Stability criteria as constraints in a fleet of ships optimisation problem

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ABSTRACT

The paper has been written within the European EUREKA Project E!2772, initiated and completed at the Faculty of Ocean Engineering & Ship Technology, Gdańsk University of Technology in the years 2001-2003. A problem has been solved concerning mathematical optimisation of a fleet of multipurpose sea-river vessels for European short-shipping regular lines, in the area of The North and Baltic Seas, on the level of marine transportation task, by the non-linear programming methods with constraints. A method is proposed which enables existing criteria of stability to be included as constraints in the optimisation model of a fleet. In the numerical examples, three typical criteria of intact stability: by IMO, PRS, and HSMB have been selected to demonstrate a post-optimisation feasibility analysis of principal parameters of ships.

Keywords : maritime transportation, computer-aided ship design, optimisation, intact stability criteria

1. INTRODUCTION

Computer aided ship design methods used at present, while offering automation of the design process, require its rationalisation and formalisation. In consequence, adequate mathematical models of the design object must be created which affect the design process by introducing a structure and terminology which unavoidably bounds reasoning to the terms of the model.

In this case a fleet of ships at the stage of owner's study is assumed to be an object and the task of optimising its main parameters is an objective of the adequate mathematical model. In consequence, the global structure of the model (further called an "optimisation model") corresponds to that proposed by operational research methods in general and non-linear programming methods (NPM) in particular [3]. Within this structure, optimisation models consist of a set of sub-models of particular properties of the object which have been recognised as significant to the predictive features of the model. Optimisation models applied to fleet/ship design are definitely synthetic in nature. This feature requires the analytical representation of particular sub-models to be relatively simple. In consequence, sub-models usually neither become isomorphic with, nor conform to the physical structure of that part of object to which they are related. Such type of models is sometimes referred to as "non-structural" [19]. NPM require for all the sub-models concerned to be formulated as constraints. Among them there are always those concerning safety of an object. In ship design, a special interest in this group is focused on the stability of ships. In naval architecture today, the stability requirements are imposed in the form of legal regulations by such institutions as IMO, classification societies, governmental organisations and other bodies. An essential part of stability regulations are stability criteria.

The paper deals with the problem of incorporating stability criteria as constraints in the optimisation model of fleet/ship design. At the initial stages of the design the principal difficulty is that the full geometry of a hull, necessary for the stability criteria to be applied, is usually unknown. A standard solution was to take into account the initial stability only, represented by the initial metacentric height GM0 [2], [4], [7], [12], [14]. The paper proposes an alternative approach, based on an idea introduced by Wiśniewski [20] and developed by Kupras [10], [11]. In this concept the full stability of ship can be accounted for by using systematic standard series of hull forms, following the methodology developed in ship resistance and power prediction.

In order to accomplish the task, an attempt has been made to define all the stability-related geometrical characteristics of a ship analytically, based on the Series 60 body forms [19]. In consequence, an arbitrary criterion of intact stability can also be defined in an analytical way and so incorporated into optimisation model as a constraint.

Stability aspects in the computer-aided modelling of ship design have been addressed on the background of the optimisation problem concerning a fleet of multipurpose sea-river vessels for European short-shipping regular lines, in the area of The North and Baltic Seas, on the level of marine transportation task, by non-linear programming methods with constraints. The problem has been undertaken within the European EUREKA Project [13] based on predictions that a significant increase of cargo transportation in Europe over the next 10 years (or probably after this period) will take place between Western Europe and the Central and East European countries.

In the numerical examples, three criteria of intact stability: IMO [6], HSMB [5], and PRS [17] have been selected, as typical of contemporary stability regulations, to demonstrate the method in a post-optimisation, feasibility analysis of principal parameters of ships.

2. PROBLEM STATEMENT

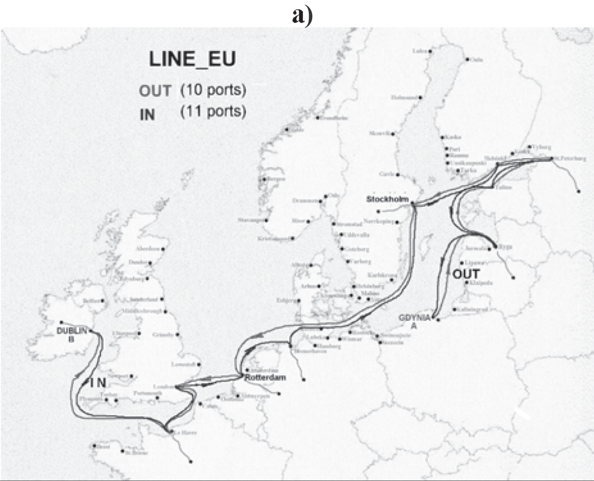
A fleet of ships consists of a number of homogeneous ships operating as a maritime transportation system in a certain environment. The transportation task for a fleet of ships is to carry goods between ports during a prescribed period of time. An optimum fleet to perform this task, given the particular (owner's) data, is a general problem under discussion. A solution to this problem needs adequate functional and mathematical models.

2.1. ***Functional model of a fleet***

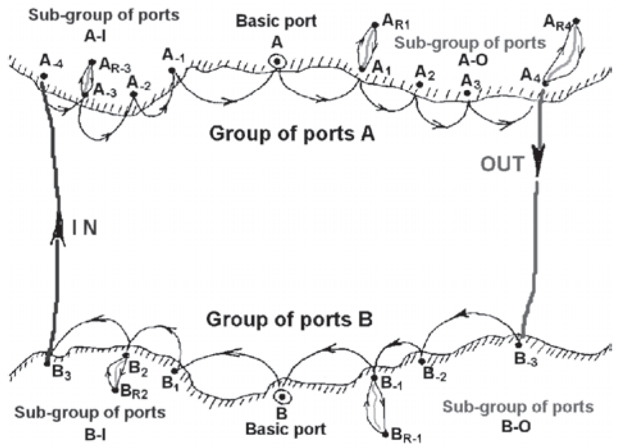
In the particular case (Tab.2.1), a (potential) shipping line connects the furthest Western and Eastern regions of Europe (**a**). A corresponding model of shipping (**b**) is called a multi-port route model linking two areas of operation A and B with the two groups of clustered sea and hinterland river ports. There are two streams of goods transportation in the model: from A to B (called OUT) and back, from B to A (called IN). Ports A-0 and B-0 are the home and destination ports. For more details about the functional model of a fleet - see [13].

Tab. 2.1. *An example of a shipping line and its graphical model*

a)



b)



2.2. ***Optimisation model of a fleet***

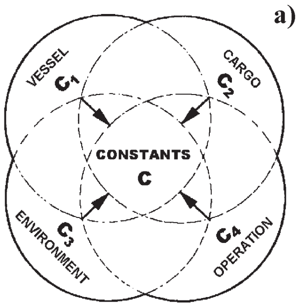
The mathematical model chosen for the fleet optimisation problem can be described as deterministic, static, continuous, single level and single objective model, imposed and solved by non-linear programming methods. A standard formulation of the model, within NPM, is as follows: given a vector **c** (or a set **C**) of constants, find such a vector of decision variables **x** that minimises a single valued objective function Q(**x**,**c**) subject to a set of inequality constraints. An adequate mathematical form of the problem is :

$$\begin{cases} \min Q(\mathbf{x};\mathbf{c}); \quad Q : \mathbf{R}^n \times \mathbf{R}^N \to \mathbf{R}^1 \\ (\mathbf{x};\mathbf{c}) \in \mathbf{\Omega} = \left\{ (\mathbf{x};\mathbf{c}) : g_j(\mathbf{x};\mathbf{c}) \le 0 \quad j = 1,2,\ldots,m \right\} \ne \varnothing \end{cases} \tag{2.1}$$

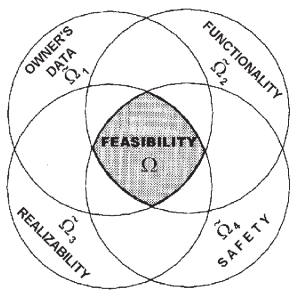
It is generally assumed that $Q(\cdot)$ and $g_j(\cdot)$ are all non-linear functions.
The conditions for existence and uniqueness of the (optimum) solution to (2.1) can be found in [3].

Tab. 2.2. *Elements of an optimisation model of a fleet of ships*

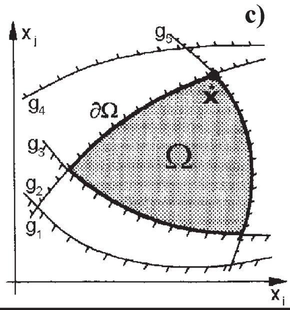
a)



b)



c)



$\mathbf{J}_B = \{2,3,5\}$

$\mathbf{J}_B^a = \{2,5\}$

$\mathbf{J}_B^p = \{3\}$

It is obvious that the optimum solution $\mathbf{x}^*$ of the problem (2.1) is, in fact, parametrised by *constants* **C** that can be classified according to different criteria (Tab.2.2a). In particular, the group $\mathbf{C}_3$ delivers constants for the stability criteria of a ship (legal environment). A crucial element of the optimisation model is a *feasible solution region* (FSR) **Ω**, because it eventually decides about an optimum solution to the problem. FSR is such a set of pairs (**x**,**c**) that all the constraints hold.

$$\mathbf{\Omega} = \{(\mathbf{x},\mathbf{c}) : g_j(\mathbf{x},\mathbf{c}) \le 0 \text{ , } j \in \mathbf{J}\} \text{ , } \mathbf{J} = \{1,2,...m\} \tag{2.2}$$

For further discussion, it is useful to classify FSR from the functional and formal points of view. In the *functional* classification, FSR defines, in fact, the notion of feasibility in ship design. Formally it can be thought of as a common part of the four groups of requirements imposed on a fleet/vessel by the environment (Tab.2.2b) and can then be written as the following product of four sets :

$$\Omega = \bigcap_{k=1}^{4} \Omega_k$$

All of them affect constraints of the model. For example, the SAFETY ($\tilde{\Omega}_4$) group contains, among other things, design restrictions concerning stability regulations.

In (2.2), an individual constraint is an inequality-type relation in which a function $g_j(\cdot)$ expresses a balance between certain (dependent) parameters of an object. In order to keep the same standard relation of inequality ($\leq$) for all constraints, the general form of $g_j(\cdot)$ has to be alternatively :

$$g_j(\mathbf{x};\mathbf{c}) = \begin{cases} p_j(\mathbf{x};\mathbf{c}) \quad - \quad \tilde{p}_j(\mathbf{x};\mathbf{c}) \quad (a) \\ \qquad\qquad\qquad \text{or} \\ \tilde{p}_j(\mathbf{x};\mathbf{c}) \quad - \quad p_j(\mathbf{x};\mathbf{c}) \quad (b) \end{cases} \quad j \in \mathbf{J} \tag{2.3}$$

where :

the $p_j(\cdot)$ parameters are those predicted by the model and $\tilde{p}_j(\cdot)$ are the corresponding ones required by $\tilde{\Omega}_k$.
In most cases the $\tilde{p}_j$ parameters are just constant figures such that $\tilde{p}_j = c_j \in \mathbf{C}$.

In the *formal* classification of FSR, attention will be focused on constraints forming the boundary of Ω.
Figure in Tab.2.2c illustrates the problem. Let us define an FSR corresponding to a single j^{th} constraints :

$$\Omega_j = \{(x;c) : g_j(x;c) \leq 0\}, \quad j \in J$$

Its boundary is then:

$$\partial\Omega_j = \{(\mathbf{x};\mathbf{c}) : g_j(\mathbf{x};\mathbf{c}) = 0\}, \quad j \in \mathbf{J}$$

For the whole feasible region one has, of course : $\mathbf{\Omega} = \bigcap \mathbf{\Omega}_j$, $j \in \mathbf{J}$

Similar relation does not, however, hold for boundaries :

$$\partial\mathbf{\Omega} \neq \bigcap \partial\mathbf{\Omega}_j, \neq \bigcup \partial\mathbf{\Omega}_j \quad \text{but} \quad \partial\mathbf{\Omega} \subset \bigcup \partial\mathbf{\Omega}_j, \; j \in \mathbf{J} \tag{2.4}$$

A j^{th} constraint is called a *boundary constraint* if its own boundary contributes to the boundary of Ω, otherwise the constraint is a *non-boundary constraint*. It is obvious that, in fact, the boundary constraints are those which determine Ω and, as such, affect the optimum solution of the problem :

$$\mathbf{\Omega} = \bigcap_{j \in \mathbf{J}} \mathbf{\Omega}_j = \bigcap_{j \in \mathbf{J}_B \subset \mathbf{J}} \mathbf{\Omega}_j \tag{2.5}$$

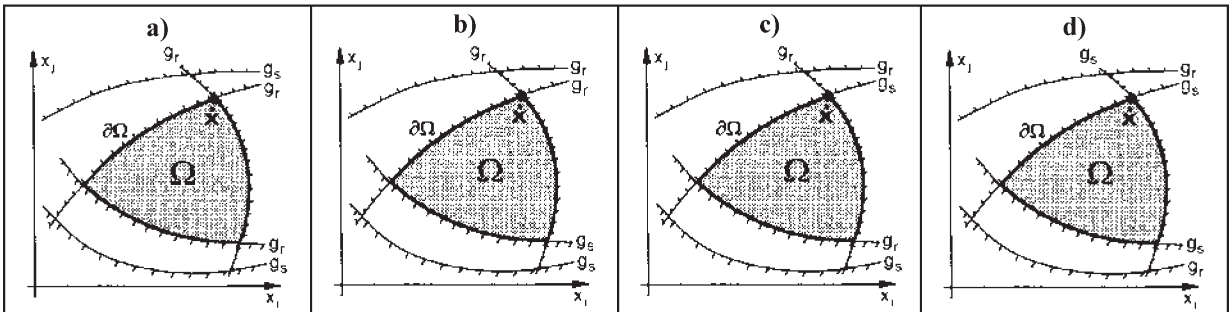
Relation (2.5) suggests an apparently equivalent formulation of (2.1) with the smaller number of constraints (boundary constraints only). Unfortunately, this is not the case because the set of boundary constraints $\mathbf{J}_B$ is not known in advance. Among the boundary constraints one can further distinguish *active* constraints and *passive* constraints. The classification refers to the location of the optimum $\mathbf{x}^*$ solution in $\mathbf{\Omega}$, which exclusively depends on the objective function $\mathbf{\Omega}(\cdot)$ used in the optimisation process. A j^{th} constraint is active in $\mathbf{x}^*$ if $\mathbf{x}^* \in \partial\mathbf{\Omega}$j ($\Rightarrow j \in \mathbf{J}_B^a$), otherwise the constraint is passive ($\Rightarrow j \in \mathbf{J}_B^p = \mathbf{J}_B \setminus \mathbf{J}_B^a$). If the optimum solution belongs to the interior of FSR ($\mathbf{x}^* \in \text{int}\mathbf{\Omega}$), all the boundary constraints are passive and no constraint affects $\mathbf{x}^*$.

Let us now introduce another classification of $\mathbf{\Omega}$ and the corresponding constraints : *stability* constraints vs. *non-stability* (remaining) constraints.

$$\mathbf{\Omega} = \mathbf{\Omega}_S \cap \mathbf{\Omega}_R \quad \Leftrightarrow \quad \mathbf{J} = \mathbf{J}_S \cup \mathbf{J}_R \tag{2.6}$$

where : $\mathbf{\Omega}_S$ is a feasible solution region with regard to stability requirements, $\mathbf{J}_S$ is a set of indices of stability constraints, $\mathbf{\Omega}_R$, $\mathbf{J}_R$, correspond to the remaining constraints accordingly. The current classification is independent of the previous one; i.e. both stability and remaining constraints can become the boundary or non-boundary constraints. Let us investigate the status of stability constraints via the relations: $\mathbf{\Omega}_S$ vs. $\mathbf{\Omega}$, $\partial\mathbf{\Omega}_S$ vs. $\partial\mathbf{\Omega}_S$, and $\mathbf{J}_S$ vs. $\mathbf{J}_B$. It is obvious that the relation $\mathbf{\Omega}_S \supseteq \mathbf{\Omega}$ is always satisfied, but similar relation does not occur for boundaries. One can distinguish here four cases (see Tab.2.3).

Tab. 2.3. *Stability constraints (g_S) vs. remaining constraints (g_R)*



a)	b)	c)	d)
$\Omega \subset \Omega_S$	$\Omega \subset \Omega_S$	$\Omega \subset \Omega_S$	$\Omega = \Omega_S$
$\partial\Omega_S \cap \partial\Omega_B = \varnothing$	$\partial\Omega_S \subset \partial\Omega$	$\partial\Omega_S \subset \partial\Omega$	$\partial\Omega_S = \partial\Omega$
$\mathbf{J}_S \cap \mathbf{J}_B = \varnothing$	$\mathbf{J}_S \in \mathbf{J}_B^p$	$\mathbf{J}_S \in \mathbf{J}_B^a$	$\mathbf{J}_S = \mathbf{J}_B$

ad. a) Stability constraints do not contribute to the FSR, so, for given stability regulations and a set of constants **C**, they are totally insignificant, regardless of the objective function Q.

ad. b) Stability constraints do contribute to the FSR, so, for given stability regulations and a set of constants **C**, they are currently insignificant but potentially significant, dependent on the objective function Q.
ad. c) Stability constraints do contribute to the FSR, so, for given stability regulations and a set of constants **C**, they are currently significant, but potentially insignificant, dependent on the objective function Q.
ad. d) Stability constraints form the boundary of the FSR, so, for given stability regulations and a set of constants **C**, they are currently and potentially significant, regardless of the objective function Q.

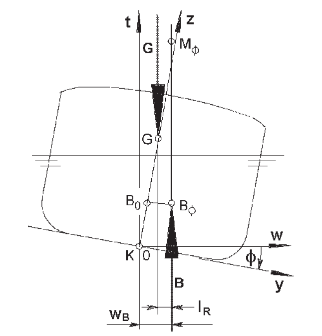
The foregoing discussion will be recalled in Chapter 5 to verify the status of stability constraints in the fleet optimisation model.

3. IDENTIFICATION OF INTACT STABILITY CRITERIA

In order to illustrate the thesis of the paper, three criteria of intact stability have been chosen, as recommended by the following institutions: International Maritime Organisation (IMO [6]), Polish Register of Shipping (PRS, [17]), and Hamburg Ship Model Basin (HSMB [5]). In the light of the discussion in [8], all these criteria can be regarded as a present day standard as far as intact stability regulations in naval architecture are concerned.

Tab.3.1 introduces the coordinate systems as well as basic notions and terminology which will be in use throughout the paper. In particular, (Y-Z) and (y-z) are, respectively, a dimensional [m] and non-dimensional [-] coordinate systems of the hull and (w-t) [-] is a non-dimensional coordinate system rotated around the x axis.

Tab. 3.1. Coordinate systems and basic terminology of intact stability of ships

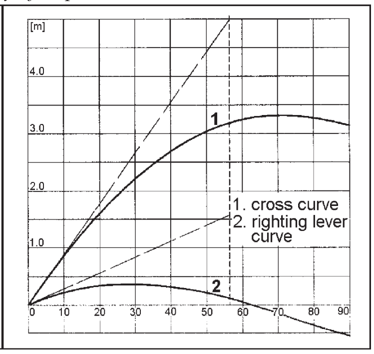


$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{T} & \\ & \frac{1}{T} \end{bmatrix} \cdot \begin{bmatrix} Y \\ Z \end{bmatrix} \quad (3.1a)$$

$$\begin{bmatrix} w \\ t \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} y \\ z \end{bmatrix} \quad (3.1b)$$

$$L_R(\phi; Z_G) = l_R(\phi; z_G) \cdot T \ [m] \quad (3.2a)$$

$$l_R(\phi; z_G) = w_B(\phi) - z_G \cdot \sin\phi \ [-] \quad (3.2b)$$



A common feature of the criteria under discussion (in fact, all the contemporary criteria of intact stability of ships) is that, as far as the righting moment of a ship is concerned, they all are totally based on the righting lever curve calculated in a calm water as the function : $\mathbf{L_R}(\phi; \mathbf{D}, \mathbf{Z_G})$ (3.2a), where : ϕ is angle of inclination and **D** [t], $\mathbf{Z_G}$ [m] are a constant displacement and coordinate of the centre of gravity **G**, describing current loading condition of a ship. In further discussion, the only loading condition accounted for is the design condition. It follows from (3.2) that all the stability-related characteristics and parameters of a ship have been normalised with regard to the design draught T. A short overview of the criteria by IMO, PRS and HSMB in an analytical form has been shown in Table 3.2.

It can be seen that the criteria by IMO and PRS involve both righting $L_R(\cdot)$ and heeling $L_H(\cdot)$ lever curves („weather-type" criteria), whereas the criteria by HSMB are based on the righting lever curve exclusively („Rahola's-type" criteria).

It follows from Tab.3.2, that, from a mathematical viewpoint, all the criteria can be formulated as combinations of certain operations on the functions $L_R(\cdot)$ and $L_H(\cdot)$: linear - such as calculating ordinates, first derivatives, and integrals or non-linear – – such as calculating characteristic angles, the weather parameter K, etc. Two examples illustrate the problem :

(i) Linear case - a dynamic righting lever curve $L_{R_2}(\cdot)$ (PRS) is defined by integration of $L_R(\cdot)$:

$$L_{R_2}(\phi) = \begin{cases} \int\limits_{-\phi_0}^{-\phi} L_{R_1}(\phi)d\phi \ + \int\limits_{0}^{-\phi_0} L_{R_1}(\phi)d\phi & \text{for} \quad \phi \le 0 \\ \int\limits_{0}^{\phi} L_{R_1}(\phi)d\phi & \text{for} \quad \phi > 0 \end{cases} \quad (3.3)$$

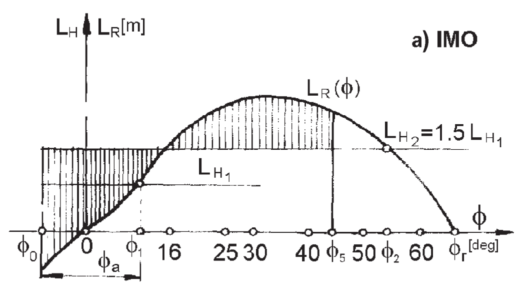
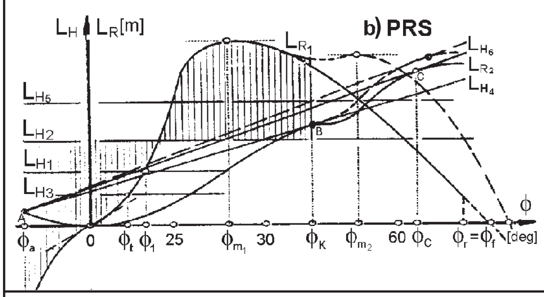
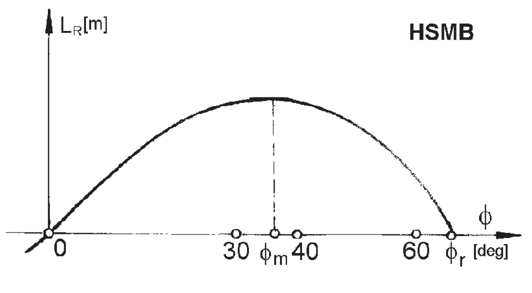
(ii) Non-linear case - angles of the first and second interception ϕ_1, ϕ_2 (IMO, PRS) are defined as the smaller and greater roots of the following non-linear equation :

$$F_1(\phi) \equiv L_{H_1}(\phi) - L_{R_1}(\phi) = 0 \ \Leftrightarrow \ \phi_1 = \min[F_1^{-1}(0)], \ \phi_2 = \max[F_1^{-1}(0)] \quad (3.4)$$

A formulation of the intact stability criteria in Tab. 3.2 differ from but are fully equivalent to those originally formulated in the referenced documents by IMO, PRS and HSMB. Among other things, a notation has been unified and sequences of the righting and heeling lever curves as well as characteristic angles have been introduced to emphasize the analytical aspects of the criteria.

In the subsequent chapters of the paper, the intact stability criteria will be examined with a special attention to those parts of them which concern the righting lever curve of a ship. This is because determination of the righting lever curve $l_R(\cdot)$, via cross curves $w_B(\cdot)$ involves the full geometry of the hull and as such decides on the reliability of the whole model.

Tab. 3.2. *Intact stability criteria by IMO [6], PRS [17] and HSMB [5]*

Righting / heeling lever curves	Stability criteria
	1. $L'_R(0) \geq 0.15$ m 2. $L_R(25) \geq 0.20$ m 3. $L_R(30) \geq 0.20$ m 4. $\int_0^{30} L_R(\phi)\,d\phi \geq 0.055$ mrad 5. $\int_0^{\phi_3} L_R(\phi)\,d\phi \geq 0.090$ mrad 6. $\int_{30}^{\phi_3} L_R(\phi)\,d\phi \geq 0.30$ m 7. $\phi_1 \leq \phi_4$ 8. $\phi_K \geq \phi_5$ 9. $\phi_r \geq 60^o$ 10. $K_{IMO} \geq 1.0$
	1. $L'_R(0) \geq 0.20$ m 2. $L_R(\phi_m) \geq 0.20$ m 3. $\phi_6 \leq \phi_7$ 4. $\phi_{m_1} \geq 25 - 0.5\cdot\Delta\phi_r$ 5. $\phi_{m_2} \geq 0.30$ 6. $\phi_K \leq \phi_C$ 7. $\phi_r \geq 60 - \Delta\phi_r$ 8. $K_{PRS} \geq 1.0$
	1. $L_R(30) \geq \frac{0.04}{C}$ 2. $L_R(\phi_m) \geq \frac{0.05}{C}$ 3. $\int_0^{30} L_R(\phi)\,d\phi \geq \frac{0.12}{C}$ 4. $\int_0^{40} L_R(\phi)\,d\phi \geq \frac{0.020}{C}$ 5. $\int_{30}^{40} L_R(\phi)\,d\phi \geq \frac{0.007}{C}$ 6. $\int_0^{\phi_r} L_R(\phi)\,d\phi \geq \frac{0.035}{C}$

Legend

Characteristic angles

ϕ_0 - initial heeling angle, ϕ_a - rolling amplitude, ϕ_f - ship flooding angle, ϕ_d - deck immersion angle

ϕ_t - turning angle, ϕ_s - superstructure flooding angle, ϕ_c - capsizing angle, $\phi_C = \min(\phi_f, \phi_c)$ - critical angle

ϕ_K - the angle for calculating K, ϕ_{m_1}, ϕ_{m_1} - 1st and 2nd maximum of L_R angles, ϕ_m maximum of L_R angle $\phi_m = \phi_{m_1}$

if $L_R(\phi_{m_1}) \geq L_R(\phi_{m_2})$ otherwise $\phi_m = \phi_{m_2}$, ϕ_1, ϕ_2 - first and second interception angles, $\phi_3 = \min(\phi_f, 40)$

$\phi_4 = \min(0.8\cdot\phi_d, 16)$, $\phi_5 = \min(\phi_f, \phi_2, 50)$, $\phi_6 = \max(\phi_1, \phi_t)$, $\phi_7 = \min(0.5\cdot\phi_d, 15)$

Weather criteria indices : $K_{IMO} = A_R / A_H\,[-]$, $K_{PRS} = L_{H_5} / L_{H_2}\,[-]$

HSMB form factor

$$C = \frac{T\cdot H'}{B^2}\cdot\sqrt{\frac{T}{z_G}}\cdot\left(\frac{C_B}{C_W}\right)^2\cdot\sqrt{\frac{100}{L}}$$, where $H' = H + h\cdot\frac{2b - B}{B}\cdot\frac{2l_h}{L}$ - a corrected depth, h - height of a hatch above deck [m], b - breadth of a hatch $(b \geq B/2)$ [m], l_h - length of hatches within 0.5 L

4. ANALYTICAL FORM OF STABILITY-RELATED CHARACTERISTICS OF A SHIP

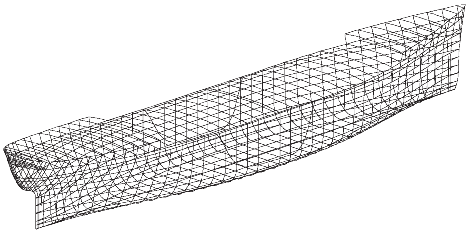
Analytical definition of the stability constraints needs the analytical definition of righting and heeling lever curves. For the reasons given earlier, the attention will focus on the righting lever curve only. This will be done by approximation or interpolation of a series of stability-related characteristics of a ship.

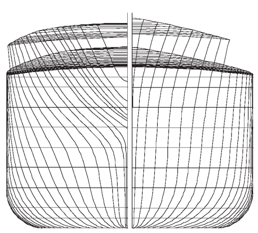
The geometrical data for the task were prepared in [1], based on the systematic calculations of cross curves for Series 60 (S60) [19] for three different block coefficients C_B and four h = H/T ratios. Three parent models of S60 had been chosen and then modified by: (i) extrapolating sections to obtain different h ratios (beyond the basic $h^0 = 1.50$), (ii) adding standard superstructures (a poop and forecastle), (iii) extracting the data concerning merely the design draught T (displacement), and (iv) making them dimensionless. The ratio $b = B/T = b^0 = 2.50$ was kept constant for all the models. Table 4.1 presents the resulting twelve models and, as an example, geometry of the M7072 model.

Table 4.2 shows a full list of ship characteristics that have to be defined analytically in order to form the stability constraints in the fleet optimisation model. All of them are dimensionless (related to T) and classified from the analytical point of view as one - (1D), two - (2D), three - (3D), and four-dimensional (4D) characteristics, taking into account a number of variables of corresponding interpolating or approximating functions.

Tab. 4.1. *The Series 60 hull models used for the approximation of cross curves and geometry of the M7072 model*

Series 60	**Series 60 Modified Models** (B/T = 2.50)				
Parent Models	**C_B \ H/T**	**1.610**	**1.726**	**1.813**	**1.900**
4210W	**0.60**	M6061	M6072	M6081	M6090
4212W	**0.70**	M7061	M7072	M7081	M7090
4214WB-4	**0.80**	M8061	M8072	M8081	M8090





Tab. 4.2. *Stability-related geometrical characteristics of a hull form and their analytical representation*

No	**Characteristics**	**Dimension**	**Generation process**	**Function**
1	Block coefficient	2D	Interpolation	$\delta = f_1(C_B, z)$
2	Vertical coordinate of the centroid B	2D	Interpolation	$z_B = f_2(C_B, z)$
3	Transversal metacentric radius	3D	Interpolation	$r_0 = f_3(C_B, b, z)$
4	Deck immersion angle	3D	Interpolation	$\phi_D = f_4(C_B, h, b)$
5	Cross curves in the (w-t) system	4D	Approximation	$w_B = f_5(\phi; C_B, h, b)$
6	Righting lever curve	4D	Approximation	$l_R = f_6(\phi, C_B, h, b; z_G)$
7	Righting lever curve	1D	Approximation	$\tilde{l}_R = f_7(\phi)$

Fig.4.1 presents three examples of 2D/3D geometrical characteristics of a hull form based on the S60 data defined by the interpolation method.

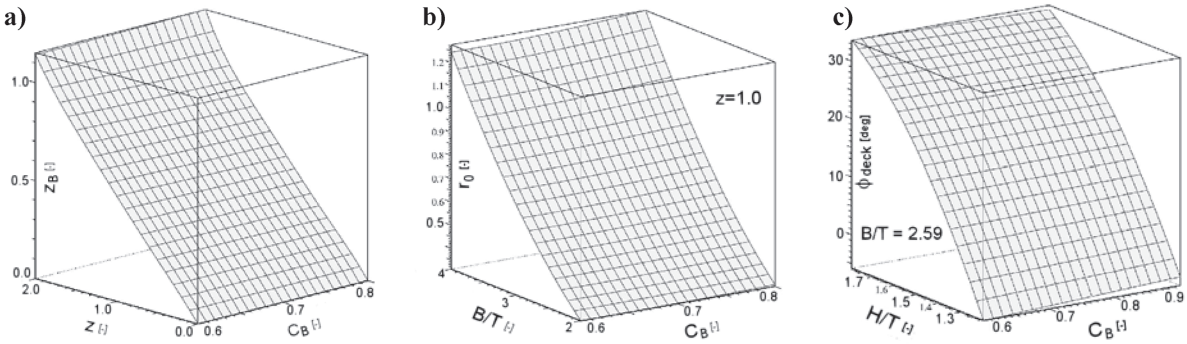


Fig. 4.1. *Graphs of analytically defined 2D / 3D geometrical characteristics of Series 60 by the interpolation method*

In the analytical definition of stability constraints, the basic characteristic is righting lever curve $l_R(\phi;\cdot)$.
A fleet optimisation model requires for the righting lever curve to be defined as the following 4D function :

$$l_R(\phi, C_B, h, b; z_G) = w_B(\phi, C_B, h, b) - z_G \cdot \sin\phi \tag{4.1}$$

The process of generation of the righting lever function $l_R(\phi;\cdot)$ (4.1) has been divided into five steps :

(i) Approximation of cross curves based on the twelve models of S60, resulting in a 3D function $w_B^0(\phi, C_B, h)$ for $b = b^0$ = const.

(ii) Correction of the $w_B^0(\cdot)$ function for the regions of small h (below the available S60 data), resulting in the $w_B^{cor}(\cdot)$ function for these special regions. This step was necessary to cover H/T ratios typical of sea-river vessels and expected when optimising a fleet of such ships.

(iii) Affine transformation of the function $w_B^0(\cdot) / w_B^{cor}(\cdot)$ with regard to $b \neq b^0$, resulting in a 4D function $w_B^1(\phi, C_B, h, b)$

(iv) Definition of the righting lever curve as a 4D function: $l_R(\phi, C_B, h, b) = w_B^1(\phi, C_B, h, b) - z_G \cdot \sin\phi$

(v) Approximation of the 4D function $l_R(\phi, C_B, h, b; z_G)$ by a 1D function (of ϕ only): $\tilde{l}_R(\phi; C_B, h, b, z_G)$ for C_B, h, b, z_G = const.

In all the steps listed above, the only problem that appears twice is that of approximation of multivariable (i) or single-variable (v) functions. The problem will be then addressed first as a separate numerical problem formulated in a compact matrix notation.

4.1. Approximation problem

Let $f(\mathbf{u})$ be a function to be approximated and $\tilde{f}(\mathbf{u})$ be its approximation, both assumed to be single valued, multi-variable functions of $\mathbf{u} = (u_1, u_2, ...u_M)$. The approximation problem is formulated as :

$$\rho(f, \tilde{f}) \to \min \tag{4.2}$$

where : $\rho(\cdot,\cdot)$ is a metric of approximation. Let the $f(\cdot)$ function be given in a discrete manner by the following sequence of data : $\{\mathbf{u}_k ; f_k\}_{k=1}^{N}$ and the $\tilde{f}(\cdot)$ function has the following linear representation :

$$\tilde{f}(\mathbf{u};\alpha) = \sum_{i=1}^{n} \alpha_i \cdot e_i(\mathbf{u}) = \alpha^T \cdot \mathbf{e}(\mathbf{u}) \quad , \quad n < N \tag{4.3}$$

where : $\alpha = (\alpha_1, \alpha_2, ...\alpha_n)^T$ a vector of unknown coefficients, $\mathbf{e}(\mathbf{u}) = [e_1(\mathbf{u}), e_2(\mathbf{u}), ...e_n(\mathbf{u})]^T$ - a basis.

For given arguments one has: $\tilde{f}_k = \tilde{f}(\mathbf{u}_k, \alpha), \ k = 1,2,..., N$ and $\tilde{\mathbf{f}} = (\tilde{f}_1, \tilde{f}_2, ..., \tilde{f}_N)^T$

Using the representation (4.3), one gets the following equation :

$$\tilde{\mathbf{f}} = \mathbf{E}^T \cdot \alpha \tag{4.4}$$

A matrix $\mathbf{E} = [E_{ij}] = [e_i(\mathbf{u}_j)]_{nxN}$ will be later called the *basis matrix*. As the metric of approximation $\rho(\cdot\,,\,\cdot)$ a weighted norm $\|\cdot\|_{2w}$ of an Euclidean distance between f and $\tilde{f}$ has been chosen, then :

$$\rho(f, \tilde{f}) = \| f - \tilde{f} \|_{2w} \tag{4.5}$$

In consequence, the approximation problem (4.2) can be formulated as the following quadratic optimisation problem :

$$\min Q(\alpha) \ , \ Q : \mathbf{R}^n \to \mathbf{R}^1 \tag{4.6a}$$

with the objective function :

$$Q(\alpha) = \| f - \tilde{f} \|_{2w}^2 = (\mathbf{f} - \mathbf{E}^T \cdot \alpha)^T \cdot \mathbf{W} \cdot (\mathbf{f} - \mathbf{E}^T \cdot \alpha) \tag{4.6b}$$

where : $\mathbf{W} = \mathrm{diag}(\mathbf{w})$, $\mathbf{w} = (w_1, w_2, ..., w_n)^T$ – a vector of weighting factors ($w_i \geq 0, \ i = 1,2,...., N$).

Now let us assume that some linear interpolatory constraints are imposed on the $\tilde{f}(\cdot)$ function :

$$I_\mu(\tilde{f}) = c_\mu \ , \ \mu = 1,2,....,m < n = \dim \alpha \tag{4.7}$$

where $I_\mu(\cdot)$ is a functional of $\tilde{f}(\cdot)$ in the general form :

$$I_\mu(\tilde{f}) = L_\mu(\tilde{f})(\mathbf{u}_\mu ; \boldsymbol{\alpha}) \quad , \quad \mu = 1,2,....,m \tag{4.8a}$$

and $L_\mu(\cdot)$ is a linear operator. For instance, $L_\mu(\cdot)$ can be a differential operator, as below :

$$L_\mu(\tilde{f}) = \frac{\partial^{\lambda_\mu} \tilde{f}(\cdot)}{\partial u_{i_\mu}^{\lambda_\mu}} \quad , \quad \lambda_\mu = 0,1,2..., \quad i_\mu = 1,2,3,.... \tag{4.8b}$$

where the sequences of indices: $\{\lambda_\mu\}$, $\{i_\mu\}$, $\mu = 1,2,...., m$ have to be defined separately.

Let $\mathbf{I}(\tilde{f}) = (I_1(\tilde{f}), I_2(\tilde{f}), ..., I_m(\tilde{f}))$ be a vector of functionals of $\tilde{f}(\cdot)$.

The interpolatory constraints can then be written as the following matrix equation :

$$\mathbf{I}(\tilde{f})(\alpha) = \mathbf{c}^T \tag{4.9}$$

where : $\mathbf{c} = (c_1, c_2, ...c_m)^T$ is a vector of given values of interpolatory constraints. Thanks to the linearity of $I(\cdot)$, one can write :

$$\mathbf{I}(\tilde{\mathbf{f}}) = \mathbf{I}(\alpha^T \cdot \mathbf{e}(\mathbf{u})) = \boldsymbol{\alpha}^T \cdot \mathbf{I}(\mathbf{e}(\mathbf{u})) = \boldsymbol{\alpha}^T \cdot \mathbf{C}^T \tag{4.10}$$

A matrix $\mathbf{C} = [c_{\mu\nu}] = [L_\mu(e_\nu)(\mathbf{u}_\mu)]$ will be later called the *constraint matrix*.

Putting (4.10) into (4.9), one obtains the interpolatory constraints as the following matrix equation :

$$\mathbf{H}(\alpha) \equiv \mathbf{C} \cdot \boldsymbol{\alpha} - \mathbf{c} = \mathbf{0} \tag{4.11}$$

The approximation problem with the additional interpolatory constraints can now be formulated as the following quadratic optimisation problem with constraints :

$$\begin{cases} \min Q(\alpha) \equiv (\mathbf{f} - \mathbf{E}^T \cdot \boldsymbol{\alpha})^T \cdot \mathbf{W} \cdot (\mathbf{f} - \mathbf{E}^T \cdot \alpha) \\ \alpha \in \Omega = \{\alpha : \mathbf{H}(\boldsymbol{\alpha}) = \mathbf{0}, \quad \dim \mathbf{0} = m < n\} \subset \mathbf{R}^n \end{cases} \tag{4.12}$$

It has been proven [3], that both problems (4.6) and (4.12), as the convex problems, do have unique solutions. Moreover, in both cases the solutions can be found analytically. In particular, for the problem (4.12), this can be done either by Lagrange's multipliers or by the penalty function method. The latter approach has been applied in the case. Let the penalty function $P(\cdot)$ be defined as a square of the norm $\|\mathbf{H}(\alpha)\|_{2\omega}$ in the form :

$$P(\boldsymbol{\alpha}) = \mathbf{H}^T(\boldsymbol{\alpha}) \cdot \omega \cdot \mathbf{H}(\alpha) \tag{4.13}$$

where: $\boldsymbol{\omega}$ = diag(ω), ω = ($\omega_1, \omega_2, ..., \omega_n$) - a vector of weighting factors for the penalty function, $\omega_i \geq 0.,\ i = 1,2,....,m$.
The optimisation problem with constraints (4.12) can now be replaced by the equivalent optimisation problem without constraints, with the modified objective function F(·) :

$$\begin{cases} \min F(\boldsymbol{\alpha}) \equiv Q(\boldsymbol{\alpha}) + P(\boldsymbol{\alpha}) \ , \ P : \mathbf{R}^n \to \mathbf{R}^1 \\ \boldsymbol{\alpha} \in \mathbf{R}^n \end{cases} \tag{4.14}$$

The solution to the problem (4.14) follows from the necessary condition of a stationary point in $\mathbf{R}^n$:

$$\nabla_\alpha F(\boldsymbol{\alpha}) = \mathbf{0} \quad \Leftrightarrow \quad \nabla_\alpha Q(\alpha) + \nabla_\alpha P(\alpha) = \mathbf{0} \tag{4.15}$$

Finally, one arrives at the equation :

$$\underbrace{(\mathbf{E} \cdot \mathbf{W} \cdot \mathbf{E}^T + \mathbf{C}^T \cdot \boldsymbol{\omega} \cdot \mathbf{C})}_{\mathbf{B}} \cdot \boldsymbol{\alpha} - \underbrace{(\mathbf{E} \cdot \mathbf{W} \cdot \mathbf{f} + \mathbf{C}^T \cdot \boldsymbol{\omega} \cdot \mathbf{g})}_{\mathbf{b}} = \mathbf{0} \tag{4.16}$$

or, in short :

$$\mathbf{B} \cdot \boldsymbol{\alpha} - \mathbf{b} = \mathbf{0} \tag{4.17}$$

Existence of the solution to the problem (4.12) assures that the matrix **B** is non-singular (det **B** ≠ 0).
So one eventually gets the desired definition of the approximating function $\tilde{f}(\cdot)$:

$$\boldsymbol{\alpha} = \mathbf{B}^{-1} \cdot \mathbf{b} \tag{4.18}$$

One can observe that: (i) the solution α is parametrized by the weighting coefficients: α = α(w;ω) and (ii) putting ω = 0, one gets the solution of an unconstrained problem (4.6a).

From the numerical point of view, solution α to the approximation problem (4.2) can be found either by solving the system of linear equations (4.17) or by solving a single matrix equation (4.18) (inverting B). The first approach has been applied in approximation of cross curves (Section 4.2) and the second one in approximation of righting lever curve (Section 4.4).

*4.2. **Approximation of cross curves***

In this case the approximated function is a cross curves function ($f = w_B^0$) determined in a discrete way for the twelve S60 models listed in Tab. 4.1. Given 11 ordinates of the function per one model, one gets N = 132 ordinates to be approximated. The following polynomial three-linear form has been chosen, as a particular representation of the approximating function $\tilde{f}(\cdot)$.

$$\tilde{f}(\mathbf{u};\boldsymbol{\alpha}) = \boldsymbol{\alpha}^T \cdot \mathbf{e}(\mathbf{u}) = \sum_{l=1}^{n} \alpha_l \cdot e_l(\mathbf{u}) = \sum_{\mu=1}^{n} \alpha_\mu \cdot e_\mu(\phi, C_B, h) = \sum_{\mu=1}^{n} \alpha_\mu \cdot \underbrace{(\phi^{i_\mu} \cdot h^{j_\mu} \cdot C_B{}^{k_\mu})}_{e_\mu(\mathbf{u})} \equiv \qquad (a)$$

$$\equiv \sum_{i=0}^{I} \sum_{j=0}^{J} \sum_{k=0}^{K} a_{i,j,k} \cdot \phi^i \cdot h^j \cdot C_B{}^k \qquad (b) \tag{4.19}$$

The equivalency of the two notations introduced follows from the correspondence of both indications. After some trials and errors, the upper limits of the indices in (4.19b) have been established as follows : $I = 5, J = 2, K = 2$. It gives the following dimension of the basis $\mathbf{e}(\cdot)$ in (4.3) : $\dim \mathbf{e}(\cdot) = n = (I+1) \cdot (J+1) \cdot (K+1) = 54$, so the condition n < N holds. The basis matrix **E** (4.4) can now be readily calculated according to the definition :

$$\mathbf{E} = [e_\mu(\mathbf{u}_\nu)] = \left[(\phi^{i_\mu})_\nu \cdot (h^{j_\mu})_\nu \cdot (C_B^{k_\mu})_\nu\right] \quad \text{for} \quad \begin{cases} \mu = 1,2,....,n \\ \nu = 1,2,....,N \end{cases} \tag{4.20}$$

The constraint imposed on the approximated function $w_B^0(\phi, \cdot)$ follows from the known property of cross curves :

$$\frac{d}{d\phi} w_B(\phi)\Big|_{\phi=0} = z_B + r_0 \equiv z_M \tag{4.21}$$

It imposes the following interpolatory constraints on the approximating function $\tilde{f}(\cdot)$:

$$\frac{\partial}{\partial \phi} \tilde{f}(\phi\,; C_B, h)\Big|_{\phi=0} = z_B(C_B) + r_0(C_B) \tag{4.22}$$

Eq. (4.22) should hold for all the models given, so the number of imposed constraints is m = 12.
The determination of the weighting coefficient matrices **W** and ω in (4.12 and 4.13) was done by the „trial and error" method.

The best results obtained are as follows :

$$w_i = \begin{cases} 1000.0 & \text{for} \quad \text{i corresponding to } \phi = 0 \\ 1.0 & - \quad \text{otherwise} \end{cases} \qquad \omega_i = 1 \ \text{ for } \ i = 1,2,...,m \tag{4.23}$$

Results of the approximation of cross curves $w_B^0(\cdot)$ for the S60 models (Tab.4.1) is shown in Fig.4.2.

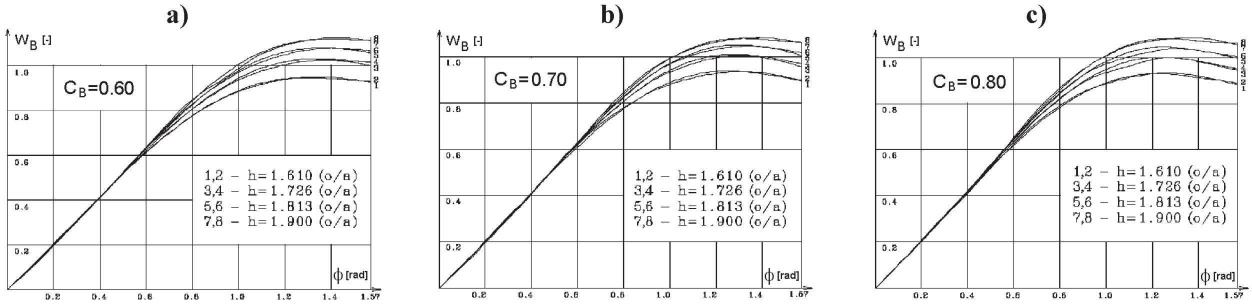
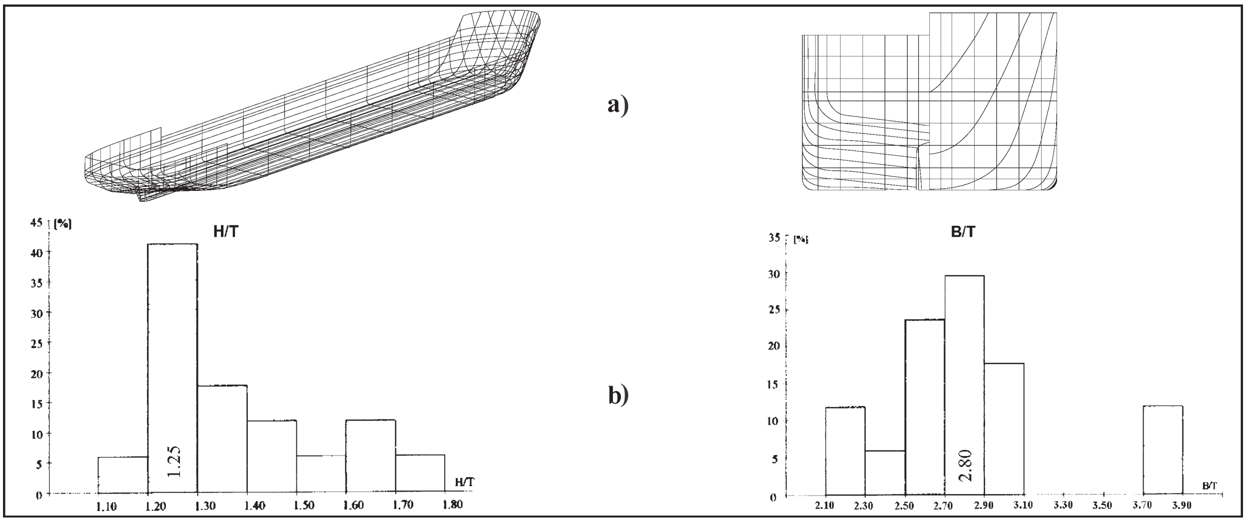


Fig. 4.2. *Comparison of the original (o) and approximated (a) cross curves of S60*

4.3. ***Correction of the cross curves function w_B^0 for small H/T ratios***

The function $w_B^0(\phi, C_B, h)$ approximating the cross curves of the S60 data is formally valid for the form parameters from the region : $(C_B, h) \in [0.60, 0.80] \times [1.61, 1.90]$, b = b^0 = 2.5. Such a range of data evidently corresponds to the standard hull form parameters of sea-going ships but does not cover the regions typical of the sea-river vessels where the block coefficient C_B reaches values as large as 0.90 and h as small as 1.1 (Tab.4.3).

Tab. 4.3. *Hull form of a sea-river vessel [18]. a - body lines, b - a sample of statistics [13]*



It turned out that a natural extrapolation of the cross curves function (4.19) obtained for S60 towards large C_B and small h=H/T is acceptable as far as C_B and not acceptable as far as h is concerned. Fig.4.3 illustrates the problem. So a special extrapolation was necessary for the function w_B^0 to be used in an optimisation model of sea-river ships. As a result, a corrected cross curves 2D function $w_B^{cor}(\phi, h)$ has been defined (by an interpolation technique), based on the boundary data of S60 (for h = 1.61) and a cross curve data for the SINE-205 sea-river vessel [18] (for h = 1.239) in the following tensor product form :

$$w_B^{cor}(\phi, h) = \mathbf{d}^T(\phi) \cdot \mathbf{g}(h) = \mathbf{e}^T(\phi) \cdot \mathbf{D} \cdot \mathbf{g}(h) \quad (4.24)$$

where : **d**(ϕ) – interpolatory characteristics, **D** - interpolatory matrix, **e**(ϕ), **g**(ϕ) – bases, dim(**d**) = dim(**g**) = 3, dim(**e**) = 5. For details - see [13].

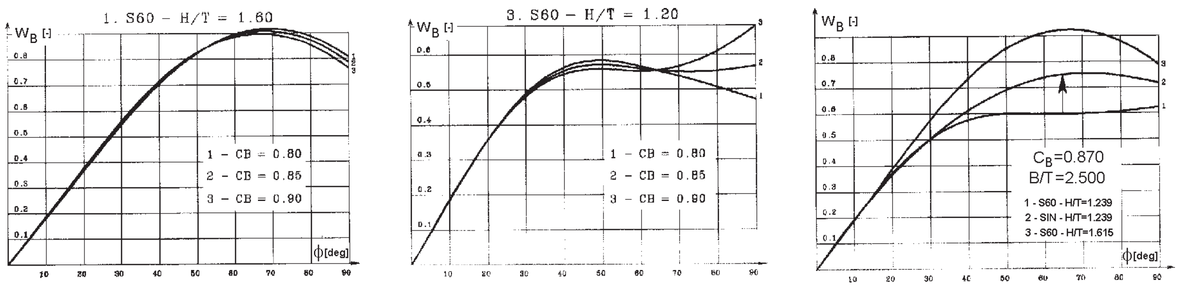


Fig. 4.3. *A natural extrapolation of the cross curves function $w_B^0(\cdot)$ of S60 towards the region of small H/T ratio*

After correction, a definition of the basic (B/T = b^0 = 2.5) cross curves function is :

$$w_B^0(\phi, C_B, h) = \begin{cases} w_B^0(\phi, C_B, h) & \text{for} \quad h \geq 1.610 \\ w_B^{cor}(\phi, h) & \text{for} \quad h < 1.610 \end{cases} \quad (4.25)$$

Fig.4.4 show comparison of the basic cross curves functions before and after correction with regard to the small H/T ratio.

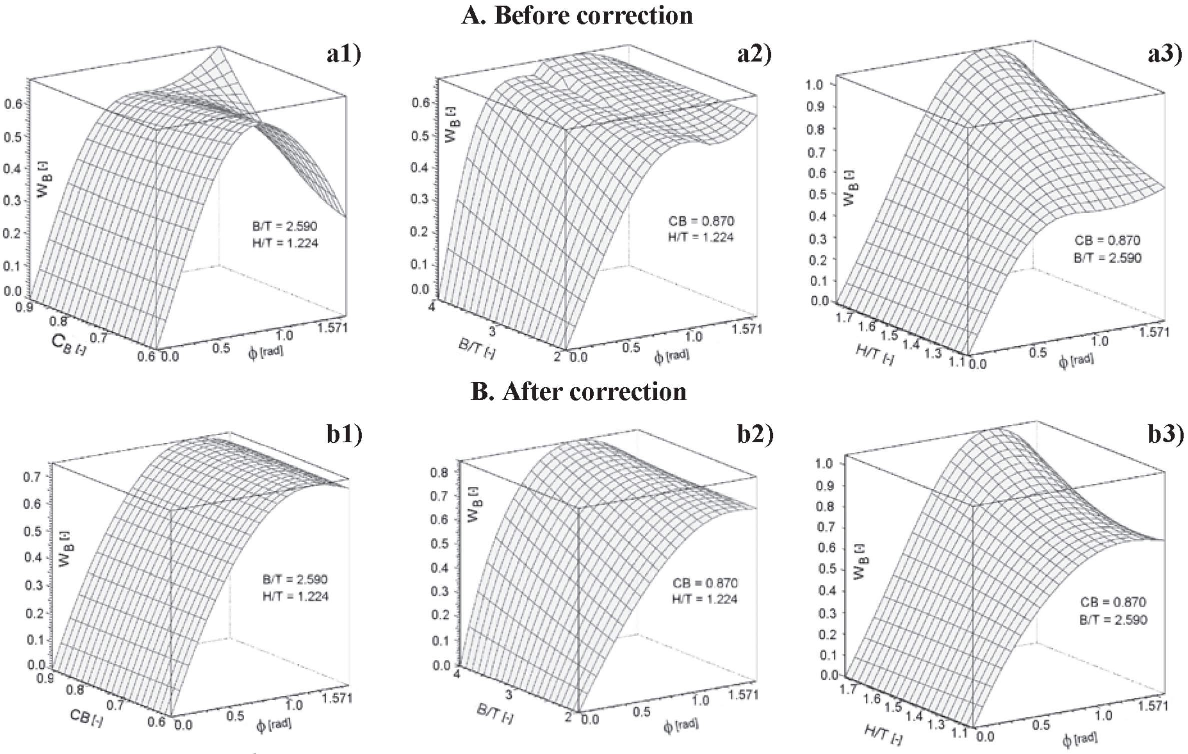
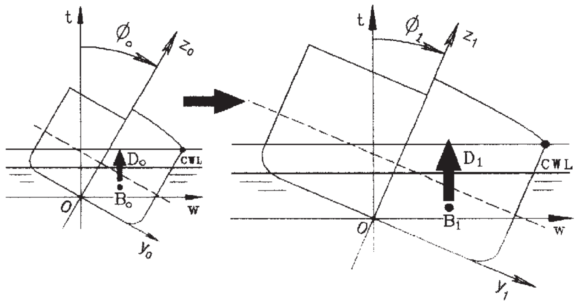


Fig. 4.4. *Basic (b = b^0=2.5) cross curves function before and after correction with regard to the small H/T ratio*

4.4. Affine transformation of the cross curves function $\mathbf{w_B^0(\cdot)}$ *with regard to B/T*

The definition of cross curves function $w_B^0(\cdot)$, obtained as the result of approximation of the Series 60 data and then corrected for small H/T ratio is only valid for B/T = b^0 = 2.5. Accounting for a new B/T ratio = $b^1 \neq b^0$ can be done by the way of affine transformation of a hull form. The situation is illustrated in Fig.4.5. A system of hull coordinates is transformed when the hull is heeled. As a result, for *equivalent* waterlines (e g. CWL or deck immersion waterline), one obtains, among other things, a changed angle of heel ($\phi_0 \rightarrow \phi_1$), changed coordinates of B ($B_0 \rightarrow B_1$) and, in consequence, the desired transformation of cross curves $w_B^0(\phi, C_B, h\ ;\ b = b^0) \rightarrow w_B^1(\phi, C_B, h, b)$.



$$\begin{bmatrix} y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \lambda_b & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} y_0 \\ z_0 \end{bmatrix} \qquad \lambda_b = b^1 / b^0$$

Fig. 4.5. *Affine transformation of coordinate system*

In order to derive a particular form of this transformation,
let us recall the inverse relation between the (y-z) and (w-t) systems (3.1b) :

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} w \\ t \end{bmatrix} \tag{4.26}$$

Using the formula (4.26) for the points B_0 and B_1 one has accordingly
the following (w-t) $\rightarrow$ (y-z) transformation of coordinates :

$$\begin{bmatrix} y_B^0 \\ z_B^0 \end{bmatrix} = \begin{bmatrix} \cos\phi_0 & -\sin\phi_0 \\ \sin\phi_0 & \cos\phi_0 \end{bmatrix} \cdot \begin{bmatrix} w_B^0 \\ t_B^0 \end{bmatrix} \tag{4.27a}$$

$$w_B^1 = y_B^1 \cdot \cos\phi_1 + z_B^1 \cdot \sin\phi_1 = (\lambda_b \cdot y_B^0) \cdot \cos\phi_1 + z_B^0 \cdot \sin\phi_1 \tag{4.27b}$$

After substitution of (4.27a) into (4.27b) one arrives at the formula :

$$w_B^1 = \cos\phi_0 \cdot \cos\phi_1 \cdot \left[w_B^0 \cdot (\lambda_b + tg\phi_0 \cdot tg\phi_1) + t_B^0 \cdot (-\lambda_b \cdot tg\phi_0 + tg\phi_1) \right] \tag{4.28}$$

Further algebraic manipulations lead to the final transformation function (the indices 1 were dropped) :

$$w_B^1(\phi, \lambda_b) = \begin{bmatrix} f_w(\phi, \lambda_b) \\ \hline f_t(\phi, \lambda_b) \end{bmatrix}^T \cdot \begin{bmatrix} w_B^0[\phi_0(\phi)] \\ t_B^0[\phi_0(\phi)] \end{bmatrix} \tag{4.29}$$

where the auxiliary functions involved in (4.29) are :

$$\begin{cases} f_w(\phi, \lambda_b) & = & \lambda_b \cdot \tau(\phi, \lambda_b) & \text{(a)} \\ f_t(\phi, \lambda_b) & = & 0.5 \cdot (1 - \lambda_b^2) \cdot \sin 2\phi \cdot \tau(\phi, \lambda_b) & \text{(b)} \\ \tau(\phi, \lambda_b) & = & \dfrac{1}{\sqrt{\cos^2\phi + \lambda_b^2 \cdot \sin^2\phi}} & \text{(c)} \\ t_B^0(\phi, \lambda_b) & = & z_B^0 - \displaystyle\int_0^{\phi_0(\phi)} w_B^0(\varphi) d\varphi & \text{(d)} \\ \phi_0(\phi, \lambda_b) & = & \operatorname{arc\,tg}(\lambda_b \cdot tg\phi) & \text{(e)} \end{cases} \tag{4.30}$$

Fig. 4.6 presents graphs of a heeling angle function (4.30e) for different $\lambda_b \in [0.25\ ,\ 2.00]$ and the functions $w_B^1(\cdot)$ (4.29) and $t_B^0(\cdot)$ (4.30 d).

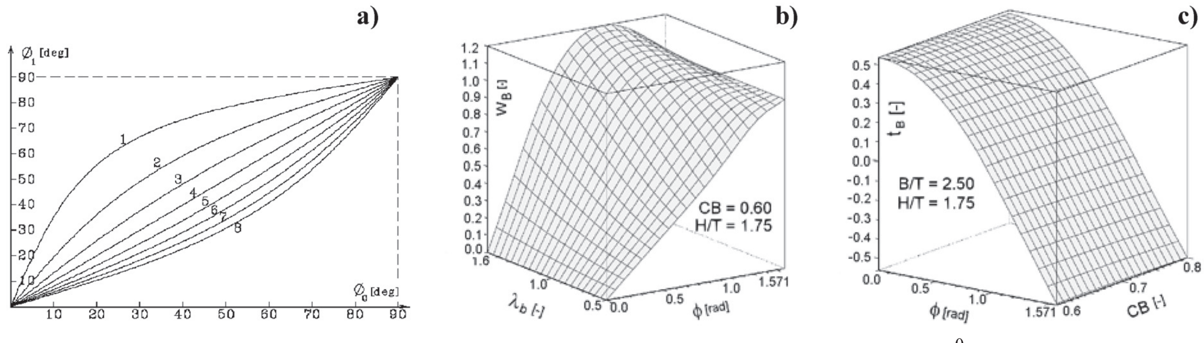


Fig. 4.6. *Affine transformation of cross curves with regard to the B/T ratio* ($\lambda_b = b/b^0$)

4.5. *Analytical definition of the righting lever curve* $l_R(\cdot)$

An analytical definition of the righting lever function for the hull form parameters $\mathbf{g} = (C_B\ , h\ , b)$:

$$l_R(\phi, \mathbf{g}; z_G) = w_B(\phi, \mathbf{g}) - z_G \cdot \sin\phi \tag{4.31}$$

can instantly be obtained based on the analytical definition of the cross curves function (4.29) :

$$w_B(\phi, \mathbf{g}) \equiv w_B^2(\phi, \mathbf{g}) = w_B^1(\phi, C_B, h) \cdot f_w(\phi, b) + t_B^1(\phi, C_B, h) \cdot f_t(\phi, b) \tag{4.32}$$

When looking at the formulae above it becomes obvious that the righting lever function so defined would be too complicated for performing various analytical operations (with regard to ϕ) when generating the stability constraints. In order to cope with this problem, a 4D function $l_R(\phi, \mathbf{g}; z_G)$ has been approximated by an 1D function $\tilde{l}_R(\phi; \mathbf{g}, z_G)$ (for $\mathbf{g}$, z_G = const.) using the known solution to the approximation problem (4.18). The idea is based on the observation that the matrix **B** depends only on the given nodes (matrices **E**, **C**), the types of constraints (matrix **C**), the weighting factors (matrices **W**, $\boldsymbol{\omega}$) and does not depend on ordinates of the approximated function or values of constraints (**c**). It follows from the above that for all the mentioned elements kept fixed, the matrix **B** and its inversion $\mathbf{B}^{-1}$ can be generated once and be used many times for different combinations of the **g** and z_G parameters, when running the optimisation procedure. Bearing in mind the need for simplicity, a linear polynomial representation has been assumed for the approximating function :

$$\tilde{f}(\mathbf{u}; \boldsymbol{\alpha}) = \boldsymbol{\alpha}^T \cdot \mathbf{e}(u) = \sum_{i=1}^{n} \alpha_i \cdot e_i(\mathbf{u}) \quad \Leftrightarrow \quad \tilde{l}_R(\phi; \boldsymbol{\alpha}) = \sum_{i=1}^{6} \alpha_i \cdot \phi^{i-1} \tag{4.33}$$

where : $\alpha = (\alpha_1, \alpha_2, ...\alpha_6)^T$ - is a solution of the approximation problem and the definition of $\tilde{l}_R(\cdot)$. Note, that $\alpha = \alpha(\mathbf{g}, z_G)$.

In analogy to approximation of cross curves, the only constraint (m = 1) imposed on the lever righting function is :

$$\frac{d}{d\phi}(\tilde{l}_R)(\phi = 0) = z_B(C_B) + r_0(C_B, b) - z_G \tag{4.34}$$

The resulting righting lever function has the desired simple form :

$$\tilde{l}_R = \tilde{l}_R(\phi; C_B, h, b; z_G) = \sum_{i=1}^{6} \alpha_i(C_B, h, b; z_G) \cdot \phi^{i-1} \tag{4.35}$$

Fig. 4.7 presents some diagrams of the righting lever curve as a 4D function $l_R(\phi, C_B, h, b; z_G)$ according to the definition (4.31).

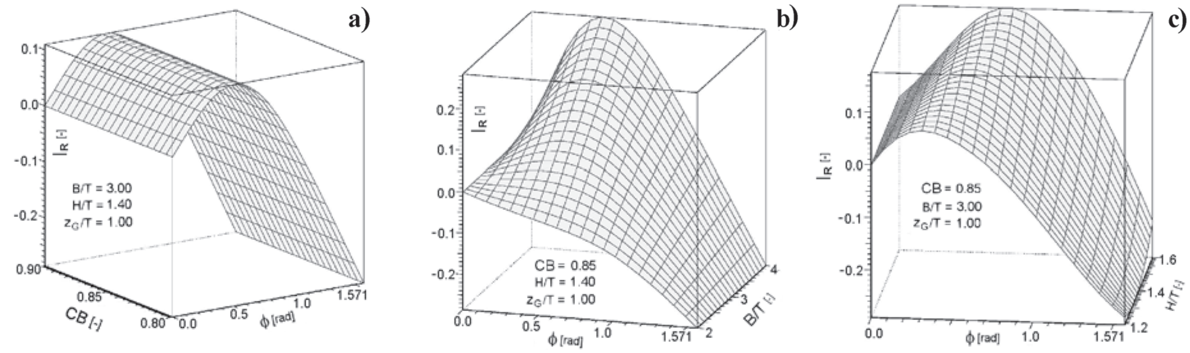


Fig. 4.7. *Righting lever curve as a multivariable function of ϕ and hull form parameters of a ship*

5. STABILITY CONSTRAINTS IN POST-OPTIMISATION STUDIES

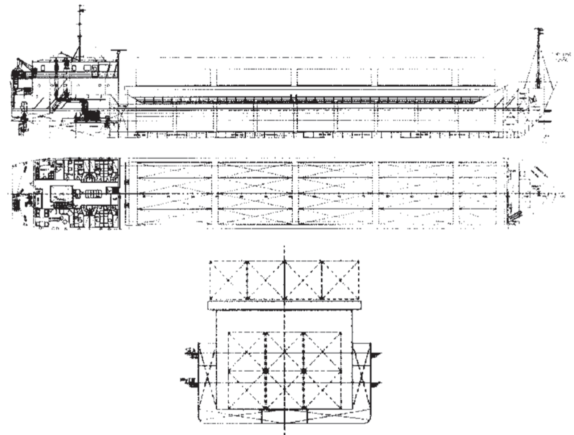
An example of post-optimisation stability-oriented study is presented on the background of the optimum solution of the fleet of ships optimisation problem. To perform the calculations, the optimisation model of a fleet has been implemented in the EUROS computer program [13].

5.1. *The data*

A typical sea-river vessel and a sample of the main fleet transportation data are shown in Table 5.1.

Tab. 5.1. *Fleet transportation problem :*

Typical sea-river vessel for short shipping in the North and Baltic Seas [18] and main transportation data [13]

Sea-river vessel	A sample of main transportation data :
	Period of operation : N = 15 years No. of ports : OUT / INP / total = 10 / 11 / 20 No. of river ports = 12 Mass of cargo OUT = 110 000.0 tons / year Mass of cargo IN = 103 000.0 tons / year Distance : OUT / IN / total: 2827 / 2959 / 5786 [NM] Max. draught / velocity in river : 2.8 m/ 14 km/h Period of calling at ports : 6 - 14 days General cargo stowage factor : 1.0 - 2.0 [m^3/t] Relative quotas of cargo type : qcc / qgc /qbc OUT = 0.8 / 0.2 / 0.0 [-] qcc /qgc / qbc IN = 0.9 / 0.1 / 0.0 [-] qcc - containerised c., qgc - general c., qbc - bulk c.

Sea-river ships, as multi-purpose vessels, carry different kind of commodities, such as containers, general cargo and bulk cargo. In the transportation data, these are accounted for by fixing relative quotas of cargo types for the whole period of fleet operation.

Referring to the terminology introduced in Table 2.2a, the overall quantities of numerical data used in the model (including those in Tab.5.1), in the four categories of constants, are: VESSEL - 154, CARGO 10, ENVIRONMENT - 374, OPERATION - 326. This sums up to 864 constants in total.

5.2. *The model and NPM algorithms*

The implemented model for fleet optimisation problem contains 12 decision variables (Tab.5.2), 70 constraints in total (including 26 stability constraints) and 3 alternative objective functions: Net Present Value (NPV), Internal Rate of Return (IRR) and Required Freight Rate (RFR). As a non-linear programming method of optimisation, a combined, double level, algorithm for constrained problems has been applied, including a penalty function shifting method by Powell and Wierzbicki, for the constrained part, and Rosebrock's method for the unconstrained part of the problem ([3],[15]).

5.3. *Numerical results*

Numerical examples of optimisation are presented in Table 5.2. The calculations have been carried out separately for three criteria of stability: IMO, PRS, and HSMB with the same data, the same objective function: (NPV) and the same starting point $\mathbf{x}^0$. The optimisation process has been repeated twice : the results of the first optimisation were used as a starting point to the second optimisation and then convergence was reached. It turned out that, from the viewpoint of the final value of the objective function, such an approach improved the results remarkably. Apart from the NPV, the results in Table 5.2 show the corresponding values of two remaining measures of merit: IRR and RFR. It can be seen from Table 5.2 that the results for the stability criteria by IMO and HSMB are identical. This will become obvious when one analyses the status of the stability constraints with regard to the feasible solutions region $\mathbf{\Omega}$.

5.4. *Feasibility analysis - the case study*

Feasibility analysis is an element of post-optimisation studies (together with parametric study and sensitivity analysis) that aims at investigating the status of a certain group of constraints (in the case - the stability constraints) versus boundary

Tab. 5.2. *Results of optimisation of a fleet of multipurpose 'sea-river' ships for three intact stability criteria. Objective function: NPV*

x	Description	Unit	Start	Intact Stability Criteria		
				IMO	PRS	HSMB
x_1	Number of ships in a fleet	[-]	4.000	3.927	4.433	3.927
x_2	Deadweight of a ship	[t]	2941.000	3178.873	3112.931	3178.873
x_3	Speed of a ship at sea	[kn]	12.000	11.075	11.059	11.075
x_4	Total container capacity of a ship	[TEU]	90.000	117.499	110.336	117.499
x_5	Length b.p. of a ship (L_{PP})	[m]	87.470	99.106	97.592	99.106
x_6	Breadth of a ship (B)	[m]	11.400	13.422	12.996	13.422
x_7	Draught of a ship (T)	[m]	4.400	4.252	4.170	4.252
x_8	Depth of a ship (H)	[m]	5.450	7.010	6.833	7.010
x_9	Block coefficient (C_B)	[-]	0.874	0.796	.827	0.796
x_{10}	No of container columns under deck	[-]	9.000	11.296	11.257	11.296
x_{11}	No of container rows under deck	[-]	3.000	4.345	3.000	4.345
x_{12}	No of container tiers under deck	[-]	2.000	2.302	2.259	2.302
NPVI	Net Present Value / Investment Cost	[%]	-10.000	28.800	18.400	28.800
IRR	Internal Rate of Return	[%]	8.23	14.540	12.934	14.540
RFR	Required Freight Rate	[$/t]	121.49	102.937	111.784	102.937

Tab. 5.3. *List of stability constraints*

No	Criterion	Type	Unit	Institution	Constraints' index in the model	
					container ship	general cargo ship
1	GM0 at river	min	[m]	-	45	-
2	GM0 at sea	min	[m]	IMO, PRS	46	59
3	GZ (30°)	min	[m]	HSMB	47	60
4	Max GZ	min	[m]	IMO, PRS, HSMB	48	61
5	Max GZ angle	min	[deg]	IMO, PRS	49	62
6	GZ range	min	[deg]	PRS	50	63
7	Critical heeling lever	min	[m]	PRS	51	64
8	Weather criterion	min	mrad	IMO	52	65
9	Turning heel angle	max	[deg]	PRS	53	-
10	Static heel angle	max	[deg]	IMO	54	66
11	Area under GZ [0, 30]	min	[mrad]	IMO, HSMB	55	67
12	Area under GZ [0, 40]	min	[mrad]	IMO, HSMB	56	68
13	Area under GZ [30, 40]	min	[mrad]	IMO, HSMB	57	69
14	Area under GZ [30, range]	min	[mrad]	HSMB	58	70

constraints on the background of the remaining constraints, significant in a vicinity of the optimum solution. It can also be thought of as a preparatory, qualitative part of sensitivity analysis. Table 5.3 presents a list of stability constraints together with their indices in the optimisation model implemented in the EUROS program.

Sea-river ships, as multipurpose craft, operate in their life as different types of vessels, both from the viewpoint of their functionality and the corresponding stability regulations. To cope with the problem three extreme situations have been recognised in the model as crucial ones :

(i) A ship operates as a „pure" container sea-going vessel. She then carries containers both in holds and on deck and is allowed to take water ballast into the double bottom and wing tanks.

(ii) A ship operates as a „pure" general cargo sea-going vessel. As such, she does not load cargo on deck but, on the other hand, she is not allowed to take water ballast into her tanks.

(iii) A ship operates as a „pure" container, river vessel. She then carriers containers both in holds and on deck but, as the total mass of ship must be smaller than that at sea due to restricted draught, the ship is not allowed to take a water ballast.

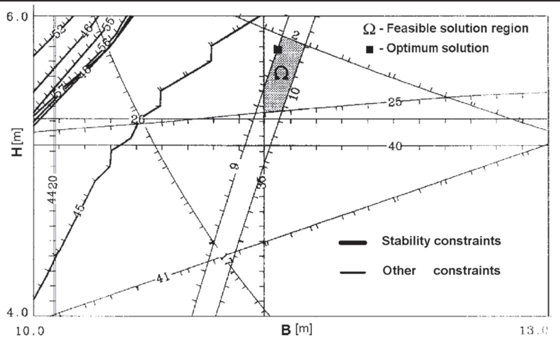
All the three situations have been accounted for by formulation of a triple set of corresponding stability constraints „acting" simultaneously. Such an approach assures that stability requirements will also be satisfied in all the intermediate loading conditions which can occur in operation. A feasibility analysis has been done in a graphical form using the method and software worked out in [9]. The results are presented in Tab. 5.4. The constraints are presented via visualisation of their contour lines in the vicinity of the optimum solution. The arguments of the graphs are ship's dimensions: B vs. H (1) and hull form ratios: B/T vs. H/T (2), the most essential ship parameters as far as the intact stability is concerned. The description in Tables 5.4 refers to the terminology introduced in Chapter 2. The stability constraints recognise three potential functional ship categories : a sea-going container carrier, a sea-going general-cargo ship and a container river ship. The feasible solution region **Ω** is shown as a darkened area and the „tufts" on the contour lines point in the unfeasible direction. The parameters of a fleet, which are not arguments of the graphs, are those in Table 5.2. The feasibility analysis is summarised in Table 5.5 where the attributes of stability constraints refer to those in Tab. 2.3.

It follows from Tab.5.5 that in the case under investigation the most restrictive stability criteria are those by PRS, where the criterion No. 50 (GZ range) appears as a boundary and active constraint.

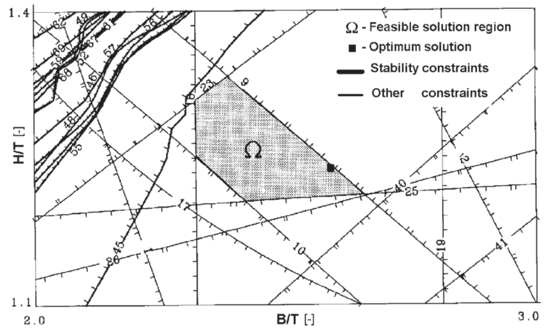
Tab. 5.4. *Feasibility analysis - the case study*

IMO

1



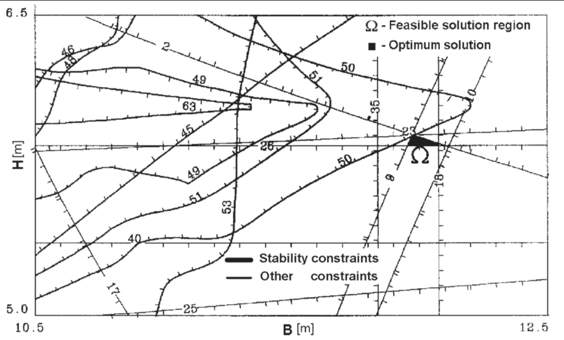
2



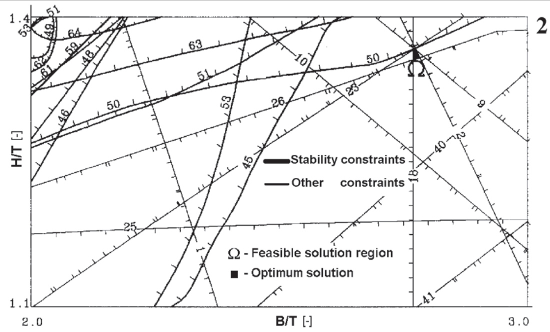
Boundary constraints (1)	Stability constraints (1)	Boundary constraints (2)	Stability constraints (2)
9 2, 10, 25, 35	**cs**: 46, 48, 52,55,56, 57 **gs**: - **cr**: 45	9 10, 18, 23, 25	**cs**: 46, 48, 49, 52, 56, 57 **gs**: 59, 62, 67, 68, 69, 81; **cr**: 45

PRS

1



2

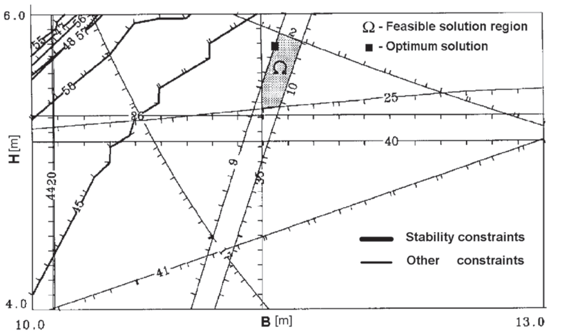


2

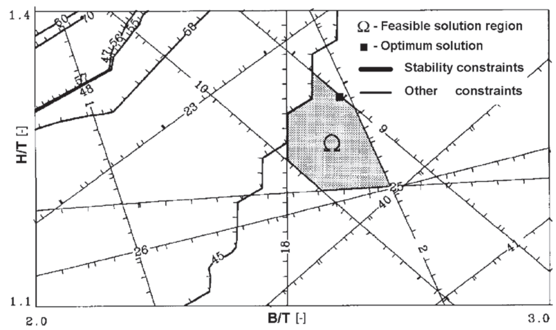
Boundary constraints (1)	Stability constraints (1)	Boundary constraints (2)	Stability constraints (2)
2, 9, **50** 23,26	**cs**: 46, 48, 49, **50**, 51,53, 63 **gs**: - **cr**: 45	2, 9, **50** 23, 26	**cs**: 46, 48, 49, **50**, 51, 53 **gs**:59, 61, 62, 64, 63 **cr**: 45

HSMB

1



2



Boundary constraints (1)	Stability constraints (1)	Boundary constraints (2)	Stability constraints (2)
9 2, 10, 25, 35	**cs**: 47, 48, 55,56,57 **gs**: - **cr**: 45	9 2, 10, 18, 25, **45**	**cs**: 47, 48, 55, 56, 57, 58 **gs**: 60, 70 **cr**: **45**

Legend : a - active constraint, p - passive constraint
cs - container ship at sea, gs - general cargo ship at sea, cr - container ship in river

Boundary constraints :

Annual mass of cargo (INP)	[t]	(min)	Vertical container clearance	[m]	(max)
Mass reserve factor	[-]	(min)	Freeboard	[m]	(min)
Mass reserve factor	[-]	(max)	Height of bow	[m]	(min)
Length of forepeak ratio	[-]	(min)	L/B ratio	[-]	(max)

Tab. 5.5. *The status of stability constraints versus boundary constraints*

	B vs. H	B/T vs. H/T
IMO	**a**	**a**
PRS	**c**	**c**
HSMB	**a**	**b**

6. SUMMARY AND CONCLUSIONS

A post-optimisation study – stability-oriented feasibility analysis has been demonstrated on the background of a marine engineering design problem – optimisation of a fleet of sea-river ships for a regular shipping line in the area of the North and Baltic Seas. Two questions can be addressed based on the results obtained: (i) why the full intact stability criteria (such as IMO or similar) should be included in the optimisation model, and (ii) how to prepare such a model to accomplish the task.

As far as the first question is concerned, the following justifications seem to be of importance :

- designers and potential customers (owners) express specific and growing interest in the safety of ships
- the stability criteria do exist as formal and legal design restrictions so they can not be neglected
- their presence in the model improves its quality, thereby making it more credible
- their incorporating into the model enables post-optimisation studies to be undertaken, such as parametric study, feasibility analysis and their quantitative extension - sensitivity analysis, with special emphasis on the economic aspects of the solution („shadow prices" and risk assessment).

As to the second question, an attempt has been made to define arbitrary stability criteria as constraints based on a complete analytical definition of all the necessary geometrical characteristics of hull form. A method has been proposed to use a systematic series of ship body forms. As an example, approximation of cross curves and accompanying characteristics of Series 60 has been demonstrated which makes it possible to determine a righting lever curve as a multi-variable function of the heeling angle and ship parameters – decision variables in the fleet optimisation model.

For a particular case under study – a fleet of sea-river ships, Series 60 turned out to be not suitable for the whole range of form parameters expected of such type of ships (very large C_B and very small H/T). Consequently some corrections have been made to fit the available characteristics of SINE-205 sea-river vessel. Such an approach, however, must be regarded as a temporary solution. A research project is under way to work out a stability-oriented, analytically defined series of hull forms for specific functional types of ships such as full cellular container ships, tankers and so on, to be used in the corresponding fleet optimisation models.

NOMENCLATURE

B / b - breadth of ship / B/T ratio
c / **C** - vector / set of constants
C_B - block coefficient
$g_j(\cdot)$ - function of a j^{th} constraint
H / h - depth of ship / H/T ratio
$\mathbf{J_B}$ / $\mathbf{J_S}$ / $\mathbf{J_R}$ - sets of indices of boundary / stability / remaining constraints
L_H - heeling lever curve
l_R / L_R - non-dimensional / dimensional [m] righting lever curve
m - number of constraints
n - number of decision variables
Q - objective function
$\mathbf{R}^n$ - n-dimensional arithmetic space
r_0 - non-dimensional transversal metacentric radius
T - design draught of a ship
w_B - cross curves function
x - decision variable vector
z_G - non-dimensional centre of gravity of a ship
ϕ - heeling angle
Ω - feasible solution region

BIBLIOGRAPHY

1. Abramowicz, T., Kozłowski K., Oleksiewicz B., Wiśniewski J.: *Stability Criteria in the Main Particulars Optimisation Algorithm for Multi-Purpose and General Cargo Ships* (in Polish). Prace Badawcze Inst. Okr. PG. Nr 1983/MR-942/84, Gdańsk 1984
2. Erichsen S.: *Optimum Capacity of Ships and Port Terminals*. The Dep. Of NA & ME, The University of Michigan, No. 123
3. Findeisen W., Szymanowski J., Wierzbicki A.: *Theory and the Calculation Methods of Optimisation* (in Polish). PWN. Warszawa, 1977
4. Gallin, C.: *Which Way Computer Aided Preliminary Ship Design and Optimisation*. ICCAS-Papers. Tokyo, 1973
5. Horman, H., Wagner D.: *Stability Criteria for Present Day Ships Designs*. The STAB 1986 Conference. Gdańsk
6. IMO A. (A.167 + A.562) *Code of Intact Stability for All Types of Ships Covered by IMO Instruments*
7. Jagoda J.: *A Method of Economic Optimisation of Cargo Ships in the Process of Preliminary Design* (in Polish). Doctor thesis. Inst. Okr. PG. 1972
8. Kobyliński, L.K., Kastner, S.: *Stability and Safety of Ships*. Volume 1: *Regulation and Operation. Elsevier Ocean Engineering Book Series*. Volume 9, 2005
9. Kozłowski. K., Michalski J., Oleksiewicz B., Szymański, T.: *Computer-Aided Visualisation of Solutions in Ship Design and Optimisation by Mathematical Programming Methods* (in Polish). Prace Badawcze W.O.iO. PG. Nr 36/92. Gdańsk, 1992
10. Kupras K., Sokołowski K.: *Calculation Methods in Preliminary Ship Design* (in Polish). Wyd. Morskie. Gdynia, 1968
11. Kupras K.: *Optimisation Method and Parametric Study in Pre--Contracted Ship Design*. I.S.P, Vol.23, No.261, 1976
12. Mandel P., Leopold R.: *Optimisation Methods Applied to Ship Design*. Trans. SNAME. 1966
13. Michalski J., Oleksiewicz B.: *Optimisation of a Fleet of Ships in a Regular Shipping on European Lines in the Area of the North and Baltic Seas* (in Polish). Prace Badawcze W.O. i O. PG, Nr 61/E/2002. Gdańsk 2002 (within the EUREKA E!2772 Project)
14. Nowacki H., Brusis F., Swift M.: *Tanker Preliminary Design - an Optimisation Problem with Constraints*. Trans. SNAME. 1970
15. Oleksiewicz B.: *Some Problems Concerning the Optimisation Modelling in the Preliminary Cargo Ship Design*. I Sympozjum Metodyki Projektowania Okrętów. Gdańsk, IX 1979
16. Pawłowski J.: *On the Application of Non-structural Models to Ship Design*. International Shipbuilding Progress, No. 332, 1982
17. PRS - *Rules for the Classification and Construction of Sea--Going Ships*. Part IV Stability. Gdańsk, 1986
18. SINE-205 - *Technical Documentation of a „Sea-River" Vessel by SINUS Ltd*. Gdańsk, 2003
19. Todd F.M.: *Some Further Experiments on Single-Screw Merchant Ship Forms* - Series 60. Trans. SNAME. 1953
20. Wiśniewski J.: *Some Propositions of a Systematic Investigation of Hull Form for the Purpose of Stability Calculations in the Preliminary Ship Design* (based on Todd's Series) (in Polish). Doctor Thesis. Wydział Budowy Okrętów PG. 1962