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A simulation model for a mooring positioning system of a vessel in presence of wind, current and waves

SUMMARY

During positioning of a floating craft by means of a mooring system different kinds of displacement of the vessel are generated due to the sea exciting forces: quasistatic displacement, first order motions (of high frequency) and second order motions (of low frequency). The corresponding loads in the lines of the anchor mooring system of the vessel are generated following the displacement of the vessel.

The motion equations of the ship equipped with the anchor mooring system are presented in the paper. Their solution is presented with regard to non-linear terms of the equations and the random exciting forces, in the form of numeric simulation in the time domain. The sample calculations were peformed for a geological vessel fitted with the 4-line positioning system.

INTRODUCTION

The mooring system , among others, is used to position the floating offshore craft. The systems can be passive or active depending on whether steering by means of the anchor winches is not or is applied. In the active system the tensions in separate mooring lines of the positioning system can be regulated depending on the actual weather conditions and actual position of the vessel in respect to a given position. The steering of the winches is performed by means of a computer system fitted with an appropriate program.

While designing a mooring positioning system the following conditions are to be fulfilled:

$$r_{T(P)\max} \leq r_{(P)admit} \tag{1}$$

$$T_{T(m)\max} \leq T_{(m)admit} \tag{2}$$

where:

- $r_{T(P)\max}$ - total, maximum, random, horizontal displacement of the floating craft (precisely - of the positioning point P at the craft) towards the to be maintained position, resulting from the total displacement
- $r_{(P)admit}$ - permissible horizontal displacement of the craft towards a given position, value of which depends first of all on the kind of work performed by the craft
- $T_{T(m)\max}$ - total, maximum, random force in the line „m" of the positioning system
- $T_{(m)admit}$ - permissible force in the line „m" of the positioning system, which depends on the kind of the line and on the applied calculation method, (compare e.g. [2])

In the case of passive systems the calculation of $r_{T(P)\max}$ and $T_{T(m)\max}$ brings about the solution of the equation system describing the quasistatic and dynamic displacements of the moored vessel under the influence of the exciting forces due to wind, current and waves. In the case of active systems it is necessary to design a system to steer the positioning system and to prepare a relevant software.

The mathematical model of the mooring positioning system of a vessel, which can be used in designing and analyzing such systems and simulating the active systems, is presented in the paper.

MOTION EQUATIONS OF A VESSEL WITH THE MOORING POSITIONING SYSTEM

The random exciting forces due to the wind, current and waves act upon the mooring positioned ship.Motions of the ship can be described by means of the following stochastic system of non-linear differential equations:

$$\begin{aligned}\left[M_{(k,l)}\right]\left\{\ddot{s}_{(l)}\right\} = \left\{F_{HM(k)}\left(\ddot{s}_{(l)}, \dot{s}_{(l)}, s_{(l)}\right)\right\} + \\ + \left\{R_{S(k)}\left(s_{(l)}\right)\right\} + \left\{F_{E(k)}\left(s_{(l)}, t\right)\right\}\end{aligned} \tag{3}$$

$$k, l = 1, 2, \ldots, 6$$

where:

- $\left[M_{(k,l)}\right]$ - matrix of generalized mass of the ship,

$\{\ddot{s}_{(l)}(t)\}$
$\{\dot{s}_{(l)}(t)\}$ - generalized vectors of random acceleration, velocity and displacement of the ship
$\{s_{(l)}(t)\}$

$\{F_{HM(k)}\}$ - vector of generalized external hydromechanical forces

$\{R_{S(k)}\}$ - vector of generalized non-linear elastic restoring forces due to positioning mooring system

$\{F_{E(k)}(s_{(l)},t)\}$ - vector of generalized stochastic exciting forces

The forces can be presented in the following form:

$$F_{E(k)}(t) = F_{(k)}(t) + R_{C(k)}(t) + R_{W(k)}(t) \quad (4)$$

$$k = 1,2,...,6$$

where:

t - time

$F_{(k)}(t)$ - exciting force due to the waves

$R_{C(k)}(t)$ - current acting force

$R_{W(k)}(t)$ - wind acting force

In the exciting forces the average quasistatic forces and random dynamic forces can be distinguished as follows :

$$F_{E(k)}(t) = \overline{F}_{ES(k)} + F_{ED(k)}(t) \quad (5)$$

$$k = 1,2,...,6$$

where:

$\overline{F}_{ES(k)}$ - average quasistatic exciting force

$F_{ED(k)}(t)$ - random dynamic exciting force

When omitting the wind and current action the dynamic random forces due to the waves that cause the motions,can be presented in form of a series as follows:

$$F_{ED(k)}(t) = \varepsilon F^{(1)}_{ED(k)}(t) + \varepsilon^2 F^{(2)}_{ED(k)}(t) + ... \quad (6)$$

$$k = 1,2,...,6$$

where:

$F^{(1)}_{ED(k)}(t)$ - generalized random dynamic first-order exciting force

$F^{(2)}_{ED(k)}(t)$ - generalized random dynamic second-order exciting force

ε - small value parameter

Resultant random ship's displacements and motions can be presented in the form of a series as follows:

$$s_{(l)}(t) = \overline{s}_{S(l)} + s_{D(l)}(t) \quad (7)$$

$$l = 1,2,...,6$$

where:

$\overline{s}_{S(l)}$ - average quasistatic displacement of the ship

$s_{D(l)}(t)$ - ship's motions

The motions $s_{D(l)}(t)$ can be expressed as follows:

$$s_{D(l)}(t) = \varepsilon s^{(1)}_{(l)}(t) + \varepsilon^2 s^{(2)}_{(l)}(t) + ... \quad (8)$$

$$l = 1,2,...,6$$

where:

$s^{(1)}_{(l)}(t)$ - random first order motions

$s^{(2)}_{(l)}(t)$ - random second order motions

Hence the ship's motion equations (3) can be split into the high frequency part (at the wave frequency) corresponding to the first order motions and low frequency part to the second order motions. In the situation the first order motions are superimposed on the second order (low frequency) motions of large displacement and velocity amplitudes.

The two parts of the equation system (3) can be expressed as follows:

- that of the second order motions (at the low frequency):

$$\left(\left[M_{(k,l)}\right] + \left[m_{(k,l)}(t)\right]\right)\left\{\ddot{s}^{(2)}_{(l)}(t)\right\} + \quad (9)$$
$$+\left(\left[B_{V(k,l)}\right] + \left[D^{(2)}_{(k,l)}\right]\right)\left\{\dot{s}^{(2)}_{(l)}(t)\right\} + \left\{R_{S(k)}\left(s^{(2)}_{(l)}(t)\right)\right\} =$$
$$= \left\{F^{(2)}_{(k)}(t)\right\} + \left\{R_{C(k)}(t)\right\} + \left\{R_{W(k)}(t)\right\}$$

$$k,l = 1,2,6$$

where:

$\left[m_{(k,l)}(t)\right]$ - matrix of generalized hydrodynamic (added) mass

$\left[B_{V(k,l)}\right]$ - matrix of linearized generalized coefficients of viscous damping forces

$\left[D^{(2)}_{(k,l)}\right]$ - matrix of low frequency coefficients of drift wave damping

$\left\{R_{S(k)}\left(s^{(2)}_{(l)}(t)\right)\right\}$ - column vector of generalized spring restoring forces due to the mooring positioning system at ship's low frequency motions

$\left\{F^{(2)}_{(k)}(t)\right\}$ - column vector of generalized wave second order exciting forces (wave drift forces)

$\left\{R_{C(k)}(t)\right\}$ - column vector of generalized sea-current forces acting on the moored vessel being in oscillatory motion

$\left\{R_{W(k)}(t)\right\}$ - column vector of generalized wind forces acting on the moored vessel being in oscillatory motion

- that of the first order motions (at the wave frequency):

$$\left(\left[M_{(k,l)}\right] + \left[m_{(k,l)}(t)\right]\right)\left\{\ddot{s}^{(2)}_{(l)}(t)\right\} + \left[b_{(k,l)}(t)\right]\left\{\dot{s}^{(2)}_{(l)}(t)\right\} +$$
$$+\left(\left[C_{H(k,l)}\right] + \left[C_{S(k,l)}\left(s^{(2)}_{(l)}(t)\right)\right]\right)\left\{s^{(2)}_{(l)}(t)\right\} = \left\{F^{(1)}_{(k)}(t)\right\}$$
$$k,l = 1,2,...,6 \quad (10)$$

where:

$\left[b_{(k,l)}(t)\right]$ - matrix of generalized potential coefficients of first-order motion damping

$\left[C_{H(k,l)}\right]$ - matrix of generalized hydrostatic coefficients of restoring forces

$\left[C_{S(k,l)}\left(s^{(2)}_{(l)}(t)\right)\right]$ - matrix of generalized linearized coefficients of elastic restoring forces due to the mooring positioning system, calculated for the ship's instantaneous position resulting from the average position $s_{S(l)}$ and low frequency motions $s^{(2)}_{(l)}$

$\left\{F^{(1)}_{(k)}(t)\right\}$ - column vector of wave first-order exciting forces

In the equation system (9) the matrix coefficients $\left[m_{(k,l)}(t)\right]$ for the low-frequency motions depend only slightly on the frequency, which was experimentally confirmed [11] The matrix is therefore non-dependent on time as far as the motions on the irregular wave are concerned.

On the other hand, the matrix coefficients $\left[m_{(k,l)}(t)\right]$ and $\left[b_{(k,l)}(t)\right]$ in the equation system (10), describing the first-order motions of the ship which performs simulaneously the second-order motions, depend upon both the wave frequency and the velocity of low frequency motions. Therefore, the matrix coefficients $\left[m_{(k,l)}(t)\right]$ and $\left[b_{(k,l)}(t)\right]$ should be known for determining the first order motions of the ship in irregular waves. In linear systems, the motions $S(t)$ generated by the random exciting force $F(t)$ can be determined on the basis of the system response $K(t)$ to the elementary exciting force as follows:

$$S(t)=\int_0^\infty K(\tau)F(t-\tau)d\tau \tag{11}$$

The theory of the impulse response function applied for determining the floating craft's motion on the irregular wave was presented in [1]. Applying the impulse response function, the potential damping of the first-order motions on the irregular wave can be expressed as follows:

$$\left[b_{(k,l)}(t)\right]\left\{\dot{s}_{(l)}^{(1)}\right\}=\int_0^\infty\left[K_{(k,l)}(\tau)\right]\left\{\dot{s}_{(l)}^{(1)}(t-\tau)\right\}d\tau \tag{12}$$
$$k,l=1,2,\ldots,6$$

and the added mass is of the following form:

$$\left[m_{(k,l)}(t)\right]=\left[m_{(k,l)}(\tilde{\omega})\right]+\frac{1}{\tilde{\omega}}\int_0^\infty\left[K_{(k,l)}(t)\sin(\tilde{\omega}t)\right]dt \tag{13}$$
$$k,l=1,2,\ldots,6$$

where:

$\left[K_{(k,l)}\right]$ - matrix of impulse transfer function

$\left[m_{(k,l)}(\tilde{\omega})\right]$ - matrix of the generalized hydrodynamic mass due to the regular wave of the frequency $\tilde{\omega}$

$\tilde{\omega}$ - arbitrary chosen regular wave frequency

τ - time interval

The form of the function $K_{(k,l)}(t)$ for floating crafts, given in [5], is as follows:

$$\left[K_{(k,l)}(t)\right]=\frac{2}{\pi}\int_0^\infty\left[b_{(k,l)}(\omega)\cos(\omega t)\right]d\omega \tag{14}$$
$$k,l=1,2,\ldots,6$$

where:

$\left[b_{(k,l)}(\omega)\right]$ - the matrix of generalized coefficients of potential damping of the first- order motions in the regular wave

Taking into consideration (12), the first-order motions of the moored vessel in the irregular wave are described with the following equation system:

$$\left(\left[M_{(k,l)}\right]+\left[m_{(k,l)}(t)\right]\right)\left\{\ddot{s}_{(l)}^{(1)}(t)\right\}+\int_0^\infty\left[K_{(k,l)}(\tau)\right]\left\{\dot{s}_{(l)}^{(1)}(t-\tau)\right\}d\tau+$$
$$+\left(\left[C_{H(k,l)}\right]+\left[C_{S(k,l)}\right]\right)\left\{s_{(l)}^{(1)}(t)\right\}=\left\{F_{(k)}^{(1)}(t)\right\} \tag{15}$$
$$k,l=1,2,\ldots,6$$

where $\left[m_{(k,l)}(t)\right]$ and $\left[K_{(k,l)}(\tau)\right]$ are defined in (13) and (14).

The hydrodynamic force coefficients $m_{(k,l)}(\omega)$ and $b_{(k,l)}(\omega)$ as well as the wave first order exciting force $F_{(k)}^{(1)}(t)$, which appear in (13) and (14), for a vessel performing only small first order motions around an average position, depend upon the wave frequency, water depth and shape of the underwater ship's body (the exciting force $F_{(k)}^{(1)}(t)$ depends also on the direction angle β_f of the wave oncoming upon the vessel). The moored vessel performs the first-order motions together with the second-order motions of large displacement and velocity amplitudes [9]. Therefore the coefficients of both the hydrodynamic and exciting forces of the first-order motions should be expressed in function of the variable displacements and second-order motion velocity as follows:

$$m_{(k,l)}(\omega,\dot{s}_{(l)}^{(2)})$$
$$b_{(k,l)}(\omega,\dot{s}_{(l)}^{(2)}) \tag{16}$$
$$F_{(k)}^{(1)}\left(s_{(l)}^{(2)},\dot{s}_{(l)}^{(2)},t\right)$$
$$k,l=1,2,6$$

The hydrodynamic force coefficients and exciting forces (16) should be determined in the $Gx_2y_2z_2$ movable reference system which moves with the low frequency motion velocity (Fig. 1).

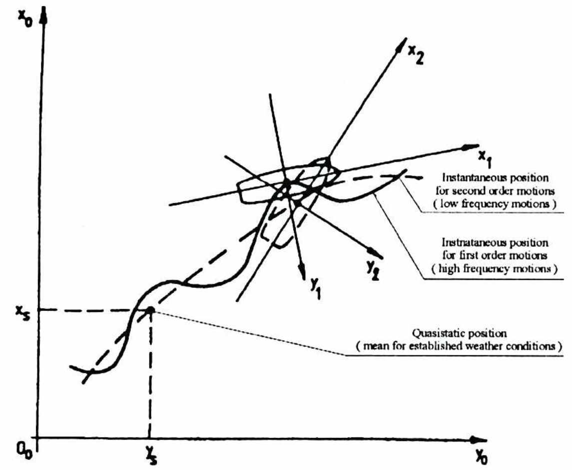


Fig. 1. The coordinate systems used for the calculation of a moored vessel

Hence, the value of the coefficients (16) would be dependent on the instantaneous position of the $Gx_2y_2z_2$ system in respect to the $0_0x_0y_0z_0$ fixed system, which could make solving the equation system (15) difficult . Therefore, the influence of the second-order motion displacement and velocity upon the first-order motion hydrodynamic forces was taken into account by expanding the functions (16) into the following Taylor series:

$$m_{(k,l)}(\omega,\dot{s}_{(l)}^{(2)})=m_{(k,l)}(\omega,0)+$$
$$+\sum_{l=1}^{6}\frac{\partial m_{(k,l)}(\omega,\dot{s}_{(l)}^{(2)})}{\partial\dot{s}_{(l)}^{(2)}}\,\dot{s}_{(l)}^{(2)}\Bigg|_{\dot{s}_{(l)}^{(2)}=0}+\ldots \tag{17}$$
$$k,l=1,2,6$$

$$b_{(k,l)}(\omega,\dot{s}_{(l)}^{(2)})=b_{(k,l)}(\omega,0)+$$
$$+\sum_{l=1}^{6}\frac{\partial b_{(k,l)}(\omega,\dot{s}_{(l)}^{(2)})}{\partial\dot{s}_{(l)}^{(2)}}\,\dot{s}_{(l)}^{(2)}\Bigg|_{\dot{s}_{(l)}^{(2)}=0}+\ldots \tag{18}$$
$$k,l=1,2,6$$

$$F^{(1)}_{(k)}\left(s^{(2)}_{(l)},\dot{s}^{(2)}_{(l)},t\right)=F^{(1)}_{(k)}(0,0,t)+ +\sum_{l=1}^{6}\frac{\partial F^{(1)}_{(k)}\left(s^{(2)}_{(l)},\dot{s}^{(2)}_{(l)},t\right)}{\partial s^{(2)}_{(l)}}\, s^{(2)}_{(l)}\Bigg|_{s^{(2)}_{(l)}=\dot{s}^{(2)}_{(l)}=0} + +\sum_{l=1}^{6}\frac{\partial F^{(1)}_{(k)}\left(s^{(2)}_{(l)},\dot{s}^{(2)}_{(l)},t\right)}{\partial \dot{s}^{(2)}_{(l)}}\, \dot{s}^{(2)}_{(l)}\Bigg|_{s^{(2)}_{(l)}=\dot{s}^{(2)}_{(l)}=0} +\ldots \qquad (19)$$

$$k,l=1,2,6$$

where:

$m_{(k,l)}(\omega,0)$ $b_{(k,l)}(\omega,0)$ $F^{(1)}_{(k)}(0,0,t)$	-	the hydrodynamic force coefficients and wave exciting forces of the ship which performs only small first order motions about her mean position, respectively

As the first order motions are described by the linear system of differential equations (10), only the first order terms, obtained by expanding the functions (16) into Taylor series (17), (18) and (19), are used. The calculation process of the partial derivatives in respect to the velocity $\dot{s}^{(2)}_{(l)}$, which appear in (17), (18) and (19), was presented in [7]. In the case of the first-order wave exciting force $F^{(1)}_{(k)}$ their differentials with regard to the displacement $s^{(2)}_{(l)}$ at $l=1,2$ equal to zero (the exciting force is not dependent upon the surge and sway). But at $l=6$ (the low frequency yaw) the differential of the force $F^{(1)}_{(k)}$ in respect to $s^{(2)}_{(6)}=\psi^{(2)}$ has no zero value because a change of the wave uprash direction occurs. The influence of $\psi^{(2)}$ upon the wave direction in respect to the vessel is taken into account when calculating the actual wave uprush angle in respect to the vessel as follows:

$$\beta_w(t)=\overline{\beta}_w+\psi^{(2)}(t) \qquad (20)$$

where:

$\overline{\beta}_w$ - wave uprush angle at the mean position of the vessel in the fixed reference system

$\psi^{(2)}$ - the low-frequency yaving

The remaining hydrodynamic force coefficients $m_{(k,l)}(\omega)$, $b_{(k,l)}(\omega)$ and the wave exciting force $F^{(1)}_{(k)}(t)$ for $k=1,2,\ldots,6$ and $l=3,4,5$, which do not appear simultaneously with the second-order motions, remain unchanged; they are the same as of the ship which performs only small first-order motions in her mean position.

ELASTIC RESTORING FORCES FROM THE MOORING POSITIONING SYSTEM

While positioning the moored vessel the sea exciting forces drive the vessel back to the mean position simultaneously generating the motions of low frequency (second-order) and of high frequency (first-order).

It means that the loads in the lines of the positioning system are of the quasistatic character resulting from the ship's mean position and random dynamic forces due to the ship motions and vibration of the lines.

The total load in the lines of the mooring positioning system can be obtained by summarizing vectorially the mean loads and the resulting amplitudes of the dynamic loads as follows:

$$\vec{T}_{T(m)}(t)=\vec{T}_{S(m)}\left(\vec{s}_{S(l)}\right)+\vec{T}^{(1)}_{D(m)}\left(\vec{s}^{(1)}_{(l)}(t)\right)+\vec{T}^{(2)}_{D(m)}\left(\vec{s}^{(2)}_{(l)}(t)\right) \qquad (21)$$

$$m=1,2,\ldots,n$$

$$l=1,2,\ldots,6$$

where:

$s_{S(l)}$ $s^{(1)}_{(l)}(t)$ $s^{(2)}_{(l)}(t)$	-	displacements of the vessel: the quasistatic and instantaneous dynamic ones due to the first and second order motions, respectively
$T_{T(m)}(t)$	-	total instantaneous load in the line „m" of the positioning system
$T_{S(m)}$ $T^{(1)}_{D(m)}(t)$ $T^{(2)}_{(m)}(t)$	-	loads in the line „m" : the mean quasistatic and instantaneous dynamic ones due to the first and second order ship motions, respectively

The static load-displacement characteristics was used to calculate the loads in the lines (its possible application for ship motion calculations was discussed in [8]).

The forces at the upper end of the line (Fig. 2) depend on the line's parametres, water depth and the position of the upper end of the line in respect to that in which the line hangs down vertically in the water.

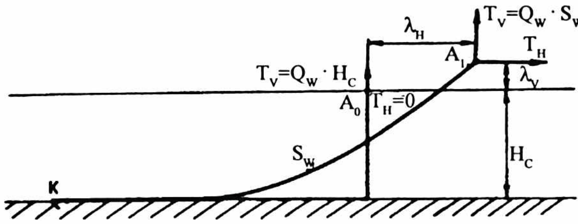


Fig. 2. Influence of the displacement of the upper end of the line on its geometry and load

It means that the changes of the line load are developed by the vertical and horizontal displacements of the upper end connected to the floating craft. The static tension-displacement characteristics of the mooring line, described in [10], is of the following form:

$$T=\sqrt{T_H^2+T_V^2}=f\left(\lambda_H,\lambda_V\,\middle|\,H_c,q_a,E\right) \qquad (22)$$

where:

T - resulting force in the line

T_H - horizontal force component in the line

T_V - vertical force component in the line

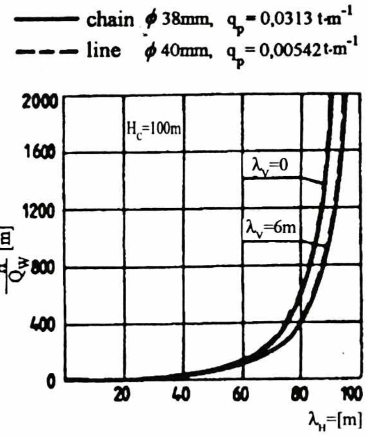
λ_H,λ_V - horizontal and vertical displacements of the upper end of the line, respectively (Fig. 2)

H_c - vertical distance from the sea-bottom to the upper end of the line

q_a - weight per 1 m of the line length in the air

E - Young's modulus

The static characteristics of the mooring line (22) is non-linear because of λ_H and λ_V (the sample characteristics are presented in Fig. 3).



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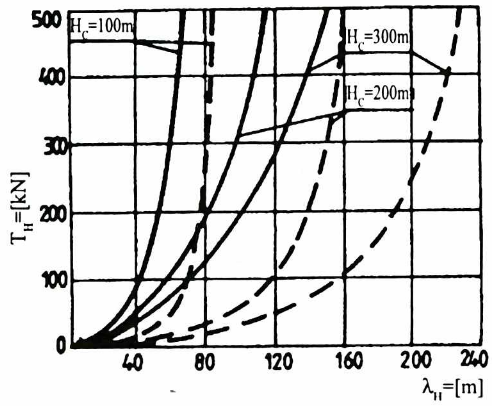

Fig. 3. Static tension-displacement characteristics of the homogeneous line

The static and dynamic load in the mooring line depends, among others, on the position of the upper end of the line. The changes of the position $\Delta\lambda_H$ and $\Delta\lambda_V$ are caused by the floating craft motions (the quasistatic displacement about the mean position and the first and second order motions):

$$\Delta\lambda_H\left(s_{S(l)}, s_{(l)}^{(1)}, s_{(l)}^{(2)}\right) \quad l = 1,2,4,5,6$$
$$\Delta\lambda_V\left(s_{S(l)}, s_{(l)}^{(1)}, s_{(l)}^{(2)}\right) \quad l = 3,4,5 \tag{23}$$

where:

$\Delta\lambda_H, \Delta\lambda_V$ - increments of the displacement of the upper end of line.

When substituting the relations (23) into (22) and summarising the components of the forces and moments from the lines, expressed in the *Gxyz* reference system connected with the vessel, the generalized resulting elastic restoring forces from the mooring positioning system are obtained as follows:

$$\left\{R_{S(k)}\right\} = \left\{\sum_{m=1}^{n} T_{(k,m)}\right\} \tag{24}$$

where:

$k = 1,2,...,6$ - direction of the generalized force

$m = 1,2,...,n$ - the number assigned to the successive line of the mooring system

The equations (22) and (24) are non-linear in respect to the displacements or motions of the moored vessel. To make the motion analysis of the moored vessel easier, the resulting force in the line and the restoring forces due to the positioning mooring system are expressed in the linearised form [10] as follows:

$$T_{L(m)} = \sum_{1}^{l} s_{(l)} e_{(l,m)} \quad l = 1,2,...,6 \tag{25}$$

$$\left\{R_{SL(k)}\right\} = \left[C_{S(k,l)}\right]\left\{s_{(l)}\right\} \quad k,l = 1,2,...,6 \tag{26}$$

where:

$T_{L(m)}$ - resulting linearized force in the line „m"

$\left\{R_{SL(k)}\right\}$ - column vector of the generalized linearized elastic restoring forces from the mooring system

$\left[C_{S(k,l)}\right]$ - matrix of the linearized generalized coefficients of the elastic restoring forces from the mooring system

$\left\{s_{(l)}\right\}$ - vector of the generalized displacements or motions (of the first- or second order) of the moored vessel

$e_{(l,m)}$ - linearization coefficients for the actual motion direction „ l " and line „m"

EXCITING FORCES

Wave exciting forces

The random wave motion (irregular) can be assumed as the finite sum of harmonic regular waves as follows:

$$\zeta(t) = \sum_{i=1}^{N} \zeta_{Ai} \cos(\omega_i t + \varepsilon_i) \tag{27}$$

where:

ζ_{Ai} - amplitude of the regular wave component

ω_i - frequency of the regular wave component

ε_i - phase angle between the wave components

Hence the wave exciting forces (6) due to irregular waves can be expressed in the following form:

- the first-order forces:

$$F_{(k)}^{(1)}(t) = \sum_{i=1}^{N} \zeta_{Ai}\left[F_{C(k)i}^{(1)} \cos(\omega_i t + \varepsilon_i) + F_{S(k)i}^{(1)} \sin(\omega_i t + \varepsilon_i)\right] \quad k = 1,2,...,6 \tag{28}$$

- the second-order forces:

$$F_{(k)}^{(2)}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} \zeta_{Ai}\zeta_{Aj}\left\{P_{(k)ij} \cos\left[(\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\right] + Q_{(k)ij} \sin\left[(\omega_i - \omega_j)t + (\varepsilon_i - \varepsilon_j)\right]\right\} \quad k = 1,2,...,6 \tag{29}$$

where:

$F_{C(k)i}^{(1)}$ - cosine-part of the exciting first-order force due to the regular wave component

$F_{S(k)i}^{(1)}$ - sine-part of the exciting first-order force due to the regular wave compoment

$P_{(k)ij}$ - second-order transfer function for the wave part of the second-order exciting force which is in phase with the low-frequency part of the square of wave ordinate

$Q_{(k)ij}$ - second order transfer function for the wave part of the second-order exciting force which is in antiphase in respect to low-frequency part of the square of wave ordinate

The regular wave amplitudes ζ_{Ai} with the frequency ω_i in the equations (27), (28), and (29) for a given sea-state are calculated as below:

$$\zeta_{Ai} = \sqrt{2S_{\zeta\zeta}(\omega_i)\Delta\omega_i} \tag{30}$$

where:

$S_{\zeta\zeta}(\omega)$ - the wave energy spectrum density function

$\Delta\omega_i$ - frequency interval of the function $S_{\zeta\zeta}(\omega)$ for which the wave component amplitude is calculated

In the case the vessel is anchored in the shallow waters with the sea current, the regular wave amplitude is calculated with the water depth and current velocity accounted for and the wave frequency taken as the encounter frequency [6].

Wind exciting forces

The aerodynamic wind forces acting on a moored ship performing low-frequency motions are expressed as follows:

$$R_{xw} = \frac{1}{2}\rho_p V_{Rw}^2 C_{xw}(\beta_{Rw}) S_x + \rho_p u(t) V_{Rw} C_{xw}^*(\beta_{Rw}) S_x$$

$$R_{yw} = \frac{1}{2}\rho_p V_{Rw}^2 C_{yw}(\beta_{Rw}) S_y + \rho_p u(t) V_{Rw} C_{yw}^*(\beta_{Rw}) S_y$$

$$M_{zw} = \frac{1}{2}\rho_p V_{Rw}^2 C_{mw}(\beta_{Rw}) L_w S_y + \rho_p u(t) V_{Rw} C_{mw}^*(\beta_{Rw}) L_w S_y \tag{31}$$

where:

ρ_p - air density

S_x, S_y - windage area of the ship above the waterline (that from the bow and lateral, respectively)

L_w - ship's length on the waterline

$C_{xw}(\beta_{Rw})$, $C_{yw}(\beta_{Rw})$, $C_{mw}(\beta_{Rw})$ - the static resistance coefficients of the ship above the waterline

$C_{xw}^*(\beta_{Rw})$, $C_{yw}^*(\beta_{Rw})$, $C_{mw}^*(\beta_{Rw})$ - the dynamic resistance coefficients of the ship above the waterline

$V_{Rw}(\bar{V}_w, \beta_w, \dot{S}_{(l)}^{(2)})$ - apparent wind velocity resulting from the mean wind velocity $\bar{V}_w$, the wind direction angle β_w in respect to the ship, and the low-frequency velocity of the ship motions

$\beta_{Rw}(\beta_{w0}, \psi^{(2)})$ - apparent wind direction angle in respect to the ship, resulting from the initial direction angle β_{w0} and the low-frequency yawing

$u(t)$ - wind velocity pulsations around the mean value $\bar{V}_w$

The pulsations $u(t)$ are calculated from the wind velocity energy spectral density function $S_w(f)$:

$$u_w(t) = \int_0^{\infty} \sqrt{4 S_W(f) df} \cos(2\pi f t + \varepsilon(f)) \tag{32}$$

where:

$S_W(f)$ - wind velocity energy spectral density function (for instance, Karman's or Davenport's spectrum)

f - frequency of wind velocity changes

$\varepsilon(f)$ - the phase angle of the several harmonic components to each other

Current exciting forces

The sea current is characterized approximately by a constant-in-time direction and the average velocity.Despite of the fact, the dynamic relation also appears in the case of a moored vessel performing the low-frequency yawing $\psi^{(2)}$. Hence, the total current reaction force is expressed as below:

$$R_{xCS} = \frac{1}{2}\rho_w LTV_{RC}^2 C_{Cx}(\beta_{RC}) + \frac{1}{2}\rho_w LTV_{RC}^2 C_{xdyn}(\beta_{RC}, \dot{\psi}^{(2)})$$

$$R_{yCS} = \frac{1}{2}\rho_w LTV_{RC}^2 C_{Cy}(\beta_{RC}) + \frac{1}{2}\rho_w LTV_{RC}^2 C_{ydyn}(\beta_{RC}, \dot{\psi}^{(2)})$$

$$M_{zCS} = \frac{1}{2}\rho_w LTV_{RC}^2 C_{Cm}(\beta_{RC}) + \frac{1}{2}\rho_w LTV_{RC}^2 C_{mdyn}(\beta_{RC}, \dot{\psi}^{(2)}) \tag{33}$$

where:

ρ_w - water density

L, T - ship's length and draught, respectively

C_{Cx}, C_{Cy}, C_{Cm} - static current reaction coefficients

$C_{xdyn}, C_{ydyn}, C_{mdyn}$ - dynamic current reaction coefficients

V_{RC} - apparent current velocity resulting from the mean current velocity $\bar{V}_C$, the current direction angle β_C in respect to the ship and the low-frequency velocity of ship motions $\dot{S}_{(l)}^{(2)}$

β_{RC} - apparent current direction angle resulting from the initial direction angle β_{C0} and of the low-frequency yawing velocity $\dot{\psi}^{(2)}$

NUMERICAL SIMULATION OF MOTIONS AND LOADS IN THE LINES OF THE POSITIONING SYSTEM

The motions and loads in the lines of the mooring positioning system of a geological vessel (Fig. 4) in presence of wind, current and waves were calculated by finding numerical solutions in the time domain of the following two systems of differential equations:

- the non-linear differential equations describing the low frequency motions of the anchored vessel at the irregular wave in presence of wind and current (9)
- the linear differential equations describing the high frequency motions of the anchored vessel at the irregular wave (15).

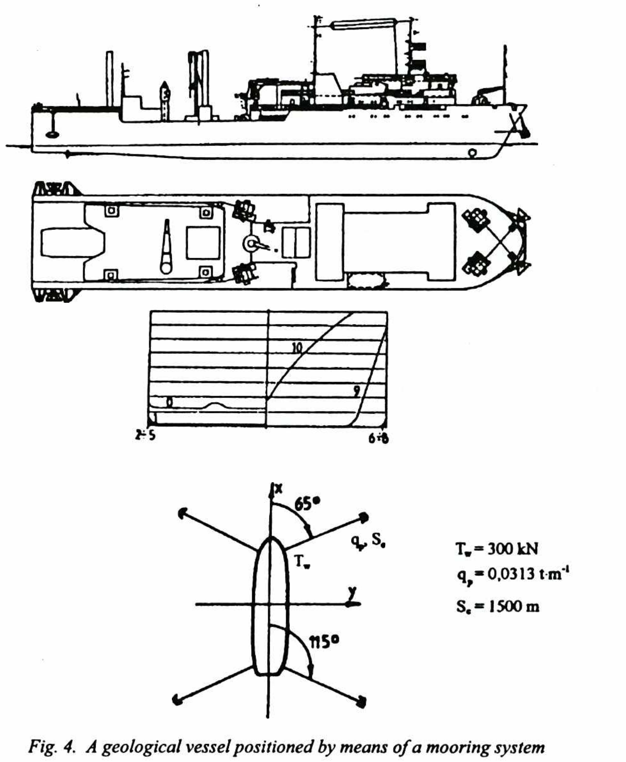

Fig. 4. A geological vessel positioned by means of a mooring system

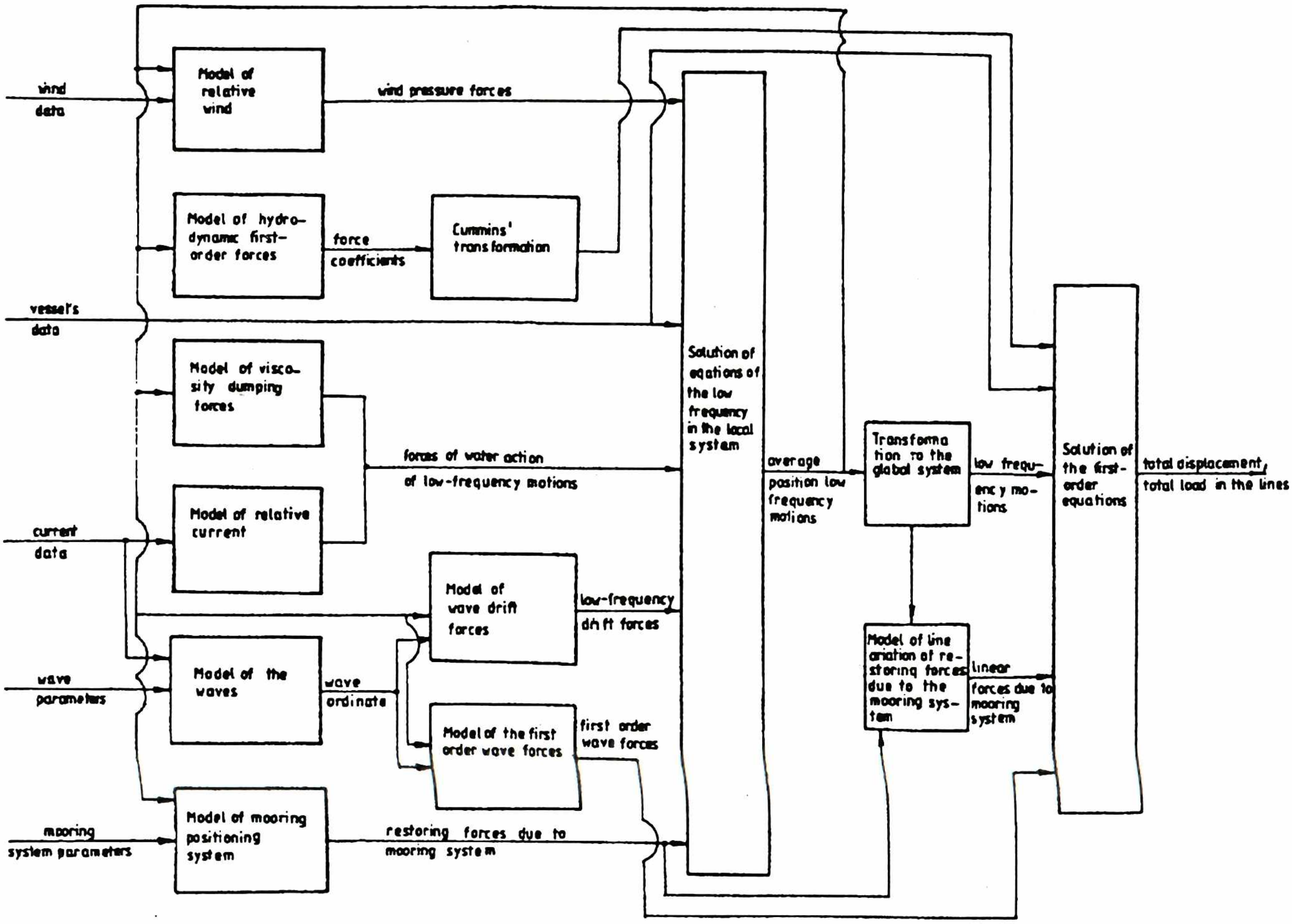

Fig. 5. Block diagram of the program for simulating the motions of a moored vessel and loads in the lines of a positioning system

From the solution in the time domain of the equation system (9) at the instant time t the following is obtained:

- the displacements $s_{S(l)}(t)$, for $l = 1,2,6$, of the vessel in respect to the average position (it concerns the short-term prognosis the environment parameters of which are of constant values and the mooring system parameters do not change; in the case the displacements $s_{S(l)}$ are quasistatic and independent of time)
- the second order (low frequency) displacements $s^{(2)}_{(l)}(t)$ and motion velocity $\dot{s}^{(2)}_{(l)}(t)$ of motion of the vessel in her mean position,- the second-order (low frequency) load $T^{(2)}_{(m)}$ in the lines of the mooring system, which consist of the average loads resulting from the mean position of the vessel and dynamic loads resulting from the frequency motions.

The high frequency motions (first order motions) of the vessel, described by the equations (15), are calculated in the time domain after the calculation at the time t the quasistatic displacements $s_{S(l)}$, low frequency motions $s^{(2)}_{(l)}(t)$ and low frequency loads in the lines of the mooring system $T^{(2)}_{(m)}$.

The exciting forces from the sea environment in the equations (9) and (15) are proportional to the square of wind and current velocity and to the wave ordinate (for the first order wave forces) and to the square of wave ordinate (for the second order drift forces).

In the presented model the current was assumed of an established mean velocity, but the wind and wave parameters variable with time. The instantaneous wind velocity is generated from the Karman spectrum [4], and the instantaneous wave elevation from the JONSWAP spectrum [3].

The equation systems (9) and (15) were solved in the time domain by means of the numeric simulation program a block diagram of which is presented in Fig. 5. In result, the total displacement of the anchored vessel $s_{T(l)}$ was obtained consisting of the quasistatic displacements $s_{S(l)}$ towards the average position and the first order $s^{(1)}_{(l)}$ and second order motions $s^{(2)}_{(l)}$ for $l = 1,2,\ldots,6$ (the second order motions are only for $l = 1,2,6$). By using the displacements the total loads in the lines of the positioning system $T_{T(m)}$ were calculated that consist of the average loads $T_{S(m)}$ and the first order dynamic loads $T^{(1)}_{D(m)}$ (from the first order motions) and second order dynamic loads $T^{(2)}_{D(m)}$ (from the second order motions).

The courses of the displacements of the moored vessel and the loads in the lines of the positioning system versus time were calculated for the following data:

mooring line system:	
- number of lines	$n = 4$
- weight per 1 m of the line length in the air	$q_p = 0{,}0313\ t \cdot m^{-1}$
- pretension in the lines	$T_w = 300\ kN$
irregular wave:	
- significant wave hight	$H_S = 2{,}64\ m$
- average characteristic period	$\overline{T} = 6{,}27\ s$
- average initial direction angle	$\beta_f = 150$ deg
- wave spectrum	JONSWAP
wind:	
- average speed	$V_w = 12{,}4\ m \cdot s^{-1}$
- average initial direction angle	$\beta_f = 90$ deg
- wind spectrum	Karman
current:	
- average speed	$V_c = 1{,}0\ m \cdot s^{-1}$
- average initial direction angle	$\beta_f = 90$ deg
- water depth	$H = 100\ m$

The simulation results are presented in Fig. 6 in the form of the total displacements of the vessel $s_{T(l)}$ for $l = 1,2,\ldots,6$ and loads in the lines $T_{T(m)}$ for $m = 1,2,3,4$, variable with time. In Fig. 7 the chosen fragmentary results of the total longitudinal motions $X_T(t)$ and loads in line No 1, $T_{T(1)}(t)$, are given.

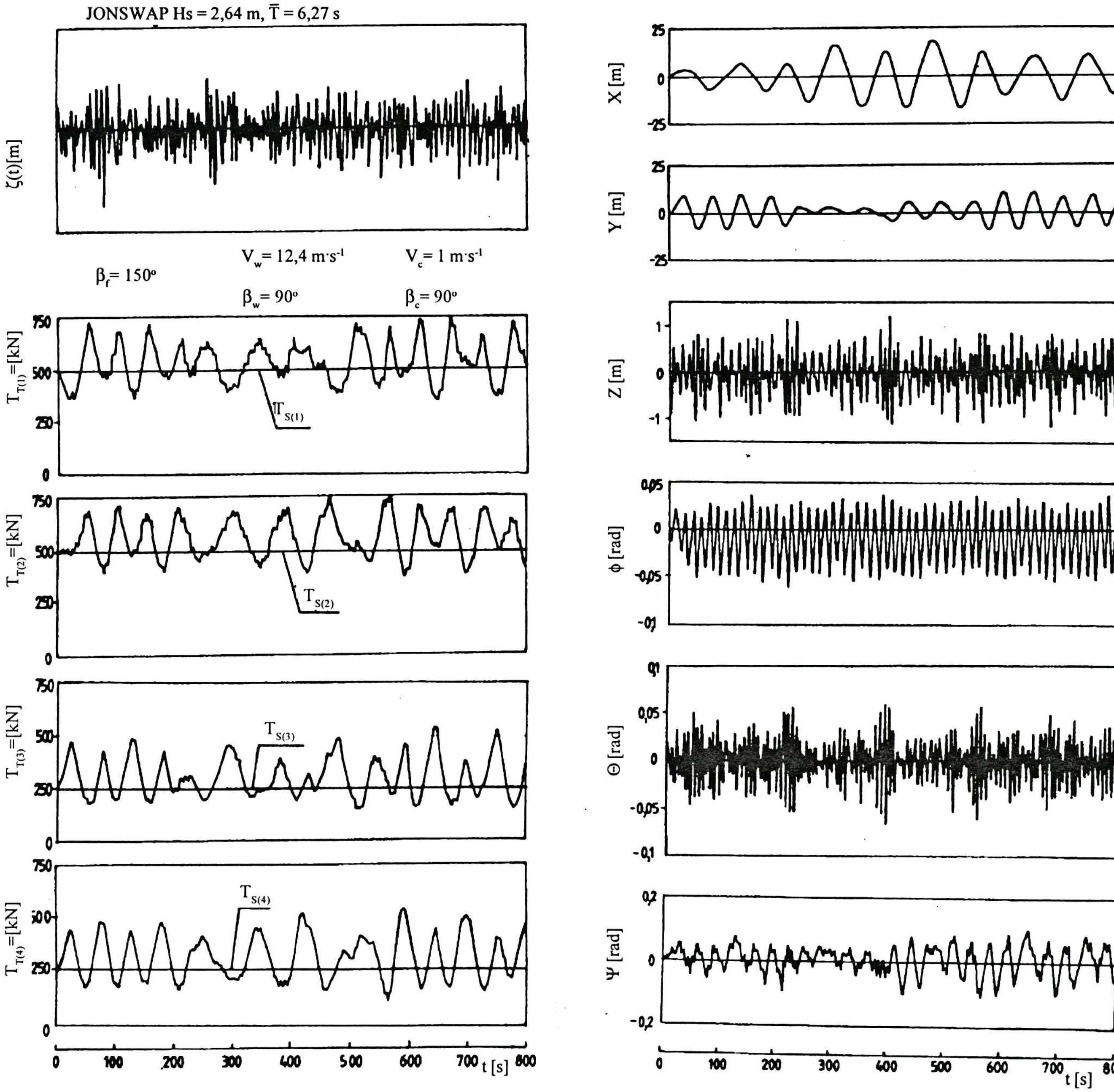

Fig. 6. Resultant motions and loads in the lines of the moored geological vessel on the irregular wave (sample calculation)

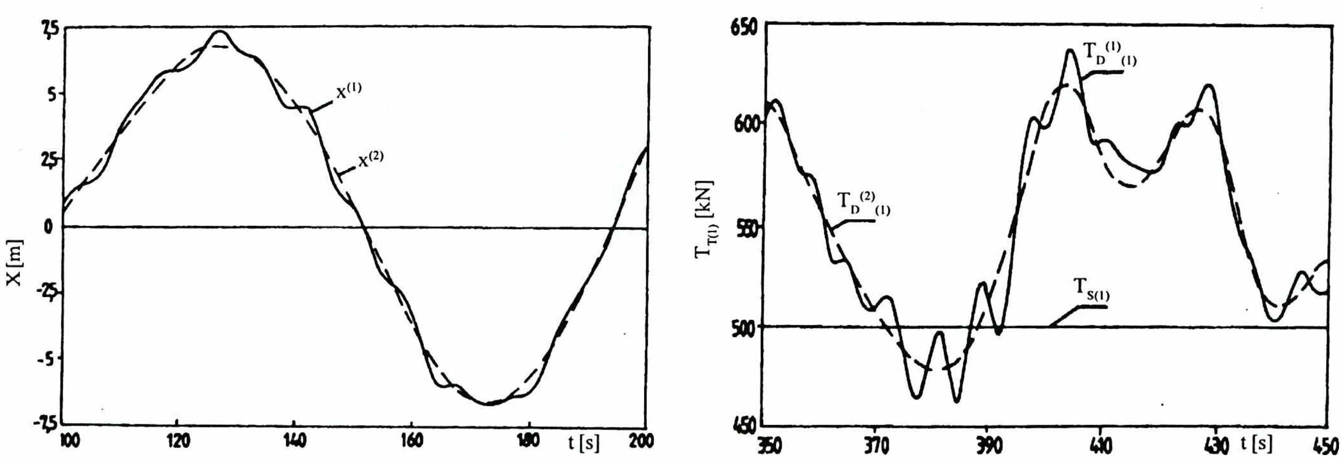

Fig. 7. High- and low frequency surge and total load in line No. 1 of the moored geological vessel (sample calculation)

CONCLUSIONS

The presented mathematical model of positioning mooring system enables:

- estimation of maximum total displacement of the vessel and of loads in the mooring lines in the assumed weather conditions
- examining of influence of the parametres of mooring system and of the sea environment on the quasistatic displacement and motions of the vessel as well as in the lines
- real time simulation of vessel's displacement and loads in the lines of positioning mooring system.

The computer program based on this model is used for design of positioning mooring systems, passive and active (a dozen or so projects of mooring systems have already been developed).

Simulation tests are carried out on the computer simulator specially constructed for this task in the Offshore and Ship Research Institute of the Technical University of Szczecin.

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NOMENCLATURE

(The symbols not explained in the text)

$\left[b_{(k,l)}\right]$ - matrix of generalized potential damping coefficients of motions

$\left\{F_{(k)}^{(1)}\right\}$ - vector of generalized wave first order exciting forces

$\left\{F_{(k)}^{(2)}\right\}$ - vector of generalized wave second order exciting forces

H_S - significant amplitude of irregular wave

$\left[m_{(k,l)}\right]$ - matrix of generalized added mass

q_P - mass of 1m of the line length in the air

Q_W - weight of 1m of the line length in the water

$\left\{R_{C(k)}\right\}$ - vector of generalized current exciting forces

$\left\{R_{W(k)}\right\}$ - vector of generalized wind exciting forces

S_C - total length of the line (from the winch to the anchor)

$\left\{\ddot{s}_{(l)}^{(1)}\right\},\left\{\dot{s}_{(l)}^{(1)}\right\},\left\{s_{(l)}^{(1)}\right\}$ - vector of acceleration, velocity and displacement of first order motions

$\left\{\ddot{s}_{(l)}^{(2)}\right\},\left\{\dot{s}_{(l)}^{(2)}\right\},\left\{s_{(l)}^{(2)}\right\}$ - vector of acceleration, velocity and displacement of second order motions

S_w - length of the hanging line section

S_{wo} - length of the hanging line section (without accounting for material extension)

$T_D^{(1)}$ - dynamic load of the line due to the first order motions

$T_D^{(2)}$ - dynamic load of the line due to the second order motions

$T_{(k,m)}$ - force component in the line „m" in the direction „k"

T_S - mean load of the line

T_T - total load of the line

T_w - pretension in the line

$\overline{T}$ - average characteristic period of the irregular waves

V_w - wind velocity

V_c - current velocity

X - surge

Y - sway

Z - heave

β_c - current direction in respect to the ship

β_w - current direction in respect to the ship

β_f - current direction in respect to the ship

ζ_A - regular wave amplitude

$\zeta_{(t)}$ - random wave elevation

Θ - pitch

ϕ - roll

Ψ - yaw

ω - regular wave frequency

Upper indices

(1) - first order parametre

(2) - second order parametre

Lover indices

m - succesive number of line

k - direction of force action

l - motion direction

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Miscellanea

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Seashore Laboratory

In 1970 the Institute of Hydro-engineering of the Polish Academy of Sciences, (IBW PAN), put into operation Seashore Laboratory in Lubiatowo at the Baltic Sea coast about 30 kms from Łeba, which carries out continuous measurements dealing with investigation of the close-to-shore waving processes. A large number of electric gauges are installed at different water depths, which measure wave parametres.

25 year long activity of the laboratory has brought important research results which are highly acknowledged by many hydro-engineering research centres all over the world. The Costal Dynamics '95 international conference gathered a.o. representatives of the research centres from Japan, China, USA, some South American countries, Russia and France.

The laboratory in Lubiatowo was the first established in Europe (the second, in Bulgaria, has just worked for several years). From time to time the laboratory hosts diver research teams from abroad to carry out special research works together with their Polish counterpartners. Some results of the research works were published in the book on „ Seashore processes of non-tidal sea. Results of an international expedition" issued by IBW PAN.