Fatigue "safe-life" criterion for metal elements under multiaxial constant and periodic loads

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ABSTRACT

Periodic stress with Cartesian components given in the form of Fourier series is considered. To account for the mean stress effect the generalised Soderberg criterion for ductile materials is employed. An equivalent stress with synchronous components is defined by means of the equivalence conditions based on the average strain energy of distortion. The fatigue "safe-life" design criterion is formulated which covers the conditions of both static strength and fatigue safety and includes material constants that have simple physical interpretation, can be determined by uniaxial tests, are related directly to the applied loads and can reflect material anisotropy.

Key words : design criteria, multiaxial loading, periodic stress, mean stress effect

INTRODUCTION

Machinery parts and structural elements are frequently subjected to simultaneous action of constant and cyclic stresses. In marine floating objects, constant stresses are mainly caused by deadweight and hydrostatic pressure, whereas those encountered in machinery systems are produced by torque, thrust, centrifugal force, etc. In the present paper also cyclic stresses induced by time-varying loads of periodic character are considered.

Each Cartesian component of the cyclic stress can be characterised by its mean value and parameters of its alternating part. It is important to remember that the total strength of an engineering element is altered if residual stresses are present. Since residual stresses have a similar influence on the fatigue behaviour of materials as that of mechanically imposed constant stresses of the same magnitude [1], no distinction will be made between constant, mean and residual stresses.

In order to ensure a fatigue safe life (theoretically infinite) under bending stress of mean value c_b and amplitude a_b two approaches can be indicated [2]. The first is based on the condition :

$$a_b^* \le F_b \tag{1}$$

where :

$$a_b^* = a_b + \psi_b c_b \tag{2}$$

is the amplitude of the equivalent zero mean stress,

$\psi_b = 0.1 \div 0.2$ - the asymmetry sensitivity index at bending
F_b - the fatigue limit under fully reversed bending.

Analogous conditions can be applied in the case of axial force or torsional moment.

In the second approach the effect of mean stress is described by a "failure diagram" or by one of the empirical equations, such as Goodman's, Gerber's, Soderberg's or Bagci's equation, depending on a given situation [2÷5]. In the following the Soderberg's equation for ductile materials is used in the form :

$$B = F_b\left(1 - \frac{c_b}{R_e}\right) \tag{3}$$

where :
B - maximum allowable stress amplitude in fatigue "safe-life" design under asymmetric bending
R_e - tensile yield strength.

BACKGROUND

Introducing the safety factor :

$$f = \frac{B}{a_b} \tag{4}$$

and partial safety factors :

$$f_s = \frac{R_e}{c_b} \qquad f_d = \frac{F_b}{a_b} \tag{5}$$

one obtains from (3) :

$$f = f_d\left(1 - f_s^{-1}\right) \tag{6}$$

So, the design criterion reads :

$$f \geq 1 \qquad (7)$$

i.e.,

$$f_s^{-1} + f_d^{-1} \leq 1 \qquad (8)$$

Here the subscript "s" stands for static and "d" for dynamic parts of the applied stress.

Obviously, the condition (1) concerns exclusively fatigue endurance of the material subjected to combined constant and cyclic loads, whereas satisfaction of (8) guarantees that not only the static strength of the material is not exceeded but also that the combination of constant and cyclic loads will not lead to fatigue failure. Another essential point is that the partial safety factors in (8) can be analysed and/or influenced separately. Therefore (1) and (2) is not used hereunder.

The explicit forms of (8) read :

$$\frac{c_b}{R_e} + \frac{a_b}{F_b} \leq 1 \qquad (9)$$

for asymmetric bending,

$$\frac{c_a}{R_e} + \frac{a_a}{F_a} \leq 1 \qquad (10)$$

for asymmetric push-pull force, and

$$\frac{c_t}{R_t} + \frac{a_t}{F_t} \leq 1 \qquad (11)$$

for asymmetric torsional moment, where :

$$R_t = \frac{1}{\sqrt{3}} R_e \qquad (12)$$

$$F_t = \frac{1}{\sqrt{3}} F_b \qquad (13)$$

applicable for steels [4].

Here the subscripts "a", "b" and "t" denote axial, bending and torsional load cases, respectively, and F_a is the fatigue limit under symmetric tension-compression.

The aim of this paper is to extend the use of eqation (8) to multiaxial non-zero mean periodic stresses. For this purpose it is necessary to determine a reduced stress, equivalent in terms of fatigue performance of the material under the multiaxial stress. For example, in the case of in-phase stress with Cartesian components :

$$\sigma_i(t) = c_i + a_i \sin \omega t \quad i = x, y, z, xy, yz, zx \qquad (14)$$

such reduced stress can be calculated for ductile metals by means of the average-distortion-energy strength hypothesis [6] as :

$$\sigma_{eq}(t) = c_{eq} + a_{eq} \sin \omega t \qquad (15)$$

where its mean value and amplitude are given by :

$$c_{eq} = \begin{bmatrix} c_x^2 + c_y^2 + c_z^2 - c_x c_y - c_y c_z + \\ - c_z c_x + 3\left(c_{xy}^2 + c_{yz}^2 + c_{zx}^2\right) \end{bmatrix}^{1/2} \qquad (16)$$

$$a_{eq} = \begin{bmatrix} a_x^2 + a_y^2 + a_z^2 - a_x a_y - a_y a_z + \\ - a_z a_x + 3\left(a_{xy}^2 + a_{yz}^2 + a_{zx}^2\right) \end{bmatrix}^{1/2} \qquad (17)$$

For the sake of brevity the stress components $\sigma_z(t)$, $\sigma_{yz}(t)$ and $\sigma_{zx}(t)$ are dropped. It is noteworthy that in the case of out-of-phase stress components :

$$\sigma_i(t) = c_i + a_i \sin(\omega t + \beta_i) \qquad i = x, y, xy \qquad (14a)$$

the average-distortion-energy strength hypothesis yields :

$$a_{eq} = \left[a_x^2 + a_y^2 - a_x a_y \cos(\beta_x - \beta_y) + 3a_{xy}^2\right]^{1/2} \qquad (17a)$$

According to the aforementioned hypothesis, the reduced stress (15) corresponds to that in an element of the specimen under uniaxial tension-compression test. Consequently, (10) can be used, which leads to the partial safety factors :

$$f_s = \frac{R_e}{c_{eq}} \qquad f_d = \frac{F_a}{a_{eq}} \qquad (18)$$

and to the fatigue "safe-life" criterion :

$$\frac{1}{R_e}\left(c_x^2 + c_y^2 - c_x c_y + 3c_{xy}^2\right)^{1/2} + \\ + \frac{1}{F_a}\left(a_x^2 + a_y^2 - a_x a_y + 3a_{xy}^2\right)^{1/2} \leq 1 \qquad (19)$$

With (12), (16) and (18) one gets :

$$f_s = \left[\frac{1}{R_e^2}\left(c_x^2 + c_y^2 - c_x c_y\right) + \frac{1}{R_t^2} c_{xy}^2\right]^{-1/2} \qquad (20)$$

Since normal stress components can be produced by loads of different mode (tension, compression, bending), and shear stress components – by torsion or shear, and the material may exhibit certain degree of anisotropy, the following modification of (20) is suggested [7] :

$$f_s = \left\{ \frac{1}{R_e^2} \begin{bmatrix} \left(\frac{R_e}{R_x} c_x\right)^2 + \left(\frac{R_e}{R_y} c_y\right)^2 + \\ - \frac{R_e}{R_x} c_x \frac{R_e}{R_y} c_y \end{bmatrix} + \\ + \frac{1}{R_t^2}\left(\frac{R_t}{R_{xy}} c_{xy}\right)^2 \right\}^{-1/2} \qquad (21)$$

Here R_x is the yield strength under static load relevant to the mean stress component c_x. The remaining material constants are defined analogously. Equation (21) gives :

$$f_s = \left[\sum_i \left(\frac{c_i}{R_i}\right)^2 - \frac{c_x c_y}{R_x R_y}\right]^{-1/2} \qquad i = x, y, xy \qquad (22)$$

With (13) and (17), the equation (23), similar to (22), can be written for the partial safety factor f_d [8] :

$$f_d = \left[\sum_i \left(\frac{a_i}{F_i}\right)^2 - \frac{a_x a_y}{F_x F_y}\right]^{-1/2} \qquad i = x, y, xy \qquad (23)$$

where F_i is the fatigue limit under fully reversed load relevant to the stress amplitude a_i. Equations (8), (22) and (23) yield the following criterion of fatigue "safe life" under combined multiaxial constant and in-phase loads [9] :

$$\left[\sum_i\left(\frac{c_i}{R_i}\right)^2-\frac{c_xc_y}{R_xR_y}\right]^{1/2}+$$

$$+\left[\sum_i\left(\frac{a_i}{F_i}\right)^2-\frac{a_xa_y}{F_xF_y}\right]^{1/2}\le 1 \qquad (24)$$

Equation (24) may be called the generalised Soderberg criterion of an infinite fatigue life under non-zero mean in-phase stress.

EQUIVALENT STRESS UNDER MULTIAXIAL CONSTANT AND PERIODIC LOADS

Let us consider the stress with Cartesian components given by Fourier series :

$$\sigma_i(t)=c_i+\sum_{p=1}^{\infty}a_{ip}\sin\left(p\,\omega_0 t+\beta_{ip}\right) \qquad i=x,y,xy \qquad (25)$$

where :

c_i - mean value of i-th stress component

a_{ip}, β_{ip} - amplitude and phase angle of p-th term in Fourier expansion of i-th stress component

$\omega_0 = 2\pi/T_0$ - fundamental circular frequency

T_0 - common period of the stress components.

Guided by the above presented criterion for the stress components (14), we shall try to model the stress components (25) by the equivalent stress components :

$$\sigma_i^{(eq)}(t)=c_i^{(eq)}+a_i^{(eq)}\sin\left(\omega_{eq}t+\varphi_i\right) \qquad i=x,y,xy \qquad (26)$$

where :

$c_i^{(eq)}$ - mean value of i-th equivalent stress component

$a_i^{(eq)},\varphi_i$ - amplitude and phase angle of i-th equivalent stress component

ω_{eq} - equivalent circular frequency.

For determination of parameters of the equivalent stress components the theory of energy transformation systems [10] is used. According to this theory, a reduced stress model and a given multiaxial stress can be regarded as equivalent in terms of fatigue life of the material if during the service life the internally and externally dissipated energies per unit volume in these two stress states are respectively equal. Under the assumption that the externally dissipated energy is proportional to the average strain energy of distortion, the following equivalence condition can be written [11] :

$$\frac{1}{T_o}\int_0^{T_o}\phi_{eq}(t)dt=\frac{1}{T_o}\int_0^{T_o}\phi(t)dt \qquad (27)$$

where :

$$\phi_{eq}(t)=\frac{1+\nu}{3E}\left[\begin{array}{l}\left(\sigma_x^{(eq)}\right)^2+\left(\sigma_y^{(eq)}\right)^2+\\-\sigma_x^{(eq)}\sigma_y^{(eq)}+3\left(\sigma_{xy}^{(eq)}\right)^2\end{array}\right]$$

is the strain energy of distortion per unit volume in the equivalent stress state, and :

$$\phi(t)=\frac{1+\nu}{3E}\left(\sigma_x^2+\sigma_y^2-\sigma_x\sigma_y+3\sigma_{xy}^2\right)$$

is that in the actual stress state, and :

E - Young modulus

ν - Poisson's ratio.

If the condition (27) is fulfilled then also the following equations are satisfied :

$$\frac{1}{T_o}\int_0^{T_o}\left[\sigma_i^{(eq)}(t)\right]^2dt=\frac{1}{T_o}\int_0^{T_o}\sigma_i^2(t)dt \qquad (28)$$

$$\frac{1}{T_o}\int_0^{T_o}\sigma_x^{(eq)}(t)\sigma_y^{(eq)}(t)dt=$$

$$=\frac{1}{T_o}\int_0^{T_o}\sigma_x(t)\sigma_y(t)dt \qquad (29)$$

When : $\omega_{eq}=k\,\omega_0$ (30)

where : k - a natural number, is assumed then (25) through (29) yield :

$$c_i^{(eq)}=c_i \qquad (31)$$

$$a_i^{(eq)}=\left(\sum_{p=1}^{\infty}a_{ip}^2\right)^{1/2} \qquad (32)$$

$$a_x^{(eq)}a_y^{(eq)}\cos\left(\varphi_x-\varphi_y\right)=$$

$$=\sum_{p=1}^{\infty}a_{xp}a_{yp}\cos\left(\beta_{xp}-\beta_{yp}\right) \qquad (33)$$

Of course, in design for an infinite fatigue life the evaluation of equivalent circular frequency can be avoided.

FATIGUE "SAFE-LIFE" CRITERION

Having determined the parameters of equivalent stress components, one can make use of the criterion (24) which, in particular, is suitable also for non-zero mean stress with synchronous components of the form (26). In this instance one obtains :

$$\left[\sum_i\left(\frac{c_i^{(eq)}}{R_i}\right)^2-\frac{c_x^{(eq)}c_y^{(eq)}}{R_xR_y}\right]^{1/2}+$$

$$+\left[\sum_i\left(\frac{a_i^{(eq)}}{F_i}\right)^2-\frac{a_x^{(eq)}a_y^{(eq)}\cos\left(\varphi_x-\varphi_y\right)}{F_xF_y}\right]^{1/2}\le 1 \qquad (34)$$

so that the fatigue "safe-life" design criterion for metal elements under multiaxial constant and periodic loads becomes :

$$\left[\sum_i\left(\frac{c_i}{R_i}\right)^2-\frac{c_xc_y}{R_xR_y}\right]^{1/2}+$$

$$+\left[\sum_i\sum_{p=1}^{\infty}\left(\frac{a_{ip}}{F_i}\right)^2-\frac{1}{F_xF_y}\sum_{p=1}^{\infty}a_{xp}a_{yp}\cos\left(\beta_{xp}-\beta_{yp}\right)\right]^{1/2}\le 1 \qquad (35)$$

Its extension to three-dimensional cases is straightforward.

CONCLUSIONS

- The fatigue "safe-life" design criterion covering the conditions of both static strength and fatigue safety of metal elements under multiaxial constant and periodic loads, was formulated.
- The presented criterion includes material constants which : have simple physical interpretation, can be determined by uniaxial tests, are directly related to the applied loads, and can reflect the material anisotropy.

NOMENCLATURE

a_a , a_b , a_t - stress amplitude under asymmetric axial force, bending moment and torsional load, respectively
a_b^* - amplitude of the equivalent zero mean stress under asymmetric bending
a_i , c_i - amplitude and mean value of i-th stress component (i = x, y, z, xy, yz, zx)
a_{eq} , c_{eq} - amplitude and mean value of the equivalent stress, respectively
$a_i^{(eq)}, c_i^{(eq)}$ - amplitude and mean value of i-th equivalent stress component, respectively
a_{ip} - amplitude of p-th harmonic in Fourier expansion of i-th stress component
B - maximum allowable stress amplitude in design for an infinite fatigue life under asymmetric bending
c_a , c_b , c_t - mean stress value under axial force, bending moment and torsional load, respectively
E - Young modulus
f - safety factor
f_d , f_s - partial safety factors
F_a , F_b , F_t - fatigue limit under fully reversed tension-compression, bending and torsion, respectively
F_i - fatigue limit under fully reversed load relevant to the stress amplitude a_i
k - natural number
R_e - tensile yield strength
R_i - yield strength relevant to the mean stress component c_i
R_t - shear yield strength
t - time
T_0 - stress period
β_i - phase angle of i-th component of the out-of-phase stress
β_{ip} - phase angle of p-th harmonic in Fourier expansion of i-th stress component
ν - Poisson's ratio
σ_i - i-th stress component
σ_{eq} - equivalent stress
$\sigma_i^{(eq)}$ - i-th equivalent stress component
φ_i - phase angle of i-th equivalent stress component
ϕ , ϕ_{eq} - strain energy of distortion per unit volume in the actual and equivalent stress states, respectively
ψ_b - asymmetry sensitivity index in bending
ω - circular frequency
ω_0 - fundamental circular frequency of the actual stress
ω_{eq} - equivalent circular frequency

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Conference

Maritime Traffic Engineering

On 20 and 21 November 2003, 10th International Conference devoted to problems of maritime traffic engineering was held in Świnoujście on the Baltic sea coast. It was organized by the Institute of Maritime Traffic Engineering, Maritime University of Szczecin.

The vast program of the Conference comprised 66 papers 6 of which were presented during the plenary session, and the remaining during 5 topical sessions.

The following papers were presented during the plenary session :

- *Extreme parameters of ships intended for entering the port of Świnoujście* - by S. Gucma and W. Ślączka (Maritime University of Szczecin)
- *Mathematical model of ship motion in canals and locks* by S. Zaikov, M. Lavrinovsky and V. Zaikov (State University for Water Communication, St. Petersburg)
- *Ship as intelligent machine* - by R. Śmierzchalski (Gdynia Maritime University)
- *On necessity of establishing a Polish Institute of Navigation following the example of similar institutions operating in neighbouring countries* - by A. Weintrit (Gdynia Maritime University)
- *The competences and duties of the officers in the charge of a navigational watch in the face of coming into operation of the ship's integrated control systems* by Z. Kopacz, W. Morgaś and J. Urbański (Naval University, Gdynia)
- *A method for the improving of location accuracy of objects with the use of teledetection data* - by J. Sanecki, A. Klewski, L. Cwojdziński, K. Maj and P. Kamieński (Military Engineering Academy, Warsaw)

The authors of the conference papers represented 15 scientific research centres including one of Slovakia and one of Russia. The greatest contribution to the conference materials (36 papers) was made by the authors of Maritime University of Szczecin. The conference became a comprehensive review of the interesting research projects concerning maritime traffic engineering, carried out in this university.