Viscoelastic lubrication of spherical slide bearing in impulsive unsteady motion

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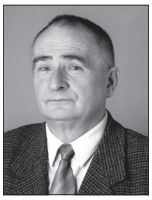
ABSTRACT



This paper presents numerical and semi-analytical solutions of oil velocity components and pressure distributions in spherical unsymmetrical gap of slide bearing. A hydrodynamic unsteady lubrication during oil flow with viscoelastic properties is here considered. In the case of various driving systems on ships the bearings with spherical journals and spherical sleeves or slide bearings with spherical bits operate often under impulsive unsteady motions. Many impurities appearing in service leads to viscoelastic properties of the oil. During service of transport machines it is necessary to adjust the shaft location respective to the sleeve in order to make optimizing the convergent lubricating film possible. Such conditions are effectively satisfied in bearings with spherical journals. The presented numerical calculations were performed by means of the Mathcad 2000 Professional Program and the method of finite differences. This method satisfies stability conditions of numerical solutions of capacity forces occurring in spherical bearings.

Key words : driving systems on ships, spherical slide bearing, unsteady impulsive viscoelastic lubrication

INTRODUCTION

Lubrication of spherical bearing under periodic motion has been considered in many papers till now [3, 4, 8, 9, 10, 11, 12]. This paper considers pressure distribution during hydrodynamic viscoelastic lubrication of spherical bearings and bearings with spherical bit, at impulsive unsteady motion. These problems have been not elaborated hitherto.

Bearing systems are commonly used in diesel engines installed in land transport machines and ships. The oil in bearing gaps in such bearing systems is contaminated mainly with dust, soot, smoke black as well as many inhibitors improving the oil properties. Transport machines usually work under unsteady impulsive vibrations. Thus the lubricating oil has often the non-Newtonian properties. Therefore in this paper viscoelastic time-dependent properties of oil are taken into account. Designing the bearings without accounting for the viscoelastic oil properties brings about to occurrence of the seizing of the bearing in kinematics pairs.

The seizing of bearings can be prevented by proper recognition of bearing operational parameters for real oils. This is very important because the seizing of ship diesel engine bearing system may lead to the catastrophe [1, 2].Therefore determination of real bearing capacity at unsteady viscoelastic lubrication has important sense.

The bearings presented in this paper have spherical journals. The spherical journals can be turned with the shaft or the form of an individual ball can be used (see Fig.1). Such ball is installed in bored end of the shaft.

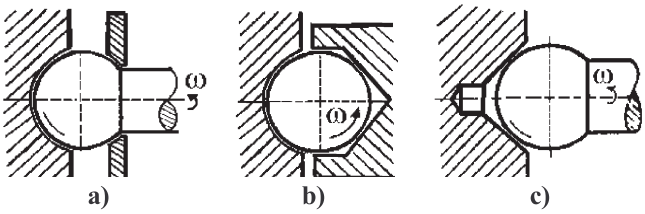


Fig.1. *Spherical bearings:* ***a)*** *spherical journal together with shaft and spherical sleeve,* ***b)*** *individual spherical ball with conical sleeve,* ***c)*** *spherical journal together with shaft and conical sleeve*

The sleeve has spherical or conical shape. The spherical sleeve is more effective than conical one, because it ensures small values of slide thrust and wear.

Usually the spherical bearing is adjustable. It makes it possible to set up the shaft with respect to the sleeve and to control the convergent lubricating film, and it is capable of transferring both axial and transverse loading, Fig.2.

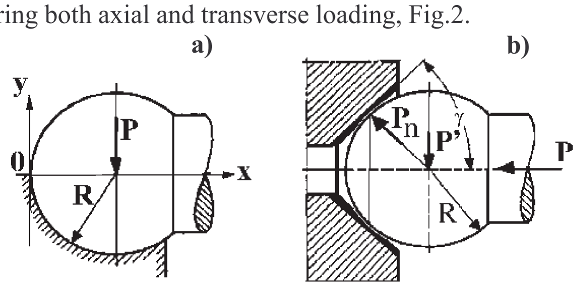


Fig. 2. *Loading of the spherical journal:* ***a)*** *transverse loading,* ***b)*** *axial and transverse loading*

GOVERNING EQUATIONS AND BOUNDARY LAYER SIMPLIFICATIONS

Lubrication of journal and sleeve in slide spherical bearing is described by oil flow. The momentum conservation equations and continuity equation describe oil flow. Moreover the second order approximation of the general constitutive equation given by Rivlin and Ericksen, can be considered. The equations can be expressed in the following form [6 , 7] :

$$\mathrm{Div}\,\mathbf{S} = \rho d\mathbf{v}/dt \quad , \quad \mathrm{div}\,\mathbf{v} = 0 \quad , \quad \mathbf{S} = -p\mathbf{I} + \eta_o \mathbf{A_1} + \alpha(\mathbf{A_1})^2 + \beta \mathbf{A_2} \tag{1}$$

where :

S	- stress tensor	ρ	- oil density
Div **S**	- stress tensor divergence	t	- time [s]
v	- velocity vector [m/s]	p	- pressure
div **v**	- velocity vector divergence	**I**	- unit tensor

$\mathbf{A}_1$ and $\mathbf{A}_2$ - two Rivlin-Ericksen strain tensors of three material constants η_o, α, β,

where :

η_o - dynamic viscosity α , β - pseudo-viscosity constans of oil.

The tensors $\mathbf{A_1}$, and $\mathbf{A_2}$ are given by the symmetric matrices defined by:

$$\begin{gathered} \mathbf{A}_1 \equiv \mathbf{L} + \mathbf{L}^T \\ \mathbf{A}_2 \equiv \mathrm{grad}\,\mathbf{a} + (\mathrm{grad}\,\mathbf{a})^T + 2\mathbf{L}^T\mathbf{L} \\ \mathbf{a} \equiv \mathbf{L}\mathbf{v} + \frac{\partial \mathbf{v}}{\partial t} \end{gathered} \tag{2}$$

where :

L	- tensor of oil velocity vector gradient $[s^{-1}]$	**a**	- acceleration vector $[m/s^2]$
$\mathbf{L}^T$	- tensor with matrix transpose $[s^{-1}]$	grad **a**	- acceleration vector gradient

It is assumed that the product of Deborah and Strouhal numbers, i.e. DeStr, and the product of Reynolds number, dimensionless radial clearance, and Strouhal number, i.e. ReψStr , are of values of the same order. Moreover DeStr >> $De \equiv \alpha\omega/\eta_o$.
where : ψ - relative radial clearance ω - angular velocity of spherical bearing journal.

The following is additionally assumed :

- the rotational motion of spherical journal with peripheral tangential velocity U = ωR
- unsymmetrical unsteady oil flow in the gap
- viscoelastic and unsteady properties of oil
- the oil density ρ of constant value
- the characteristic value of the bearing gap height, ε
- no slip at the bearing surfaces
- R - radius of spherical journal.

By neglecting the terms of the radial clearance $\psi \equiv \varepsilon/R \cong 10^{-3}$ in the governing equations expressed in the spherical coordinates φ, r, ϑ, and by taking into account the above mentioned assumptions the following is obtained :

$$\frac{\partial v_\varphi}{\partial t} = -\frac{1}{\rho R \sin\frac{\vartheta}{R}} \frac{\partial p}{\partial \varphi} + \frac{\eta_o}{\rho} \frac{\partial}{\partial r}\left(\frac{\partial v_\varphi}{\partial r}\right) + \frac{\beta}{\rho} \frac{\partial^3 v_\varphi}{\partial t \partial r^2} \tag{3}$$

$$0 = \frac{\partial p}{\partial r} \tag{4}$$

$$\frac{\partial v_\vartheta}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial \vartheta} + \frac{\eta_o}{\rho} \frac{\partial}{\partial r}\left(\frac{\partial v_\vartheta}{\partial r}\right) + \frac{\beta}{\rho} \frac{\partial^3 v_\vartheta}{\partial t \partial r^2} \tag{5}$$

$$\frac{\partial v_\varphi}{\partial \varphi} + R \sin\left(\frac{\vartheta}{R}\right) \frac{\partial v_r}{\partial r} + \frac{\partial}{\partial \vartheta}\left[R v_\vartheta \sin\left(\frac{\vartheta}{R}\right)\right] = 0 \tag{6}$$

where : $0 \le \varphi \le 2\pi c_1$, $0 < c_1 < 1$, $b_m \equiv \pi R/8 \le \vartheta \le \pi R/2 \equiv b_s$, $0 \le r \le h$, h - gap height.

Symbols v_φ, v_r, v_ϑ denote oil velocity components in the circumferential, gap-height and meridianal directions of the spherical journal, respectively. The terms multiplied by the coefficient β in the right hand sides of (3) , (5) denote influence of time-variable viscoelastic oil properties on the bearing operational parameters. The terms in the left hand sides of (3) , (5) describe influence of accelerations which occur during the impulsive motion, on the bearing lubrication. The relationships between dimensional and dimensionless quantities are assumed in the following form :

$$r = \varepsilon r_1 \;,\; \vartheta = R\vartheta_1 \;,\; t = t_o t_1 \;,\; h = \varepsilon h_1 \;,\; v_\varphi = U v_{\varphi 1} \;,\; v_r \equiv U\psi v_{r1}$$
$$v_\vartheta \equiv U v_{\vartheta 1} \;,\; p = p_o p_1 \;,\; p_o \equiv U\eta_o R/\varepsilon^2 \tag{7}$$

and Reynolds number, modified Reynolds number and Strouhal - Deborah number are as follows :

$$Re \equiv \rho U\varepsilon/\eta_o \;,\; Re\psi \equiv \rho\omega\varepsilon^2/\eta_o \;,\; Str \equiv R/(Ut_o) \;,\; De \equiv \beta U/(\eta_o R) \tag{8}$$

hence :

$$DeStr = \beta/(\eta_o t_o) \equiv Des \;,\; Re\psi Str = \rho\varepsilon^2/(\eta_o t_o) \equiv Res \tag{8a}$$

For the oil containing inhibitors the constant β/t_o is always $0 < \beta/t_o < \eta_o$ and of values usually in the range from 0.0001 to 0.1000 Pas^2. The dimensionless symbols have lower index "1". Hence the equations (1) ÷ (6) obtain the following dimensionless form :

$$Res\frac{\partial v_{\varphi 1}}{\partial t_1} = -\frac{1}{\sin\vartheta_1}\frac{\partial p_1}{\partial\varphi} + \frac{\partial}{\partial r_1}\left(\frac{\partial v_{\varphi 1}}{\partial r_1}\right) + Des\frac{\partial^3 v_{\varphi 1}}{\partial t_1 \partial r_1^2} \tag{9}$$

$$0 = \frac{\partial p_1}{\partial r_1} \tag{10}$$

$$Res\frac{\partial v_{\vartheta 1}}{\partial t_1} = -\frac{\partial p_1}{\partial\vartheta_1} + \frac{\partial}{\partial r_1}\left(\frac{\partial v_{\vartheta 1}}{\partial r_1}\right) + Des\frac{\partial^3 v_{\vartheta 1}}{\partial t_1 \partial r_1^2} \tag{11}$$

$$\frac{\partial v_{\varphi 1}}{\partial\varphi} + \sin(\vartheta_1)\frac{\partial v_{r1}}{\partial r_1} + \frac{\partial}{\partial\vartheta_1}\left[v_{\vartheta 1}\sin(\vartheta_1)\right] = 0 \tag{12}$$

where : $0 \le \varphi \le 2\pi c_1$, $0 \le c_1 \le 1$, $\pi/8 \le \vartheta_1 \le \pi/2$, $0 \le r_1 \le h_1$

THE METHOD OF INTEGRATION

For lubrication at impulsive motion a new dimensionless variable was introduced [4] :

$$\chi \equiv r_1 N \;,\; N \equiv \frac{1}{2}\sqrt{\frac{Res}{t_1}} \;,\; t_1 > 0 \;,\; 0 < \frac{Des}{t_1} < 1 \tag{13}$$

and the solutions in the form of the following convergent series, were assumed :

$$v_{\varphi 1} = v_{\varphi 0\Sigma}(\chi,\varphi,\vartheta_1) + \frac{Des}{t_1}v_{\varphi 1\Sigma}(\chi,\varphi,\vartheta_1) + \left(\frac{Des}{t_1}\right)^2 v_{\varphi 2\Sigma}(\chi,\varphi,\vartheta_1) + \dots\dots\dots \tag{14}$$

$$v_{\vartheta 1} = v_{\vartheta 0\Sigma}(\chi,\varphi,\vartheta_1) + \frac{Des}{t_1}v_{\vartheta 1\Sigma}(\chi,\varphi,\vartheta_1) + \left(\frac{Des}{t_1}\right)^2 v_{\vartheta 2\Sigma}(\chi,\varphi,\vartheta_1) + \dots\dots\dots \tag{15}$$

$$v_{r1} = v_{r0\Sigma}(\chi,\varphi,\vartheta_1) + \frac{Des}{t_1}v_{r1\Sigma}(\chi,\varphi,\vartheta_1) + \left(\frac{Des}{t_1}\right)^2 v_{r2\Sigma}(\chi,\varphi,\vartheta_1) + \dots\dots\dots \tag{16}$$

$$p_1 = p_{10}(\varphi,\vartheta_1,t_1) + \frac{Des}{t_1}p_{11}(\varphi,\vartheta_1,t_1) + \left(\frac{Des}{t_1}\right)^2 p_{12}(\varphi,\vartheta_1,t_1) + \dots\dots\dots \tag{17}$$

where : $t_1 > 0$, $0 < Des << 1$, $(Des/t_1) < 1$

The first terms of the series (14), (17) describe oil flow parameters in impulsive unsteady motion, with neglecting the visco-elastic properties. The second, third, etc terms in the series (14), (17) describe the corrections of oil flow parameters, caused by the time-changeable viscoelastic oil properties. In (9) - (11) the derivatives with respect to the variables t_1, r_1, can be replaced by the derivatives with respect to the variable χ only, by using the following relationships :

$$\frac{\partial}{\partial t_1} = \frac{\partial}{\partial\chi}\frac{\partial\chi}{\partial t_1} = -\frac{1}{4}\sqrt{Res}\,\frac{r_1}{t_1\sqrt{t_1}}\frac{\partial}{\partial\chi} = -\frac{\chi}{2t_1}\frac{\partial}{\partial\chi}$$
$$\frac{\partial^2}{\partial r_1^2} = \frac{\partial}{\partial r_1}\left(\frac{\partial}{\partial r_1}\right) = \frac{\partial}{\partial\chi}\left(\frac{\partial}{\partial\chi}\frac{\partial\chi}{\partial r_1}\right)\frac{\partial\chi}{\partial r_1} = \frac{Res}{4t_1}\frac{\partial^2}{\partial\chi^2} \tag{18}$$

$$\frac{\partial^3}{\partial t_1 \partial r_1^2} = \frac{\partial}{\partial t_1}\left(\frac{Res}{4t_1}\frac{\partial^2}{\partial \chi^2}\right) = -\frac{Res}{4t_1^2}\frac{\partial^2}{\partial \chi^2} + \frac{Res}{4t_1}\frac{\partial}{\partial \chi}\left(\frac{\partial^2}{\partial \chi^2}\right)\frac{\partial \chi}{\partial t_1} = -\frac{Res}{4t_1^2}\left(\frac{\partial^2}{\partial \chi^2} + \frac{\chi}{2}\frac{\partial^3}{\partial \chi^3}\right) \tag{19}$$

Next, the series (14)-(17) were put into the changed set of the equations (9)-(12) where the variables t_1, r_1 were replaced by the variable χ. And, the terms multiplied by the parameter $(Des/t_1)^k$ of the same power values k , for k = 0,1,2, ... , were respectively equalled to each other.Thus the following sequence of the sets of ordinary second-order differential equations, was obtained [5] :

$$(v_{i0\Sigma})^{(2)} + 2\chi(v_{i0\Sigma})^{(1)} = \frac{1}{N_i^2}\frac{\partial p_{10}}{\partial \alpha_i} \tag{20}$$

$$(v_{i1\Sigma})^{(2)} + 2\chi(v_{i1\Sigma})^{(1)} + 4(v_{i1\Sigma}) = \frac{1}{N_i^2}\frac{\partial p_{11}}{\partial \alpha_i} + (v_{i0\Sigma})^{(2)} + \frac{1}{2}\chi(v_{i0\Sigma})^{(2)} \tag{21}$$

$$(v_{i2\Sigma})^{(2)} + 2\chi(v_{i2\Sigma})^{(1)} + 8(v_{i2\Sigma}) = \frac{1}{N_i^2}\frac{\partial p_{12}}{\partial \alpha_i} + 2(v_{i1\Sigma})^{(2)} + \frac{1}{2}\chi(v_{i1\Sigma})^{(3)} \tag{22}$$

and so on,

where : $i = \varphi , \vartheta$, $\alpha_\varphi \equiv \varphi$, $\alpha_\vartheta \equiv \vartheta_1$

The upper indices: (1), (2), (3), ... denote : the first, second, third, etc. derivative with respect to the variable χ, and :

$$(N_\varphi)^2 \equiv N^2 \sin(\vartheta_1) \quad , \quad N_\vartheta \equiv N \tag{23}$$

GENERAL SOLUTIONS AND VALIDITY OF BOUNDARY CONDITIONS

The general solutions of the equations (20) for : $i = \varphi , \vartheta$; have the form :

$$v_{i0\Sigma}(\chi) = C_{i1}v_{01}(\chi) + C_{i2}v_{02}(\chi) + v_{i\,03}(\chi) \tag{24}$$

where : C_{i1} , C_{i2} - integration constants.

The following particular solutions of homogeneous and non - homogeneous differential equations were obtained :

$$v_{01}(\chi) = \int_0^\chi e^{-\chi_1^2} d\chi_1 \quad , \quad v_{02}(\chi) = 1 \tag{25}$$

$$v_{i03}(\chi) = -\frac{1}{N_i^2}\frac{\partial p_{10}}{\partial \alpha_i}\left[\int_0^\chi e^{\chi_1^2} v_{01}(\chi_1) d\chi_1 - v_{01}(\chi)\int_0^\chi e^{\chi_1^2} d\chi_1\right] \tag{26}$$

where : $0 \le \chi_1 \le \chi \equiv r_1 N$.

For $t_1 \to 0$, $N \to \infty$, thus $\chi \to \infty$. For $t_1 \to \infty$, $N \to 0$ hence for $r_1 > 0$ will be $\chi \to 0$. For $t_1 > 0$ and $r_1 = 0$ also $\chi = 0$. The following limits are true :

$$\begin{aligned}
&v_{01}(\chi) = \pi^{0.5}/2 && \text{for : } \chi \to \infty \;,\; t_1 \to 0 \;,\; N \to \infty \\
&v_{01}(\chi) = 0 && \text{for : } \chi \to 0 \;,\; r_1 = 0 \;,\; 0 < t_1 < t_2 < \infty \;,\; N > 0 \\
&v_{01}(\chi) = 0 && \text{for : } \chi \to 0 \;,\; r_1 > 0 \;,\; t_1 \to \infty \;,\; N \to 0 \\
&v_{i03}(\chi) = 0 && \text{for : } \chi \to 0 \;,\; r_1 = 0 \;,\; 0 < t_1 < t_2 < \infty \;,\; N > 0 \quad \text{where : } i = \varphi , \vartheta \\
&v_{\varphi 03}(\chi) = -\frac{r_1^2}{2\sin\vartheta_1}\frac{\partial p_{10}}{\partial \varphi} && \text{for : } \chi \to 0 \;,\; r_1 > 0 \;,\; t_1 \to \infty \;,\; N \to 0 \\
&v_{\vartheta 03}(\chi) = -\frac{r_1^2}{2}\frac{\partial p_{10}}{\partial \vartheta_1} && \text{for : } \chi \to 0 \;,\; r_1 > 0 \;,\; t_1 \to \infty \;,\; N \to 0
\end{aligned} \tag{27}$$

The spherical journal moves only in the circumferential direction φ. Hence the oil velocity components on the journal surface in this direction are equal to the peripheral velocity of the spherical journal surface. The oil velocity component on the spherical journal surface in the meridianal direction ϑ equals zero because the spherical journal is motionless in ϑ- direction. The oil flow around the journal is assumed viscous. Hence on the journal surface the oil velocity component in the gap height direction equals zero. Therefore the following boundary conditions are valid:

$$\begin{gathered}
v_{\varphi 0\Sigma}(\chi = 0) = \sin\vartheta_1 \quad , \quad v_{\vartheta 0\Sigma}(\chi = 0) = 0 \quad , \quad v_{r0\Sigma}(\chi = 0) = 0 \\
\text{for : } r_1 = 0 \Leftrightarrow \chi = 0 \text{ and } 0 < t_1 < t_2 < \infty \quad , \quad N > 0
\end{gathered} \tag{28}$$

The spherical sleeve surface is motionless in both circumferenial and meridianal directions. But the spherical sleeve performs any impulsive displacements in the gap height direction. Hence the gap height changes along with time. Thus the oil velocity components on the sleeve surface are equal to zero in both circumferential and meridianal directions. The oil velocity component in the gap height direction *r* is equal to the first derivative of the gap height with respect to time. Hence the following boundary conditions are valid :

$$v_{\varphi 0\Sigma}(\chi = M) = 0 \quad , \quad v_{\vartheta 0\Sigma}(\chi = M) = 0 \quad , \quad v_{r0\Sigma}(\chi = M) = Str\partial h_1/\partial t_1$$
$$\text{for} : r_1 \to h_1 \Leftrightarrow \chi \to Nh_1 \equiv M \quad , \quad 0 < t_1 < t_2 < \infty \quad , \quad N > 0 \tag{29}$$

where : $h = \varepsilon h_1$ - gap height , h_1 - dimensionless gap height , $Str \equiv 1/\omega t_o$

Imposing the conditions (28) , (29) on the solution (24) one obtains :

$$\begin{aligned}
&C_{\varphi 1}v_{01}(\chi = 0) + C_{\varphi 2} + v_{\varphi 03}(\chi = 0) = \sin\vartheta_1 && \text{for} : r_1 = 0\\
&C_{\varphi 1}v_{01}(\chi = M) + C_{\varphi 2} + v_{\varphi 03}(\chi = M) = 0 && \text{for} : r_1 = h_1\\
&C_{\vartheta 1}v_{01}(\chi = 0) + C_{\vartheta 2} + v_{\vartheta 03}(\chi = 0) = 0 && \text{for} : r_1 = 0\\
&C_{\vartheta 1}v_{01}(\chi = M) + C_{\vartheta 2} + v_{\vartheta 03}(\chi = M) = 0 && \text{for} : r_1 = h_1
\end{aligned} \tag{30}$$

By taking into account the limits (27) the following solutions of the set of the equations (30), are obtained :

$$C_{\varphi 1} = -\frac{\sin\vartheta_1 + v_{\varphi 03}(M)}{v_{01}(M)} \quad , \quad C_{\vartheta 1} = -\frac{v_{\vartheta 03}(M)}{v_{01}} \quad , \quad C_{\varphi 2} = \sin\vartheta_1 \quad , \quad C_{\vartheta 2} = 0 \tag{31}$$

Now, into the right hand side of (21) the solution (24), (25), (26), (31) is inserted.
Thus the general solution of (21) obtains the following form :

$$v_{i1\Sigma}(\chi) = C_{i3}v_{11}(\chi) + C_{i4}v_{12}(\chi) + v_{i13}(\chi) \quad \text{for} : i = \varphi , \vartheta \tag{32}$$

where : C_{i3} , C_{i4} - integration constants.

The particular solutions are as follows :

$$v_{11}(\chi) = \chi e^{-\chi^2} \quad , \quad v_{12}(\chi) = \chi e^{-\chi^2}\int_\delta^\chi \frac{1}{\chi_1^2} e^{-\chi_1^2} d\chi_1 \tag{33}$$

$$\begin{aligned}
v_{i13}(\chi, C_{i1}) = {}& v_{11}(\chi)\int_0^\chi \left\{ C_{i1}\,\chi_1(\chi_1+2) - \left(1+\frac{\chi_1}{2}\right)e^{\chi_1^2}\frac{d^2}{d\chi_1^2}\left[v_{i03}(\chi_1)\right] + \frac{1}{N_i^2}\frac{\partial p_{11}}{\partial\alpha_i} \right\} v_{12}(\chi_1)\,d\chi_1 + \\
& + v_{12}(\chi)\int_0^\chi \left\{ \left(1+\frac{\chi_1}{2}\right)e^{\chi_1^2}\frac{d^2}{d\chi_1^2}\left[v_{i03}(\chi_1)\right] + \frac{1}{N_i^2}\frac{\partial p_{11}}{\partial\alpha_i} - C_{i1}\chi_1(\chi_1+2) \right\} v_{11}(\chi_1)\,d\chi_1
\end{aligned} \tag{34}$$

$$\text{for} : i = \varphi , \vartheta \quad , \quad 0 < \delta \le \chi_1 \le \chi$$

The solutions (32) represent the corrections of the oil velocity components due to the viscoelastic oil properties.
By virtue of the solutions (33) and (34), for : $\chi \to 0$, $r_1 \to 0$, $N > 0$, it follows :

$$\lim_{\chi\to 0, N>0} v_{12}(\chi) = \lim_{\chi\to 0, N>0} \chi e^{-\chi^2}\int_\delta^\chi \frac{1}{\chi_1^2} e^{\chi_1^2} d\chi_1 = -1 \tag{35}$$

The following limits are true :

$$\begin{aligned}
&v_{11}(\chi) = 0 && \text{for} : \chi \to 0 \;,\; r_1 = 0 \;,\; 0 < t_1 < t_2 < \infty \;,\; N > 0\\
&v_{12}(\chi) = -1 && \text{for} : \chi \to 0 \;,\; r_1 = 0 \;,\; 0 < t_1 < t_2 < \infty \;,\; N > 0\\
&v_{i13}(\chi) = 0 && \text{for} : \chi \to 0 \;,\; r_1 = 0 \;,\; 0 < t_1 < t_2 < \infty \;,\; N > 0 \quad \text{where} : i = \varphi , \vartheta
\end{aligned} \tag{36}$$

The corrections of oil velocity components can not violate the boundary conditions (28),(29) which are assumed on the journal and sleeve surfaces in the circumferential and meridianal directions. Therefore, the following boundary conditions were applied to the corrections of oil velocity components :

$$\begin{aligned}
&v_{\varphi 1\Sigma}(\chi = 0) = 0 \;,\; v_{\vartheta 1\Sigma}(\chi = 0) = 0 \quad \text{for} : r_1 = 0 \Leftrightarrow \chi = 0 \;,\; 0 < t_1 < t_2 < \infty \;,\; N > 0\\
&v_{\varphi 1\Sigma}(\chi = M) = 0 \;,\; v_{\vartheta 1\Sigma}(\chi = M) = 0 \quad \text{for} : r_1 \to h_1 \Leftrightarrow \chi \to Nh_1 \equiv M \;,\; 0 < t_1 < t_2 < \infty \;,\; N > 0
\end{aligned} \tag{37}$$

Imposing conditions (37) on the general solution (32) one gets :

$$\begin{aligned}
&C_{\varphi 3}v_{11}(\chi = 0) + C_{\varphi 4}v_{21}(\chi = 0) + v_{\varphi 13}(\chi = 0) = 0 && \text{for} : r_1 = 0\\
&C_{\varphi 3}v_{11}(\chi = M) + C_{\varphi 4}v_{21}(\chi = M) + v_{\varphi 13}(\chi = M) = 0 && \text{for} : r_1 = h_1
\end{aligned} \tag{38}$$

$$C_{\vartheta 3}v_{11}(\chi=0)+C_{\vartheta 4}v_{21}(\chi=0)+v_{\vartheta 13}(\chi=0)=0 \quad \text{for}: r_1=0$$
$$C_{\vartheta 3}v_{11}(\chi=M)+C_{\vartheta 4}v_{21}(\chi=M)+v_{\vartheta 13}(\chi=M)=0 \quad \text{for}: r_1=h_1 \tag{39}$$

By taking into account the limits (36), the following solutions of the set of the equations (38),(39) were obtained :

$$C_{i3}=\frac{v_{i13}(\chi=h_1N)}{v_{11}(\chi=h_1N)}\ ,\ C_{i4}=0 \qquad \text{for}: i=\varphi\ ,\ \vartheta \tag{40}$$

where : $0\le\chi_1\le h_1N$, $N=\frac{1}{2}\sqrt{\frac{\mathrm{Re}\,\psi\,\mathrm{Str}}{t_1}}$, $0<t_1<\infty$, $0\le r_1\le h_1$, $b_{m1}\le\vartheta_1\le b_{s1}$, $0<\varphi<2\pi c_1$, $0\le c_1<\infty$

NEWTONIAN UNSTEADY LUBRICATION

By neglecting the viscoelastic properties of oil, by virtue of solutions (24) and constants (31), the particular velocity components of oil in φ - and ϑ - directions for non steady flow obtained the following dimensionless form :

$$v_{\varphi 0\Sigma}(\varphi,r_1,\vartheta_1,t_1)=\sin\vartheta_1-\left\{\sin\vartheta_1-\frac{\sqrt{\pi}}{2N^2\sin\vartheta_1}\frac{\partial p_{10}}{\partial\varphi}\left[Y(\chi=Nh_1)\right]\right\}\cdot\frac{\mathrm{erf}(r_1N)}{\mathrm{erf}(h_1N)}+$$
$$-\frac{\sqrt{\pi}}{2N^2\sin\vartheta_1}\frac{\partial p_{10}}{\partial\varphi}Y(\chi=Nr_1) \tag{41}$$

$$v_{\vartheta 0\Sigma}(\varphi,r_1,\vartheta_1,t_1)=\frac{\sqrt{\pi}}{2N^2}\frac{\partial p_{10}}{\partial\vartheta_1}\left[Y(\chi=Nh_1)\right]\cdot\frac{\mathrm{erf}(r_1N)}{\mathrm{erf}(h_1N)}-\frac{\sqrt{\pi}}{2N^2}\frac{\partial p_{10}}{\partial\vartheta_1}Y(\chi=Nr_1) \tag{42}$$

where :

$$Y(\chi)\equiv\int_0^{\chi}e^{\chi_1^2}\mathrm{erf}\chi_1 d\chi_1-\mathrm{erf}(h_1N)\int_0^{\chi}e^{\chi_1^2}d\chi_1 \tag{43}$$

$$N\equiv\frac{1}{2}\sqrt{\frac{\mathrm{Re}\,s}{t_1}}\ ,\ \mathrm{erf}(\chi_1)=\frac{2}{\sqrt{\pi}}\int_0^{\chi_1}e^{-\chi_2^2}d\chi_2 \tag{44}$$

for :

$0\le t_1<\infty$, $0\le r_1\le h_1$, $b_{m1}\le\vartheta_1\le b_{s1}$, $0<\varphi<2\pi c_1$, $0\le c_1<\infty$, $0\le\chi_2\le\chi_1\le\chi\equiv r_1N\le h_1N\equiv M$, $h_1=h_1(\varphi,\vartheta_1,t_1)$

The oil velocity components (41),(42) were put into the continuity equation (12) and both sides of this equation were integrated with respect to the variable r_1. The oil velocity component $v_{r0\Sigma}$ in the gap height direction equals zero on the spherical journal surface. Therefore by imposing the boundary condition $v_{r0\Sigma}=0$ for $r_1=0$, the oil velocity component in the gap height direction obtained the following form :

$$v_{r0\Sigma}(\varphi,r_1,\vartheta_1,t_1)=-\frac{Ne^{-h_1^2N^2}}{\mathrm{erf}(h_1N)}\left[\frac{\partial h_1}{\partial\varphi}-\frac{\sqrt{\pi}}{2}\left(\frac{1}{\sin^2\vartheta_1}\frac{\partial h_1}{\partial\varphi}\frac{\partial p_{10}}{\partial\varphi}+\frac{\partial h_1}{\partial\vartheta_1}\frac{\partial p_{10}}{\partial\vartheta_1}\right)\frac{1}{N}\int_0^{h_1N}e^{\chi^2}\mathrm{erf}\chi d\chi\right]\int_0^{r_1}\frac{\mathrm{erf}(r_2N)}{\mathrm{erf}(h_1N)}dr_2+$$
$$-\frac{\sqrt{\pi}}{2}\left(\frac{1}{\sin^2\vartheta_1}\frac{\partial^2p_{10}}{\partial\varphi^2}+\frac{\partial^2p_{10}}{\partial\vartheta_1^2}+\frac{\partial p_{10}}{\partial\vartheta_1}\cot\vartheta_1\right)\left\{\frac{1}{N^2}Y(\chi=h_1N)\int_0^{r_1}\frac{\mathrm{erf}(r_2N)}{\mathrm{erf}(h_1N)}dr_2-\int_0^{r_1}Y(\chi=r_2N)dr_2\right\} \tag{45}$$

where : $0\le t_1<\infty$, $0\le r_2\le r_1\le h_1$, $b_{m1}\le\vartheta_1\le b_{s1}$, $0<\varphi<2\pi c_1$, $0\le c_1<\infty$, $0\le\chi_2\le\chi_1\le\chi\equiv r_1N\le h_1N\equiv M$

The oil velocity component $v_{r0\Sigma}$ in the gap height direction does not equal zero on the sleeve surface. Therefore by integrating the continuity equation (12) and imposing the boundary condition (29) for $r_1=h_1$ on the velocity component in the gap height direction, the following equation was obtained :

$$\frac{\partial}{\partial\varphi}\int_0^{h_1}v_{\varphi 0\Sigma}dr_1+\frac{\partial}{\partial\vartheta_1}\int_0^{h_1}\sin\vartheta_1 v_{\vartheta 0\Sigma}dr_1=-\mathrm{Str}\frac{\partial h_1}{\partial t_1}\sin\vartheta_1 \tag{46}$$

If the expressions (41)÷(42) are put into (46) the following modified Reynolds equation is yielded :

$$\frac{\sqrt{\pi}}{2N^2}\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\varphi}\left\{\left[\frac{\int_0^{h_1}\mathrm{erf}(r_1N)dr_1}{\mathrm{erf}(h_1N)}Y(\chi=Nh_1)-\int_0^{h_1}Y(\chi=Nr_1)dr_1\right]\frac{\partial p_{10}}{\partial\varphi}\right\}+$$

$$+\frac{\sqrt{\pi}}{2N^2}\frac{\partial}{\partial \vartheta_1}\left\{\left[\frac{\int_0^{h_1}\mathrm{erf}(r_1 N)dr_1}{\mathrm{erf}(h_1 N)}Y(\chi = Nh_1) - \int_0^{h_1} Y(\chi = Nr_1)dr_1\right]\frac{\partial p_{10}}{\partial \vartheta_1}\sin\vartheta_1\right\} = \tag{47}$$

$$= -(\sin\vartheta_1)\frac{\partial}{\partial\varphi}\left(\int_0^{h_1}\left[1-\frac{\mathrm{erf}(r_1 N)}{\mathrm{erf}(h_1 N)}\right]dr_1\right) - \mathrm{Str}\frac{\partial h_1}{\partial t_1}(\sin\vartheta_1)$$

where :

$0 \le r_2 \le r_1 \le h_1$, $0 \le \varphi < 2\pi c_1$, $0 \le c_1 < 1$, $0 \le \vartheta_1 < \pi/2$, $0 \le t_1 < \infty$

$0 \le \chi_2 \le \chi_1 \le h_1 N$, $0 \le N(t_1) = 0.5(\mathrm{Res}/t_1)^{0.5} < \infty$

The modified Reynolds equation (47) determines an unknown pressure function $p_{10}(\varphi, \vartheta_1, t_1)$. If t_1 tends to infinity, i.e. N tends to zero, then the equation (47) tends to the classical Reynolds equation. To explain this fact the following limits were calculated :

$$\lim_{N\to 0}\frac{\sqrt{\pi}}{2N^2}Y(\chi = h_1 N) \equiv \lim_{N\to 0}\frac{\sqrt{\pi}}{2N^2}\left[\int_0^{h_1 N}\exp(\chi^2)\mathrm{erf}(\chi)d\chi - \mathrm{erf}(h_1 N)\int_0^{h_1 N}\exp(\chi^2)d\chi\right] =$$

$$= \lim_{N\to 0}\frac{1}{N^2}\left\{\int_0^{h_1 N}\left[\exp(\chi^2)\int_0^{\chi}\exp(-\chi_1^2)d\chi_1\right]d\chi - \left(\int_0^{h_1 N}\exp(-\chi^2)d\chi\right)\left(\int_0^{h_1 N}\exp(\chi^2)d\chi\right)\right\} = \tag{48}$$

$$\overset{H}{=} -\lim_{N\to 0}\frac{h_1\int_0^{Nh_1}\exp(\chi^2)d\chi}{2N\exp(h_1^2 N^2)} \overset{H}{=} -\frac{h_1^2}{2}\lim_{N\to 0}\frac{\exp(h_1^2 N^2)}{\exp(h_1^2 N^2) + 2h_1^2 N^2 \exp(h_1^2 N^2)} = -\frac{h_1^2}{2}$$

and, analogously :

$$\lim_{N\to 0}\frac{\sqrt{\pi}}{2N^2}Y(\chi = Nr_1) = -\frac{r_1^2}{2} \tag{49}$$

as well as :

$$\lim_{N\to 0}\frac{\mathrm{erf}(r_1 N)}{\mathrm{erf}(h_1 N)} = \frac{r_1}{h_1} \tag{50}$$

Thus the equation (46) for $N \to 0$ tends to the following form :

$$\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\varphi}\left\{\left[\left(-\frac{h_1^2}{2}\right)\int_0^{h_1}\frac{r_1}{h_1}dr_1 - \int_0^{h_1}\left(-\frac{r_1^2}{2}\right)dr_1\right]\frac{\partial p_{10}}{\partial\varphi}\right\} + \frac{\partial}{\partial\vartheta_1}\left\{\left[\left(-\frac{h_1^2}{2}\right)\sin\vartheta_1\int_0^{h_1}\frac{r_1}{h_1}dr_1 + \right.\right.$$

$$\left.\left. - \int_0^{h_1}\left(-\frac{r_1^2}{2}\right)\sin\vartheta_1 dr_1\right]\frac{\partial p_{10}}{\partial\vartheta_1}\right\} = -(\sin\vartheta_1)\frac{\partial}{\partial\varphi}\left[\int_0^{h_1}\left(1-\frac{r_1}{h_1}\right)dr_1\right] - \mathrm{Str}\frac{\partial h_1}{\partial t_1}(\sin\vartheta_1) \tag{51}$$

Finally, after calculations, the following form of the classical Reynolds equations of flow in the spherical coordinates was obtained :

$$\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\varphi}\left(h_1^3\frac{\partial p_{10}}{\partial\varphi}\right) + \frac{\partial}{\partial\vartheta_1}\left(h_1^3\frac{\partial p_{10}}{\partial\vartheta_1}\sin\vartheta_1\right) = 6\frac{\partial h_1}{\partial\varphi}\sin\vartheta_1 + 12\,\mathrm{Str}\frac{\partial h_1}{\partial t_1}\sin\vartheta_1 \tag{52}$$

for : $0 \le \varphi < 2\pi c_1$, $0 \le c_1 < 1$, $0 \le \vartheta_1 < \pi/2$

The time-dependent average gap height with perturbations has the following form :

$$h_1 = (h_o/\varepsilon)[1 + s_1 \cdot \exp(-t_o t_1 \omega_o)] \tag{53}$$

where :

$$h_o(\varphi,\vartheta_1) \equiv \Delta\varepsilon_1\cos\varphi\,\sin\vartheta_1 + \Delta\varepsilon_2\sin\varphi\,\sin\vartheta_1 - \Delta\varepsilon_3\cos\vartheta_1 - R +$$
$$+ [(\Delta\varepsilon_1\cos\varphi\,\sin\vartheta_1 + \Delta\varepsilon_2\sin\varphi\,\sin\vartheta_1 - \Delta\varepsilon_3\cos\vartheta_1)^2 + (R + \varepsilon_{min})(R + 2D + \varepsilon_{min})]^{0.5} \tag{54}$$

The coefficient *s* controls changes of the gap height during the impulsive motion. If s > 0 the gap height increases, if s < 0, the gap height decreases. The symbol ω_o denotes an angular velocity expressed in $[s^{-1}]$, describing impulsive changes of perturbations in the unsteady oil flow in the bearing gap in its height direction. If Strouhal number tends to zero the equation (52) tends to the classical Reynolds equation for stationary flow.

The centre of the spherical journal was assumed in the point O(0,0,0) and the centre of the spherical sleeve in the point O_s $(x - \Delta\varepsilon_1,\ y - \Delta\varepsilon_2,\ z + \Delta\varepsilon_3)$. The eccentricity was determined by the value D (Fig.3). The lubrication region Ω indicated in Fig.3 was defined as follows : $0 \le \varphi \le \pi$, $\pi R/8 \le \alpha_3 \equiv \vartheta \le \pi R/2$.

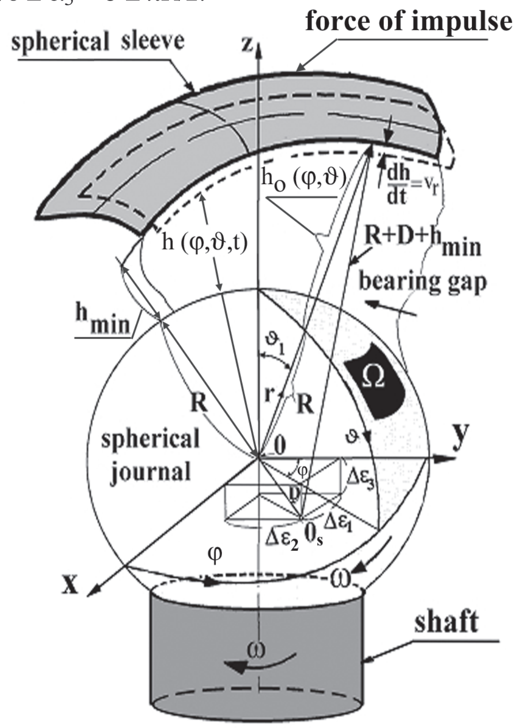


Fig.3. *Schematic diagram of the gap height and eccentricities*

VISCOELASTIC UNSTEADY EFFECTS

The particular corrections (32) of the oil velocity components in φ – and ϑ_1 – directions, caused by the viscoelastic, fluid properties and unsteady fluid flow, were multiplied by the factor $DeStr/t_1$. By using the expressions (32), (33), (34), (40) and boundary conditions (37), the corrections of the oil velocity components (32) obtained the following form :

$$\begin{aligned}
\frac{Des}{t_1} v_{\varphi 1\Sigma}(\varphi,\vartheta_1,r_1,t_1) &= \frac{4\beta}{\rho\varepsilon^2}\frac{N r_1 e^{-r_1^2 N^2}}{\sin\vartheta_1}\left\{\frac{\partial p_{11}}{\partial\varphi}\left[\int_{r_1N}^{h_1N}\chi Y_1(\chi)d\chi + \right.\right.\\
&+\left.\frac{N^2}{2}\left(h_1^2 - r_1^2\right)Y_1(\chi = h_1N)\right] + \frac{\partial p_{10}}{\partial\varphi}\left[\int_{r_1N}^{h_1N} Y_2(\chi)\,d\chi - Y_1(\chi = h_1N)\int_0^{h_1N}\chi e^{-\chi^2}Y_2(\chi)d\chi + \right.\\
&+\left. Y_1(\chi = r_1N)\int_0^{r_1N}\chi e^{-\chi^2}Y_2(\chi)d\chi\right] - \frac{2}{\sqrt{\pi}\,erf(h_1N)}\frac{\partial p_{10}}{\partial\varphi}\left[\int_{r_1N}^{h_1N} Y_1(\chi)Y_3(\chi)Y(\chi)d\chi + \right.\\
&\left.\left.- Y_1(\chi = h_1N)\int_0^{h_1N} Y_3(\chi)Y(\chi)d\chi + Y_1(\chi = r_1N)\int_0^{r_1N} Y_3(\chi)Y(\chi)d\chi\right]\right\} + \\
&- \frac{8\beta N^2 r_1 e^{-r_1^2N^2}\sin\vartheta_1}{\sqrt{\pi}\rho\varepsilon^2 erf(h_1N)}\left[Y_1(\chi = h_1N)\int_0^{h_1N} Y_3(\chi)d\chi - Y_1(\chi = r_1N)\int_0^{r_1N} Y_3(\chi)d\chi - \int_{r_1N}^{h_1N} Y_1(\chi)Y_3(\chi)d\chi\right]
\end{aligned} \tag{55}$$

$$\begin{aligned}
\frac{Des}{t_1} v_{\vartheta 1\Sigma}(\varphi,\vartheta_1,r_1,t_1) &= \frac{4\beta}{\rho\varepsilon^2} N r_1 e^{-r_1^2N^2}\left\{\frac{\partial p_{11}}{\partial\vartheta_1}\left[\int_{r_1N}^{h_1N}\chi Y_1(\chi)d\chi + \frac{N^2}{2}\left(h_1^2 - r_1^2\right)Y_1\left[(\chi = h_1N)\right]\right] + \right.\\
&+ \frac{\partial p_{10}}{\partial\vartheta_1}\left[\int_{r_1N}^{h_1N} Y_2(\chi)d\chi - Y_1(\chi = h_1N)\int_0^{h_1N}\chi e^{-\chi^2}Y_2(\chi)d\chi + Y_1(\chi = r_1N)\int_0^{r_1N}\chi e^{-\chi^2}Y_2(\chi)d\chi\right] + \\
&\left.- \frac{2}{\sqrt{\pi}\,erf(h_1N)}\frac{\partial p_{10}}{\partial\vartheta_1}\left[\int_{r_1N}^{h_1N} Y_1(\chi)Y_3(\chi)Y(\chi)d\chi - Y_1(\chi = h_1N)\int_0^{h_1N} Y_3(\chi)Y(\chi)d\chi + Y_1(\chi = r_1N)\int_0^{r_1N} Y_3(\chi)Y(\chi)d\chi\right]\right\}
\end{aligned} \tag{56}$$

with :

$$Y_1(\chi)\equiv\int_{\delta}^{\chi}\frac{1}{\chi_1^2}e^{\chi_1^2}d\chi_1 \quad , \quad Y_2(\chi)\equiv\left(\chi+\frac{\chi^2}{2}\right)\left(2\chi e^{-\chi^2}\int_0^{\chi}e^{\chi_1^2}d\chi_1-1\right) \quad , \quad Y_3(\chi)\equiv\chi^2(\chi+2)e^{-\chi^2} \tag{57}$$

whereas : $0 \le t_1 < \infty$, $0 \le r_2 \le r_1 \le h_1$, $b_{m1} \le \vartheta_1 \le b_{s1}$, $0 < \varphi < 2\pi c_1$, $0 \le c_1 < \infty$, $0 \le \chi_1 \le \chi \equiv r_1 N \le h_1 N \equiv M$

The corrections of the oil velocity components (55) , (56) were put into the continuity equation (12) and both sides of this equation were integrated with respect to the variable r_1. From the viscous oil properties it follows that the corrections of the oil velocity components in the gap height direction equal zero on the journal surface for $r_1 = 0$.Thus the corrections of oil velocity components in the gap height direction obtained the form :

$$v_{r1\Sigma}(\varphi,\vartheta_1,r_1,t_1)=-\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\varphi}\left(\int_0^{r_1}v_{\varphi1\Sigma}(\varphi,\vartheta_1,r_1,t_1)\,dr_1\right)-\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\vartheta_1}\left(\int_0^{r_1}(\sin\vartheta_1)v_{\vartheta1\Sigma}(\varphi,\vartheta_1,r_1,t_1)\,dr_1\right) \tag{58}$$

The corrections of the oil velocity components can not violate the boundary conditions (28) , (29) assumed on the journal and sleeve surfaces in the gap height direction.

Hence the corrections of the oil velocity component in the gap height direction equal zero on the sleeve surface for $r_1 = h_1$.

By imposing this condition on the solution (58) the following modified Reynolds equation was obtained :

$$\begin{aligned}
&\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\varphi}\left\{\frac{\partial p_{11}}{\partial\varphi}\left[\int_0^{h_1}r_1e^{-r_1^2N^2}Z_1(r_1)dr_1+Z_2(h_1)\right]\right\}+\frac{\partial}{\partial\vartheta_1}\left\{\frac{\partial p_{11}}{\partial\vartheta_1}\sin\vartheta_1\left[\int_0^{h_1}r_1e^{-r_1^2N^2}Z_1(r_1)dr_1+Z_2(h_1)\right]\right\}=\\
&=\frac{2N\sin\vartheta_1}{\operatorname{erf}(Nh_1)}\int_0^{h_1}r_1e^{-r_1^2N^2}Z_3(r_1)dr_1+\frac{2}{\sqrt{\pi}\operatorname{erf}(h_1N)\sin\vartheta_1}\frac{\partial}{\partial\varphi}\left\{\frac{\partial p_{10}}{\partial\varphi}\left[\int_0^{h_1}r_1e^{-r_1^2N^2}Z_4(r_1)dr_1-Z_5(h_1)\right]\right\}+\\
&+\frac{2}{\sqrt{\pi}\operatorname{erf}(h_1N)}\frac{\partial}{\partial\vartheta_1}\left\{\frac{\partial p_{10}}{\partial\vartheta_1}\sin\vartheta_1\left[\int_0^{h_1}r_1e^{-r_1^2N^2}Z_4(r_1)dr_1-Z_5(h_1)\right]\right\}+\\
&-\frac{1}{\sin\vartheta_1}\frac{\partial}{\partial\varphi}\left\{\frac{\partial p_{10}}{\partial\varphi}\left[\int_0^{h_1}r_1e^{-r_1^2N^2}Z_6(r_1)dr_1+Z_7(h_1)\right]\right\}-\frac{\partial}{\partial\vartheta_1}\left\{\frac{\partial p_{10}}{\partial\vartheta_1}\sin\vartheta_1\left[\int_0^{h_1}r_1e^{-r_1^2N^2}Z_6(r_1)dr_1+Z_7(h_1)\right]\right\}
\end{aligned} \tag{59}$$

where:

$$Z_1(r_1)\equiv\int_{r_1N}^{h_1N}\chi Y_1(\chi)d\chi-r_1^2Y_1(\chi=h_1N) \quad , \quad Z_2(h_1)=\frac{1}{4}(1-e^{-h_1^2N^2})h_1^2Y_1(\chi=Nh_1) \tag{60}$$

$$Z_3(r_1)\equiv Y_1(\chi=r_1N)\int_0^{h_1N}Y_3(\chi)d\chi-Y_1(\chi=r_1N)\int_0^{r_1N}Y_3(\chi)d\chi-\int_{r_1N}^{h_1N}Y_1(\chi)Y_3(\chi)d\chi \tag{61}$$

$$Z_4(r_1)=\int_{r_1N}^{h_1N}Y_1(\chi)Y_3(\chi)Y(\chi)d\chi+Y_1(\chi=r_1N)\int_0^{r_1N}Y_1(\chi)Y_3(\chi)Y(\chi)d\chi \tag{62}$$

$$Z_5(h_1)\equiv\frac{1-e^{-h_1^2N^2}}{2N^2}Y_1(\chi=h_1N)\int_0^{h_1N}Y_1(\chi)Y_3(\chi)Y(\chi)d\chi \tag{63}$$

$$Z_6(r_1)\equiv\int_{r_1N}^{h_1N}Y_2(\chi)d\chi+Y_1(\chi=r_1N)\int_0^{h_1N}Y_2(\chi)\,\chi e^{-\chi^2}d\chi \tag{64}$$

$$Z_7(h_1)\equiv\frac{1-e^{-h_1^2N^2}}{2N^2}Y_1(\chi=h_1N)\int_0^{h_1N}Y_2(\chi)\,\chi e^{-\chi^2}d\chi \tag{65}$$

and : $0 \le r_2 \le r_1 \le h_1$, $0 \le \varphi < 2\pi c_1$, $0 \le c_1 < 1$, $0 \le \vartheta_1 < \pi/2$, $0 \le t_1 < \infty$
$0 \le \chi_1 \le \chi \le h_1N$, $0 \le N(t_1) = 0.5(Res/t_1)^{0.5} < \infty$

The modified Reynolds equation (59) determines an unknown function $p_{11}(\varphi, \vartheta_1, t_1)$ of the pressure corrections due to the viscoelastic properties of oil in unsteady conditions.

NUMERICAL CALCULATIONS

The dimensionless pressure distribution p_{10} and its dimensionless corrections p_{11}, p_{12} , ... are determined in the lubrication region Ω by virtue of the modified Reynolds equations (47) , (59) and by taking into account the gap height (53) , (54). On the boundary of the region Ω the dimensional pressure and its corrections have values of the atmospheric pressure p_{at}. The region Ω indicated as a section of the bowl of the sphere (Fig.3), is defined by the following inequalities: $\Omega : 0 \le \varphi \le \pi,\ \pi R/8 \le \vartheta \le \pi R/2$.

Numerical calculations were performed for :

- ✪ the radius of spherical journal R = 0.08 [m]
- ✪ the angular velocity of the perturbations of bearing sleeve $\omega_o = 0.2\ [s^{-1}]$

- the characteristic time $t_o = 0.001$ [s]
- the characteristic value of the radial clearance $\Psi \equiv \varepsilon/R = 0.001$.

And, the following bearing eccentricities $\Delta\varepsilon_1$=20 [μm] , $\Delta\varepsilon_2$= 2 [μm] , $\Delta\varepsilon_3$= 1[μm] , the oil viscosity η_o = 0.03 [Pas] , the pseudoviscosity coefficient β = 0.0006 [Pas^2], the oil density ρ = 950 [kg/m^3], the rotational velocity of the spherical journal, n = 1500 [rev/min], and the average minimum gap height ε_{min} = 4 [μm], were assumed.

The numerical calculations were performed by using the Mathcad 11 Program and the finite difference method. The obtained pressure distributions for the dimensionless time values $t_1 = 1$, $t_1 = 10$, $t_1 = 100$, $t_1 = 1000$, $t_1 = 10000$, $t_1 = \infty$, and $s = \pm 1/4$, are presented in Fig.4 and 5.

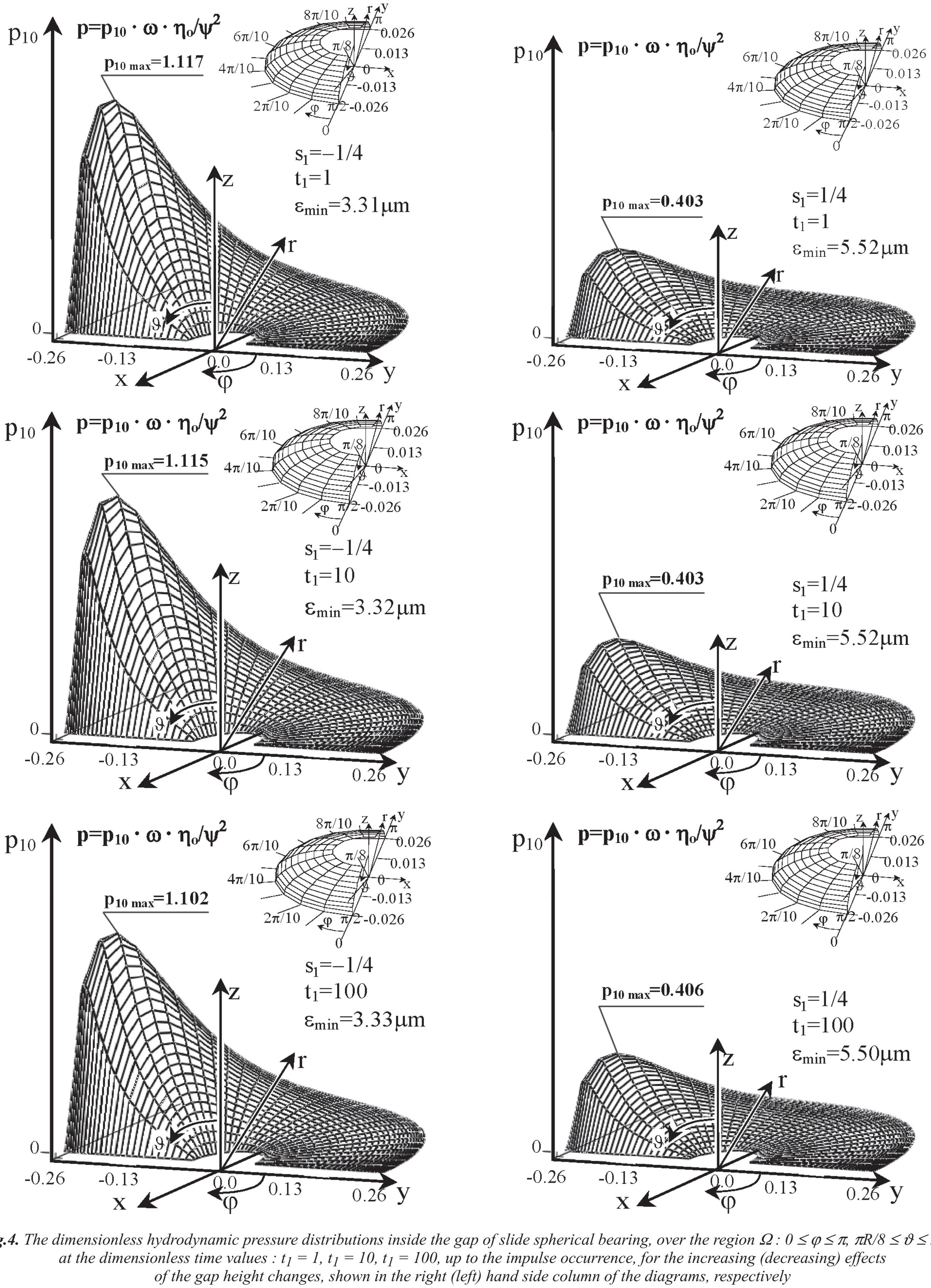


Fig.4. *The dimensionless hydrodynamic pressure distributions inside the gap of slide spherical bearing, over the region $\Omega : 0 \le \varphi \le \pi$, $\pi R/8 \le \vartheta \le \pi R/2$, at the dimensionless time values : $t_1 = 1$, $t_1 = 10$, $t_1 = 100$, up to the impulse occurrence, for the increasing (decreasing) effects of the gap height changes, shown in the right (left) hand side column of the diagrams, respectively*

In order to obtain real values of time it is necessary to multiply the dimensionless values t_1 by the characteristic time t_o = 0.001 s. For example t_1 = 1000 denotes the time of 1s after impulse occurrence. In order to obtain realistic dimensional pressure values the dimensionless pressure values indicated in Fig.4 and 5 are to be multiplied by the dimensional coefficient $UR\eta_o/\varepsilon^2$.

The pressure distributions shown on the right-hand sides of Fig.4 and 5 were obtained for the increasing of the gap height, caused by the impulse effects. In this case the longer the time up to the impulse, the more gap height decreases and pressure increases. The pressure distributions shown on the left-hand side of Fig.4 and 5 were obtained for the decreasing of the gap height, caused by the impulse effects. In this case the longer the time up to the impulse, the more gap height increases and pressure decreases.

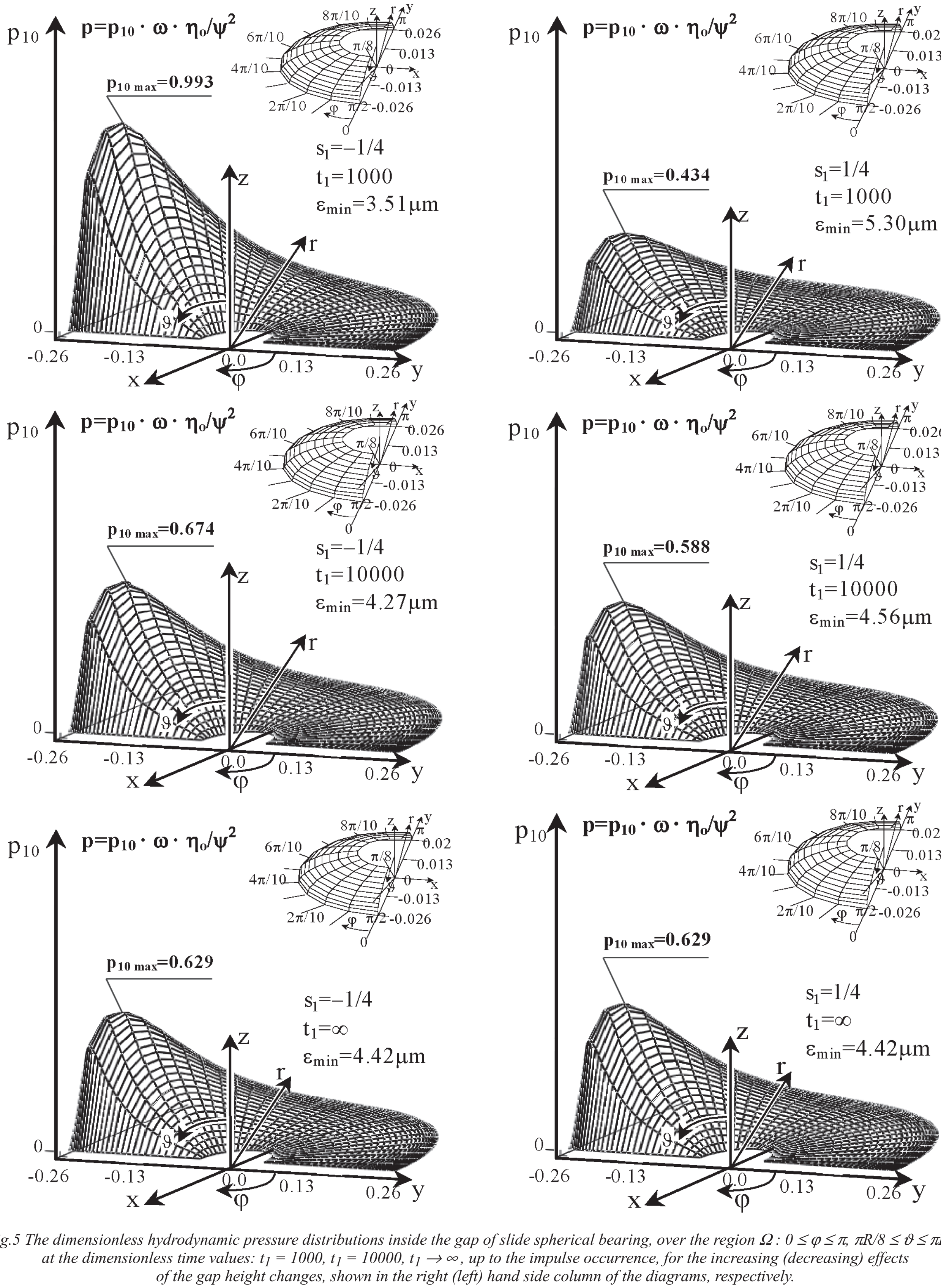


Fig.5 The dimensionless hydrodynamic pressure distributions inside the gap of slide spherical bearing, over the region $\Omega : 0 \le \varphi \le \pi$, $\pi R/8 \le \vartheta \le \pi R/2$, at the dimensionless time values: $t_1 = 1000$, $t_1 = 10000$, $t_1 \to \infty$, up to the impulse occurrence, for the increasing (decreasing) effects of the gap height changes, shown in the right (left) hand side column of the diagrams, respectively.

If the time interval up to the impulse occurrence is sufficiently large i.e. for $t_1 \to \infty$, then the pressure distributions for the increasing ($s_1 > 0$) and decreasing ($s_1 < 0$) effects of the gap height changes caused by the impulse, tend to the identical pressure distributions (Fig.5). Such limit pressure distribution can be also obtained from the classical Reynolds equation (52).

For the dimensionless time values : $t_1 = 1$, $t_1 = 10$, $t_1 = 100$, $t_1 = 1000$, $t_1 = 10\,000$, $t_1 = \infty$, i.e. for : t = 0.001s, t = 0.010 s, t = 0.100 s, t = 1.000 s, t = 10.000 s, t = ∞ s, after impulse occurrence the maximum pressure distributions for $s_1 < 0$ have the following dimensional values, respectively :

$$1.117\frac{\omega\eta_o}{\psi^2}\ ,\ 1.115\frac{\omega\eta_o}{\psi^2}\ ,\ 1.102\frac{\omega\eta_o}{\psi^2}\ ,\ 0.993\frac{\omega\eta_o}{\psi^2}\ ,\ 0.674\frac{\omega\eta_o}{\psi^2}\ ,\ 0.629\frac{\omega\eta_o}{\psi^2} \tag{66}$$

For the dimensionless time values : $t_1 = 1$, $t_1 = 10$, $t_1 = 100$, $t_1 = 1000$, $t_1 = 10\,000$, $t_1 = \infty$, i.e. for : t = 0.001s, t = 0.010 s, t = 0.100 s, t = 1.000 s, t = 10.000 s, t = ∞ s, after impulse occurrence the maximum pressure distributions for $s_1 > 0$ have the following dimensional values, respectively :

$$0.403\frac{\omega\eta_o}{\psi^2}\ ,\ 0.403\frac{\omega\eta_o}{\psi^2}\ ,\ 0.406\frac{\omega\eta_o}{\psi^2}\ ,\ 0.434\frac{\omega\eta_o}{\psi^2}\ ,\ 0.588\frac{\omega\eta_o}{\psi^2}\ ,\ 0.629\frac{\omega\eta_o}{\psi^2} \tag{67}$$

CONCLUSIONS

- The pressure distribution changes at the instant of impulse occurrence are caused mainly by the bearing gap height changes and viscoelastic oil properties.
- The gap height changes during impulsive motion and the viscoelastic oil properties may either increase or decrease the pressure distribution and load-carrying capacity of spherical bearings in contrast to those of the same bearing but free from impulse effects.
- The pressure distribution changes and load-carrying capacity values at the instant of impulse occurrence attain about 40 percent of those appearing in the spherical bearing free from impulse effects.
- The pressure distribution changes caused by the viscoelastic oil properties can attain only about 10 percent of those appearing in the instant of impulse occurrence.
- Just after the impulse occurrence the influences of the viscoelastic oil properties on the pressure and capacity changes quickly tend to zero. This is the moment in which the largest values of wear may be expected.

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NOMENCLATURE

a - acceleration vector [m/s]
$\mathbf{A}_1$ - strain tensor, [1/s]
$\mathbf{A}_2$ - dimensional tensor, [s^{-2}]
b_{m1}, b_{s1} - dimensionless origin and end coordinate of lubrication surface in meridianal direction
c_1 - dimensionless end coordinate of lubrication surface in circumferential direction
$C_{i1}, C_{i2}, C_{i3}, C_{i4}$ - integral constant
D - dimensional eccentricity, [m]
De - Deborah Number
Des = DeStr - Deborah and Strouhal numbers
erf - special integral function
h - gap height, [m]
h_o - dimensional gap height for spherical journal and spherical sleeve, [m]
h_1 - dimensionless gap height
h_{min} - average gap height minimum, [m]
$M = Nh_1$ - time depended dimensionless function
N - time depended dimensionless function
O - centre of the journal
O_s - centre of the sleeve
p - dimensional pressure, [Pa]
p_1 - dimensionless pressure
p_{10} - dimensionless pressure for Newtonian (classical) unsteady oil flow
$p_{11}, p_{12}, \ldots$ - dimensionless pressure corrections caused by viscoelastic oil properties in unsteady flow
r - dimensional radial coordinate, [m]
r_1 - dimensionless radial coordinate
R - radius of the journal, [m]
Re - Reynolds Number
s_1 - dimensionless coefficient of gap height changes caused by the impulse
S - stress tensor, [Pa]
Str - Strouhal Number
t - dimensional time, [s]
t_1 - dimensionless time
t_o - characteristic value of the dimensional time, [s]
U - peripheral velocity of spherical journal, [m/s]
v_r - dimensional oil velocity component in radial direction, [m/s]
v_{r1} - total dimensionless oil velocity component in radial direction
$v_{r0\Sigma}$ - dimensionless oil velocity component in radial direction for Newtonian (classical) unsteady oil flow
$v_{r1\Sigma}, v_{r2\Sigma}, \ldots$ - dimensionless corrections of oil velocity components in radial direction caused by the viscoelastic oil properties in unsteady flow
v_ϑ - dimensional oil velocity component in meridianal direction, [m/s]
$v_{\vartheta 1}$ - total dimensionless oil velocity component in meridianal direction
$v_{\vartheta 0\Sigma}$ - dimensionless oil velocity component in meridianal direction for Newtonian (classical) unsteady oil flow
$v_{\vartheta 1\Sigma}, v_{\vartheta 2\Sigma}, \ldots$ - dimensionless corrections of oil velocity components in meridianal direction caused by the viscoelastic oil properties in unsteady flow
v_φ - dimensional oil velocity component in circumferential direction, [m/s]
$v_{\varphi 1}$ - total dimensionless oil velocity component in circumferential direction
$v_{\varphi 0\Sigma}$ - dimensionless oil velocity component in circumferential direction for Newtonian (classical) unsteady oil flow
$v_{\varphi 1\Sigma}, v_{\varphi 2\Sigma}, \ldots$ - dimensionless corrections of oil velocity components in circumferential direction caused by the viscoelastic oil properties in unsteady flow
Y, Y_1, Y_2 - dimensionless functions
Z_1, Z_2 - dimensionless functions
α - pseudo viscosity coefficient, [Pas^2]
β - pseudo viscosity coefficient, [Pas^2]
ε - radial clearance, [m]
$\Delta\varepsilon_1, \Delta\varepsilon_2, \Delta\varepsilon_3$ - components of the sleeve centre, [m]
η_o - dynamic viscosity of the oil, [Pas]
ϑ - meridianal direction
φ - circumferential direction
χ - time depended dimensionless variable
ψ - dimensionless radial clearance
ω - angular velocity of the journal, [1/s]
ω_o - angular velocity of the impulsive changes caused by the perturbations in unsteady conditions, [1/s]
Ω - lubrication surface, [m^2]

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Conference

Hydroacoustics

From 25 to 28 May 2004 the yearly
21st Symposium on Hydroacoustics

was held in Jurata, a touristic resort at Hel Peninsula. It was organized under the auspices of the Acoustics Committee, Polish Academy of Sciences, and the Polish Acoustical Society. The symposium was hosted by Naval University of Gdynia and Gdańsk University of Technology.

The symposia in question have been aimed at providing an opportunity for direct exchange of experience and information among teams dealing with hydroacoustics and related subjects.

The scope of the 21st Symposium
covered the following items :

- acoustic wave propagation in sea water
- hydroacoustic noise
- non-linear acoustics in water environment
- ultrasonic transducers
- signal processing
- hydroacoustic devices and systems
- other related problems.

Presentation of 31 papers was performed during four plenary sessions and four topical sessions.The following papers were presented during the plenary sessions :

- *Golay's codes sequences in ultrasonography* – by A. Nowicki, I. Trots, W. Secomski, J. Litniewski (Institute of Fundamental Technological Research, Polish Academy of Scieces, Warszawa)
- *Acoustic reconnaisance of fish and environmental background in demersal zone in Southern Baltic* – by A. Orłowski (Sea Fisheries Institute, Gdynia)
- *Underwater ship passport* – by I. Gloza (Naval University of Gdynia)
- *Stability of mechanical and dielectric parameters in pzt based ceramics* – by J. Ilczuk, J. Bluszcz, R. Zachariasz (University of Silesia, Sosnowiec)
- *Directional sonobuoy system for detection of submarines* by R. Salamon (Gdańsk University of Technology).

Most of the presented papers were prepared by 47 authors representing 10 Polish universities and scientific centres, including the following : of Gdańsk University of Technology - 8 papers, of institutes of Polish Academy of Sciences - 6 papers, of Naval University of Gdynia - 5 papers. A group of foreign authors consisted of : 3 authors of Sevchenko Research Institute of Applied Physical Problems, Minsk, Belarus, 1 author of University of Victoria, Canada, 1 author of Institute of Applied Physics of Nizhny Novogorod, and 2 authors of Nizhny Novogorod State University, Russia, 1 author of Institute of Marine Sciences of Mersin, Turkey.