Semi-Markovian models of the process of technical state changes of technical objects

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ABSTRACT

The most important problem of operation of all technical objects (devices) is the problem of rational (optimum) decision control of the process of technical state changes of the objects. Such control can be realized when a model of the process of technical state changes is applied. In view of this, a formal description of the operational process of technical objects was presented in this paper, as well as it was justified that the model of the process may be a semi-Markovian process of finite set of states. Examples of 4-and 3-state semi--Markovian process were given as a model of the above mentioned process of state changes of any device. Difficulties associated with forming such model, which arise from application of the theory of semi-Markovian processes, were indicated. Therefore the proposed model is the semi-Markovian process whose values are technical states occurring in the phase of operation of many technical objects of practical importance (e.g. combustion piston engines, turbines, displacement and rotary compressors, impeller pumps etc).

Key words: model, technical object, semi-Markovian process, technical state

INTRODUCTION

The process of technical state changes of any technical object is the process of occurence of successive technical states casually linked during time *t*, which appear one by one in such a way that the successive state occurs after a determined duration time interval of the directly preceding state. The duration time intervals of particular states are random variables of continuous realizations and finite expected values.

The process of technical state changes of every technical object belongs to the most important processes occurring in the phase of its operation. Course of the process should be rational, i.e. that whose realization results from an assumed optimization criterion. Such criterion may be e.g. expected value of operational cost of a given technical object or instantaneous availablity factor. The last criterion is important when in any instant of operation of particular technical objects, t, a task can be assigned to them for realization of which their reliable operation is necessary [7 , 8]. The control which makes the optimum course of the process of occurrence of successive technical states possible, can be realized when the model of the process, which allows for application one of the decision theories, is elaborated. In the case of such technical objects as combustion piston engines and turbines, positive-displacement and axial-flow compressors two theories are of importance : the statistical decision theory and the theory of controlled semi--Markovian processes [8, 10, 11, 12, 13]. Recently the theory of controlled semi-Markovian processes is more and more often applied with success to solving different problems of durability, reliability and decision-based operational control of various technical objects. The theory may also find use in solving similar problems associated with operation of many technical objects including those dealing with control of the process of technical state changes of the objects. For this reason the semi--Markovian model of the process has been proposed and its practical applicability justified.

PREMISES FOR ELABORATION OF SEMI--MARKOVIAN MODEL OF TECHNICAL STATE CHANGES OF OBJECTS

From the definition of semi-Markovian process it results [12, 17, 20, 21] that such process is stochastic and of a discrete set of states, and its realizations are functions of constant intervals (i.e. those having uniform values within operation time intervals being random variables), and right-hand continuous ones. From the definition it also results that the process is determined when its initial distribution $P_i = P\{Y(0) = s_i\}$ as well as its matrix function $\mathbf{Q}(t) = [Q_{ij}]$ whose elements are probabilities of transfer of the process from the state s_i to the state s_j, during the time not greater than t ($i \neq j$; $i,j = 1, 2,..., k$), are known. The elements $Q_{ij}[(i \neq j \; ; \; i,j = 1, 2, ... , k)]$ of the matrix $\mathbf{Q}(t)$ are non-decreasing functions of the variable *t* [12 , 17]. Non-zero elements of the matrix can be interpreted as follows :

$$Q_{ij}(t)=P\{W(\tau_{n+1}) = s_j \;\; ; \;\; \tau_{n+1} - \tau_n < t \,|\, W(\tau_n) = s_i\} \;\; ;$$
$$s_i , s_j \in S \;\; ; \;\; i , j = 1, 2, 3, 4 \;\; ; \;\; i \neq j \qquad (1)$$

The semi-Markovian model of real process of occurrence of successive technical states of any technical object can be formed only when determination of states of the process is possible in such a way as to obtain duration time of the existing state in the instant τ_n as well as the state available in the instant τ_{n+1} , stochastically independent on the preceding states and their duration times [6 , 12]. Hence in the modelling which is supposed to lead to elaboration of a semi-Markovian model of the technical state changing process of given technical objects the analysis of changes of technical states occurring in real operational conditions of the objects should be accounted for.

In the case of such technical objects as self-ignition engines, piston compressors, impeller pumps etc it has been observed that prediction of technical state changes of the objects can be performed if actual technical state and future conditions of their operation are known [5, 6, 7]. This fact which simultaneously means that technical state changes of such objects are not strictly dependent on their time of operation, can be highlighted by means of the following hypothesis **H1** :

the technical state changing process of any technical object (*understood as a random time function whose values are random variables representing existing technical states*), **which occurs in a rational operation system** (*i.e. in such operation system where operational cost calculations are carried out*), **is the process of asymptotically independent values because its arbitrary state considered in any instant τ_n (n = 0 , 1 ,…, m ; $\tau_0 < \tau_1 < ... < \tau_m$) significantly depends on the state directly preceding it, but not on the states which have occurred earlier and their duration time intervals.**

It should be observed that the hypothesis does not contain any contradictions which could falsify it in a logical sense before testing it.

The consequences of the hypothesis are as follows [5 , 7] :

- the probabilities (p_{ij} ; $i \neq j$; $i,j \in N$) of transfer of the state changing process of any technical object from any actual state of the object, s_i, to any next state s_j do not depend on the states in which the process has been before
- the unconditional duration time intervals of the particular states s_i of the technical state changing process of a technical object are the random stochastically independent variables (T_i ; $i \in N$)
- the duration time intervals of any possible state s_i of the technical state changing process of a technical object, provided that the next state will be one of the remaining states s_j of the process, are the random stochastically independent variables (T_{ij} ; $i \neq j$; $i,j \in N$).

The above mentioned consequences reveal the probabilistic law of changing the technical states of any technical object. They are not contradictory to each other, and their logical veracity is doubtless. Therefore there is no obstacle to consider the consequences as one common consequence K1, in order to use it for empirical verification of the formulated hypothesis H1. Such verification consists in experimental investigation of veracity of the enumerated consequences taken as one common consequence K1.

The hypothesis can be supported by the more detailed hypothesis H2 dealing with all technical objects fitted with tribological units (e.g. self-ignition engines, piston compressors etc). The process of changing their technical states is that in which the duration time intervals of each of the states are random variables. Particular realizations of the random variables depend on many factors, a.o. on wear quantity of the tribological units of the objects. For the objects in question (e.g. self-ignition engines) it was observed that wear of their sliding tribological units is weakly correlated with time [3, 4, 9, 14, 18, 26]. The observation is important because serviceability of such objects mainly depend on technical state (i.e. wear) of their tribological units. This made it possible to predict technical state of such objects by taking into account only their current state as well as service conditions without accounting for the states occurred before. In order to highlight the fact the following hypothesis **H2** can be given :

the state of any sliding tribological unit of any technical object, as well as its duration time significantly depend on the preceding state but not on those occurred before and on their duration time intervals because its load and both wear rate and wear increments induced by the load are the processes of asymptotically independent values.

The statement contained in the hypothesis results from two obvious facts :

- ★ there is a close relationship between the loading of tribological units of different technical objects (e.g. self-ignition engines, piston compressors etc) and their wear [16, 18, 26, 27]
- ★ in a longer operation period of any technical object (e.g. self-ignition engines, piston compressors etc) there is no monotically increasing load changes of their tribological units, hence the service loading of the units can be assumed stationary [2, 4, 15, 22, 23, 24, 25].

In order to verify the presented hypothesis H2 and to determine if it is true, it is necessary to predict the consequences whose occurrence is possible to be empirically checked.

The consequences K2 which can be concluded from the hypothesis (with accounting for loading features of the objects and their sliding tribological units) were presented in [6].

Verification of the presented hypothesis H1 and H2 by means of experimental testing the veracity of the consequences K1 and K2 can be performed with the use of the same reasoning methods which were presented in [3, 4, 7].

VARIANTS OF SEMI-MARKOVIAN MODEL OF TECHNICAL STATE CHANGES OF OBJECTS

From the presented hypotheses H1 and H2 it results that models of the technical state changing process $\{W(t) : t \geq 0\}$ of many technical objects such as e.g. self-ignition engines can be the stochastical processes of discrete set of states and continuous time being the duration time of distinguished technical states of the objects. The considered models of the technical state changing process of any technical object can be mathematically expressed by the functions mapping the set of instants, $T \in R_+$, into the set of technical states, S. Hence to elaborate such model it is necessary to determine a finite set of technical state changes of the objects in question. Assuming the serviceability of technical objects as the criterion for distinguishing the states one can distinguish the set of classes (subsets) of technical states (shortly called „states"), S. The below given set can be deemed a set of the states of practical operational importance [10] :

$$S = \{s_{i\,;}\ i = 1, 2, 3, 4\} \tag{2}$$

which have the following interpretation :

- ♦ s_1 – the state of full serviceability, i.e. the technical state of any technical object, which makes it possible to use the object within the whole range of loads to which it was adjusted in the phases of its designing and manufacturing

- s_2 – the state of partial serviceability, i.e. the technical state of any technical object, which makes it possible to fulfil all its tasks (as in s_1) but at lower values of operational indices (e.g. at a lower overall efficiency, hence – in the case of self-ignition engine – at a greater specific fuel oil consumption)
- s_3 – the state of limited unserviceability, i.e. the technical state of any technical object, which makes it possible to fulfil only some of its tasks (e.g. such state which – in the case of self-ignition engine – precludes it from working in accordance with maximum continuous rating characteristics, and in the case of piston compressor – precludes it from filling the air receiver up to a required air pressure, etc)
- s_4 – the state of full unserviceability, i.e. the technical state of any technical object, which precludes the object from fulfilling any task of the set of the tasks to which it was adjusted in the phases of its designing and manufacturing (for instance such state of engine, which precludes it from work in accordance with maximum continuous rating characteristics.

Elements of the set $S = \{s_i ; i = 1, 2, 3, 4\}$ are values of the process $\{W(t) : t \geq 0\}$ composed of the states successively occurring one by one, $s_i \in S$, and being casually related to each other.

In the case of many technical objects (such as self-ignition combustion engines, gas turbines, piston compressors, etc,) the distinguishing of the states $s_i \in S$ $(i = 1, 2, 3, 4)$ is so much important because their use is crucial when they are in the state s_1 or in the state s_2. However in the second case the objects should be used for as-short-as-possible period only after which they should be renewed to bring them back to the state s_1. The states are values of the process $W(t) : t \geq 0\}$which is fully determined if its functional matrix is known [9 , 12] :

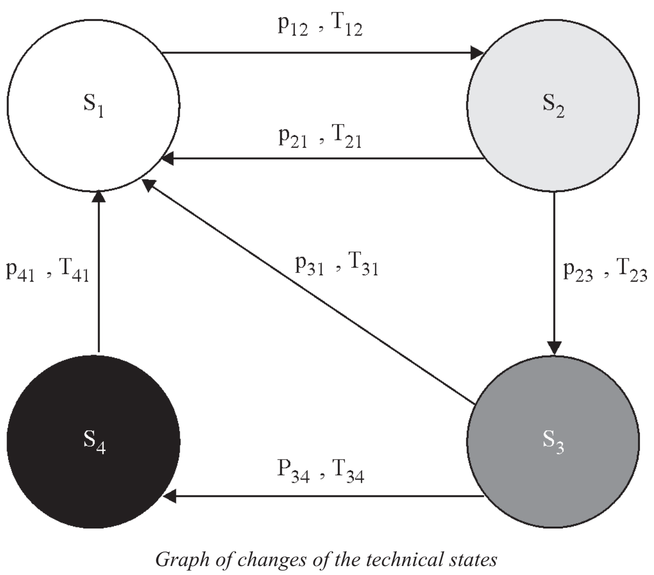
$$\mathbf{Q}(t) = [Q_{ij}(t)] \tag{3}$$

as well as its initial distribution is given :

$$p_i = P\{W(0) = s_i\} \qquad s_i \in S \; ; \; i = 1, 2, 3, 4 \tag{4}$$

Depending on an assumed strategy of maintaining the technical objects in the technical states making it possible to realize their tasks, diferent realization variants of the process $\{W(t) : t \geq 0\}$ may be taken into account.

In the first variant a technical state change can occur in compliance with the graph of technical state changes presented in the Figure.



Graph of changes of the technical states $s_i \in S(i = 1, 2, 3, 4)$ of the process $\{W(t): t \geq 0\}$

The following initial distribution of the process $\{W(t) : t \geq 0\}$ can be assumed :

$$p_1 = P\{W(0) = s_1\}=1$$
$$p_i = P\{W(0) = s_i\} = 0 \quad \text{for} \quad i = 2, 3, 4 \tag{5}$$

and its functional matrix in the following form complying with the graph of Figure :

$$\mathbf{Q}(t) = \begin{bmatrix} 0 & Q_{12}(t) & 0 & 0 \\ Q_{21}(t) & 0 & Q_{23}(t) & 0 \\ Q_{31}(t) & 0 & 0 & Q_{34}(t) \\ Q_{41}(t) & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

The matrix (6) represents changes of the states $s_i \in S(i = 1, 2, 3, 4)$ of the process $\{W(t) : t \geq 0\}$. The probabilities $P_j(j = 1, 2, 3, 4)$ of the event that a given technical object will be in the states $s_i \in S(i = 1, 2, 3, 4)$ determine a chance of fulfilling the task by the object. It is obvious that the user of every technical object is interested in the object to be in the state s_1 as long as possible. The chance of lasting any technical object in this state is determined by the probability determined for an appropriately long time of operation. It means that the limiting distribution of the process should be determined.

From the semi-Markovian process theory it results [12, 17, 21] that the probabilities of changes of states of any technical object are determined by the probabilities p_{ij} of the Markov's chain $\{W(\tau_n) : n = 0, 1, 2,...\}$ introduced into the process $\{W(t) : t \geq 0\}$. The probabilities form the following matrix of transfer probabilities :

$$\mathbf{P} = [p_{ij} \;\; ; \;\; i,j = 1, 2, 3, 4] \tag{7}$$

where :

$$p_{ij} = P\{W(\tau_{n+1}) = s_j \mid W(\tau_n) = s_i\} = \lim_{t\to\infty} Q_{ij}(t)$$

The matrix (7) makes determining the limiting distribution of the process $\{W(t) : t \geq 0$ possible. From the matrix (6) it results that the matrix (7) has the following form :

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ p_{21} & 0 & p_{23} & 0 \\ p_{31} & 0 & 0 & p_{34} \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

From the theorem given in [12] it results that the limiting distribution of the considered process exists :

$$P_j = \lim_{t\to\infty} P\{W(t) = s_j\} = $$
$$= \lim_{t\to\infty} P\{W(t) = s_j / W(0) = s_i\} \tag{9}$$

which is determined by the expression :

$$P_j = \frac{\pi_j E(T_j)}{\sum_{k=1}^{4} \pi_k E(T_k)} \quad ; \quad j = 1, 2, 3, 4 \tag{10}$$

and the limiting distribution π_j $(j = 1, 2, 3, 4)$ of the introduced Markov's chain $\{W(\tau_n) : n = 0, 1, 2,...\}$ fulfils the following equations :

$$[\pi_1, \pi_2, \pi_3, \pi_4] \begin{bmatrix} 0 & 1 & 0 & 0 \\ p_{21} & 0 & p_{23} & 0 \\ p_{31} & 0 & 0 & p_{34} \\ 1 & 0 & 0 & 0 \end{bmatrix} = [\pi_1, \pi_2, \pi_3, \pi_4] \tag{11}$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \qquad (12)$$

Making use of the relationships (10) ÷ (12) one obtains the following formulae :

$$P_1 = E(T_1)M^{-1} \quad ; \quad P_2 = E(T_2)M^{-1}$$
$$P_3 = p_{23}E(T_3)M^{-1} \quad ; \quad P_4 = p_{23}p_{34}E(T_4)M^{-1} \qquad (13)$$

and :

$$M = E(T_1) + E(T_2) + p_{23}E(T_3) + p_{23}p_{34}E(T_4)$$

where :

$E(T_j)$ - expected value of duration time of the state $s_j \in S$ (j = 1, 2, 3, 4)
p_{ij} - probability of transfer of the process $\{W(t) : t \geq 0\}$ from the state s_i to the state s_j ($s_i , s_j \in S$; i,j = 1, 2, 3, 4 ; i ≠ j).

The particular probabilities P_j(j = 1, 2, 3, 4) given by the formulae (13) have the following interpretation :

$$P_1 = \lim_{t\to\infty} P\{W(t) = s_1\} \quad ; \quad P_2 = \lim_{t\to\infty} P\{W(t) = s_2\}$$
$$P_3 = \lim_{t\to\infty} P\{W(t) = s_3\} \quad ; \quad P_4 = \lim_{t\to\infty} P\{W(t) = s_4\}$$

In the presented variant, such situations are accounted for in which the user is able to risk to fulfil his task in the state s_2 of the technical object and even to risk to fulfil some tasks in the state s_3 of the object.

The second variant deals with the case when, within an assumed operational strategy of technical objects, the distinguishing between the states s_1 and s_2 is of no importance. Then it is possible to consider the simpler process $W(t) : t \geq 0\}$ of technical state changes of the objects, namely the model having the set of states :

$$S = \{s_1, s_2, s_3\} \qquad (14)$$

as well as their following interpretation [6, 7, 8] :

* the state of full serviceability, s_1, which makes it possible to use the object in any conditions and in any range of loads, to which it was adjusted in the phases of its designing and manufacturing
* the state of partial serviceability, s_2, which makes it possible to use the object in limited conditions and in a range of loads lower than those to which it was adjusted in the phases of its designing and manufacturing
* the state of unserviceability, s_3, which does not make it possible to use the object in accordance with the purpose it was intended for (e.g due to its failure, performing maintenance operations on its subassemblies etc).

A graph of technical state changes of such process $W(t) : t \geq 0\}$ of objects of the kind as well as its example course were presented in [6]. Hence the process is the three-state one of continuous realizations (i.e. time-continuous). It may be assumed that if anyone of the states s_2 or s_3 does not occur then every technical object remains in the state s_1. And, the set of the technical states $S = \{s_1, s_2, s_3\}$ can be considered as the set of values of the stochastic process $\{W(t) : t \geq 0\}$ of realizations constant within intervals, and being right-hand continuous.

The initial distribution of the considered process having transfer graph [6 , 8] is given by the formula :

$$P_i = P\{W(0) = s_i\} = \begin{cases} 1 & \text{for } i = 1 \\ 0 & \text{for } i = 2,3 \end{cases} \qquad (15)$$

and its functional matrix, if the function $Q_{32}(t)$ is different from zero ($Q_{32}(t) \neq 0$), is the following :

$$\mathbf{Q}(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & Q_{23}(t) \\ Q_{31}(t) & Q_{32}(t) & 0 \end{bmatrix} \qquad (16)$$

In operational practice of technical objects any partial renewal strategy of their technical states should not be applied. Use of such strategy may lead to occurrence of the unserviceability state s_3 and, as a result, – e.g. to occurrence of a high economical loss. It means that renewals can be applied in special cases only, e.g. those resulting from impossibility to discontinue realization of the undertaken task and to complete the technical state renewal of the object. Therefore operation process of every technical object may be rational when the function $Q_{32}(t) = 0$. In this case the matrix (16) obtains the form :

$$\mathbf{Q}(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) \\ Q_{21}(t) & 0 & Q_{23}(t) \\ Q_{31}(t) & 0 & 0 \end{bmatrix} \qquad (17)$$

And, like in the case of the earlier considered processes, for the presented process $\{W(t) : t \geq 0\}$ having the functional matrix given by (17), the following limiting distribution can be determined :

$$P_1 = \frac{E(T_1)}{H} \; ; \; P_2 = \frac{p_{12}E(T_2)}{H}$$
$$P_3 = \frac{(1 - p_{12}p_{21})(E(T_3)}{H} \qquad (18)$$

and:

$$H = E(T_1) + p_{12}E(T_2) + (1 - p_{12}p_{21})E(T_3)$$

where :

P_1, P_2, P_3 - probabilities of the event that a given technical object is in the state : s_1, s_2, s_3, respectively
p_{ij} - probability of transfer of the process $\{W(t) : t \geq 0\}$ from the state s_i to the state s_j
$E(T_j)$ - expected value of duration time of the state s_j.

The particular probabilities P_j(j = 1, 2, 3, 4) given by (18) can be interpreted as follows :

$$P_1 = \lim_{t\to\infty} P\{W(t) = s_1\} \quad ; \quad P_2 = \lim_{t\to\infty} P\{W(t) = s_2\}$$
$$P_3 = \lim_{t\to\infty} P\{W(t) = s_3\}$$

The presented probabilities which determine possible occurrence of particular states of any technical object, are essential in making operational decisions [1, 7, 8, 10, 13].

SUMMARY

- In the presented considerations it is shown that the process of technical state changes of different technical objects (self--ignition engines, gas turbines, piston compressors, impeller pumps etc) is a process continuous over time and states. Because technical state of every technical object is subject to continuous changes hence it is possible to consider the sets composed of infinite number of technical states.
- Identification of all technical states of any technical object is neither possible nor purposeful for both technical and economical reasons. Therefore a need arises to split the set of technical states into a finite number of classes (subsets) of technical states.

- Assuming the serviceability of technical objects to be a splitting criterion one can distinguish (in the simplest case) the following classes of of their technical states : the full serviceability state s_1, the partial serviceability state s_2, and the unserviceability state s_3.
- The set of the states $S = \{s_1, s_2, s_3\}$ may be considered as the set of values of the simplest stochastic process $\{W(t) : t \geq 0\}$ whose realizations are constant within intervals and right-hand continuous. The process, a model of real process of technical state changes of objects, is mathematically described by a function mapping the set of the instants T ($T \in R_+$) into the set of technical states S. Elaboration of such a model adequate to the process of technical state changes of the objects is indispensable for rational control of the process.
- The model presented in the form of the stochastic process $\{W(t) : t \geq 0\}$ is the simplest and fulfilling the two conditions :
  - it works like the original, i.e. it realizes analogical functions
  - it makes it possible to reveal – on the basis of analysis of its structure and functioning mode – the new, hidden features of the processes of technical state changes of the objects in question, which are represented by this model, namely : the reliability indices expressed by the probabilities, P_j, of lasting the process in distinguished states.
- From the presented hypotheses it results that the process of technical state changes of the objects in question can be investigated by means of the models formed as semi-Markovian processes. Therefore the process $\{W(t) : t \geq 0\}$ can be considered as a semi-Markovian process of real processes of technical state changes of many technical objects.
- The hypotheses should be verified for each kind of technical objects, e.g. self-ignition engines, piston compressors, impeller pumps etc, as it may happen that for a given kind of technical objects operating in assumed service conditions it would not be possible to use the models of technical state changes in the form of semi-Markovian processes.

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