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Determination of the natural ship hull vibration modes by means of the Vlasov functions

SUMMARY

The general description of a method of ship hull vibration analysis is presented. Based on the Vlasov's theory the differential vibration equations were formulated of a ship hull modelled as a system of linear, elastic orthotropic shells of multi-circuit cross-sections, floating on the ideal fluid. An algorithm for the generation of the motion equations and boundary conditions, as well as for the numerical integration of the motion equations was elaborated.

The method enables to determine the hull response to excitation caused by the system of harmonic forces as well as to calculate the natural modes and natural frequencies of vibration. Examples of numerical calculations compared with experimental results are included for two types of hulls.

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INTRODUCTION

In the design practice the following types of computation modelsare usually applied to the description of ship hull vibrations [1]:

- the bar (beam) models based on the assumption of a non-deformable hull transverse cross-section, or its projection onto the plane perpendicular to the hull longitudinal axis
- the multi-plate and shell models usually based on the finite element method.

The basic advantages of the beam models are:

- simplicity
- low costs of computations against those based on more complex models
- description clarity leading to understandable classification of vibrations
- consistence of calculation results of basic resonance frequencies with those based on other models. The consistence also occurs in the cases where the cross-section deformability is accounted for by means of more exact models. The bar models, though the simplest in applications, provide results close to reality only in the case of slender enough structures and in respect to lower frequencies of vibration. They appear unreliable for higher frequencies.

The multi-plate and shell models make a more exact description possible, but they are much more expensive in use. Their application is connected with long computation time, use of a large operation memory computer and laborious, often prone to errors, process of data preparation. Physical description loses the clarity presented by the beam theory, therefore interpretation of calculation results is more difficult. Calculation accuracy not always can be improved by increasing the number of finite elements, and application of different finite elements sometimes leads to nonconsistent results. The results not always are reliable in respect to higher vibration frequencies.

The drawbacks of the above mentioned models made a team led by Prof.J.Więckowski (deceased in 1984) of the Technical University of Gdańsk, begin searching for another computational model, more exact than the bar-beam models and more transparent than the discrete, multid.o.f. models most often based on plate and shell finite elements. The frame-shell structure model proposed by W.Z.Vlasov in the 1930s was selected for static calculations of multicell, thin-walled structures of closed transverse cross-sections [2].

THEORY ASSUMPTIONS

The system of prismatic, connected to each other, thin-walled segments built of the orthotropic, linearly elastic plates or shells mutually joined along their parallel edges called *the nodal lines*, was assumed as the ship hull structural model. The contours of transverse cross-sections of each segment consist of a finite number of polygons whose corners are called *the cross-section nodes* (see Fig.2 and 4).

The so called *half-moment theory*, based on the known stress-strain relationship for two dimensional state of stress was applied to formulate motion equations of particular segments. In the theory the resulting direct forces in the longitudinal and transverse directions, the resulting shear forces in the shell plane and the bending moment along circumferential direction, are taken into account from among all internal forces which act onto a given shell element, being reduced to the middle plane resultants. However the bending moment in the longitudinal direction and twisting moment are neglected as being small. Moreover it is assumed that the structure can be loaded by a system of the external forces directly applied to shell surfaces as well as to the end cross-sections of segments. The forces are explicit functions of the time t and the location coordinates z,s.

The Vlasov's theory consists in representing the projections of the displacement vector of an arbitrarily chosen point of the structure cross-section circumference, effected onto the axis of the local system of the coordinates b, s, n (Fig.1), in the form of the polynomials of two separated variables as follows:

$$u = \sum_{i=1}^{N} v_i \varphi_i \qquad v = \sum_{k=1}^{R} \vartheta_k \psi_k \qquad w = \sum_{k=1}^{R} \vartheta_k \chi_k \tag{1}$$

where:

$\varphi_i, \psi_k, \chi_k$ - *the shape functions* assumed known, representing constraints imposed upon system's motion

v_i, ϑ_k - *the location functions* of the coordinate z along the longitudinal structure axis and the time t , to be the searched magnitudes.

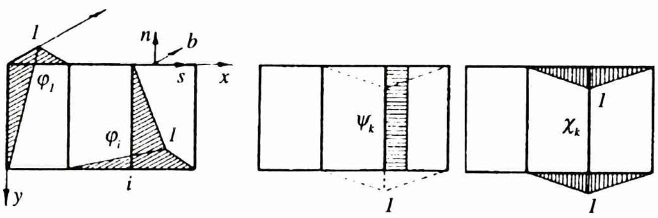

Fig.1. Vlasov's shape functions and local coordinate system for ship hull cross-section

The linear shape functions of *the first type,* proposed by Vlasov, were used in this work.The functions φ_i which describe the longitudinal displacements, take the value equal 1 in i-th node of the cross-section (i = 1, 2,...N) and the value equal 0 in all the remaining nodes. The coupled-in-pairs functions ψ_k, χ_k (k = 1, 2, ...R) which describe the transverse displacements, are generated by shifting the mutually connected contour nodes of the structure cross-section by the value equal 1 in the circumference coordinate direction. Simultaneously the contour is considered as a planar frame built of hinged rigid bars.

The number $N+R$ of the generalized coordinates v_i, ϑ_k used for description is to be equal to the number of degrees of freedom of one cross-section of a structure.

MOTION EQUATIONS

Making equal zero the variation of a functional which expresses the work done by the internal elastic forces and external forces applied to a given structural segment, provides the following set of motion equations of the segment, written in matrix form [3]:

$$\mathbf{A}\,\ddot{\mathrm{z}} = \mathbf{B}\,\mathrm{z}'' + \mathbf{C}\,\mathrm{z}' + \mathbf{D}\,\mathrm{z} - \mathrm{f} \tag{2}$$

which is the set of second-order, partial differential equations, as well as the boundary conditions as follows:

$$\mathbf{M}_{\mathbf{A}}\,\hat{\mathrm{z}}_A = \mathrm{p}_A \qquad \mathbf{M}_{\mathbf{B}}\,\hat{\mathrm{z}}_B = \mathrm{p}_B \tag{3}$$

which is the set of first-order differential equations; where:

A, B, C, D - the square matrices of the constant coefficients depending on the material properties (density, moduli of elasticity, shear modulus, Poisson's ratio) and dimensions of particular shell elements and their location within segment's cross-section

z - the column matrix of the searched generalized coordinates v_i, ϑ_k

f - the column matrix of generalized vibration excitation forces

M - the rectangular matrix with the constant coefficients depending on the material properties and cross-section geometry

$\hat{\mathrm{z}}$ - the column matrix of the generalized coordinates v_i, ϑ_k and their first derivatives in respect to the location coordinate z

p - the column matrix of the generalized forces applied to the segment's end cross-sections of $z = z_A$ and $z = z_B$.

Note:
Dots over symbols stand for the derivatives in respect to the time t
Upper apostrophes stand for the derivatives in respect to the location variable z along the structure axis
The lower indices A, B stand for the left and right end cross-section (edge) of a segment, respectively.

If a vibrating structure floats partly immersed in an inviscous, incompressible liquid, the liquid reaction forces applied to surface of the structure can be taken into account in the motion equations by adding,to the inertia matrix **A**, the appropriately formed, square matrix $\tilde{\mathbf{A}}$ of constant coefficients, called the added mass matrix. This does not change the general form of the equations and does not influence their way of integration. Methods of calculation of the added mass matrix coefficients, expressed by Vlasov's coordinates, is given in [4].

Description of vibrations of the entire structure consists in formulating the differential equations of motion for each of the segments separately, and then coupling the obtained equations by using boundary conditions. Details of the procedure can be found in [3].

INTEGRATION OF EQUATIONS

The set of the partial differential equations describing segment's motion, if the vibrations are harmonic or excitation forces oscillate harmonically, can be transformed by substituting $\mathrm{z} = \mathrm{x}\cos\omega t$ into a set of ordinary differential equations as follows:

$$\mathbf{B}\,\mathrm{x}'' + \mathbf{C}\,\mathrm{x}' + \mathbf{G}\,\mathrm{x} = \mathrm{g} \tag{4}$$

where:

$$\mathbf{G} = \omega^2 \mathbf{A} + \mathbf{D}$$

$$\mathrm{g} = \mathrm{f}\cos\omega t$$

ω - vibration frequency.

A functional matrix of the following form can be found:

$$\mathrm{x}_j(z) = \mathbf{X}_j(z)\,\mathrm{c}_j \tag{5}$$

which is the solution of the set of ordinary differential equations obtained for j-th segment of the structure in question, where:

$\mathbf{X}_j$ - the integral matrix of the set of the differential equations describing j-th segment's motion (j = 1, 2,...,M)

M - number of structural segments

c_j - the column matrix of integration constants for j-th segment.

Determination of the constants is connected with the need of inversion of a square matrix having the size equal to the double number of the generalized coordinates v_i, ϑ_k ,assumed for all segments of the computation model.

To integrate the equations, the Francis iterative method was used which makes it possible to continually control the accuracy solution, as well as to get, by steering the program appropriately, more or less exact solutions subject to an assumed time of calculation. Details of the method can be found in [5].

Several computer programs for vibration analysis of multi-cell, thin-walled structures composed of shell segments with multi-circuit, closed cross-sections [6], were elaborated at the Department of Ship Structural Mechanics and Hull Structures, Technical University of Gdańsk. Operation of the computer programs consists in automatic generation of the matrices of the coefficients of differential motion equations and boundary conditions after putting-in the data on dimensions, material contants and external structural loads, and integrating the composed equations.

The programs make it possible to calculate:

- the natural frequencies and the main vibration modes of a structural system
- the frequencies of the excitation forces at which resonance vibrations of the system are generated, as well as:
- the structural response to excitations generated by a known system of harmonic forces.

EXAMPLES OF COMPUTATIONS

Transverse vibration of ro-ro ship hull

In 1976 ÷ 1980 five ro-ro ships of B 481 shipyard series number were built in Gdańsk Shipyard. The ships of multi-cell, closed hull structure, 182 m long, of 16 000 t displacement at 6.8 m draught, without transverse bulkheads in the midship part, with flat-bottom stern , demonstrated an excessible level of vibrations in service. The need to disclose causes of the vibrations and eliminate them induced the builder to perform measurements onboard consecutive ships of the series, as well as to carry out hull vibration analyses with the use of various computational models.

The simplified ship hull model used for the calculations based on the Vlasov's hypothesis is shown in Fig.2. The model consists of five prismatic segments having contours partly adhering to each other. Each of the segments is built of flat orthotropic shells of different thicknesses and material constants. Detail information on principles of modelling the real, stiffened deck and side structures by using orthotropic shells, as well as dimensions and material properties of particular shells are given in [7].

The natural frequencies of the hull transverse vibrations in the vertical plane, calculated by means of five different methods, as well as several first resonance frequencies experimentally determined with the use of a vibration exciter, are given in the table.The measurements were carried out on real structures of two sister ships at sea.

Natural frequencies of hull transverse vibrations of B 481 ship [s^{-1}]

Number of nodes	Computational model					Measurements	
	A	B	C	D	E	B 481-3	B 481- 4
2	9.92	9.63	9.23	9.01	9.14	9.17	9.15
3	20.11	18.95	17.70	16.76	16.95	16.72	16.55
4	31.63	29.22	27.08	25.13	23.83	23.00	21.36
5	44.40	38.64	36.65	33.93	29.51	28.35	29.43
6	55.82	48.28	45.45	41.78	36.86	-	34.14
7	67.23	57.28	55.40	50.74	44.93	37.01	-
8	80.01	65.66	63.90	58.01	48.13	-	-
8	-	-	-	-	53.23	-	-

Notes:

A, C, D - the calculations performed by Ship Design and Research Centre (CTO) by means of a computer program based on the stiff finite element method [8]. The particular calculation versions differ from each other in determining the added mass and shear influence [9]

B - the results of the calculations based on the Timoshenko beam model, carried out by Ship Research Institute, Technical University of Szczecin, cited in [9]

E - the hull resonance frequencies determined on the basis of the Vlasov frame-shell model.

During the measurements at sea, carried out on the real ship hull, the first modes of transverse structural vibrations were experimentally determined by means of vibration sensors placed along the middle deck axis [10]. The results of the measurements are compared with the calculated values in Fig.3.

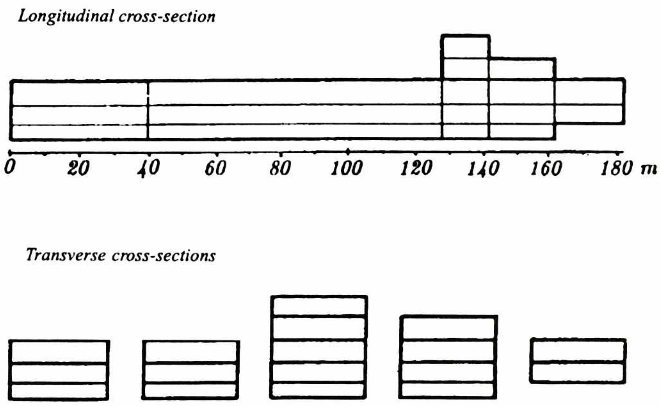

Fig.2.The simplified ship hull model

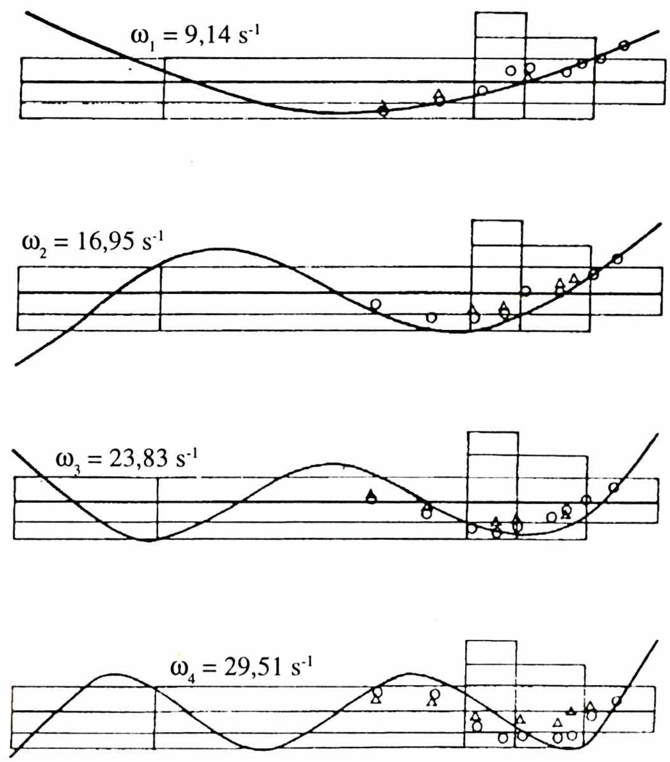

Fig.3.B-481 hull vibration resonance frequencies $\omega_1, \omega_2, \omega_3, \omega_4$ and modes calculated on the basis of the Vlasov frame-shell model, compared with the results of full scale experiments

Note:

Small circle and triangle marks-the results of two different vibration measurements

Continuous lines - the calculated vibration modes (amplitudes of vertical displacements of the middle deck axis).

The comparison of the results of calculations and experiments confirm the widely known observation that the beam model is unreliable in ship hull vibration calculations as far as higher order frequencies are concerned. The frame-shell theory, more exact than the beam theories, made it possible to calculate accurately two additional frequencies and basic modes of vibration.

Transverse and transverse-torsional vibrations of containership hull

The basic natural vibration frequencies and the corresponding main modes of transverse and transverse-torsional hull vibrations of the 2700 TEU containership, designed by Gdańsk Shipyard in 1992, were determined by using the above mentioned computer programs [6].

The ship of 65 000 t displacement at 12.3 m draught, 227 m long, is ,in her midship region, of the open hull structure strengthened by transverse bulkheads.

The shell structure composed of six segments with contours partly adhering to each other was assumed as the calculation model shown in Fig.4. The stiffened plates of ship hull bottom and sides were transformed into orthotropic shells in compliance with the principles given in [7].

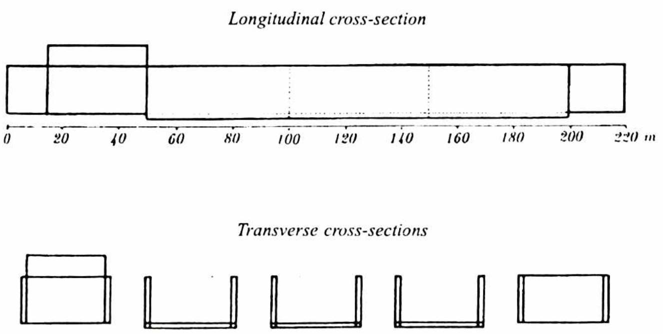

Fig.4. Scheme of 2700 TEU containership calculation model

The resonance frequencies of hull transverse vibrations in the vertical plane, calculated on the basis of the Vlasov's theory, are [Hz]:

$$\omega_1 = 1.19 \qquad \omega_2 = 2.12 \qquad \omega_3 = 2.94 \qquad \omega_4 = 4.27$$

The approximate methods [1] based on vibration measurements of the container ships of a similar displacement value, provide similar results:

$$\omega_1 = 0.75 \text{ to } 1.6 \qquad \omega_2 = 1.5 \text{ to } 3 \qquad \omega_3 = 2.4 \text{ to } 4.2 \qquad \omega_4 = 3.1 \text{ to } 5.5$$

An analysis of transverse-torsional vibrations of the investigated hull, based on the Vlasov theory revealed additionally the following resonance vibration frequencies:

$$\omega_1 = 1.92 \qquad \omega_2 = 3.11 \qquad \omega_3 = 3.71$$

The basic vibration modes correspond to the calculated frequencies.Twisting the structural cross-sections is the dominant mode in the first two cases, in the third one - diagonal bending distinctly coupled with twisting.The vibration mode (displacement amplitudes of the left hull bottom edge) as well as the displacements and deformations of four selected structural cross-sections, which correspond to the frequency $\omega_3 = 3.71$ Hz , are illustrated in Fig. 5.

The first two cross-sections stiffened by transverse bulkheads do not reveal any evident deformations. Third cross-section (z = 125 m, at the half-length of hold) , not stiffened, apart from the rotation and translational displacement, manifests also deformations.

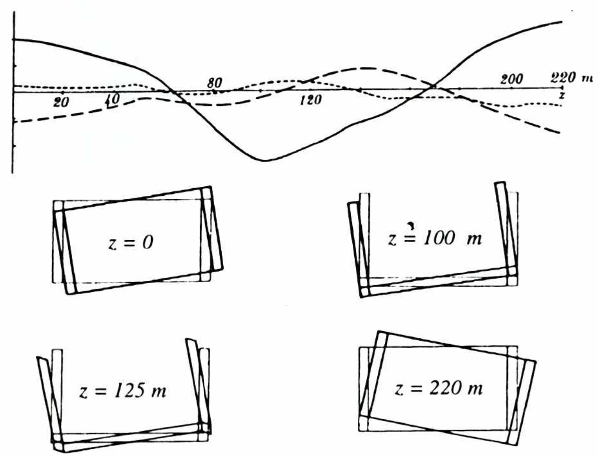

Fig.5. The hull vibration mode (displacement amplitudes of the left hull bottom edge), displacements and deformations of four structural cross-sections, at $\omega_3 = 3.71$ Hz , of 2700 TEU containership

Note:

——	the transverse lateral displacements
- - - -	the transverse vertical displacements
....	the longitudinal displacements

CONCLUSIONS

● The Vlasov frame-shell model with the linear shape functions applied to the analysis of vibrations of a prismatic thin-walled structure of rectangular cross-section, shows the results close to those obtained from the bar models of Bernoulli-Euler (for longitudinal vibrations), Saint Venant-Bredt model (for torsional vibrations) and the Timoshenko beam model (for transverse vibrations).

● Moreover it discloses an additional series of natural frequencies and natural modes of vibrations corresponding to the exactly determined forms of deformation of the structural cross-sections.

● The model used to describe vibrations of multi-cell structures, as well as of the structure composed of joint segments of different cross-sections, gives more accurate results than the beam models, because it takes into account the effects of warping and transverse deformations of the structure cross-sections.

● The presented model, cheap in use, clear for interpreting the results and providing more information about the character of the vibration motion than the beam models provide, can be useful for the vibration analysis of some kinds of thin-walled structures at the early stage of design.

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Appraised by Janusz Kolenda, Prof.,D.Sc.,M.E.