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Fatigue limit-state design criteria for beams

SUMMARY

Taking advantage of the equivalent state of stress in beams determined in [2], in this paper the limit-state design criteria in fatigue design for infinite life and a finite life till crack initiation are formulated. The cases of periodic and stationary random stress produced by simultaneous bending, twisting and tension-compression are considered.

INTRODUCTION

Design calculations of ship hull and offshore structures are often based on beam models. In the general case of dynamic loading beams can be simultaneously subjected to alternate bending, twisting and tension-compression. As a result, normal and tangential stress components occur. Unfortunately no general consensus has been reached on the best theory or criteria of multiaxial fatigue [1]. In this paper the limit-state design criteria are formulated under assumption that the original state of stress can be approximated with the equivalent state of stress with synchronous components. The transformation of the original stress components into the equivalent synchronous ones is presented in [2].

LIMIT-STATE DESIGN CRITERIA FOR BEAMS UNDER PERIODIC LOADING

Most of the fatigue data for structural steels are available in the form of S-N curves. It is assumed that the S-N curves for tension-compression, alternate bending and twisting are given as:

$$N_i \sigma_i^{m_i} = K_i \qquad i = a\,,\ b\,,\ t \tag{1}$$

for

$$Z_i < \sigma_i \le L_i \tag{2}$$

where indices a, b and t are r elated to axial loading, bending and twisting respectively and :

- N_i - number of stress cycles to cause crack initiation under i-th simple loading
- σ_i - stress amplitude under i-th simple loading
- m_i , K_i - fatigue strength exponent and fatigue strength coefficient for i-th simple loading
- Z_i - fatigue limit for i-th simple loading
- L_i - maximum stress amplitude satisfying (1).

In the fatigue design three principal approaches can be indicated, namely the design for infinite life, the design for a finite life till crack initiation and the design with certain crack growth accepted. The latter is not discussed here.

When the normal and tangential stress components are approximated with the equivalent in-phase components [2]:

$$\tilde{\sigma}_i^{(eq)}(t) = \sigma_i^{(eq)} \sin \omega_{eq} t \tag{3}$$

the design for infinite life requires the condition:

$$f = \left[\left(\frac{\sigma_a^{(eq)}}{Z_a} + \frac{\sigma_b^{(eq)}}{Z_b} \right)^2 + \left(\frac{\sigma_t^{(eq)}}{Z_t} \right)^2 \right]^{-1/2} \ge 1 \tag{4}$$

to be met [3]. Here f is the fatigue safety factor in the safe region of the basic variable space whereas that in the design for finite life till crack initiation can be expressed as:

$$n = \frac{T}{T_d} \tag{5}$$

for the failure sub-region:

$$f < 1 \le l \tag{6}$$

where T_d is the design life and T is the life-time till crack initiation according to [4]:

$$T = \frac{2\pi}{\omega_{eq}} N \qquad N = \left\{ \left[\frac{\left(\sigma_a^{(eq)}\right)^{m_a}}{K_a} + \frac{\left(\sigma_b^{(eq)}\right)^{m_b}}{K_b} \right]^2 + \left[\frac{\left(\sigma_t^{(eq)}\right)^{m_t}}{K_t} \right]^2 \right\}^{-1/2} \tag{7}$$

In (6) l is the limiting factor given according to [4] as:

$$l = \left[\left(\frac{\sigma_a^{(eq)}}{L_a} + \frac{\sigma_b^{(eq)}}{L_b} \right)^2 + \left(\frac{\sigma_t^{(eq)}}{L_t} \right)^2 \right]^{-1/2} \tag{8}$$

Introducing the dimensionless safety margin according to [5]:

$$\mu = 1 - \frac{1}{f} \tag{9}$$

in the design for infinite life and :

$$\mu = 1 - \frac{1}{n} \tag{10}$$

in the design for finite life, the limit-state design criterion can be formulated as:

$$\mu = 0 \tag{11}$$

i.e. respectively:

$$\left[\left(\frac{\sigma_a^{(eq)}}{Z_a} + \frac{\sigma_b^{(eq)}}{Z_b} \right)^2 + \left(\frac{\sigma_t^{(eq)}}{Z_t} \right)^2 \right]^{1/2} = 1 \tag{12}$$

$$\frac{\omega_{eq} T_d}{2\pi} \left\{ \left[\frac{\left(\sigma_a^{(eq)}\right)^{m_a}}{K_a} + \frac{\left(\sigma_b^{(eq)}\right)^{m_b}}{K_b} \right]^2 + \left[\frac{\left(\sigma_t^{(eq)}\right)^{m_t}}{K_t} \right]^2 \right\}^{1/2} = 1 \tag{13}$$

If the fatigue strength assessment in the design for infinite life is to be based on Miner's rule of the cumulative damage [6] and the stress in each block of the stress range spectrum expected during the envisaged service life is periodic and such that $l \geq 1$, the limit-state design criterion becomes:

$$\frac{1}{2\pi} \sum_{r=1}^{w} (\omega_{eq})_r (T_d)_r N_r^{-1} = 1 \tag{14}$$

where:

- w - total number of blocks of the stress range spectrum fulfilling the condition (6)
- $(\omega_{eq})_r$ - equivalent circular frequency in r-th block
- $(T_d)_r$ - duration time of r-th block
- N_r - value of N in r-th block.

LIMIT-STATE DESIGN CRITERIA FOR BEAMS UNDER STATIONARY RANDOM LOADING

In this case the normal and tangential stress components are approximated according to [2] by:

$$\tilde{\sigma}_i^{(eq)}(t) = \sigma_i^{(eq)} \sin\left(\omega_{eq} t + \varphi_i\right) \qquad i = a\ ,\ b\ ,\ t \tag{15}$$

where $\sigma_i^{(eq)}$ are the random amplitudes with zero expected values:

$$e_i = E\left\{\sigma_i^{(eq)}\right\} = 0 \tag{16}$$

and variances:

$$v_i = E\left\{\left(\sigma_i^{(eq)}\right)^2\right\} \tag{17}$$

To formulate the limit-state design criteria the variance of the dimensionless safety margin:

$$v_\mu = E\left\{(\mu - e_\mu)^2\right\} \tag{18}$$

must be calculated. In (18) e_μ is the expected value of the dimensionless safety margin. Since the margins (9) and (10) are nonlinear if expressed in random variables $\sigma_i^{(eq)}$ their expected values will be here approximated according to [7] as:

$$e_\mu = E\left\{\mu\left(\sigma_i^{(eq)}\right)\right\} \cong \mu\left(e_i\right) \tag{19}$$

From (4), (5), (9), (10), (16), (18), and (19) one gets:

$$e_\mu = 1 \qquad v_\mu = E\left\{ \left(\frac{\sigma_a^{(eq)}}{Z_a} + \frac{\sigma_b^{(eq)}}{Z_b} \right)^2 + \left(\frac{\sigma_t^{(eq)}}{Z_t} \right)^2 \right\} \tag{20}$$

in the safe region and:

$$e_\mu = 1 \qquad v_\mu = \left(\frac{\omega_{eq} T_d}{2\pi} \right)^2 E\left\{ \left[\frac{\left(\sigma_a^{(eq)}\right)^{m_a}}{K_a} + \frac{\left(\sigma_b^{(eq)}\right)^{m_b}}{K_b} \right]^2 + \left[\frac{\left(\sigma_t^{(eq)}\right)^{m_t}}{K_t} \right]^2 \right\} \tag{21}$$

in the failure subregion. It is assumed [2] that $\sigma_i^{(eq)}$ are statistically independent. Then (16), (20), and (21) yield:

$$v_\mu = \sum_i \frac{v_i}{Z_i^2} \tag{22}$$

and

$$v_\mu = \left(\frac{\omega_{eq} T_d}{2\pi} \right)^2 \left[\sum_i \frac{1}{K_i^2} E\left\{\left(\sigma_i^{(eq)}\right)^{2m_i}\right\} + \frac{2}{K_a K_b} E\left\{\left(\sigma_a^{(eq)}\right)^{m_a}\right\} E\left\{\left(\sigma_b^{(eq)}\right)^{m_b}\right\} \right] \tag{23}$$

In order to perform distribution-free calculations the central moments in (23) can be approximated by means of the following relationships (24) for Gaussian variables [8]:

$$E\left\{\left(\sigma_i^{(eq)}\right)^{2m_i}\right\} = 1*3***(2m_i - 1) v_i^{m_i}$$

$$E\left\{\left(\sigma_i^{(eq)}\right)^{m_i}\right\} = 1*3***(m_i - 1) s_i^{m_i} \tag{24}$$

if m_i is an even number,

$$E\left\{\left(\sigma_i^{(eq)}\right)^{m_i}\right\} = 0$$

if m_i is an odd number, where:

$$s_i = \left(v_i\right)^{1/2} \tag{25}$$

is the standard deviation of the amplitude of i-th component in equivalent state of stress. Hence (23) can be written as follows:

$$v_\mu = \left(\frac{\omega_{eq} T_d}{2\pi} \right)^2 \sum_i \frac{1*3***(2m_i - 1)}{K_i^2} v_i^{m_i} \tag{26}$$

if m_a and/or m_b is an odd number, and as:

$$v_\mu = \left(\frac{\omega_{eq} T_d}{2\pi} \right)^2 \left[\sum_i \frac{1*3***(2m_i - 1)}{K_i^2} v_i^{m_i} + \frac{2}{K_a K_b} 1*3***(m_a - 1)*1*3***(m_b - 1) s_a^{m_a} s_b^{m_b} \right] \tag{27}$$

if m_a and m_b are even numbers.
From (11), (20) and (21) follows the limit-state criterion:

$$s_\mu = 1 \tag{28}$$

where:

$$s_\mu = \left(v_\mu\right)^{1/2} \tag{29}$$

is the standard deviation of the dimensionless safety margin. In particular (28) takes the form:

$$\left(\sum_i \frac{v_i}{Z_i^2}\right)^{1/2} = 1 \tag{30}$$

in the safe region, and in the case of odd fatigue strength coefficient (s):

$$\frac{\omega_{eq} T_d}{2\pi}\left[\sum_i \frac{1*3***(2m_i-1)}{K_i^2} v_i^{m_i}\right]^{1/2} = 1 \tag{31}$$

in the failure region. Equation (31) can be applied if the limit state (30) is exceeded so that:

$$\left(\sum_i \frac{v_i}{Z_i^2}\right)^{1/2} > 1 \geq \left(\sum_i \frac{v_i}{L_i^2}\right)^{1/2} \tag{32}$$

The application of Miner's rule or any other cumulative damage theory is straightforward.

CONCLUDING REMARKS

In engineering calculations the above presented limit-state design criteria should be adjusted by factors increasing the safety margin according to consequences of failure, data uncertainties etc [9]. Additional factors should be included in each limit state formulation to reflect errors resulting from the applied assumptions.

NOMENCLATURE

- e_i - expected value of the amplitude of i-th component in the equivalent state of stress (i=a, b, t - for the component produced by axial force , bending moment, and torsional moment, respectively)
- e_μ - expected value of the dimensionless safety margin
- E_i - Young modulus, shear modulus
- f - fatigue safety factor in the design for an infinite life
- K_i, m_i - fatigue strength coefficient and fatigue strength exponent in log-linear equation of S-N curve for i-th simple loading
- l - limiting factor for the stress with in-phase components which determines the upper boundary of the high-cycle fatigue subregion
- L_i - maximum stress amplitude satisfying log-linear equation of S-N curve for i-th simple loading (beyond which low-cycle fatigue may have to be taken into consideration)
- n - fatigue safety factor in the design for a finite life till crack initiation
- N - number of cycles to cause crack initiation under the stress with in-phase sinusoidal components
- s_i - standard deviation of the amplitude of i-th component in the equivalent state of stress
- s_μ - standard deviation of the dimensionless safety margin
- t - time
- T - time to crack initiation under the stress with in-phase sinusoidal components
- T_d - design life
- v_i - variance of the amplitude of i-th component in the equivalent state of stress
- v_μ - variance of the dimensionless safety margin
- Z_i - fatigue limit for i-th simple loading
- η_i - coefficient of internal viscous damping of the material, associated with i-th component
- μ - dimensionless safety margin
- σ_i - stress amplitude in the equation (1)
- $\tilde{\sigma}_i^{(eq)}(t)$, $\sigma_i^{(eq)}$ - i-th stress component and its amplitude in the equivalent state of stress
- ω_{eq} - equivalent circular frequency

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