Roll response of ship fitted with passive stabilizing tank

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ABSTRACT

Physical and mathematical models of roll motions of a ship equipped with a roll stabilizing tank of working liquid free surface, is presented. Elaboration of the physical model was based on the idea of two mutually coupled mathematical pendulae. On the basis of the physical model, motion equations of the ship with the tank were determined and solved. A way of using the achieved solutions is shown, as well as calculation formulae for coefficients of the motion equations, directly related to the main parameters of the ship and tank, are presented. Such form of the coefficients enhances possiblility of application of the equations and their solutions in ship design practice. Some examples of the use of the solutions for analysis of stabilizing effectiveness of a designed tank for a given ship, are also attached. Moreover, guidelines for correct design of the stabilizing tanks having free surface of liquid, based on the proposed physical model of the ship-tank system, are offerred. It is also indicated that on the basis of the presented results it would be possible to search for ways to make operation of the stabilizing tanks in question more effective.

Key words : seaworthiness, ship safety, ship hydromechanics, stability of floating units

INTRODUCTION

Out of the oscillation motions performed by a ship in waves the side roll (heeling) undergoes stabilization most often. The stabilization means first of all a limitation of roll amplitude values, sometimes associated with increasing its period. The stabilizing tanks of free surface of liquid, which are further considered in this paper, are simple devices intended for limiting roll motions.

A disadvantage of the existing methods describing work of stabilizing tanks is their low usefulness for designing. The equations applied in the methods are rather complicated [8, 10, 12, 13 , 14] as they are usually aimed at correct describing real motions of ship fitted with tank, and not at achieving practical design recommendations.

In this paper is presented such physical model of the ship with stabilizing tank under rolling and mathematical model based on it, as to obtain motion equations of a possibly simple form more useful for formulating important design recommendations. Coefficients of such equations are directly associated with ship and tank parameters. The obtained form of solutions makes it possible to get access to these parameters of ship and tank whose selection is crucial for ensuring expected stabilizing effects.

The simplifications introduced to obtain an appropriate form of the equations and their solutions do not impair to an evident degree the quality of the description of ship motions, based on them.

FREE - SURFACE STABILIZING TANKS

The basic types of free-surface stabilizing tanks presented in Fig.1 , 2, and Fig.3 and 4 exemplify their possible location on ships. An appropriate adjustment of natural period of motions of liquid contained in tank to that of rolling ship or of wave acting on ship, is of a dominant influence on stabilizing effect. The way of changing the motion period of liquid in such tanks consists in changing level of the liquid, which also leads to changing its amount.

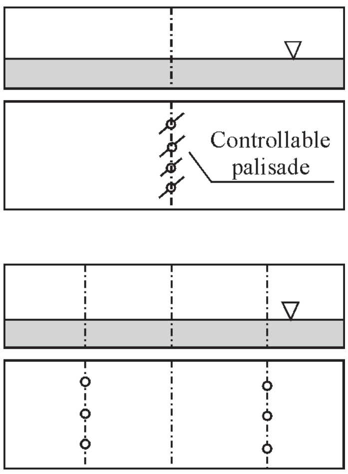
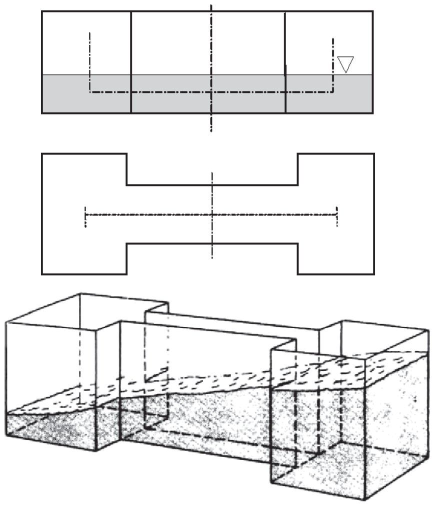
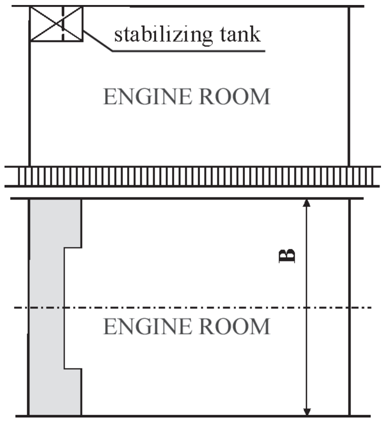


Fig. 1. *Types of cubicoidal stabilizing tanks [16]*



Rys. 2. *"Flume" stabilizing tank*



Rys. 3. *Location of "Flume" tank onboard a dry cargo ship*

Rys. 4. *Stabilizing tank in the form of container installed on deck of a fishing ship [2]*

Behaviour of a ship with stabilizing tank can be limited to consideration of motions of the system of two degrees of freedom (DOF), namely : for ship – side rolling angles, for tank – – translations of centre of mass of the liquid contained in it. The liquid mass centre translation can be represented by the average slope angle of its surface relative to ship, against its initial position. Hence description of motions of the ship-tank system amounts to the known problem of motion of a two-DOF system, whose solutions are based on the Lagrange 2nd kind equations. However in order to form such equations a "physical" model of the ship-tank system, i.e. physical representation of the ship, of the tank and their mutual coupling should be first determined. A form of external excitation of motions of the system, due to wave action on ship, should be also given.

PHYSICAL MODEL OF SHIP - TANK SYSTEM

A rolling ship is usually represented by a physical pendulum of an inclination moment dependent on ship's metacentric height. The mathematical pendulum is the simplest and lightest out of possible physical ones of a given inclination moment and period. The system of two mutually coupled pendulae is a known mechanical analogy of the two-DOF oscillating systems. It was assumed that the ship with stabilizing tank can be represented by the system of two mathematical pendulae appropriately selected and mutually coupled. The main oscillation axis of the pendulae corresponds to location of ship's rolling axis. The location of ship's rolling axis was assumed constant, known and not necessarily identical with that of ship's mass centre [1]. A schematic diagram of the physical model is shown in Fig.7, whereas Fig.5 and 6 highlight its association with the main parameters of ship and tank. The necessary modelling principles according to which the elements of the pendulae and their coupling parameters have been determined, are given by the formulae (1) below. The main coupling parameter of the pendulae is the distance between their rotation axes; i.e. the segment $\overline{O'_S m'}$ in Fig.7. The point O'_S represents the location of the ship's rolling axis, O_S.

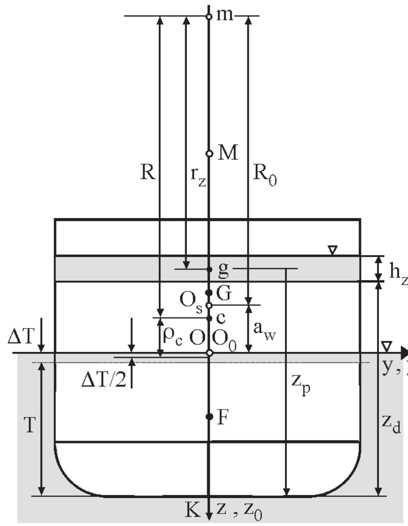


Fig. 5. *Ship with stabilizing tank in the upright position*

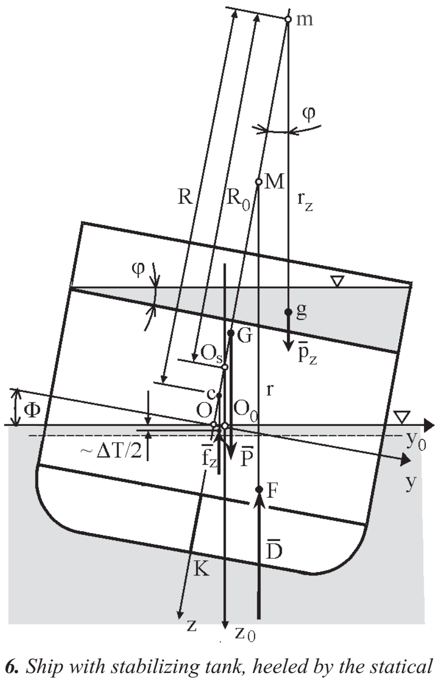
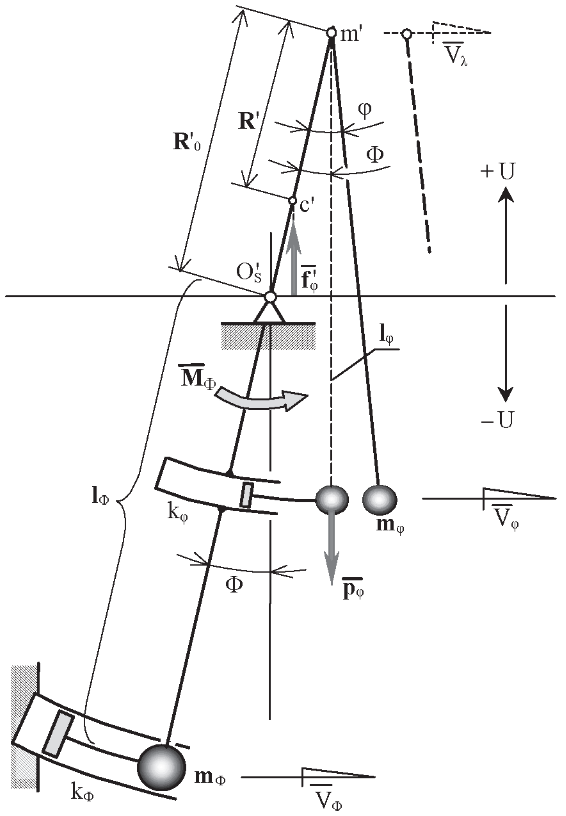


Fig. 6. *Ship with stabilizing tank, heeled by the statical angle φ*



Modelling principles

$$l_\Phi = \frac{\tau_\Phi^2 g}{4\pi^2} \qquad l_\varphi = \frac{\tau_\varphi^2 g}{4\pi^2}$$

$$m_\Phi = \frac{m_s h_s}{l_\Phi} \qquad m_\varphi = \frac{m_z r_z}{l_\varphi}$$

$$R' = R\frac{m_z}{m_\varphi} \qquad R'_0 = R_0\sqrt{\frac{l_\varphi}{r_z}}$$

$$\overline{f'_\varphi} = -m_\varphi g$$

$$M_\Phi = Dh\alpha_m \sin\omega_\lambda t$$

(1)

***Fig. 7.** Physical model of a ship with stabilizing tank*

Modelling principles

The system of pendulae shown in Fig.7 represents a physical model of a ship with stabilizing tank having free surface of liquid. The model can be deemed correct if statical and dynamical response of the system to external excitation moment is the same as that for the real ship with tank. For the pendulum of ship and tank the following can be written :

$$l_\Phi = \frac{r_{sm}^2}{h_s} \quad \text{or} \quad l_\Phi = \frac{\tau_\Phi^2\, g}{4\pi^2} \tag{2}$$

$$l_\varphi = \frac{r_{zm}^2}{r_z} \quad \text{or} \quad l_\varphi = \frac{\tau_\varphi^2\, g}{4\pi^2} \tag{3}$$

where :

- τ_Φ - period of natural oscillation of ship's mathematical pendulum, and of ship itself
- l_Φ - length of ship mathematical pendulum
- r_{sm} - radius of ship mass inertia
- r_{zm} - radius of mass inertia of liquid in tank
- r_z - metacentric radius of tank
- g - acceleration of gravity
- h_s - initial metacentric height of ship without tank, $\overline{GM_s}$
- τ_φ - period of natural oscillation of tank mathematical pendulum, and of motion of liquid in the tank
- $l\varphi$ - length of tank mathematical pendulum.

The static condition can be expressed by the relationships (4) and (5). The expressions (4) concern the equality of the moments due to heeling for the ship itself, the tank itself and, respectively, for the pendulae being their physical models. By using them the value of the mass m_Φ - for the ship mathematical pendulum, and of the mass m_φ - for the tank pendulum, can be obtained :

$$m_s h_s = m_\Phi\, l_\Phi \;,\; m_\Phi = \frac{m_s\, h_s}{l_\Phi}$$
$$m_z\, r_z = m_\varphi\, l_\varphi \;,\; m_\varphi = \frac{m_z\, r_z}{l_\varphi} \tag{4}$$

where :

- m_s - ship mass (without tank)
- m_z - tank mass, i.e. mass of liquid contained in the tank.

The total static condition has to account for similarity of the righting moments for the heeled ship with tank and for the system of pendulae inclined by the same angle. Rigidity of the ship with tank and of the pendulae can be expressed by using the quantities shown in Fig.5, Fig.6 and Fig.7, which provides the following equality :

$$m_s h_s - m_z R = m_\Phi l_\Phi - m_\varphi R' = (m_s + m_z)h - m_z r_z \tag{5}$$

where :

- $R = \overline{mc}$ - the distance between the tank metacentre and the metacentre of the ship's immersed volume increment ΔV
- $R' = \overline{m'c'}$ - the distance, in the physical model , which corresponds to R of the ship
- h - initial metacentric height of the ship with tank without any correction for liquid free surface
- $m_z r_z = i_x\rho$ - correction for free surface of liquid contained in tank.

To the system of pendulae the same moment due to water in tank must be applied as that applied to the ship. Knowing that : $m_s h_s = m_\Phi l_\Phi$ one obtains, acc.to (5) : $m_z R = m_\varphi R'$. It corresponds to the moment of couple of forces. For the ship with tank these are the forces : $\overline{p}_z = \overline{f}_z = m_z\overline{g}$ (Fig.6), and respectively for the system of pendulae $\overline{p_\varphi} = \overline{f_\varphi'} = m_\varphi \overline{g}$ (Fig.7).

Hence in the physical model the value of R' amounts to :

$$R' = R\,\frac{m_z}{m_\varphi} \quad , \quad R = z_p + r_z - T - \Delta T/2 - \rho_c \qquad (6)$$
$$\rho_c = \Delta I_{wx} / \Delta V$$

where :

ρ_c - metacentric radius of displacement layer of ΔT in thickness [7]

ΔI_{wx} - increment of transverse inertia moment of waterplane area for a given ΔT.

The dynamic condition first of all consists in the equality of the inertia moment of the ship and that of its pendulum, as well as the mass inertia moment of liquid in the tank and that of its pendulum. Moreover has to be fulfilled the equality of kinetic energy of the ship with tank and that of the system of pendulae when the same roll inducing moments are applied to them.

Hence the following is obtained for the ship and tank :

$$m_s r_{sm}^2 = m_\Phi l_\Phi^2 \quad , \quad m_z r_{zm}^2 = m_\varphi l_\varphi^2 \qquad (7)$$

All the elements in that expression have been already determined by the statical condition. By means of the expressions (2), (3) and (4) it is easy to check that the above given relationship is satisfied. Kinetic energy of the ship with tank will be equal to that of the system of pendulae, if the following relationship is satisfied :

$$m_s r_{sm}^2 + m_z R_0^2 + m_z r_{zm}^2 = m_\Phi l_\Phi^2 + m_\varphi R_0'^2 + m_\varphi l_\varphi^2$$

where :

R_0 - the distance between tank's metacentre and ship's rolling axis, $\overline{mO_s}$

R_0' - the distance between the suspension point of the tank mathematical pendulum and the oscillation axis of the system of pendulae, $\overline{m'O'_s}$.

From (7) it results: $m_z R_0^2 = m_\varphi R_0'^2$, hence, after taking into account (4), one obtains the following expression for R_0' :

$$R_0' = R_0 \sqrt{\frac{l_\varphi}{r_z}} \quad \text{or :} \quad R_0' = R_0 \frac{r_{zm}}{r_z} \qquad (8)$$
$$R_0 = z_p + r_z - T - \Delta T + a_w$$

The quantity a_w appearing in (8) determines the distance of the rolling axis from the waterplane (Fig. 5) [1]. It concerns the draft T and the mass centre location coordinate z_G for the ship with "frozen" liquid in the tank.

General information on effects of operation of a tank selected for a given ship can be achieved on the basis of the roll trasfer function of ship with tank. Therefore the roll is induced by the heeling moment resulting from the action of regular plane beam wave on the motionless ship. Wave frequencies to be selected should properly cover the whole range of encounter frequencies possible to occur in service of the given ship. The roll inducing moment due to regular plane wave is of the following form [3,11] :

$$M_\lambda = D\, h\, \alpha_m \sin \omega_\lambda t \quad , \quad \alpha_m = \kappa_B\, \kappa_T\, \alpha_\lambda \qquad (9)$$

where :

M_λ - ship roll inducing moment

D - ship buoyance force

h - ship metacentric height

α_λ , α_m - amplitude and effective amplitude of wave slope angle, respectively

ω_λ - wave frequency

t - time

κ_B , κ_T - wave slope angle corrective coefficients dependent on B/λ i T/λ , where λ stands for wave length.

In the expression (9) the buoyance force D and metacentric height *h* concern the ship with tank. It may take also another form if influence of sway on motions of liquid in the tank is accounted for. To take into account the sway it was assumed that it influence solely motions of liquid in the tank [15], and is equivalent to the horizontal component of orbital motion of the ship in real waves. The radius of the motion is equal to a half of wave height associated with the amplitude of its effective slope angle, α_m. And, for the horizontal oscillations the following is valid :

$$y_\lambda(t) = -\frac{H_m}{2}\alpha_\lambda(t) \quad , \quad \alpha_\lambda(t) = \alpha_m \sin\omega_\lambda t$$

therefore :

$$\dot{y}_\lambda(t) \equiv V_\lambda = -\frac{g}{\omega_\lambda}\alpha_m \cos\omega_\lambda t \qquad (10)$$

where :

$\dot{y}_\lambda$ - ship sway

H_m - wave height corresponding to the effective slope angle α_m

V_λ - horizontal transverse component of ship sway-induced velocity.

In the physical model the accounting for sway is equivalent to the applying of the horizontal oscillations V_λ, to the pivoting axis of the tank pendulum (Fig.7). The so obtained motion equations of the tank mathematical pendulum, at neglecting the damping, are as follows :

$$\ddot{\varphi} + \frac{g}{l_\varphi}\varphi = \frac{H_m}{2\,l_\varphi}\omega_\lambda^2 \sin\omega_\lambda t$$
$$\ddot{\varphi} + \omega_\varphi^2 \varphi = \omega_\varphi^2 \alpha_m \sin\omega_\lambda t \qquad (11)$$

It can be observed that instead to take into account ship sway in the physical model it is possible to apply, to the tank pendulum, the excitation moment whose normalized form is given by the right-hand side of the relationship (11). The same value of the moment was used in compliance with the Watanabe's method, in [15].

MATHEMATICAL MODEL

The mathematical model is equivalent to the set of motion equations of ship with stabilizing tank, achieved on the basis of the above presented physical model. A searched solution is the roll transfer function further used for analyzing the behaviour of ship with tank. Solutions are searched for within the frame of linear approach.

Derivation of motion equations

The Lagrange 2nd kind equations are used for derivation of motion equations :

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_j}\right) - \frac{\partial E}{\partial q_j} + \frac{\partial \theta}{\partial \dot{q}_j} + \frac{\partial U}{\partial q_j} = M_{q_j} \qquad (12)$$

In the equation the angle Φ - for the ship's pendulum and the angle φ - for the tank's pendulum corresponds , in the physical model, to the quantities q_j , namely q_1 and q_2 , respectively [9, 12, 13].

Particular kinds of energy considered in the physical model are the following : E - kinetic energy, U - potential energy and Θ - dissipation energy. In Fig.7, the horizontal line passing through the axis of the system of pendulae represents the plane of zero- value potential energy of the system.

The kinetic energy of the system of pendulae is of the form :

$$\begin{aligned} E = {} & m_\Phi l_\Phi^2 \frac{\dot\Phi^2}{2} + m_\varphi (l_\varphi - R'_0)^2 \frac{\dot\Phi^2}{2} + \\ & + m_\varphi (l_\varphi - R'_0) l_\varphi \dot\Phi \dot\varphi + m_\varphi (l_\varphi - R'_0) \dot\Phi V_\lambda + \\ & + m_\varphi l_\lambda \dot\varphi V_\lambda + m_\varphi l_\varphi^2 \frac{\dot\varphi^2}{2} + m_\varphi \frac{V_\lambda^2}{2} \end{aligned} \tag{13}$$

The potential and dissipation energies are as follows :

$$\begin{aligned} U = {} & [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] \frac{\Phi^2}{2} + \\ & + m_\varphi g l_\varphi \Phi \varphi + m_\varphi g l_\varphi \frac{\varphi^2}{2} \end{aligned} \tag{14}$$

$$\Theta = \frac{1}{2} k_\Phi \dot\Phi^2 + \frac{1}{2} k_\varphi \dot\varphi^2 \tag{15}$$

The physical model of tank and ship represents a two-DOF object, hence the expression (12) is the set of two equations. One of them is the equation of derivatives of relevant energies respective to Φ, (q_1), and another - respective to φ, (q_2). The wave-induced moment $M_{qj} \equiv M_{q1} = M_\lambda$, complying with the equation, is introduced to the right-hand side of the ship roll equation. As the passive tank is considered, $M_{qj} \equiv M_{q2} = 0$ is introduced to the right-hand side of the equation of motions of liquid in the tank. After calculation of energy differentials, replacements and ordering the equations, one obtains :

$$\begin{cases} [m_\Phi l_\Phi^2 + m_\varphi (R'_0 - l_\varphi)^2] \ddot\Phi + k_\Phi \dot\Phi + \\ \quad + [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] \Phi + \\ \quad - m_\varphi (R'_0 - l_\varphi) l_\varphi \ddot\varphi + m_\varphi g l_\varphi \varphi + \\ \quad - m_\varphi (R'_0 - l_\varphi) \dot V_\lambda = D h \alpha_m \sin \omega_\lambda t \\ m_\varphi l_\varphi^2 \ddot\varphi + k_\varphi \dot\varphi + m_\varphi g l_\varphi \varphi - m_\varphi (R'_0 - l_\varphi) l_\varphi \ddot\Phi + \\ \quad + m_\varphi g l_\varphi \Phi + m_\varphi l_\varphi \dot V_\lambda = 0 \end{cases} \tag{16}$$

The final form of the equations is obtained by introducing the value $\dot V_\lambda = g\alpha_m \sin\omega_\lambda t$ into (16), with making use of (10).

Therefore :

$$\begin{cases} [m_\Phi l_\Phi^2 + m_\varphi (R'_0 - l_\varphi)^2] \ddot\Phi + k_\Phi \dot\Phi + \\ \quad + [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] \Phi - m_\varphi (R'_0 - l_\varphi) l_\varphi \ddot\varphi + \\ \quad + m_\varphi g l_\varphi \varphi = [D h + m_\varphi g (R'_0 - l_\varphi)] \alpha_m \sin \omega_\lambda t \\ m_\varphi l_\varphi^2 \ddot\varphi + k_\varphi \dot\varphi + m_\varphi g l_\varphi \varphi - m_\varphi (R'_0 - l_\varphi) l_\varphi \ddot\Phi + \\ \quad + m_\varphi g l_\varphi \Phi = - m_\varphi l_\varphi g \alpha_m \sin \omega_\lambda t \end{cases} \tag{17}$$

Under the made assumptions the expression (17) represents the full form of the roll motion equations of the ship fitted with passive stabilizing tank. The only simplification represents not taking into account the mutual influence of ship's oscillations and motions of liquid in tank, due to damping. If an influence of sway is neglected the motion equations take the following form :

$$\begin{cases} [m_\Phi l_\Phi^2 + m_\varphi (R'_0 - l_\varphi)^2] \ddot\Phi + k_\Phi \dot\Phi + \\ \quad + [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] \Phi - m_\varphi (R'_0 - l_\varphi) l_\varphi \ddot\varphi + \\ \quad + m_\varphi g l_\varphi \varphi = D h \alpha_m \sin \omega_\lambda t \\ m_\varphi l_\varphi^2 \ddot\varphi + k_\varphi \dot\varphi + m_\varphi g l_\varphi \varphi + \\ \quad - m_\varphi (R'_0 - l_\varphi) l_\varphi \ddot\Phi + m_\varphi g l_\varphi \Phi = 0 \end{cases} \tag{18}$$

Each of the coefficients appearing in the motion equations should be clearly defined by ship and tank parameters. The equations (17) and (18) derived on the basis of the proposed physical model, satisfy the postulate. Their normalized form is as follows :

$$\begin{cases} \ddot\Phi + \mu_{\Phi 1} \dot\Phi + \omega_{\Phi 1}^2 \Phi - \beta_1 \ddot\varphi + \\ \quad + \gamma_1 \omega_\Phi^2 \varphi = \omega_{D1}^2 \alpha_m \sin \omega_\lambda t \\ \ddot\varphi + \mu_\varphi \dot\varphi + \omega_\varphi^2 \varphi - b \ddot\Phi + \omega_\varphi^2 \Phi = \\ \quad = - \omega_\varphi^2 \alpha_m \sin \omega_\lambda t \qquad \text{– accounting for sway} \end{cases} \tag{19}$$

$$\begin{cases} \ddot\Phi + \mu_{\Phi 1} \dot\Phi + \omega_{\Phi 1}^2 \Phi - \beta_1 \ddot\varphi + \\ \quad + \gamma_1 \omega_\Phi^2 \varphi = \omega_{D2}^2 \alpha_m \sin \omega_\lambda t \\ \ddot\varphi + \mu_\varphi \dot\varphi + \omega_\varphi^2 \varphi - b \ddot\Phi + \\ \quad + \omega_\varphi^2 \Phi = 0 \qquad \text{– not accounting for sway} \end{cases} \tag{20}$$

Next, the expression appearing at $\ddot\Phi$ in the equations (18) was marked K and the following notation was applied :

$$\begin{gathered} [m_\Phi l_\Phi^2 + m_\varphi (R'_0 - l_\varphi)^2] = K \quad , \quad k_\Phi / K = \mu_{\Phi 1} \\ [m_\Phi g l_\Phi - m_\varphi g (R' - l_\varphi)] / K = \omega_{\Phi 1}^2 \\ m_\varphi (R'_0 - l_\varphi) l_\varphi / K = \beta_1 \quad , \quad m_\varphi g l_\varphi / K = \gamma_1 \omega_\Phi^2 \\ [D h + m_\varphi g (R'_0 - l_\varphi)] / K = \omega_{D1}^2 \quad , \quad D h / K = \omega_{D2}^2 \\ k_\varphi / m_\varphi l_\varphi^2 = \mu_\varphi \quad , \quad m_\varphi g l_\varphi / m_\varphi l_\varphi^2 = g / l_\varphi = \omega_\varphi^2 \\ m_\varphi (R'_0 - l_\varphi) l_\varphi / m_\varphi l_\varphi^2 = (R'_0 - l_\varphi) / l_\varphi = b \end{gathered} \tag{21}$$

The usually available initial data deal separately with ship itself and tank itself. Then $m_\Phi l_\Phi^2$ is given istead of K. Hence the coefficients in the equations (19) should be expressed by means of :

$$\begin{gathered} \mu_{\Phi 1} = \frac{\mu_\Phi}{K'} \quad , \quad \omega_{\Phi 1}^2 = \frac{\omega_\Phi^2}{K''} \quad , \quad \beta_1 = \frac{\beta}{K'} \\ \gamma_1 = \frac{\gamma}{K'} \quad , \quad \omega_{D1}^2 = \frac{\omega_\Phi^2}{K'} \quad , \quad \omega_{D2}^2 = \omega_\Phi^2 \frac{K'''}{K'} \end{gathered} \tag{22}$$

where :

$$\mu_\Phi = k_\Phi / m_\Phi l_\Phi^2 \quad , \quad \omega_\Phi^2 = m_\Phi g l_\Phi / m_\Phi l_\Phi^2 = g / l_\Phi^2$$

The quantities μ_Φ, ω_Φ^2, β, γ, K', K'', K''', expressed respectively by the physical model parameters and the ship and tank parameters, have the same form as that given in the expressions (23), which makes it possible to examine structure of each of the coefficients appearing in the motion equations.

PHYSICAL MODEL	SHIP
$\mu_\Phi = \frac{k_\Phi}{m_\Phi l_\Phi^2}$	$\mu_\Phi = \frac{k_\Phi}{I_x + i_{xx}} = \frac{k_\Phi}{m_s r_{sm}^2}$
$\omega_\Phi^2 = \frac{m_\Phi g l_\Phi}{m_\Phi l_\Phi^2} = \frac{g}{l_\Phi}$	$\omega_\Phi^2 = \frac{m_s g h_s}{I_x + i_{xx}} = \frac{g h_s}{r_{sm}^2}$
$\gamma = \frac{m_\varphi l_\varphi}{m_\Phi l_\Phi}$	$\gamma = \xi \frac{r_z}{h_s}$ (23)
$\beta = \frac{m_\varphi}{m_\Phi} \frac{(R'_0 - l_\varphi)}{l_\Phi} \frac{l_\varphi}{l_\Phi}$	$\beta = \gamma \left(\frac{R_0 k_0}{\sqrt{l_\Phi r_z}} - 1 \right) \frac{1}{k_0^2}$
$b = \frac{R'_0}{l_\varphi} - 1$	$b = \frac{R_0 k_0}{\sqrt{l_\Phi r_z}} - 1$

PHYSICAL MODEL | SHIP

$$K' = 1 + \frac{m_\varphi}{m_\Phi}\,\frac{(R_0' - l_\varphi)^2}{l_\Phi^2} \qquad\Big|\qquad K' = 1 + \beta^2\,\frac{k_0^2}{\gamma}$$

$$K'' = \frac{K'}{1 - \frac{m_\varphi}{m_\Phi}\,\frac{(R' - l_\varphi)}{l_\Phi}} \qquad\Big|\qquad K'' = \frac{K'}{1 - \gamma\left(\frac{R}{r_z} - 1\right)}$$

$$K''' = \frac{m_s + m_z}{m_\Phi}\,\frac{h}{l_\Phi} \qquad\Big|\qquad K''' = (1 + \xi)\,\frac{h}{h_s} \tag{23}$$

$$R' = R\,\frac{l_\varphi}{r_z} \qquad\Big|\qquad R = z_p + r_z - T - \Delta T/2 - \rho_c$$

$$R_0' = R_0\sqrt{\frac{l_\varphi}{r_z}} \qquad\Big|\qquad R_0 = z_p + r_z - T - \Delta T + a_w$$

$$h = h_s + \frac{\xi}{1+\xi}\left(T + \frac{\Delta T}{2} + \rho_c - z_p - h_s\right)$$

$$\rho_c = r_s\,(1 - \delta/\alpha)\,\frac{2}{2 + (\delta/\alpha)\,\xi} \qquad\Big|\qquad a_w = 0.43\,[(T + \Delta T) - z_G] + 0.1B$$

$$\xi = m_z/m_s \;,\quad k_0 = \omega_\varphi/\omega_\Phi \;,\quad \omega_0 = \omega_\lambda/\omega_\Phi \;,\quad \mu_{\varphi 0} = \mu_\varphi/\omega_\Phi$$

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for : $\Delta T/2 \approx 0$ and $\rho_c \approx 0$ it yields :

$$R \approx z_p + r_z - T \qquad\Big|\qquad R_0 \approx R + a_w$$

$$h \approx h_s + \frac{\xi}{1+\xi}\,(T - z_p - h_s)$$

Solutions of the motion equations, transfer functions

The general form of ship roll transfer functions obtained as a result of solving the equations (19) and (20), is the following :

$$\frac{\Phi}{\alpha_m} = K_S\sqrt{\frac{A^2 + B^2}{C^2 + D^2}} \tag{24}$$

For (19) where sway is accounted for, one obtains :

$$A = \mu_{\varphi 0}\,k_0\,\omega_0$$

$$B = k_0^2\,(1 + \gamma) - \omega_0^2\,(1 - \beta\,k_0^2)$$

$$C = -K'\,(1/K'' - \omega_0^2)(k_0^2 - \omega_0^2) + $$
$$+(\gamma + \beta\omega_0^2)(k_0^2 + b\,\omega_0^2) + \mu_{\Phi 0}\,\mu_{\varphi 0}\,k_0\,\omega_0^2$$

$$D = K'\,\mu_{\varphi 0}\,k_0\,\omega_0\,(1/K'' - \omega_0^2) + \mu_{\Phi 0}\,\omega_0\,(k_0^2 - \omega_0^2)$$

$$K_s = 1$$

For (20), without accounting for sway, it is :

$$A = \mu_{\varphi 0}\,k_0\,\omega_0 \;,\quad B = k_0^2 - \omega_0^2$$

C and D as for (20) , $K_s = K''' = \frac{m_s + m_z}{m_\Phi}\,\frac{h}{l_\Phi}$

The presented expressions which make it possible to calculate transfer functions for the ship fitted with stabilizing tank, directly provide the following options :

- ✦ to follow changes appearing in solutions depending on an applied variant of the equations
- ✦ to determine in which way changes of the main parameters of tank and ship influence the calculated functions.

In the equations based on ship and tank data axcess to variables and parameters is easy and direct. It is possible to directly change such parameters as: z_p - tank position, k_0 - tuning factor of natural frequencies of tank and ship, r_z -metacentric radius of tank , i.e. its type, ξ - i.e. amount of liquid in tank, D_s and h_s - i.e. ship loading state.

The assumed form of the data and auxiliary values, (23), including the variables :

$\omega_0 = \omega_\lambda/\omega_\Phi$ - dimensionless excitation frequency, an independent variable

$\mu_{\varphi 0} = \mu_\varphi/\omega_\Phi$ - dimensionless damping coefficient of motions of liquid in tank, the parameter for sucessive realizations of functions of a given type

$k_0 = \omega_\varphi/\omega_\Phi$ - relative natural frequency of oscillations of liquid in tank, possible to be used as an intermediate parameter

$\xi = m_z/m_s$ - relative mass of liquid in tank, possible to be used as an intermediate parameter;

all of them make it possible to use a single, simple set of initial data, regardless of simplification level of the equations in question.

APPLICATION OF THE MATHEMATICAL MODEL

The obtained roll transfer functions make it possible to follow influence of the main ship and tank parameters on ship roll amplitudes, and thus to ensure stabilization effectiveness of a given tank. To achieve courses of relevant functions the data of the ship and tank considered in [16] were selected. A correct course of the achieved functions serves also as a check of correction of the formulae based on the presented physical and mathematical models.

The ship with stabilizing tank acc.to [16] :

Ship parameters :

- length b.p.	L_{bp} = 266.00 m
- breadth	B = 42.50 m
- draught	T = 10.85 m
- mass displacement	$\Delta \equiv m_s$ = 97800 t
- height of centre of gravity	z_{Gs} = 15.40 m
- initial metacentric height (without tank)	$GM_s \equiv h_s$ = 3.576 m
- block coefficient	δ = 0.797
- waterplane coefficient	α = 0.831
- natural roll period (natural frequency)	τ_Φ = 17.10 s (ω_Φ = 0.37 1/s)
- roll damping coefficient	μ_Φ = 0.070 1/s.

Tank parameters (see Fig.1) :

- length	l_z = 13.60 m
- breadth	b_z = 40.89 m

- design height $h_k = 4.79$ m
- natural frequency of motions of liquid in tank $\omega_\varphi = 0.36$ 1/s
- level of liquid in tank $h_z = 2.20$ m
- mass of liquid in tank at $\rho = 1.00$ t/m^3 $m_z = 1224$ t
- tank bottom height over the plane of reference $z_d = 20.00$ m
- tuning factor $k_0 = \omega_\varphi/\omega_\Phi = 0.97$
- relative mass of liquid in tank $\xi = m_z/m_s = 0.0125$

The diagrams shown in Fig.8 and 9 illustrate the roll transfer functions of the ship with tank. The dimensionless damping coefficient of motions of liquid in tank, $\mu_{\varphi 0}$, is a parameter for particular curves. Fig.8 deals with the full set of equations in which sway is accounted for, and Fig.9 - the full set but without taking into account those oscillations. The presented diagrams are usually used for choosing an optimum value of the damping coefficient $\mu_{\varphi 0}$, and after that – for evaluating the maximum stabilizing effect [5, 12, 15]. A value of the damping coefficient for tank is so selected as in the middle, "design" range of the diagrams the difference between the roll amplitude of the non-stabilized ship, which corresponds to the value $\mu_{\varphi 0} = \infty$, and a selected value e.g. $\mu_{\varphi 0} = 0.6$; were the greatest. The value of this difference, determined for the maximum amplitude of the non-stabilized ship, hence for the excitation resonant frequency, is the stabilizing effect searched for. A value of $\mu_{\varphi 0}$ should be selected in such a way as in a range far from resonance e.g. outside the characteristic nodes of the transfer functions' diagram, an increment of amplitudes, usually appearing there for stabilized ship, were the smallest. The diagrams also show that for $\mu_{\varphi 0} = 0$, which is possible theoretically only, such excitation frequency exists at which the stabilizing effect amounts to 100%. The stabilizing effect expressed in percent, is the ratio of the roll amplitude reduction and non-stabilized amplitude.

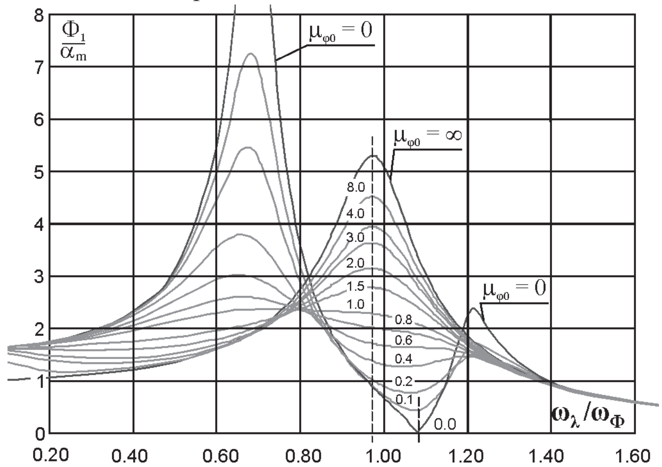

Fig. 8. Diagrams of roll transfer functions with taking into account sway

Fig.8 it can be observed that when sway is accounted for, the zero value of roll amplitude occurs at a frequency other than resonant one of non-stabilized ship. It means that the maximum amount of roll energy is absorbed by the tank from the ship at that other frequency. Hence in the case if sway is accounted for no phenomenon of double resonance can be directly observed.

Influence of changing the tank parameters on course of roll transfer function

It was assumed that in the preliminary design stage it is more convenient to make use of the full-form equations but without taking into account sway. Information this way obtained is more transparent and more useful for making relevant estimations and comparisons.

In Fig.10 based on Fig.9 , only the courses for the dimensionless damping coefficients of liquid in tank, $\mu_{\varphi 0} = \infty$ and $\mu_{\varphi 0} = 0.8$, are left. Fig.10 is further used asa a basis for making comparisons of tank stabilizing effectiveness depending on changes introduced to its location or other parameters. Fig.11 shows in which way the roll reduction effect is influenced by a change in vertical location of the stabilizing tank. Next, in Fig.12 it is presented to which extent a change of tank metacentric radius impairs ship roll amplitudes. From the presented diagrams it can be easily observed that their courses are almost in the same way unfavourably influenced by the decreasing of

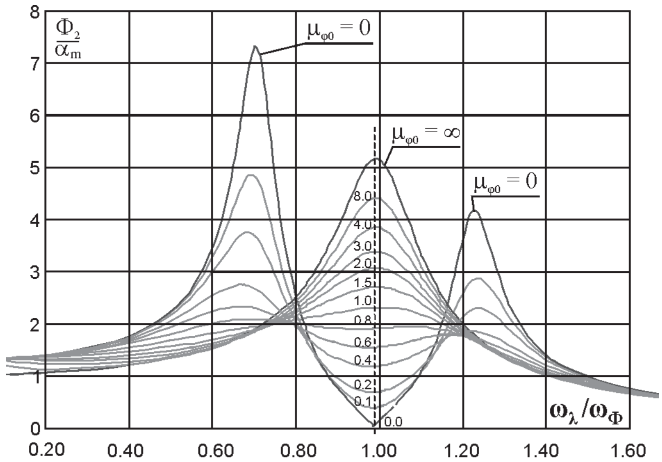

***Fig.9.** Diagrams of roll transfer functions without taking into account sway*

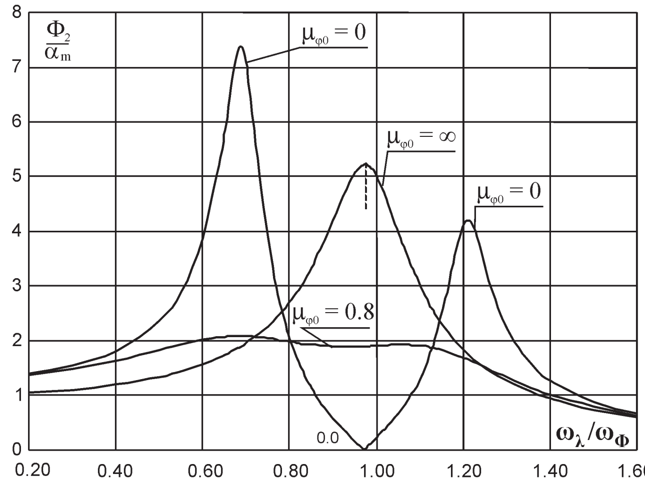

***Fig. 10.** Main transfer functions, without accounting for sway*
$k_0 = 0.97$; $\xi = 0.013$; $z_p = 20.00$ m ; $r_z = 63.33$ m

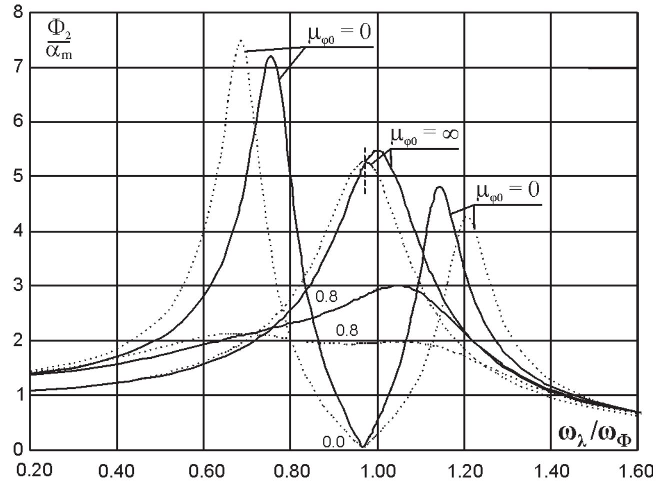

***Fig. 11.** Influence of the vertical coordinate of tank location,*
——— $z_p = 2.00$ m ; --------- $z_p = 20.00$ m (Fig. 10)

tank elevation and the decreasing of tank metacentric radius. In both the cases the stabilizing effect is lower because the roll amplitudes of the stabilized ship are greater. By making use of the physical model (Fig.5, Fig.6, Fig.7) it can be observed that in both cases this is caused by changing the distance between the tank pendulum suspension point and the pivoting point of the whole system of pendulae of the physical model.

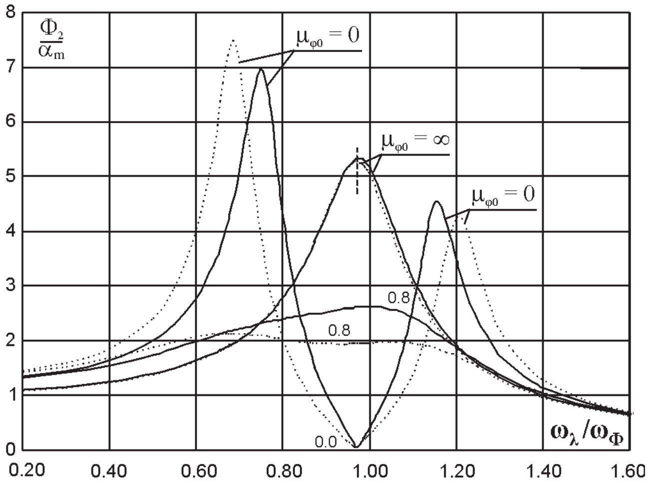

Fig. 12. *Influence of the tank metacentric radius - change of type of tank ——— r_z = 46.73 m ; ---------- r_z = 63.33 m (Fig. 10)*

- ⇨ Generally, it can be stated that the higher located the passive stabilizing tank metacentre over the ship rolling axis, the greater the stabilizing effects.This is an important guideline for designers.
- ⇨ Another important conclusion is : that type of selected passive stabilizing tank is better whose metacentric radius is the greatest, at maintaining the remaining parameters, such as working liquid mass and its natural period of motion in tank, constant.
- ⇨ It is proposed to assume the ratio of the length of the tank metacentric radius and that of the tank mathematical pendulum as a relative measure of tank stabilizing quality, independent on tank dimensions and mass of its working liquid, and expressed as follows :

$$C_Z = r_Z/l_\varphi \tag{25}$$

For tanks of perpendicular sides the quantity C_Z is practically constant, independent on their dimensions and the working liquid level h_Z . For instance for the cubicoid tank and "Flume" tank it amounts to: $C_{Z1} \cong 0.82$ and $C_{Z2} \cong 0.67$, respectively (the C_{Z2} value deals with the "Flume" tank having the dimensions of the middle throat equal to a half of tank length and a half of tank breadth, respectively). C_Z values start to change distinctly for greater values of the liquid level in tank, h_Z , (i.e. for $h_Z > 0.2b_Z$) which is associated with the necessity of using more complex relationships in determining the natural periods of working liquid motions. Basing on the above given statements one can postulate the following thesis :

Effectiveness of the passive stabilizing tanks of free liquid surface increases when the ratio of their metacentric radius and length of mathematical pendulum increases; the pendulum length is determined by the natural period of working liquid in tank. The ratio in question can be called
"the stabilizing effectiveness of tank of a given type".

The presented relationships open an interesting area for searching for optimum solutions of passive stabilizing tanks having free working liquid surface,
and also for further research on ship roll stabilization.

RECAPITULATION AND CONCLUSIONS

- ❍ The presented mathematical model can be deemed well adjusted for designing the passive stabilizing tank to be installed in ship. The elaborated calculation method can be used as a tool for aiding preliminary design stage of a ship fitted with passive stabilizing tank.
- ❍ For correct design of stabilizing tanks the following three principles formulated on the basis of the presented model of ship with stabilizing tank, are important :
  - ❖ the higher located metacentre of passive stabilizing tank over ship rolling axis, the greater expected stabilizing effects
  - ❖ that type of passive stabilizing tank is better whose metacentric radius is the greatest, at maintaining the remaining parameters such as working liquid mass and natural period of motions of liquid, constant
  - ❖ the higher the ratio of the tank metacentric radius and the mathematical pendulum length determined by natural period of motions of liquid in tank, the higher the stabilizing effectiveness of a selected type of passive stabilizing tank with free liquid surface.

Moreover, it should be added that :

- ➢ in spite of the simplifications resulting from the assumed physical model, the achieved solutions provide sufficiently exact description of roll of a ship fitted with stabilizing tank. The wide range of the simplifications was introduced in order to reach "design usufulness" of the elaborated equations and their solutions
- ➢ the way of accounting for sway in the obtained equations arouses some reservations with regard to the introduced important simplifications. Nonetheless it was proved that in the case of real sea conditions and accounting for sway the so called double resonance did not directly appear for a ship with stabilizing tank. For this reason its stabilizing effect may appear lower than that expected.

NOMENCLATURE

a_W - distance between rolling axis and waterplane, [m]
D, D_s - buoyance force of ship with and without tank, respectively, [kN]
h, h_s - initial metacentric height, not corrected regarding free-surface influence, for ship with and without tank, respectively, [m]
k_Φ, μ_Φ and $\mu_{\Phi0}$ - ship roll damping coefficient, kg m^2/s, 1/s, and [-]
k_φ, μ_φ and $\mu_{\varphi0}$ - damping coefficient of motions of liquid in tank, [kg m^2/s, 1/s] and [-]
$k_0 = \omega_\varphi/\omega_\Phi$ - relative natural frequency of motions of liquid in tank, tuning factor, [-]
l_Φ - length of ship mathematical pendulum, [m]
l_φ - length of tank mathematical pendulum, [m]
M_λ - ship roll excitation moment, [Nm]
m_s - ship mass, [t]
m_z - mass of liquid in tank, [t]
m_Φ - mass of ship mathematical pendulum, [t]
m_φ - mass of tank mathematical pendulum, [t]
R - distance between metacentre of tank and metacentre of increment of ship immersed volume, corresponding to mass of liquid in tank, [m]
R' - distance in physical model corresponding to R of ship, [m]
R_0 - distance between tank metacentre and ship rolling axis, [m]
R'_0 - distance between tank pendulum suspension point and physical model pivoting axis, [m]
r_s - metacentric radius of ship without tank, [m]

r_{sm} - ship mass inertia radius, [m]
r_z - tank metacentric radius, [m]
r_{zm} - mass inertia radius of liquid in tank, [m]
t - time, [s]
T - draft, [m]
V_λ - horizontal component of velocity of mass of liquid in tank, due to ship sway, [m/s]
y_λ - ship sway, [m]
z_G - height of mass centre of ship with tank, over reference plane, [m]
z_{Gs} - height of mass centre of ship without tank, over reference plane, [m]
z_p - height of mass centre of liquid in tank, over reference plane, [m]
α - waterplane coefficient, [-]
α_λ, α_m - amplitude and effective amplitude of wave slope angle, respectively, [rad, °]
δ - ship block coefficent, [-]
κ_B, κ_T - correction coefficients of wave slope angle, dependent on B/λ and T/λ, where : λ - length of ship roll inducing wave
$\xi = m_z/m_s$ - relative mass of liquid in tank, [-]
ρ_c - metacentric radius of displacement layer of ΔT thickness, [m]
τ_Φ - natural oscillation period of ship pendulum and real ship, [s]
τ_φ - natural oscillation period of tank pendulum and of liquid's motions in tank, [s]
Φ - ship heeling angle, [rad, °]
φ - angular displacement of mass centre of liquid in tank, respective to ship, (mean slope angle of liquid surface in tank, respective to ship), [rad, °]
$\omega_0 = \omega_\lambda/\omega_\Phi$ - relative excitation frequency, [-]
ω_λ - wave frequency, [1/s]
ω_Φ - ship roll natural frequency, [1/s]
ω_φ - natural frequency of liquid's motions in tank, [1/s]

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Conference

E Scientific Seminar of Regional Group E

of the Section on Exploitation Foundations

On 24 June 2004 Gdynia Maritime University (GMU) hosted the 2nd – in-this-year scientific seminar of the Regional Group of the Section on Exploitation Foundations, Machine Building Committee, Polish Academy of Sciences.

Four papers whose authors came from the scientific staff of GMU Mechanical Faculty, were presented during the seminar :

★ *Results of service investigations of transverse sliding bearings lubricated with the use of non-Newtonian oils* by A. Miszczak, D.Sc.
★ *Problems associated with modeling NOx emission from ship two-stroke engine* – by J. Kowalski, M.Sc.
★ *Influence of lubricating oil contamination on wear of elements working in mixed friction conditions* by A. Młynarczyk, M.Sc., Eng.
★ *Selected problems associated with diagnostics of one--stage refrigerating system* – by T. Hajduk, M.Sc.

The seminar was ended by presentation of the laboratory facilities of GMU Mechanical Faculty to the seminar participants.

Photo: C. Spigarski