On hydrodynamic forces acting on the ship in large motions

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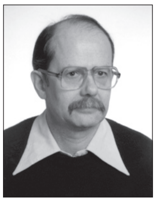
ABSTRACT



Present state of mathematical description of ship dynamic non-linear behaviour is presented in this paper with a view to avoiding excessive complications in solving the problem. The non-linearity concerns first of all Froude-Krilov forces and damping forces occurring after entering ship's deck into water or those resulting from drag of bilge keels. And, to the remaining, accompanying and diffraction forces the linear extrapolation has been applied.

Key words : hydrodynamic forces, ship motions of large amplitudes, added forces

INTRODUCTION

The paper presents current state of mathematical description of ship dynamic non-linear behaviour. The subject matter concerns large-amplitude motions of ship in waves, considered within the frame of ship roll non-linear theory. The split into the accompanying forces, Froude-Krilov forces and diffraction forces is assumed still valid. The Froude-Krilov forces are calculated with taking into account changeable wetted area of ship hull surface. However to calculate the accompanying and diffraction forces a method of extrapolation of linear solutions is applied.

FROUDE-KRILOV FORCES

Froude-Krilov forces are obtained by integrating pressures in waving water not disturbed by ship's presence in it. In the non-linear theory the integration is performed over wetted surface area which results from an instantaneous position of ship hull relative to wave surface. The pressure *p* is defined by the following expression :

$$p - p_a = \rho g(z - \zeta e^{-kz}) \tag{1}$$

The Froude-Krilov resultant force $\vec{F}$ and moment $\vec{M}$ can be obtained with the use of the integration formulae :

$$\vec{F} = -\int_S \vec{n}(p - p_a)dS = -\int_V \left(\vec{i}\frac{\partial p}{\partial x} + \vec{j}\frac{\partial p}{\partial y} + \vec{k}\frac{\partial p}{\partial z} \right) dV \tag{2}$$

$$\vec{M} = -\int_S \vec{r} \times \vec{n}(p - p_a)dS =$$

$$= -\int_V \left[\vec{i}\left(y\frac{\partial p}{\partial z} - z\frac{\partial p}{\partial y} \right) + \vec{j}\left(z\frac{\partial p}{\partial x} - x\frac{\partial p}{\partial z} \right) + \vec{k}\left(x\frac{\partial p}{\partial y} - y\frac{\partial p}{\partial x} \right) \right] dV \tag{3}$$

The forces are usually calculated by means of the above presented surface integrals. In practice the wetted area is divided into a finite number of the directed area elements $\vec{n}\,\Delta S$, and integration is replaced by summation. The integration is rather troublesome as to determine the normal versor for every element ΔS is necessary. However the sufrace integrals can be replaced by scalar volume integrals in accordance with Gauss--Ostrogradski formula, which make determination of the normal versors not necessary. A way of calculation of the Froude-Krilov volume integrals was presented in [1], and realized in [2].

ADDED FORCES

The added forces acting on ship result from the forced oscillatory motion of ship in still water. They are determined by means of the formula :

$$F_i = -m_{ij}\ddot{u}_j - N_{ij}\dot{u}_j \qquad i, j = 1,2,...6 \tag{4}$$

In the formula the Einstein's convention of summation is applied. F_i , for i = 1, 2, ... 3, stands for x, y, z components of added forces, and for i = 4, 5, 6 it stands for components of moments of added forces (relative to ship's centre of gravity in the ship-fixed coordinate system). For j = 1, 2, 3 the velocities $\dot{u}_j$ stand for the components υ_x , υ_y , υ_z of the oscillation velocity of ship's centre of gravity relative to its mean location, whereas for j = 4, 5, 6 the velocities stand for the ship's angular velocity components ω_x , ω_y , ω_z (defined in the moving system). The accelerations $\ddot{u}_j$ for j = 1, 2, 3 stand for the acceleration components a_x , a_y , a_z of ship's gravity centre oscillations relative to its mean location, whereas for j = = 4, 5, 6 they stand for the components $\dot{\omega}_x$, $\dot{\omega}_y$, $\dot{\omega}_z$.

The added masses m_{ij} and damping coefficients N_{ij}, called the hydrodynamic coefficients, are constant provided the ship in question performs oscillations of constant frequency. Then they functionally depend on the oscillation frequency ω. If ship's motion is non-harmonic the added masses and damping coefficients do not have any constants and using them is senseless. In such case the hydrodynamic coefficients can be determined by using the Fourier transform method. To this end it is enough to consider the addede force component F_j resulting from j-th DOF oscillation as the Fourier transform is a linear operation.

Assuming that the ship hull oscillations occur at the amplitude u_A :

$$u_j(t) = u_A \sin \omega t \tag{5}$$

one obtains the added force :

$$F(t) = u_A \omega^2 m(\omega) \sin \omega t - u_A \omega N(\omega) \cos \omega t \tag{6}$$

The hydrodynamic force can be expressed by means of the function of response to the step excitation [3] :

$$F(t) = -\int_0^\infty r_1(\tau)\dot{u}(t-\tau)d\tau - \int_0^\infty r_2(\tau)\ddot{u}(t-\tau)d\tau \tag{7}$$

The relationship is also valid for harmonic motion. It is possible to introduce (5) to it, and next to compare it with (6) :

$$\begin{aligned} m(\omega) &= \int_0^\infty r_2(\tau)\cos\omega\tau d\tau - \frac{1}{\omega}\int_0^\infty r_1(\tau)\sin\omega\tau d\tau \\ N(\omega) &= \omega\int_0^\infty r_2(\tau)\sin\omega\tau d\tau + \int_0^\infty r_1(\tau)\cos\omega\tau d\tau \end{aligned} \tag{8}$$

Moreover, the following relationships are valid :

$$\lim_{\omega\to\infty} m(\omega) = m_\infty \qquad \lim_{\omega\to\infty} N(\omega) = 0 \tag{9}$$

The function $r_2(\tau)$ must have the following forms in order the relationships (8) to have the limits given by (9) :

$$r_2(\tau) = m_\infty \delta(\tau) \tag{10}$$

where : $\delta(\tau)$ - Dirac's delta functions.

However the function $r_1(\tau)$ must be limited and decaying along with τ increasing. Then the integral :

$$\int_0^\infty r_1(\tau)\cos\omega\tau d\tau$$

decreases along with ω increasing,with the rate of about $1/\omega$ (its exact decaying rate is unknown). On accounting for the above mentioned comments and introducing the notion : $r(\tau) \equiv$ $\equiv r_1(\tau)$, the formulae (8) are transformed into the following :

$$\begin{aligned} m(\omega) &= m_\infty - \frac{1}{\omega}\int_0^\infty r(\tau)\sin\omega\tau d\tau \\ N(\omega) &= \int_0^\infty r(\tau)\cos\omega\tau d\tau \end{aligned} \tag{11}$$

The inverse Fourier transform makes it possible to achieve an unknown response function $r(\tau)$:

$$\begin{aligned} r(\tau) &= \frac{2}{\pi}\int_0^\infty [m_\infty - m(\omega)]\omega\sin\omega\tau d\omega \\ r(\tau) &= \frac{2}{\pi}\int_0^\infty N(\omega)\cos\omega\tau d\omega \end{aligned} \tag{12}$$

From the second formula (12) one obtains the relationship for the initial value of $r(\tau)$:

$$r(\tau = 0) = \frac{2}{\pi}\int_0^\infty N(\omega)d\omega \tag{13}$$

From (12) it results that knowledge of one of the hydromechanical functions : $m(\omega)$ or $N(\omega)$ within the whole frequency range is sufficient to find the response functions $r(\tau)$. Also, it can be observed that after determination of one of the hydromechanical functions and calculation of $r(\tau)$ the other function can be determined by means of the relevant formula (11).

It is also possible to determine radiation forces in harmonic motion. On the basis of (7) the following is yielded :

$$F(t) = -m_\infty \ddot{u}(t) - \int_0^\infty r(\tau)\dot{u}(t-\tau)d\tau \tag{14}$$

The formula (14) is valid for any DOF. The integral term appearing in the formula is the so called *memory effect* which fast decays along with τ increasing. Its decaying rate depends on that of $r(\tau)$, hence it depends on the decaying rate of the damping coefficient $N(\omega)$ along with ω increasing. The above presented theory, called Cummins model, is generally related to irregular waves.

The formula (14) is generally valid for small harmonic motions of ship, performed around its equilibrium position in still water, within linear range. The linearity assumption is practically valid until the bilge does not emerge from and the deck does not immerse into water. This makes it possible to extend validity of the formula into the case of large motions, as for practical reasons it is sufficient to be limited only to large roll amplitudes and small amplitudes of the remaining motions. Different calculation procedures for added forces at large heeling angles can be met (Fig.1).

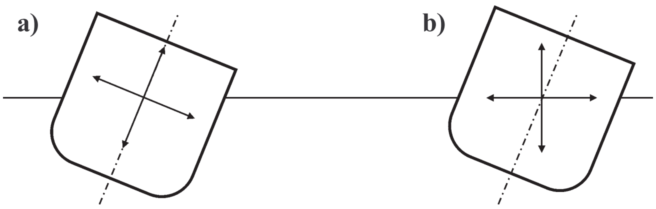


Fig. 1. *Ship motions possible to be performed around its equilibrium position in still water*

The added masses m_{ij} and the damping coefficients N_{ij} are numerically determined for the different draughts *z* and heeling angles Φ [4] :

$$\begin{aligned} m_{ij} &= m_{ij}(z,\Phi) \\ N_{ij} &= N_{ij}(z,\Phi) \end{aligned} \tag{15}$$

The forced oscillatory motion shown in Fig.1a is more favourable because the heeling angle Φ is of a negligible influence on added forces.

WATER ON THE DECK

It is assumed that the added forces determined by (14) can be also applied to heeling angles greater than the angle of deck entrance into water. However in this case the additional forces due to presence of water on the deck should be taken into account. Unfortunately in the existing computer programs the problem has been neglected though results of model tests have indicated that the additional forces due to water on the deck would be significant [7]. It is only possible to roughly estimate them as the problem is insolvable strictly.

The forces in question clearly depend on direction of motion. If the deck emerges into water they practically equal zero, and if the water flows out from the deck they become significant. The forces can be estimated by applying the momentum conservation law, and the way of their estimation is highlighted in Fig.2 [8].

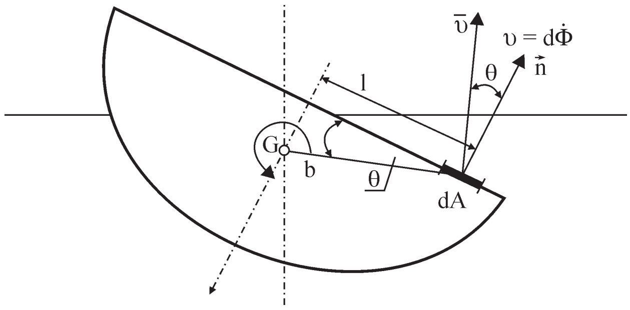


Fig. 2. Mutual relations between the deck area element dA, the normal to the deck, $\vec{n}$, and the deck area element velocity $\overline{\upsilon}$

The momentum conservation law was used along the direction $\vec{n}$ for the water head over the deck area element dA. The water is subject to a change of momentum with time, which, according to 2nd principle of dynamics, is equal to the deck reaction R_n :

$$\frac{d}{dt}(m\upsilon_n) = R_n \tag{16}$$

or :

$$\frac{dm}{dt}\upsilon_n + m\frac{d\upsilon_n}{dt} = R_n \tag{17}$$

The continuity equation for the water head over the deck area element dA subject to momentum change, can be written as follows :

$$\frac{dm}{dt} = \rho\upsilon_n dA \tag{18}$$

Taking into account the following auxiliary relationships (Fig. 2) :

$$\upsilon = b\,\dot{\Phi} \quad , \quad \upsilon_n = \upsilon\cos\theta$$

$$\upsilon_n = b\,\dot{\Phi}\cos\theta = l\dot{\Phi} \quad , \quad \frac{d\upsilon_n}{dt} = l\ddot{\Phi}$$

one can transform (17) into the following :

$$R_n = \rho\dot{\Phi}^2 l^2 dA + \rho\ddot{\Phi} l dV \tag{19}$$

The equation (19) expresses a unit force acting on the ship deck during water running off the deck. The elementary moment of the force R_n relative to x - axis is expressed by the following relationship:

$$dK = \rho\dot{\Phi}^2 l^3 dA + \rho\ddot{\Phi} l^2 dV \tag{20}$$

The additional moment due to water running off the deck can be obtained by integrating (20) over the wetted deck area A and the volume of the water appearing on the deck, V :

$$K = \dot{\Phi}^2\rho\int_A l^3 dA + \ddot{\Phi}\rho\int_V l^2 dV \tag{21}$$

The approximate formula (21) for the additional moment due to water running off the deck can be presented as the sum of two elements : the damping moment and the added mass moment :

$$K = I_3\dot{\Phi}^2 + i_X\ddot{\Phi} \tag{22}$$

where :

I_3 - 3rd order moment of the wetted deck area
i_x - mass inertia moment of the water appearing on the deck, respective to the ship central axis x.

DIFFRACTION FORCES

The diffraction forces result from hull-induced disturbances of pressure distribution in waving water. In the linear theory the forces in question are calculated on the basis of wave action on motionless ship. The assumption is valid in the case of small motions only. Nevertheless it is also extended into large motions. The diffraction forces acting on a ship in irregular waves at large motion amplitudes are determined by superposing the forces resulting from particular harmonic components. It results from the assumption, common for added and diffraction forces, in which the ship is further considered, despite large amplitudes of motion, as a linear object respective to those forces. The assumption makes calculations much simpler as it allows for using the characteristics of the ship in upright position, which are independent on time and instantaneous positions of the ship. Otherwise it would be necessary to determine them in every time step and for every instantaneous wetted area of hull surface, which is a very difficult task.

If to denote, by F_{Dsij} and F_{Dcij}, respectively the sinusoidal and cosinusoidal parts of the amplitude of the diffraction force associated with i-th DOF and resulting from j-th harmonic component, then the generalized diffraction force F_{Di} can be expressed as follows:

$$F_{Di}(t) = \sum_j \begin{bmatrix} F_{Dcij}\cos(\omega_{Ej}t - \varepsilon_j) + \\ + F_{Dsij}\sin(\omega_{Ej}t - \varepsilon_j) \end{bmatrix} \tag{23}$$

where the indicated summation is performed over all harmonic components.

CONCLUSIONS

- The radiation and diffraction forces are smaller than Froude-Krilov ones approximately by one order of magnitude [5]; they influence first of all the phase shift angles between waves and ship motions, and – to a much smaller degree – amplitudes of the motions [6].
- On the basis of the subject-matter literature it can be stated that the differences between the linear theory of ship motions and the non-linear one are not large, and that they do not significantly influence solutions of ship motion equations if only motions of water relative to ship are assumed to occur within the range of ship's sides.

NOMENCLATURE

a	- acceleration component of ship gravity centre oscilation
A	- ship deck wetted area
b	- distance between the deck surface area element dA and the ship gravity centre G
$\vec{F}$	- hydrodynamic Froude-Krilov resultant force acting on ship hull
F_i	- components of added forces
F_{Di}	- diffraction force component
F_{Dcij}	- cosinusoidal component of diffraction force amplitude
F_{Dsij}	- sinusoidal component of diffraction force amplitude
g	- gravity acceleration
$\vec{i}, \vec{j}, \vec{k}$	- versors
k	- wave number
K	- additional moment resulting from the water running off the deck
l	- distance between the deck surface area element dA and the ship plane of symmetry
m	- mass of water on the ship deck surface element dA
m_{ij}	- ship added masses
m_∞	- added mass corresponding to the angular frequency ω tending to infinity

$\vec{M}$ - hydrodynamic Froude-Krilov resultant moment acting on ship hull
$\vec{n}$ - unit vector normal to ship surface
N_{ij} - ship damping coefficients
p - pressure inside wave
p_a - atmospheric pressure
r - response to the step excitation
$\vec{r}$ - tracing vector
R_n - unit force acting on the deck surface area element dA
S - instantaneous ship hull surface wetted area closed by free surface of wave
t - time
u - oscillation of ship gravity centre
u_A - ship hull oscillation amplitude
u_j - oscillation of ship gravity centre in the j-th direction respective to its mean position
υ - velocity of the deck surface area element dA
υ_n - projection of the velocity $\bar{\upsilon}$ onto direction of the normal $\vec{n}$
V - instantaneous volume of immersed part of ship hull; volume of water on the deck
x, y, z - spatial coordinates
ε_j - phase shift angle of harmonic component
ξ - wave profile ordinate
θ - angle between vectors of the velocity $\bar{\upsilon}$ and the normal $\vec{n}$
ρ - water density
τ - time shift angle
Φ - ship heeling angle
$\dot{\Phi}, \ddot{\Phi}$ - ship roll angular velocity and acceleration, respectively
ω - oscillation frequency
ω_E - encounter frequency of the ship

Acronims

CTO - Ship Design and Research Centre
DOF - degrees of freedom
ISC - International Shipbuilding Conference
PRS - Polish Register of Shipping

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Conference

10th Jubilee Seminar

On 18 February 2004 the 10th, successive Seminar on

Drives and Control Systems

was held in Gdańsk. The series of such seminars accompanying the Fairs of Manufacturers, Subcontractors and Providers of Power Units and Control Systems, have been each year organized by Gdańsk University of Technology since 1995. This has been a good occasion for direct mutual contacts of representatives of scientific centres working at development of this engineering branch and the manufacturers and dealers of such technical devices, which next brings about novel solutions based on results of R&D projects.

As usual, the Seminar was devoted to the problems of the following areas :

- drives and control systems of machines
- automation and dynamic behaviour of driving systems for shipbuilding and power industries
- electronic devices applicable to driving and control systems
- automation of electric drives
- applications of hydraulic and pneumatic drives
- applications of control and signal processing methods.

The problems were discussed during three sessions :

A - *Mechanical, hydraulic and pneumatic drives* (12 papers)
B - *Automation of electric drives* (6 papers)
C - *Applications of control and signal processing methods* (7 papers),

and, during one poster session another 22 topics were presented.

Moreover, the Seminar was accommpanied with the meetings organized in the frame of the workshops devoted to the following topics :

- I - *Control techniques for linear electro-hydraulical servodrives*. (Technical University of Poznań)
- II - *Advanced simulation techniques for converter drives. Application of Tcad package*. (Gdańsk University of Technology)
- III - *Automation of ship electric power system*. (Gdynia Maritime University)
- IV - *Electronic microsystems - their design, diagnostics and integration*. (Gdańsk University of Technology in cooperation with TASC company).