

LESZEK KONIECZNY, D.Sc., N.A.  
 Ship Design and Research Centre (CTO)  
 Ship Structures Division  
 Gdańsk

# Design thickness determination of the plates exposed to wheel loading with an arbitrary unknown footprint

## SUMMARY

*The paper contains results of pilot calculations of the plates exposed to pressure or wheel loading, which are compared with the data found in literature sources. The permanent set of the typical deck plates, generated during the entire loading cycle, was also calculated.*

*The investigation was aimed at the determination of a practically useful relationship between concentrated load and permanent set of plate.*

*This is a partial result of the research carried out by the CTO, Gdańsk, on request of the Polish Register of Shipping (PRS).*

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## INTRODUCTION

The rule formula, given in [1], for thickness determination of the deck plates subjected to wheel loading is based generally on the classical linear theory of elastic plates, where the state in which material attains yield point in the most strained point of plate is assumed the limit state. In the formula, appropriately lower design permissible stresses should be assumed to account for safety margin, e.g. 160 MPa ( for normal strength steel ) as it is the case with stiffeners.

The rule permissible stresses for plates exposed to wheel loading are at least twice higher than that, i.e. 320 MPa ( in sea-going conditions) and 370 MPa ( in harbour conditions). In the case when sea-going conditions are decisive, it leads to a deck plate thickness lower almost by 30 % than that obtained from the usually applied permissible stress level. It is probably justified by positive service experience with such deck designs. However the design method has no physical sense since the stress level exceeding the material yield point of 235 MPa by 36% is permitted there which is physically not obtainable. In this case it is not possible to evaluate a real structural safety margin, which makes rational designing more difficult.

This research is aimed at the determination and theoretical justification of a formula for design thickness of the plates exposed to wheel loading with an arbitrary unknown footprint. The maximum permissible level of the plate permanent set was assumed the limit state criterion as in the case of plates with pressure loading [2]. The problem can be solved only by means of numerical calculations as large deflections of plates and material plastic deformations should be taken into account. Such calculations were performed with the use of SILICON GRAPHICS Indy 4600 work station and PATRAN-NASTRAN software system.

## DESIGN CRITERIA FOR PLATES EXPOSED TO WHEEL LOADING

In the design of plates for lateral load the permanent set in the centre of the plate panel framed with stiffeners and girders is the most important parameter. An allowable value of the permanent set is determined on the basis of the ultimate strength criterion or serviceability of plates. In the case when compressive loading prevails ( acting in the plate plane ) it is necessary to limit the permanent set to ensure an appropriate ultimate strength of the plate. However if pressure or lateral forces are dominant, an allowable value of the permanent set is determined by service requirements, e.g. these connected with handling and shipping vehicles. In such case the permanent set limitation is the main criterion. In any case it is necessary to know a relationship between lateral loading and permanent set of plate. In the previous work [2] the permanent set criterion was implemented to plates with pressure loading. It has been found that the results of the research are also applicable to the plates with concentrated lateral loads.

For design purposes, concentrated random loads ( which can occur once in the service time ) and movable loads are distinguished. Although the point of application of the former ones may be random, the worst position - midway between stiffeners - is assumed for them ( this is the case of „single - location loads” - SL ). The movable loads can occur at least several times over ship's life and with different points of application ( this is the case of „ multiple-location loads” - ML).

The wheel loading belongs naturally to the latter load class. The maximum value of the loads can be determined if the maximum axle ( or wheel) load or tyre inflation pressure of a vehicle in question is known. The maximum plate loading can be then calculated on the basis of an assumed permanent set limit value. When applying safety factor ( to account for accidental overloading in service ) the design load for the plates in question can be obtained.

# PILOT CALCULATIONS OF Laterally LOADED PLATE PANELS

The pilot calculations were aimed at comparison of the results achieved with the use of PATRAN-NASTRAN software with those obtained by means of other programs, published in [6].

## Ultimate strength of the bulkhead under lateral pressure and its comparison with the limit loads obtained from approximate formulas for rigid-plastic material

The calculation model, shown in Fig. 1, comprises a quarter of the plate panel together with the adjacent longitudinal stiffener. The partition into finite elements (FEs) was assumed in accordance with [6]. The bulkhead plating thickness was 11.5 mm and the steel properties as follows:

$$E = 210\,000 \text{ MPa}, R_e = 264 \text{ MPa}, \nu = 0.3$$

The uniformly distributed pressure load ( applied to the unstiffened side of the plate ) grew monotonically till its ultimate value. The plate was rigidly supported along its shorter edge BD, and its deflection angle assumed zero at the edges AB and BD. The edges could shift uniformly. At the edges AC and CD the boundary conditions resulting from double symmetry were valid.

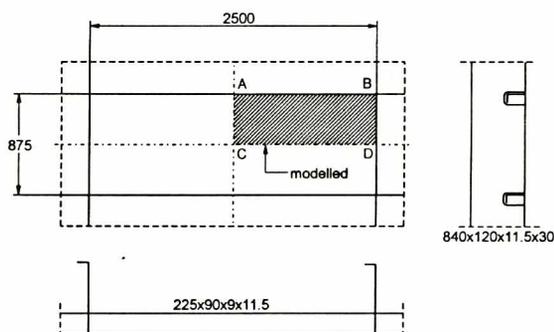


Fig. 1. Part of the bulkhead structure considered in calculations

The relationship between the uniform pressure  $p$  and the deflection of the stiffener  $w_A$  ( in the point A ) and that of the plate  $w_C$  ( in the point C ), obtained on the basis of the calculations by means of NASTRAN and - for comparison - the relevant relationships according to [6] are shown in Fig. 2. The difference  $w_C - w_A$ , viz. the plate deflection in respect to that of stiffener, ( denoted „plate” in the figure ) is also given. The calculation results illustrate well the large influence of the tensile membrane stresses on the ultimate load of the stiffened plate. The load- carrying capacity of the plate was not exhausted until the ultimate strength of the stiffeners was reached ( the classical ultimate strength assessment methods of the isolated plate are decisively too conservative ). However the ultimate strength of the bulkhead can be assessed ( lower limit ) on the basis of the ultimate strength of the stiffener ( with the included effective plate length ) considered as the fixed-end beam.

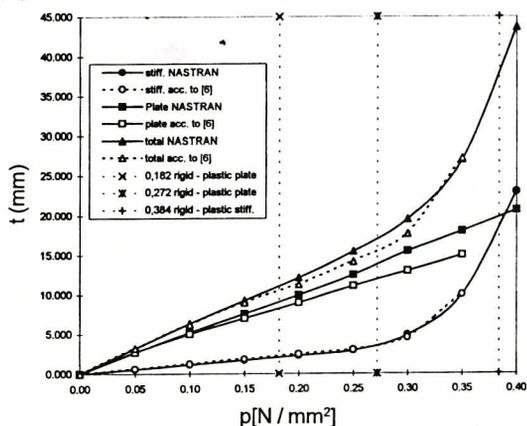


Fig. 2. Deflection of the bulkhead versus pressure

As far as the comparison between the results obtained from NASTRAN and those from the program used in [6] is concerned, the triangular plate FEs applied in [6] yield plate deflections lower ( in the case of the plate clamped along its boundary ) than those obtained from the calculations by using NASTRAN where the rectangular FEs are implemented.

## Non-linear analysis of the deck ro-ro ship exposed to wheel loading

The natural size model of the ro-ro ship deck, used in [6] to investigate permanent set of the plating subjected to wheel loading, is shown in Fig. 3a. Results of the investigations are utilized for comparison in this research. Here a quarter of the model was considered with free support along its contour, exposed to tyre pressure applied over the plate mid-region. The material properties were assumed identical as in the case of the above described bulkhead. In Fig. 3b the assumed load area and FE mesh of the centre plate is presented. Calculations were carried out for the entire loading-unloading cycle to determine permanent set of the loaded (centre) plate.

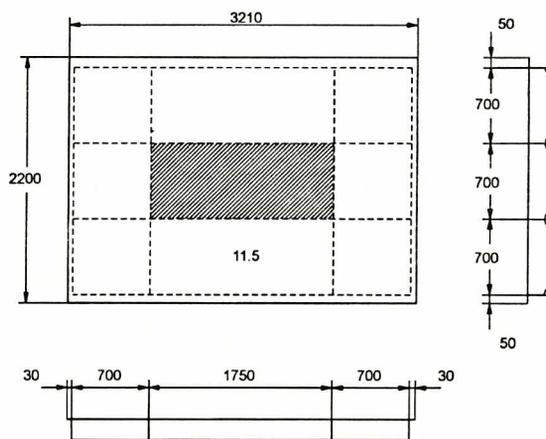


Fig. 3a. Model of ro-ro ship deck

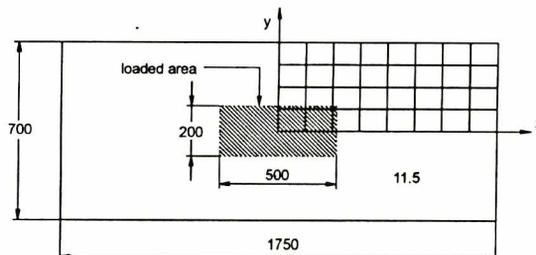


Fig. 3b. Load area and FE mesh of the centre plate panel

Additionally, calculations of the centre plate panel taken as being isolated out of the model were performed for the same boundary conditions as assumed in [6] ( the plate pinned at the restrained contour ).

In Fig. 4 calculation results are presented in the form of the relationships of the maximum plate relative deflections  $w/t$  and the resultant load  $P$ .

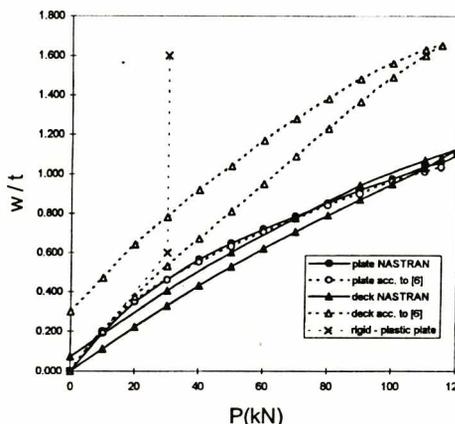


Fig. 4. Deck structure relative deflections  $w/t$  versus the resultant wheel loading  $P$

In the same figure the results of calculation of the isolated plate panel, published in [6], are also given for comparison. Results of the calculation with the use of NASTRAN practically match those published in [6] ( marked with small circles in Fig. 4.) because in both cases the identical isolated plate panel with the restrained contour is considered.

However quite different situation occurs when the calculation results of the entire deck model, obtained from NASTRAN, are compared with those of the isolated plate panel pinned at the restrained contour, according to [6] (represented by triangles in Fig. 4). The deflections differ by several tens percent already within the linear-elastic range when influence of contour sliding is negligible. The permanent set values differ even several - fold. It was caused by assuming the pinned support condition at the isolated plate contour for the calculations presented in [6]. It means that in the case of the plates exposed to concentrated loads the influence of the bending-twisting stiffness of the surrounding plates and stiffeners can not be neglected. That is why the deck model shown in Fig. 3a was taken, instead of that of the isolated plate, into further considerations.

Also in this case the ultimate strength assessment of the plate by applying the classical limit load analysis (rigid-plastic material) is useless.

### SYSTEMATIC CALCULATIONS OF PERMANENT SET OF THE PLATES SUBJECTED TO WHEEL LOADING

For purposes of the work, the method and denotations are adopted used by Hughes in the publications [3,4,5] on the design of plates laterally loaded, in compliance with the permissible permanent set criterion. The aim of the work is to verify the application range of Hughes' method by using computerized FEM analyses and with the assumed large deflections and perfectly elastic material.

It is assumed that a load is spread over the rectangular area of  $e \times f$  dimensions and that the side  $e$  is always parallel to the shorter plate side  $b$ , as shown in Fig. 5. The geometrical mean value  $e_m$  of the dimensions  $e$  and  $f$  is assumed as the wheel footprint measure:

$$e_m = \sqrt{ef} \tag{1}$$

As the resultant permanent set value of plates does not depend on the  $e/f$  ratio thus both footprint dimensions are of the same importance. According to (1) the footprint area is  $e_m^2$  the refore:

$$e_m = \sqrt{P/p_t} \tag{2}$$

The dimensionless load concentration parameter  $\lambda$  is defined as follows:

$$\lambda = \frac{e_m}{b} \tag{3}$$

The dimensionless load parameter  $Q_p$  is defined in the case of concentrated loads as below:

$$Q_p = \frac{PE}{b^2 \sigma_y} \tag{4}$$

The systematic calculations of plate permanent set were performed for the quarter of the ro-ro ship's deck model presented in Fig. 3a, exposed to the wheel loading shown in Fig. 5. After each load application the structure was unloaded and then loaded again with the consecutive load. The whole cycle of three loadings and unloadings was repeated two times to control influence of the repetitions on permanent set magnitude. The detailed input data and results are given in Tab. 1.

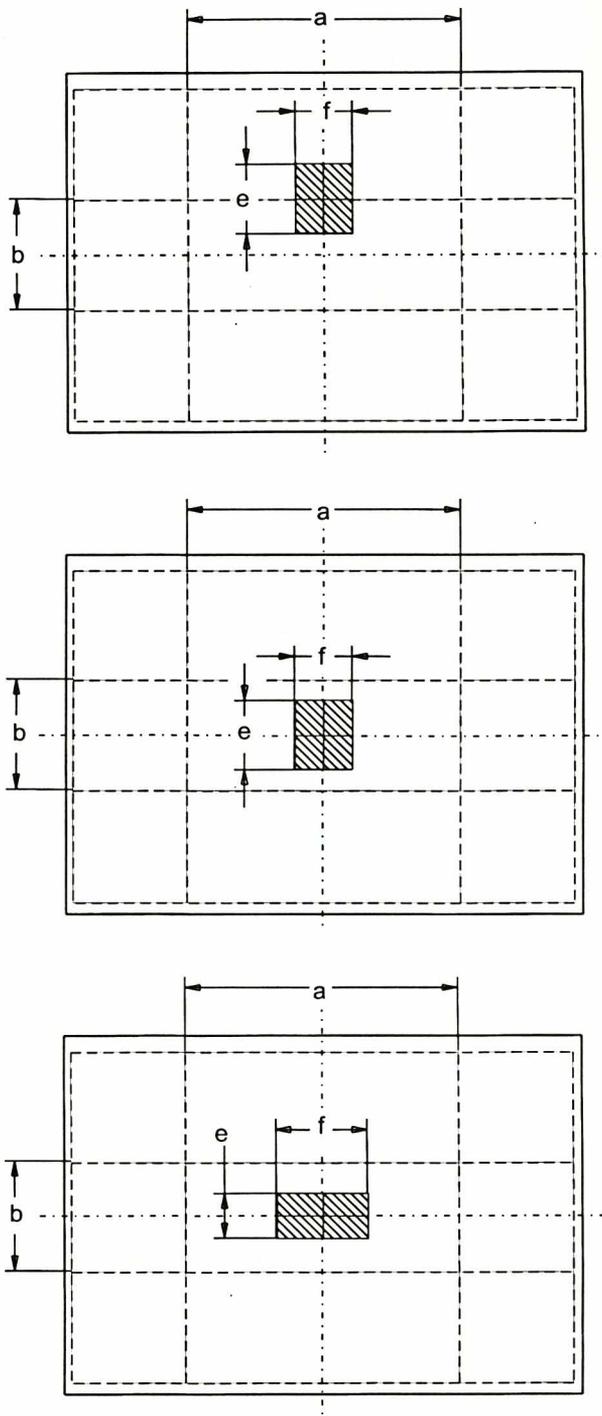


Fig. 5. The wheel loading assumed in the plate permanent set calculations

Tab. 1. Calculation results of the plate panel 1:  $\beta = 2.12$ ;  $a = 1750$  mm;  $b = 700$  mm;  $t = 11.5$  mm

No.	$p_t$ [kPa]	P [kN]	$w_p$ [mm]	$\lambda$ [-]	Q [-]	$Q_p$ [-]	r [-]
1	2	3	4	5	6	7	8
1.1	696.5	139.8	0.817	0.64	0.97	0.894	1.085
1.2	696.5	234.5	5.107	0.829	1.29	1.5	0.86
1.3	1762	139.8	2.811	0.4	1.18	0.984	1.32
1.4	3172	139.8	9.419	0.3	1.45	0.984	1.62

The data of the models were assumed identical with those of the models tested by Sandvik [3, 6]. Efforts were also made to apply the similar loading of the models. Apart from the calculations performed for the load cases used in the cited research the additional calculations were performed for an almost concentrated load ( with  $\lambda < 0.6$  ) to verify the range of application of the method described in the following chapter.

In Fig. 6 the centre plate permanent set values, calculated with the use of NASTRAN, are presented and compared with those obtained by Sandvik ( the maximum value corresponds to the stationary deflection values ). To assess the design stationary values the deflections multiplied by the factor 1.13 are given in col. 4, Tab. 1. On the basis of independent calculations a dependence of the permanent set and the uniform pressure applied to the isolated plate was established and , with the use of it, the values of  $Q_c$ , given in col. 6, Tab. 1, corresponding to the permanent set due to wheel loading (col. 4), were determined.

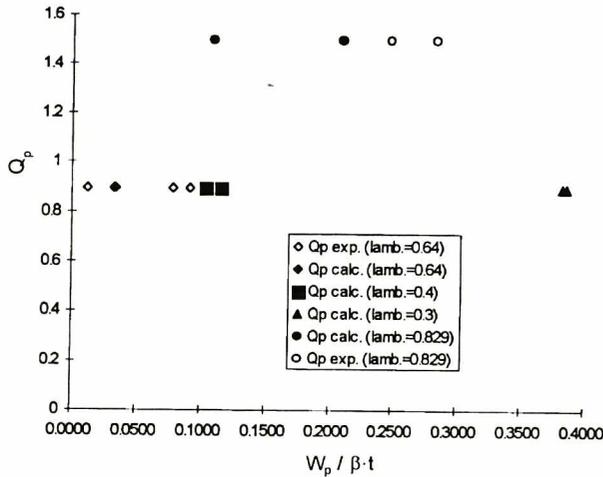


Fig. 6. The dimensionless concentrated load versus permanent set parameter for the plate of  $a/b = 2.5$ ;  $\beta = 2.12$  (comparison of the calculation and test results)

## APPLICATION OF THE PERMISSIBLE PERMANENT SET CRITERION TO THE DESIGN OF PLATES EXPOSED TO WHEEL LOADING

When a sufficiently large concentrated load moves across plate panel, plastic deformations of the plate occur along loading path and the final maximum plastic deformation is larger than that in the case of the application of the same load only once in the middle of the plate. As wheel loading can be applied in any place, therefore finally almost every part of the plate panel deforms plastically, at least over the surface layer. During ship operation the tyre diameter and breadth change. Double wheels prevail which, in the case of their move along the longer side of the plate ( with the contour stiffener between the wheels ), can cause the plate to rotate plastically along that side and the permanent deflections to increase farther on. Although load values are not always equal to the maximum ones assumed in design, yet the plastic deformations reach gradually the stationary distribution and the maximum permanent set reaches the final value corresponding to the design loading. This is the final value which is important in design. **The value depends on the plate parameters  $\alpha$  and  $\beta$  as well as on the load concentration parameter  $\lambda$  and does not depend either on the shape of the loaded area or the  $ef$  ratio.**

Owing to the cumulative nature of the plastic deformations a hypothesis can be accepted that **the final distribution of the plastic deflections caused by wheel loading is principally similar to that due to uniform pressure.**

If the hypothesis stands, the relationship between the load coefficient and the plate permanent set will be similar to that governing in the case of the uniform pressure load. Therefore the stationary permanent set value corresponding to the design value of the wheel load coefficient  $Q_p$  will be the same as for the equivalent uniform pressure  $p_e$  which the load coefficient  $Q_c$  due to the same pressure corresponds to:

$$Q_c = \frac{p_e E}{\sigma_y^2} \quad (5)$$

or after introduction of the notation:

$$r = \frac{Q_c}{Q_p} = \frac{p_e b^2}{P} = \frac{b^2 p_e}{e_m^2 p_t}$$

$$Q_c = r Q_p \quad (6)$$

**The relationship between the concentrated load  $P$  of the plate and its resultant stationary plastic deflection can be assumed the same as that concerning the plate exposed to uniform pressure, in accordance with the approximate formulas published in [3], [4] or [2], in which  $Q_c$  instead of  $Q$  should be substituted.**

In compliance with the hypothesis it is assumed that the stationary permanent set distributions for both loading cases are similar and the maximum value of the deflections mainly depends on the load concentration parameter  $\lambda$ , and only slightly on the parameters  $\alpha$  and  $\beta$  which characterize the plate, because the dependence on the parameters is already accounted for in the formulas concerning the permanent set of the plates exposed to uniform pressure. On the basis of the general considerations [3] it can be demonstrated that, in the case of sufficiently large values of  $\lambda$ , the following relationship is valid:

$$r = \frac{1}{\lambda^2} \quad (7)$$

and that  $r = 2$  when  $\lambda = 0$  ( perfectly concentrated load ).

The relationship  $r(\lambda)$  can be determined by experiments. In the available literature sources [3], [6] the author succeeded to find only the information about Sandvik's experiments with two natural scale stiffened plates of the typical slenderness factor values (  $\beta = 2.12$  and 1.8). In Fig. 3a the model used in the first series of the experiments is presented ( in the second series the applied stiffener spacing was two times smaller). The model was many times loaded by wheels of a fork lift truck while increasing load level. In the intermediate states the loading was repeated two times only: one truck run perpendicularly to the plate stiffener and one run along the stiffener ( with the stiffener between the double wheels ). On the other hand at the maximum load level the truck ran additionally 10 times there and back in different directions to determine a stationary permanent set value of the plate. The results are available for two maximum load values of model 1 and four maximum load values of model 2. The ratio of the stationary permanent set value and the permanent set value reached after the second run is of importance. The ratio for model 1 is equal to 1.13, but 1.2 for model 2.

On the basis of theoretical considerations and the above described experiments O. Hughes [3] proposed the relationship  $r(\lambda)$  in the following form:

$$r = \frac{0.88}{0.44 + \lambda^2} \quad (8)$$

In Fig. 7 the curve according to (8) is presented for comparison together with the values of  $r$  calculated from the measured permanent set values. Since the experiments dealt only with the values  $\lambda \geq 0.64$ , it was necessary to verify the relationship (8) also for  $\lambda < 0.6$ . In Fig.7 the calculation results already presented in Tab. 1, obtained from NASTRAN, are additionally introduced also for  $\lambda < 0.6$ .

If the factor  $r$  value is known, the plate thickness necessary for  $p_e$  value, [the equivalent uniform pressure in compliance with (5) and (6)] can be calculated, accounting for the approximate formula given in [2], as follows:

$$t = \frac{18}{\sqrt{\sigma}} \frac{s \sqrt{p_e}}{l} \left( 1 - 0.27 \frac{s}{l} \right)^2 \quad (9)$$

where:

$p_e$  - equivalent uniform pressure [kPa]

$s$  - stiffener spacing [m]

$l$  - stiffener span [m]

$\sigma$  - permissible stresses [kPa]

The formula given in [2], proposed on the basis of the systematic calculations of the permanent set of the plates under pressure load, ensures obtaining results in compliance with the PRS rules requirements.

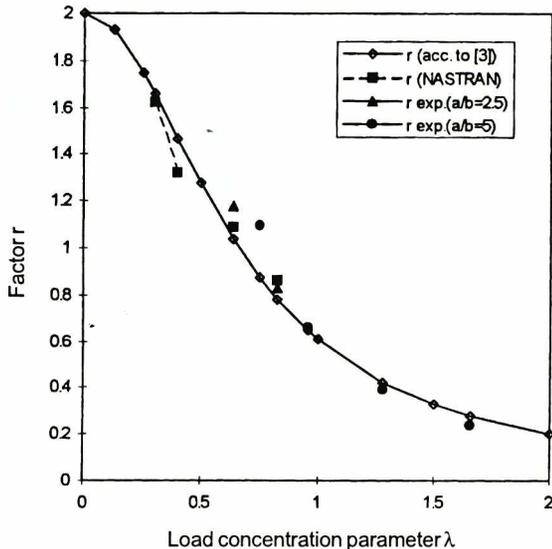


Fig. 7. Comparison of the factor  $r$  values

## CONCLUSIONS

In the paper the permissible permanent set criterion was applied to the design of plates under vehicle wheel loading. The movable wheel loading can be substituted by the equivalent uniform pressure when using Hughes' formula (8) to determine the factor  $r$  by which the concentrated load coefficient  $Q_p$  should be multiplied to get the uniform pressure load coefficient  $Q_e$ .

The factor  $r$  depends mainly on the load concentration and almost does not depend on the plate slenderness and side ratio as well as wheel footprint shape. The presented results, obtained from systematic calculations by applying PATRAN-NASTRAN software, justify a statement that the Hughes' formula for the factor  $r$  can be used also in the case of  $\lambda < 0.6$ .

The permanent set calculations described in the paper were carried out for the plates with the ratio  $a/b = 2.5$ . Therefore such systematic calculations should be continued for other values of  $a/b$  ratio to eventually elaborate a corrected version of Hughes' formula for  $r$  factor in the low value range of  $\lambda$ . The corrected formula for  $r$ , recently published [5], is not applicable for practical use for  $\lambda < 0.6$  as it yields the plate thickness values not complying with the rule requirements.

## NOMENCLATURE

$a$  - plate length [mm]       $b$  - plate breadth [mm]

$e$  - load area dimension parallel to the plate breadth  $b$

$e_m$  - average wheel load area dimension

$E$  - Young's modulus of plate material [MPa]

$f$  - load area dimension parallel to the plate length  $a$

$l$  - stiffener span [mm]\*

$p$  - plate design lateral pressure [MPa]\*

$p_e$  - equivalent uniform pressure [MPa]\*

$p_t$  - tyre inflation pressure [MPa]\*

$P$  - resultant wheel load [N]\*

$Q = p \frac{E}{\sigma_y^2}$  - dimensionless plate lateral load coefficient due to the pressure  $p$

$Q_e$  - dimensionless equivalent lateral load

$Q_p$  - dimensionless P concentrated load coefficient

$r$  - dimensionless factor

$R_e$  - yield point of plate material [MPa]

$s$  - stiffener spacing [mm]\*       $t$  - plate thickness [mm]

$w$  - maximum plate deflection [mm]

$w_p$  - plate permanent set due to lateral loading [mm]

$\alpha = \frac{a}{b}$  - plate slenderness

$\beta = \frac{b}{t} \sqrt{\frac{\sigma_y}{E}}$  - dimensionless plate slenderness

$\lambda$  - load concentration parameter [-]

$\nu$  - Poisson's ratio of material [-]

$\sigma$  - permissible stresses for plates in bending [MPa]

$\sigma_y$  - yield point of plate material [MPa]

\*) unless another physical unit is specified in the text

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Appraised by Tadeusz Jastrzębski, Assoc.Prof., D.Sc., N.A.

## Current reports



TECHNICAL UNIVERSITY OF GDAŃSK  
FACULTY OF OCEAN ENGINEERING  
AND SHIP TECHNOLOGY

## A novel propeller blade pitch control system

A novel design of propeller blade pitch control system was elaborated in the Chair of Ship Deck Equipment and Systems, Technical University of Gdańsk. The system is applicable to small ships powered with 100 to 400 kW engine. Its novelty is the use of a special crossed helical - toothed gear instead of a hydraulic servo-motor usually applied in the traditional design of such mechanisms.

The following features are characteristic of the new solution:

- substantially lower initial costs
- stable pitch setting
- possible manual pitch setting
- holding set pitch without energy supply to the control system.

The features make the new design useful for small floating craft.

Józef Krępa, prof., D.Sc., and Czesław Dymarski, D.Sc., are the authors of the design.