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Fatigue strength assessment of beams under variable combined periodic stress

SUMMARY

The paper deals with the fatigue life prediction of beams under variable periodic stress due to combined bending moment, torsional moment and axial force. It is assumed that the envisaged service life can be considered as a set of states and that the stress components in each state are given in the form of Fourier series.

The equivalent state of stress with in-phase constant-amplitude components are determined on the basis of Cempel's theory of energy transforming systems. It enables to estimate the time to fatigue failure without application of any cycle counting method and hypothesis of fatigue damage cumulation.

INTRODUCTION

Ship hull structures are subjected to steady and cyclic stresses. One of the major sources of cyclic stress is the seaway load. Consequently , the fatigue strength assessment of hull structures must account for various states which can be expected during ship's lifetime. For this purpose Miner's rule of fatigue damage cumulation and one of the cycle counting methods are frequently used. However in many applications the Miner summation may be biased ,quite often in an unpredictable manner, leading to large uncertainties in the fatigue strength calculations [1]. Moreover for more complex load processes it is not clear what should be meant by a cycle and the predicted results depend on the particular cycle countig method chosen [2].

To avoid the shortcomings, Cempel's theory of energy transforming systems [3] can be employed which links the lifetime of the system with the dissipated energy. It makes periodic stress modelling by the equivalent sinusoidal stress possible provided the dissipated energies in both cases are equal [4]. An analogous approach is applied in the paper to beams subjected to variable periodic stress due to the combined bending moment, torsional moment and axial force. It is assumed that the dissipated energy can be calculated separately for each stress component and each load state. The low-cycle fatigue is not considered.

EQUIVALENT STATE OF STRESS UNDER VARIABLE CONDITIONS

If for a given beam the states S_1 , S_2 , ... of combined periodic stress are envisaged it is convenient to determine the equivalent states S'_1 , S'_2 , ...of stress with the following components [4]:

$$\sigma_{ri}(t) = \sigma_{rio} \sin \omega_r t \qquad i = a, b, t \quad r = 1, 2 \ldots \tag{1}$$

where:

$\sigma_{ra}, \sigma_{rb}, \sigma_{rt}$ - stress components in S'_r-th state due to axial force, bending moment and torsional moment, respectively

The amplitudes σ_{rio} and the frequency ω_r can be calculated after expanding the actual stress components in each state S_1 , S_2, ... into Fourier series in accordance with the formulae (2) given in [4]:

$$\sigma_{rio} = \left\{ \frac{8}{k_r^2 T_{or}} \int_0^{T_{or}} \left[\sum_{p=1}^{\infty} \sigma_{rip} \sin\left(p\omega_{or} t + \alpha_{rip}\right) \right]^2 \left[\sum_{p=1}^{\infty} p\sigma_{rip} \cos\left(p\omega_{or} t + \alpha_{rip}\right) \right]^2 dt \right\}^{\frac{1}{4}}$$

$$\omega_r = k_r \omega_{or} \qquad \omega_{or} = \frac{2\pi}{T_{or}} \qquad k_r = Round(\kappa_r) \tag{3}$$

$$\kappa_r = \left[\frac{\sum_i \sum_{p=1}^{\infty} \left(p\sigma_{rip}\right)^2 \frac{\eta_i}{E_i^2}}{\sum_i \sum_{p=1}^{\infty} \sigma_{rip}^2 \frac{\eta_i}{E_i^2}} \right]^{\frac{1}{2}} \tag{4}$$

where:

σ_{rip}, α_{rip} - amplitude and phase angle of p-th term after expanding i-th stress component in S_r-th state into Fourier series

T_{or} - common period of the stress components in S_r-th state

k_r - natural number obtained by rounding the real number κ_r

$E_i,\ \eta_i$ - material constants appearing in (10)

The strain components corresponding to the components (1) are denoted as follows:

$$\varepsilon_{ri}(t) = \varepsilon_{rio} \sin(\omega_r t + \beta_{ri}). \tag{5}$$

Equations (2) to (4) are derived in [4] by means of Cempel's theory with the energies externally and internally dissipated by the vibrating beam taken into account.In the similar way it is possible to model the states S'_1, S'_2, ... by the equivalent state of stress S with the components:

$$\sigma_i(t) = \sigma_{io} \sin \omega t. \tag{6}$$

In order to determine the amplitudes σ_{io} and the frequency ω the number of equivalent stress cycles in S_r-th state of a duration T_r

$$n_r = \frac{\omega_r T_r}{2\pi} \tag{7}$$

is assumed a natural number and the externally dissipated energy due to the strain components (5) proportional to the sum of the integrals of ε_{ri}. Since the integrals of the components (5) over the time T_r are zero the externally dissipated energy in S_r-th state, D'_r, is assumed proportional to the following sum:

$$D'_r \propto \sum_i \int_0^{T_r} \varepsilon_{ri}^2 dt \tag{8}$$

The internally dissipated energy by i-th stress component in S_r-th state, D''_{ri}, can be expressed as follows [5]:

$$D''_{ri} = \int_0^{T_r} \sigma_{ri} \dot{\varepsilon}_{ri} dt \tag{9}$$

When applying Kelvin-Voigt's model of a material of the beam:

$$\sigma_{ri} = E_i \varepsilon_{ri} + \eta_i \dot{\varepsilon}_{ri} \tag{10}$$

the equation (9) yields:

$$D''_{ri} = I_1 + I_2$$

where:

E_i - Young or shear modulus

η_i - internal viscous damping coefficient of the material, corresponding to i-th strain component

$$I_1 = \int_0^{T_r} \eta_i \dot{\varepsilon}_{ri}^2 dt$$

$$I_2 = \int_0^{T_r} E_i \varepsilon_{ri} \dot{\varepsilon}_{ri} dt = 0$$

Similarly as in the case of the strain integrals above, the integrals:

$$I'_2 = \int_0^{T_r} (E_i \varepsilon_{ri} \dot{\varepsilon}_{ri})^2 dt$$

are taken, instead of I_2, for further consideration. Hence, under the assumption that the total internally dissipated energy in S_r-th state, D''_r, can be calculated as follows:

$$D''_r = \sum_i D''_{ri} \tag{11}$$

one gets the following equivalence conditions;

$$\int_0^T \varepsilon_i^2 dt = \sum_r \int_0^{T_r} \varepsilon_{ri}^2 dt \tag{12}$$

$$\int_0^T \eta_i \dot{\varepsilon}_i^2 dt = \sum_r \int_0^{T_r} \eta_i \dot{\varepsilon}_{ri}^2 dt \tag{13}$$

$$\int_0^T (E_i \varepsilon_i \dot{\varepsilon}_i)^2 dt = \sum_r \int_0^{T_r} (E_i \varepsilon_{ri} \dot{\varepsilon}_{ri})^2 dt \tag{14}$$

and the alternative conditions resulting from (8) and (11):

$$\sum_i \int_0^T \varepsilon_i^2 dt = \sum_i \sum_r \int_0^{T_r} \varepsilon_{ri}^2 dt \tag{15}$$

$$\sum_i \int_0^T \eta_i \dot{\varepsilon}_i^2 dt = \sum_i \sum_r \int_0^{T_r} \eta_i \dot{\varepsilon}_{ri}^2 dt \tag{16}$$

$$\sum_i \int_0^T (E_i \varepsilon_i \dot{\varepsilon}_i^2)^2 dt = \sum_i \sum_r \int_0^{T_r} (E_i \varepsilon_{ri} \dot{\varepsilon}_{ri}^2)^2 dt \tag{17}$$

where:

$$\varepsilon_i = \varepsilon_{io} \sin(\omega t + \varphi_i) \tag{18}$$

denotes i-th strain component in the state S, and:

$$T = \sum_r T_r \tag{19}$$

In the following, the conditions (12), (14) and (16) are used. So, for the natural number:

$$n = \frac{\omega T}{2\pi} \tag{20}$$

(7), (12), (14), (16), (18) and (20) lead to:

$$\varepsilon_{io}^2 T = \sum_r \varepsilon_{rio}^2 T_r \tag{21}$$

$$\omega^2 \varepsilon_{io}^4 T = \sum_r \omega_r^2 \varepsilon_{rio}^4 T_r \tag{22}$$

$$\omega^2 T \sum_i \eta_i \varepsilon_{io}^2 = \sum_i \sum_r \eta_i (\omega_r \varepsilon_{rio})^2 T_r \tag{23}$$

From (21) and (23) the frequency ω' is obtained:

$$\omega' = \left[\frac{\sum_i \sum_r \eta_i (\omega_r \varepsilon_{rio})^2 T_r}{\sum_i \sum_r \eta_i \varepsilon_{rio}^2 T_r} \right]^{1/2} \tag{24}$$

close to the frequency ω, and the real number:

$$n' = \frac{\omega' T}{2\pi} \tag{25}$$

close to the natural number:

$$n = Round\ (n') \tag{26}$$

Combining (20), (25) and (26) gives

$$\omega = \frac{2\pi}{T} Round \left(\frac{\omega' T}{2\pi} \right) \tag{27}$$

Equation (22) yields:

$$\varepsilon_{io} = \left(\frac{\sum_r \omega_r^2 \varepsilon_{rio}^4 T_r}{\omega^2 T} \right)^{1/4} \tag{28}$$

In accordance with (10) and (18) i-th stress component in the state S:

$$\sigma_i = E_i \varepsilon_i + \eta_i \dot{\varepsilon}_i \tag{29}$$

has the amplitude:

$$\sigma_{io} = \varepsilon_{io} \left[E_i^2 + (\eta_i \omega)^2 \right]^{1/2} \tag{30}$$

Since:

$$\sigma_i \cong E_i \varepsilon_i \qquad \sigma_{io} \cong E_i \varepsilon_{io} \tag{31}$$

(12), (14), (16), (24) and (28) may be replaced in practical calulations with:

$$\int_0^T \sigma_i^2 dt = \sum_r \int_0^{T_r} \sigma_{ri}^2 dt \tag{32}$$

$$\int_0^T (\sigma_i \dot{\sigma}_i)^2 dt = \sum_r \int_0^{T_r} (\sigma_{ri} \dot{\sigma}_{ri})^2 dt \tag{33}$$

$$\sum_i \frac{\eta_i}{E_i^2} \int_0^T \dot{\sigma}_i^2 dt = \sum_i \sum_r \frac{\eta_i}{E_i^2} \int_0^{T_r} \dot{\sigma}_{ri}^2 dt \tag{34}$$

$$\omega' = \left[\frac{\sum_i \sum_r \frac{\eta_i}{E_i^2} (\omega_r \sigma_{rio})^2 T_r}{\sum_i \sum_r \frac{\eta_i}{E_i^2} \sigma_{rio}^2 T_r} \right]^{1/2} \tag{35}$$

$$\sigma_{io} = \left(\frac{\sum_r \omega_r^2 \sigma_{rio}^4 T_r}{\omega^2 T} \right)^{1/4} \tag{36}$$

If the difference between ω and ω' is negligible, i.e. if :

$$\omega \cong \omega' \tag{37}$$

one gets the following formulae for the parametres of the equivalent stress components under variable conditions:

$$\sigma_{io} = \left[\frac{\sum_r \omega_r^2 \sigma_{rio}^4 T_r \sum_i \sum_r \frac{\eta_i}{E_i^2} \sigma_{rio}^2 T_r}{\sum_r T_r \sum_i \sum_r \frac{\eta_i}{E_i^2} (\omega_r \sigma_{rio})^2 T_r} \right]^{1/4} \tag{38}$$

$$\omega = \left[\frac{\sum_i \sum_r \frac{\eta_i}{E_i^2} (\omega_r \sigma_{rio})^2 T_r}{\sum_i \sum_r \frac{\eta_i}{E_i^2} \sigma_{rio}^2 T_r} \right]^{1/2} \tag{39}$$

DESIGN CRITERIA

In the fatigue design of the structural elements subjected to a simple loading, the S-N curves :

$$N_i \sigma_{io}^{m_i} = K_i \tag{40}$$

valid for:

$$Z_i < \sigma_{io} \le L_i \tag{41}$$

are usually applied, where:

- N_i - number of cycles to failure at constant stress amplitude σ_{io}
- K_i , m_i - material dependent constants
- Z_i - fatigue limit at i-th simple loading
- L_i - maximum stress amplitude satisfying (40)

The criterion of an infinite fatigue life reads:

$$\sigma_{io} \le Z_i$$

The formulae presented in [6] can be used to formulate the analogous relationships in the considered case. Thus, an infinite fatigue life of a beam under variable combined periodic stress can be obtained if in each load/stress state the equivalent stress (1) fulfils the following inequality:

$$\left[\left(\frac{\sigma_{rao}}{Z_a} + \frac{\sigma_{rbo}}{Z_b} \right)^2 + \left(\frac{\sigma_{rto}}{Z_t} \right)^2 \right]^{-1/2} \ge 1 \tag{42}$$

If such states that:

$$\left[\left(\frac{\sigma_{rao}}{Z_a} + \frac{\sigma_{rbo}}{Z_b} \right)^2 + \left(\frac{\sigma_{rto}}{Z_t} \right)^2 \right]^{-1/2} < 1 \le \left[\left(\frac{\sigma_{rao}}{L_a} + \frac{\sigma_{rbo}}{L_b} \right)^2 + \left(\frac{\sigma_{rto}}{L_t} \right)^2 \right]^{-1/2} \tag{43}$$

are expected the high-cycle fatigue failure can occur. Making use of the equivalent stress (6) the time to failure can be estimated in the case, as follows [6]:

$$T_f = \frac{2\pi}{\omega}\left[\left(\frac{\sigma_{ao}^{m_a}}{K_a}+\frac{\sigma_{bo}^{m_b}}{K_b}\right)^2+\left(\frac{\sigma_{to}^{m_t}}{K_t}\right)^2\right]^{-1/2} \qquad (44)$$

and the following design criterion can be written:

$$M = T_f - T > 0 \qquad (45)$$

where:

M - arbitrary safety margin
T - summary duration of these states
ω, σ_{io} - frequency and amplitudes of the equivalent stress components, respectively, calculated from (38) and (39) for these states

Equation (44) cannot be used if neither (42) nor (43) is satisfied in one or more states, because in such case the low-cycle fatigue may have to be taken into consideration.

CONCLUSIONS

The presented assessment procedure of the fatigue strength under variable conditions differs from the known methods. In particular it includes:

- Fourier analysis of periodic stress components
- approximation of complex states of periodic stress by the equivalent stress states with in-phase components
- modelling the set of periodic stress states by one equivalent stress state with in-phase components.

However experimental investigations are needed to verify its accuracy.

The superposition of normal stress components resulting from in-plane and out-of-plane bending can be analyzed in the similar way as the components due to bending and axial force. It means the procedure may be employed in the fatigue calculations of ship's hull structures subjected to vertical and horizontal wave bending loads.

NOMENCLATURE

D'_r, D''_r - externally and internally dissipated energies in r-th stress state
M - safety margin
T - summary duration of the considered stress states
T_f - time to failure under variable conditions
T_r - duration of r-th stress state
Z_i - fatigue limit at i-th simple loading (i=a,b,t : tension-compression, alternate bending, twisting, respectively)
ε_i, ε_{io} - i-th equivalent strain component and its amplitude for the set of periodic stress states
ε_{ri}, ε_{rio} - i-th equivalent strain component and its amplitude for r-th state of periodic stress
σ_i, σ_{io} - i-th equivalent stress component and its amplitude for the set of periodic stress states
σ_{ri}, σ_{rio} - i-th equivalent stress component and its amplitude for r-th state of periodic stress
ω - circular frequency of the equivalent stress for the set of periodic stress states
ω_r - circular frequency of the equivalent stress for r-th state of periodic stress

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Conferences

CTO

Modern shipbuilding materials

On 14 May 1996 the seminar on „ Modern Shipbuilding Materials " was held as a part of celebration of 25th anniversary of the Ship Design and Research Centre (CTO), Gdańsk. It was arranged by the Ship Materials, Corrosion and Environment Protection Division of CTO.

The following four papers were presented of authors or co-authors employed in the Division:

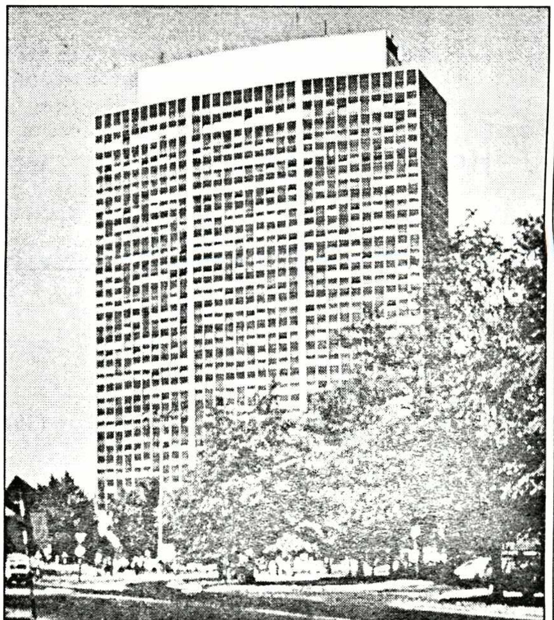
1. „ Trends of shipbuilding material development " by G. Szydłowska - Herburt , D.Sc.
2. „ New materials and anti-corrosion techniques in marine building " by A. Zieliński, Prof., D.Sc.
3. „ New trends in anti-corrosion protection of ships by means of paint coatings " by J. Birn, D.Sc., A. Baraniak, M.Sc.
4. „ Technical requirements for shipbuilding GRP composites in the light of international standards, ship classification rules and EU guidelines " by St. Szpak-Szpakowski, M. Sc.

The subject area of the seminar attracted many specialists. Representatives of all Polish shipbuilding and shiprepair yards, several ship equipment producers and supervising institutions : Polish Register of Shipping and Marine and Tropical Medicine Institute took part in the seminar.

Ecological aspects, especially those dealing with the utilization of wastage of composites and shipbuliding paints and prevention of shipyard basins against pollution were most disscussed apart from material application technical problems.

Questions raised by shipbuilding practitioners during the disscussion will be taken into account in future research plans of the Division.

The participants admitted organizing such meetings periodically to be worthwile as they contribute in integrating the professional circle and bring industrial problems nearer to the research centres.



The main building of Ship Design and Research Centre (CTO) in Gdańsk