtures on the phenomenon of free convection of heat and mass. It leads to establishing the conditions of maintaining the microclimate inside hyperbaric complexes on research ships.

A method of determination of the heat comfort temperature in function of the most important hyperbaric environment parameters as well as mathematical model of ventilating and air-conditioning processes of the diving complex were elaborated.

The hyperbaric pressure chamber applicable to hyperbaric environment research work was designed and constructed by the Division's team in cooperation with the Diving Equipment and Underwater Technique Division, Polish Naval Academy in Gdynia. The investigations can be performed in the atmosphere of compressed air, helium, carbon dioxide, air-helium mixtures.

Research methods for the following problems were elaborated :

- ⇨ free heat convection around elements of various shapes
- ⇨ evaporation of water from free surfaces
- ⇨ determination of thermal conductance coefficient
- ⇨ determination of diffusion resistance of fabrics.

Results of the research work are utilized to improving the ventilation and air-conditioning models of hyperbaric objects.

In the Department a stand is installed containing the pressure chamber for hyperbaric tests under 5 MPa working pressure and temperature up to 50°C. Its end covers are equipped with helium-tight electric passages which makes performing research in pure helium atmosphere or gas mixtures of helium content possible. The stand is equipped with the gas tank (16 MPa) and gas manifold which enables to prepare gas mixtures. A gas analyzing system contains the following devices : Infralyt (CO_2), Permolyt (O_2) and G.C.H.F 18 gas chromatograph.

The test stand can be used to :

- model testing the heat and mass exchange processes
- research on pressure and temperature influence on water evaporation in various gas atmospheres
- research on thermodynamics of real gases and their physical properties
- research on the thermal conductance coefficient of the materials filled with different gases
- research on influence of helium on operation of various measuring instruments.

Tadeusz Kozak, D.Sc., M.E. and Anna Majchrzycka D.Sc., M.E. are the main authors of the performed research works.