JANUSZ KOLENDA,
Contract. Prof.,D.Sc., M.E.
Technical University of Gdańsk
Faculty of Ocean Engineering
and Ship Technology

Influence of phase shift between sinusoidal loadings on the fatigue safety of beams

SUMMARY

The paper deals with the stress-based fatigue analysis of beams fabricated from constructional steels and subjected to combined bending, tension-compression, and twisting. A complex state of stress with sinusoidal components of equal frequency and arbitrary phase angles is considered. Two functionals of safety are defined the minimum values of which in the time domain determine the safety factor in the safe range of the basic variable space and the number of stress cycles to failure in the high-cycle fatigue range.

As an example the influence of the phase shift between bending moment and axial force on the safety factor and the number of cycles to failure is analyzed. It is shown that the more pronounced is the effect, the closer to π/2 is the phase shift, the smaller is the difference in values of the partial safety factors and the higher are the exponents in equations of S-N curves.

INTRODUCTION

Beam models are frequently used in vibration analysis of ship hulls. In general, alternate bending, twisting and tension-compression of beams may have to be taken into consideration. The same refers to propeller-induced vibrations of tail shafts.

Since excessive vibrations may lead to fatigue damages in ships [1, 2], more attention should be focussed on numerous factors that influence cyclic life of beams. In this paper the influence of phase shifts between sinusoidal stress components on fatigue safety of beams is considered.

FATIGUE SAFETY FACTORS

Most of the fatigue data for constructional steels is available in the form of S-N curves which are the plots of exponential relations between stress range S (or stress amnplitude) and the number of stress cycles to cause failure. In particular, the following relationships may be applied (see e.g. [3]):

- for alternate bending:

$$N_g\sigma_g^{m_g} = K_g \;,\quad Z_{go} < \sigma_g \le L_g \tag{1}$$

- for symmetrical tension-compression:

$$N_r\sigma_r^{m_r} = K_r \;,\quad Z_{rc} < \sigma_r \le L_r \tag{2}$$

- for twisting:

$$N_s\tau^{m_s} = K_s \;,\quad Z_{so} < \tau \le L_s \tag{3}$$

where:

N_g, N_r, N_s - number of cycles to failure

σ_g, σ_r, τ - stress amplitudes

K_g, K_r, K_s - fatigue strength coefficients

m_g, m_r, m_s - fatigue strength exponents

Z_{go}, Z_{rc}, Z_{so} - fatigue limits

L_g, L_r, L_s - stress amplitudes beyond which low-cycle fatigue phenomena occur (see e.g. [4])

In the safe ranges:

$$\sigma_g \le Z_{go},\quad \sigma_r \le Z_{rc},\quad \tau \le Z_{so} \tag{4}$$

the safety factor is respectively:

$$f_g = \frac{Z_{go}}{\sigma_g},\quad f_r = \frac{Z_{rc}}{\sigma_r},\quad f_s = \frac{Z_{so}}{\tau} \tag{5}$$

When the fatigue limits are exceeded the safety factors can be calculated as:

$$n_g = \frac{N_g}{N_o},\quad n_r = \frac{N_r}{N_o},\quad n_s = \frac{N_s}{N_o} \tag{6}$$

where N_o is the required number of stress cycles to achieve a given design life.

In the following, the fatigue safety of beams subjected to combined bending, tension-compression, and twisting in the safe range of the basic variable space and in the high-cycle fatigue range is considered. In the case the safety factors (5) and (6) become the partial safety factors. With the use of (1) to (3) equation (6) can be rewritten as :

$$n_g = K_g\left(N_o\sigma_g^{m_g}\right)^{-1},\quad n_r = K_r\left(N_o\sigma_r^{m_r}\right)^{-1},\quad n_s = K_s\left(N_o\tau^{m_s}\right)^{-1}$$

In the complex state of stress with in-phase components:

$$\tilde{\sigma}_g = \sigma_g \sin\omega t,\ \tilde{\sigma}_r = \sigma_r \sin\omega t,\ \tilde{\tau} = \tau \sin\omega t \quad (8)$$

the formula for the safety factor is given in [5] as:

$$f = \left[\left(f_r^{-1} + f_g^{-1}\right)^2 + f_s^{-2}\right]^{-1/2} \quad (9)$$

for the safe range:

$$f \geq 1 \quad (10)$$

and in [6] as:

$$n = \left[\left(n_r^{-1} + n_g^{-1}\right)^2 + n_s^{-2}\right]^{-1/2} \quad (11)$$

for the high-cycle fatigue range:

$$f < 1 \leq l \quad (12)$$

In (12) l is the limiting factor for the high-cycle fatigue range given by:

$$l = \left[\left(l_r^{-1} + l_g^{-1}\right)^2 + l_s^{-2}\right]^{-1/2} \quad (13)$$

where:

$$l_g = \frac{L_g}{\sigma_g},\ l_r = \frac{L_r}{\sigma_r},\ l_s = \frac{L_s}{\tau} \quad (14)$$

are the partial limiting factors. The number of cycles to failure amounts to:

$$N = nN_o \quad (15).$$

FORMULATION OF THE PROBLEM

In this paper the complex state of stress with components (16):

$$\tilde{\sigma}_g = \sigma_g \sin(\omega t + \beta_g),\ \tilde{\sigma}_r = \sigma_r \sin(\omega t + \beta_r),\ \tilde{\tau} = \tau \sin(\omega t + \beta_s)$$

is considered where $\beta_g, \beta_r, \beta_s$ are the phase angles.

In order to determine the safety factor and the number of cycles to failure with the phase angles taken into account, the complex state of stress can be transformed into a hypothetically equivalent uniaxial state of stress by means of a selected criterion of multiaxial fatigue failure. Such criteria are discussed e.g. in [7].

An alternative calculation procedure is proposed in [8] for the general state of stress with out-of-phase sinusoidal components. This procedure is applied to the stress given by equation (16). For this purpose functionals of safety corresponding to (16) are to be determined.

FUNCTIONALS OF SAFETY

Replacing the stress amplitudes in (5), (7), and (14) with the stress components (8), the following variable factors can be defined:

$$\tilde{f} = \left[\left(\tilde{f}_r^{-1} + \tilde{f}_g^{-1}\right)^2 + \tilde{f}_s^{-2}\right]^{-1/2} \quad (17)$$

$$\tilde{l} = \left[\left(\tilde{l}_r^{-1} + \tilde{l}_g^{-1}\right)^2 + \tilde{l}_s^{-2}\right]^{-1/2} \quad (18)$$

$$\tilde{n} = \left[\left(\tilde{n}_r^{-1} + \tilde{n}_g^{-1}\right)^2 + \tilde{n}_s^{-2}\right]^{-1/2} \quad (19)$$

where:

$$\tilde{f}_g = \frac{Z_{go}}{\tilde{\sigma}_g},\ \tilde{f}_r = \frac{Z_{rc}}{\tilde{\sigma}_r},\ \tilde{f}_s = \frac{Z_{so}}{\tilde{\tau}} \quad (20)$$

$$\tilde{l}_g = \frac{L_g}{\tilde{\sigma}_g},\ \tilde{l}_r = \frac{L_r}{\tilde{\sigma}_r},\ \tilde{l}_s = \frac{L_s}{\tilde{\tau}} \quad (21)$$

$$\tilde{n}_g = K_g\left(N_o\tilde{\sigma}_g^{m_g}\right)^{-1},\ \tilde{n}_r = K_r\left(N_o\tilde{\sigma}_r^{m_r}\right)^{-1},\ \tilde{n}_s = K_s\left(N_o\tilde{\tau}^{m_s}\right)^{-1} \quad (22)$$

From equations (5), (7) to (9), (11), (13), (14), and (17) to (22) it is seen that factors $\tilde{f},\ \tilde{l},\ \tilde{n}$ vary from:

$$\tilde{f}_{\min} = f \quad (23)$$

$$\tilde{l}_{\min} = l \quad (24)$$

$$\tilde{n}_{\min} = n \quad (25)$$

at $\sin\omega t = \pm 1$ to ∞ at $\sin\omega t = 0$.

It means that the factors f, l and n represent the minimum values of the variable factors $\tilde{f},\ \tilde{l},\ \tilde{n}$ in the time domain. Hence the conclusions can be drawn that:

- The variable factor $\tilde{f}$ can be regarded as a functional of safety the minimum value of which in the time domain can be taken as a measure of safety in the safe range:

$$\tilde{f}_{\min} \geq 1 \quad (26)$$

and that:

- The variable factor $\tilde{n}$ can be regarded as a functional of safety the minimum of which in the time domain can be taken as a measure of safety in the high-cycle fatigue range:

$$\tilde{f}_{\min} < 1 \leq \tilde{l}_{\min} \quad (27).$$

CALCULATION PROCEDURE

It is assumed that equations (23) to (25) written for the in-phase stress components (8) hold true also for the out-of-phase stress components (16).

Consequently, in order to evaluate the safety factor of a beam subjected to the stress with the components (16), equations (16) and (20) must be substituted into (17) and the minimum value of the functional $\tilde{f}$ in the time domain be calculated. If this value is not smaller than unity, the calculations will thus be completed. In the opposite case, one has to calculate the minimum value of the variable limiting factor $\tilde{l}$ in the time domain with (16) and (21) to be substituted. Should it happen that $\tilde{l}_{\min} < 1$, the proposed calculation procedure can not be used. In the case of $\tilde{l}_{\min} \geq 1$ (16) and (22) must be substituted into (19) and the minimum value of the functional $\tilde{n}$ in the time domain be calculated. Then, according to (15) and (25), the number of cycles to failure can be determined, as follows:

$$N = \tilde{n}_{\min} N_o \quad (28).$$

EXAMPLE

As an example, the influence of the phase shift β between bending moment and axial force on the fatigue safety is considered. The stress components are:

$$\tilde{\sigma}_g = \sigma_g \sin\omega t,\ \tilde{\sigma}_r = \sigma_r \sin(\omega t + \beta),\ \tilde{\tau} = 0 \quad (29).$$

The analyzed effect can be presented as the minimum values of the quotients:

$$\frac{\tilde{f}}{f} = \frac{f_r^{-1} + f_g^{-1}}{\left|\tilde{f}_r^{-1} \pm \tilde{f}_g^{-1}\right|} \quad (30)$$

and

$$\frac{\tilde{n}}{n} = \frac{n_r^{-1} + n_g^{-1}}{\left|\tilde{n}_r^{-1} \pm \tilde{n}_g^{-1}\right|} \quad (31)$$

Equation (30) refers to the safe range:

$$\left\{\left|\tilde{f}_r^{-1} \pm \tilde{f}_g^{-1}\right|^{-1}\right\}_{\min} \geq 1 \qquad (32)$$

and (31) corresponds to the high-cycle fatigue range (33):

$$\left\{\left|\tilde{f}_r^{-1} \pm \tilde{f}_g^{-1}\right|^{-1}\right\}_{\min} < 1 \leq \left\{\left|\tilde{l}_r^{-1} \pm \tilde{l}_g^{-1}\right|^{-1}\right\}_{\min} \qquad (33)$$

In order to account for the stress in outer fibres on both sides of the beam cross-section with respect to the neutral axis, in the equations (30) to (33) the sign «-» is also put in front of $\tilde{f}_g^{-1}$, $\tilde{n}_g^{-1}$.

According to equations (5),(7), (20), (22) and (29), one gets:

$$\frac{\tilde{f}}{f} = \frac{f_r^{-1} + f_g^{-1}}{\left|f_r^{-1}\sin(\omega t + \beta) \pm f_g^{-1}\sin\omega t\right|} \qquad (34)$$

$$\frac{\tilde{n}}{n} = \frac{n_r^{-1} + n_g^{-1}}{\left|n_r^{-1}\left[\sin(\omega t + \beta)\right]^{m_r} \pm n_g^{-1}(\sin\omega t)^{m_g}\right|} \qquad (35)$$

The results of calculations are depicted in Fig. 1 and 2.

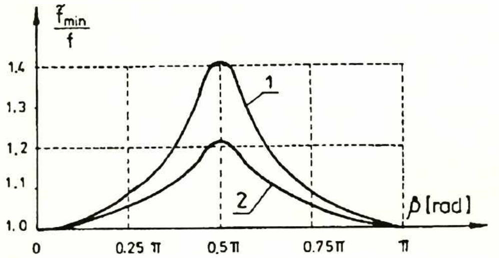
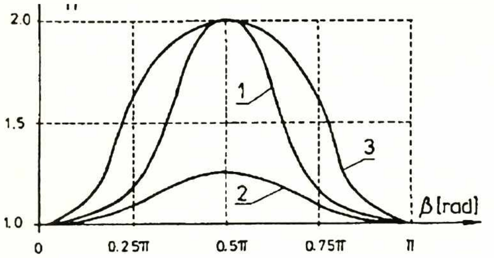


Fig. 1 Influence of the phase shift between bending moment and axial force on the safety factor in the safe range for:

① $\frac{f_g}{f_r} = 1$ ② $\frac{f_g}{f_r} = 0{,}25$ and 4

Fig. 2 Influence of the phase shift between bending moment and axial force on the safety factor in the high-cycle fatigue range for:



① $\frac{n_g}{n_r} = 1,\ m_g = m_r = 2$ ② $\frac{n_g}{n_r} = 0{,}25$ and 4 , $m_g = m_r = 2$

③ $\frac{n_g}{n_r} = 1,\ m_g = m_r = 6$

CONCLUSIONS

From Fig. 1 and 2 it follows that the phase shift between bending moment and axial force is advantageous. The closer to $\pi/2$ is the phase shift, the smaller is the difference in values of the partial safety factors and the higher are the fatigue strength exponents, the more pronounced is the effect. In comparison to beams under in-phase bending and tension-compression with equal partial safety factors, beams subjected to the very same loadings but shifted in phase by $\pi/2$ have the safety factor greater by 41% in the safe range and by 100% in the high-cycle fatigue range. Therefore, the fatigue lives of beams under in-phase and out-of-phase bending and tension-compression may differ even by an order of magnitude.

NOMENCLATURE

f	- safety factor in the safe range of the basic variable space
$\tilde{f}$	- functional of safety in the safe range
$f_g,\ f_r,\ f_s$	- safety factors in the safe ranges under alternate bending, tension-compression and twisting, respectively; partial safety factors in the safe range
$\tilde{f}_g,\ \tilde{f}_r,\ \tilde{f}_s$	- variable partial safety factors in the safe range
$K_g,\ K_r,\ K_s$	- fatigue strength coefficients
l	- limiting factor for the high-cycle fatigue range
$\tilde{l}$	- variable limiting factor
$l_g,\ l_r,\ l_s$	- partial limiting factors
$\tilde{l}_g,\ \tilde{l}_r,\ \tilde{l}_s$	- variable partial limiting factors
$L_g,\ L_r,\ L_s$	- maximum stress amplitudes satisfying equations (1) to (3)
$m_g,\ m_r,\ m_s$	- fatigue strength exponents
n	- safety factor in the high-cycle fatigue range
$\tilde{n}$	- functional of safety in the high-cycle fatigue range
$n_g,\ n_r,\ n_s$	- safety factor in the high-cycle fatigue ranges under alternate bending, tension-compression, and twisting, respectively; partial safety factors in the high-cycle fatigue range
$\tilde{n}_g,\ \tilde{n}_r,\ \tilde{n}_s$	- variable partial safety factors in the high-cycle fatigue range
$N_g,\ N_r,\ N_s$	- number of stress cycles to cause failure under alternate bending, tension-compression, and twisting, respectively
N_o	- required number of stress cycles to achieve a given design life
N	- number of stress cycles to cause failure under combined load
t	- time
$Z_{go},\ Z_{rc},\ Z_{so}$	- fatigue limit for alternate bending, symmetrical tension-compression and twisting, respectively
$\beta_g,\ \beta_r,\ \beta_s,\ \beta$	- phase angles
$\tilde{\sigma}_g,\ \tilde{\sigma}_r,\ \tilde{\tau}$	- stress component produced by alternate bending, tension compression, and twisting, respectively
$\sigma_g,\ \sigma_r,\ \tau$	- amplitudes of the stress components
ω	- angular frequency

BIBLIOGRAPHY

1. Jeffrey N. E., KendrickA. M. (Ed.): Proceedings of the 12th Int. Ship and Offshore Structures Congress, Vol. 1, St. Johns, Canada, September 1994
2. Kerneur J. : „Statistics on tail-shaft damages". Bull. Techn. du Bureau Veritas, 1980, No. 2
3. Kocańda S., Szala J.: „Fundamentals of fatigue calculations" (in Polish). PWN, Warszawa, 1985 r.
4. Kocańda S. , Kocańda A. : „Low-cycle fatigue strength of metals" (in Polish). PWN, Warszawa, 1989 r.
5. Günter W. (Ed.) : „Schwingfestgkeit". Deutscher Verlag für Grundstoffindustrie, Leipzig, 1973
6. Kolenda J.: „Stress-based lifetime prediction method for beams in a complex state of cyclic stress". Marine Technology Transaction 1994, No 4
7. Łagoda E., Macha E. : „Spectral analysis of the criteria for multiaxial random fatigue" (in Polish). Zeszyty Naukowe Politechniki Świętokrzyskiej, Mechanika, 1992 r., nr 48
8. Kolenda J. : „On fatigue strength under out-of-phase sinusoidal loadings". Engineering Transaction , 42, 1994, No 1-2