Viscoelastic unsteady lubrication of radial slide journal bearing at impulsive motion

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ABSTRACT

This paper presents an analytical solution of velocity components of unsymmetrical oil flow and pressure distribution in radial journal bearing gap for hydrodynamic unsteady lubrication with viscoelastic oil. Numerical calculations are performed in Mathcad 11 Professional Program, with taking into account the method of finite differences. This method satisfies stability conditions of numerical solutions of partial differential equations and values of capacity forces occurring in cylindrical bearings. Exact calculations of pressure in journal bearing and its load capacity may be useful to prevent from premature wear tribological units of self ignition engines, especially those applied in ships.

Key words : viscoelastic unsteady lubrication, analytical and numerical calculation, capacity forces, hydrodynamic pressure

INTRODUCTION

Correctness assessment of functioning the machines used for driving systems of various transport means e.g. diesel engines in which journal friction units are installed, depends to a large extent on assumed computational models, correct estimation of assumed appropriate simplifications, if necessary, and then on an assumed numerical method for the determining of operational parameters. During maneouvres of sea-going ships as well as when driving cars many frequent changes of engine operational parameters occur, especially of its rotational speed and loading. Similarly it happens during sailing the ship in rough weather when non-stationary loads on its propulsion engine happen.

Both car vehicles and majority of sea-going ships are equipped with a propulsion system of a single self-ignition combustion engine. Seizure of trigological system of such engine is equivalent to depriving the vehicle or ship of its propulsion and thereby of its serviceability [3]. If such an event occurs during storm emergency situations or even a catastrophy may happen [4]. One of the ways to avoid such failures is to apply a lubricating oil of required properties, especially of appropriate viscosity and lubricity.

Therefore the main aim of this work is to present analytical-numerical calculations of distributions of hydrodynamic pressure values occurring during non-stationary impulse lubrication of bearing surfaces at viscoelastic oil flow. Viscoelastic properties are characteristic for all oils which contain various bettering admixtures or in which some impurities such as lead salts, soot or dust, happen. All such impurities and admixtures are typical for land and sea transport.

Taking into account the above mentioned observations it is necessary to precisely analyze the influence of non-stationary load impulses which are transferred through a propulsion system to a slide friction unit and result in such characteristic quantity of the bearing as its load-carrying capacity determined on the basis of pressure distributions.

In Fig.1 the structure and loading scheme of a slide journal bearing is presented.

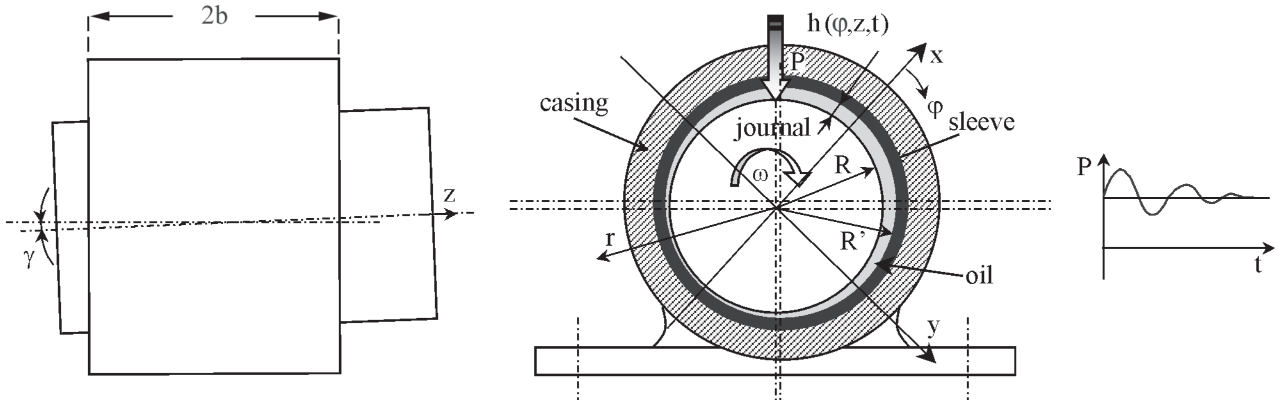

Fig. 1. Structure and loading scheme of a radial slide journal bearing and its characteristic dimensions

As shown in the scheme the bearing sleeve axis may undergo a skew respective to the journal axis. The quantity is described by the angle γ. In such case the oil gap height depends on the tangential variable φ and the longitudinal coordinate z. In the case of non-stationary impulse loads the oil gap height additionally depends on time t.

THEORETICAL MODEL

Oil flow through the cylindrical gap height of radial slide bearing is described by the momentum conservation equations and continuity equation [1, 2, 6, 7, 8, 10, 11, 15]. Additionally, Rivlin-Ericksen constitutive relationships were assumed. The equations in question are of the following form :

$$\text{Div}\,\mathbf{S} = \rho d\mathbf{v}/dt \quad , \quad \text{div}\,\mathbf{v} = 0 \quad , \quad \mathbf{S} = -p\mathbf{I} + \eta_o\mathbf{A}_1 + \alpha(\mathbf{A}_1)^2 + \beta\mathbf{A}_2 \tag{1}$$

where :

$\mathbf{S}$	- stress tensor	ρ	- oil density
Div $\mathbf{S}$	- stress tensor divergence	t	- time
$\mathbf{v}$	- velocity vector	p	- pressure
div $\mathbf{v}$	- velocity vector divergence	$\mathbf{I}$	- unit tensor

$\mathbf{A}_1$ and $\mathbf{A}_2$ - two Rivlin-Ericksen strain tensors of three material constants η_o, α, β,

where :

η_o - dynamic viscosity α, β - pseudo-viscosity constans of oil

The coordinates of $\mathbf{A}_1$, $\mathbf{A}_2$ tensors are described by the symmetrical matrices defined as follows :

$$\begin{gathered} \mathbf{A}_1 \equiv \mathbf{L} + \mathbf{L}^T \\ \mathbf{A}_2 \equiv \text{grad}\,\mathbf{a} + (\text{grad}\,\mathbf{a})^T + 2\mathbf{L}^T\mathbf{L} \\ \mathbf{a} \equiv \mathbf{L}\mathbf{v} + \frac{\partial \mathbf{v}}{\partial t} \end{gathered} \tag{2}$$

where :

$\mathbf{L}$	- tensor of oil velocity vector gradient s^{-1}	$\mathbf{a}$	- acceleration vector
$\mathbf{L}^T$	- tensor with matrix transpose s^{-1}	grad $\mathbf{a}$	- acceleration vector gradient

The product of the Deborah and Strouhal numbers, marked DeStr, is assumedof the same order as the product of the Reynold's number, relative radial clearance and Strouhal number, marked ReψStr. Moreover : DeStr >> De $\equiv \alpha\omega/\eta_o$. where : ψ - relative radial clearence, and ω - angular speed of cylindrical bearing journal.

The following is additionally assumed :

- rotational motion of the journal with the tangential speed $U = \omega R$
- unsymmetrical, non-stationary oil flow through bearing gap height
- non-stationary viscoelastic properties of oil
- constant oil density ρ
- the characteristic gap height $h(\varphi,z,t)$, in the cylindrical bearing
- no slip between bearing surfaces
- R - radius of cylindrical journal
- 2b - length of the bearing in question.

Neglecting the terms for the relative radial clearance $\psi \equiv \varepsilon/R \cong 10^{-3}$ in the basic equations defined in the cylindrical coordinate frame : φ, r, z, as well as taking into account the above mentioned assumptions, one can obtain :

$$\frac{\partial v_\varphi}{\partial t} = -\frac{1}{\rho R}\frac{\partial p}{\partial \varphi} + \frac{\eta_o}{\rho}\frac{\partial}{\partial r}\left(\frac{\partial v_\varphi}{\partial r}\right) + \frac{\beta}{\rho}\frac{\partial^3 v_\varphi}{\partial t \partial r^2} \tag{3}$$

$$0 = \frac{\partial p}{\partial r} \tag{4}$$

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\eta_o}{\rho}\frac{\partial}{\partial r}\left(\frac{\partial v_z}{\partial r}\right) + \frac{\beta}{\rho}\frac{\partial^3 v_z}{\partial t \partial r^2} \tag{5}$$

$$\frac{1}{R}\frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} = 0 \tag{6}$$

for : $0 \le \varphi < 2\pi$, $-b \le z \le +b$, $0 \le r \le h$, where : h - characteristic gap height.

The symbols: v_φ, v_r, v_z represent the respective oil velocity vector components : tangentially directed, that along gap height, and longitudinally directed. The following relationships between the dimensional and dimensionless quantities are assumed [12 , 13] :

$$\begin{gathered} r = R(1+\psi r_1) \; , \; z = bz_1 \; , \; t = t_o t_1 \; , \; h = \varepsilon h_1 \; , \; v_\varphi = U v_{\varphi 1} \; , \; v_r \equiv U\psi\, v_{r1} \\ v_z \equiv (U/L_1)\, v_{z1} \; , \; p = p_o p_1 \; , \; p_o \equiv U\eta_o R/\varepsilon^2 \; , \; L_1 = b/R \end{gathered} \tag{7}$$

Reynolds number, modified Reynolds number, Strouhal and Deborah number are assumed in the following forms :

$$Re \equiv \rho U\varepsilon/\eta_o \ , \ Re\psi \equiv \rho\omega\varepsilon^2/\eta_o \ , \ Str \equiv R/(Ut_o) \ , \ De \equiv \beta U/(\eta_o R) \tag{8}$$

hence :

$$DeStr = \beta/(\eta_o t_o) \equiv Des \ , \ Re\psi Str = \rho\varepsilon^2/(\eta_o t_o) \equiv Res \tag{8a}$$

For the commonly applied inhibitors the coefficient β satisfies the inequality: $0 < \beta/t_o < \eta_o$. Values of the coefficient β vary from 0.000001 $Pa{\cdot}s^2$ to 0.01 $Pa{\cdot}s^2$. The dimensionless symbols are marked with the lower index "1". The equations (3) ÷ (6) take the dimensionless form :

$$Res\frac{\partial v_{\varphi 1}}{\partial t_1} = -\frac{\partial p_1}{\partial \varphi} + \frac{\partial}{\partial r_1}\left(\frac{\partial v_{\varphi 1}}{\partial r_1}\right) + Des\frac{\partial^3 v_{\varphi 1}}{\partial t_1 \partial r_1^2} \tag{9}$$

$$0 = \frac{\partial p_1}{\partial r_1} \tag{10}$$

$$Res\frac{\partial v_{z1}}{\partial t_1} = -\frac{\partial p_1}{\partial z_1} + \frac{\partial}{\partial r_1}\left(\frac{\partial v_{z1}}{\partial r_1}\right) + Des\frac{\partial^3 v_{z1}}{\partial t_1 \partial r_1^2} \tag{11}$$

$$\frac{\partial v_{\varphi 1}}{\partial \varphi} + \frac{\partial v_{r1}}{\partial r_1} + \frac{1}{L_1^2}\frac{\partial v_{z1}}{\partial z_1} = 0 \tag{12}$$

for : $0 \le \varphi < 2\pi$, $-1 \le z_1 \le +1$, $0 \le r_1 \le h_1$

GENERAL AND PARTICULAR SOLUTIONS

A new variable is now introduced :

$$\chi \equiv r_1 N \ , \ N \equiv \frac{1}{2}\sqrt{\frac{Res}{t_1}} \ , \ t_1 > 0 \ , \ 0 < \frac{Des}{t_1} < 1 \tag{13}$$

and solutions are assumed to have the form of the following convergent series [5] :

$$v_{\varphi 1} = v_{\varphi 0\Sigma}(\chi,\varphi,z_1) + \frac{Des}{t_1}v_{\varphi 1\Sigma}(\chi,\varphi,z_1) + \left(\frac{Des}{t_1}\right)^2 v_{\varphi 2\Sigma}(\chi,\varphi,z_1) + ... \tag{14}$$

$$v_{z1} = v_{z0\Sigma}(\chi,\varphi,z_1) + \frac{Des}{t_1}v_{z1\Sigma}(\chi,\varphi,z_1) + \left(\frac{Des}{t_1}\right)^2 v_{z2\Sigma}(\chi,\varphi,z_1) + ... \tag{15}$$

$$v_{r1} = v_{r0\Sigma}(\chi,\varphi,z_1) + \frac{Des}{t_1}v_{r1\Sigma}(\chi,\varphi,z_1) + \left(\frac{Des}{t_1}\right)^2 v_{r2\Sigma}(\chi,\varphi,z_1) + ... \tag{16}$$

$$p_1 = p_{10}(\varphi,z_1,t_1) + \frac{Des}{t_1}p_{11}(\varphi,z_1,t_1) + \left(\frac{Des}{t_1}\right)^2 p_{12}(\varphi,z_1,t_1) + ... \tag{17}$$

for : $t_1 > 0$, $0 < Des << 1$, $(Des/t_1) < 1$

In the equations (9) ÷ (11) the derivatives respective to the variables t_1, r_1 are substituted for the derivatives relative to the variable χ, by using the relationships given in App. 1. Therefore the variables t_1, r_1 are substituted for the variable χ. Now the series (14) ÷ (17) are introduced to the equation system (9) ÷ (11). Next, the terms multiplied by the parameters of the same power, $(Des/t_1)^k$, are successively compared for k = 0, 1, 2, ... Hence the following sequence of ordinary differential equations is obtained:

$$\frac{d^2 v_{\varphi 0\Sigma}}{d\chi^2} + 2\chi\frac{dv_{\varphi 0\Sigma}}{d\chi} = \frac{1}{N^2}\frac{\partial p_{10}}{\partial \varphi} \tag{18}$$

$$\frac{d^2 v_{z0\Sigma}}{d\chi^2} + 2\chi\frac{dv_{z0\Sigma}}{d\chi} = \frac{1}{N^2}\frac{\partial p_{10}}{\partial z_1} \tag{19}$$

$$\frac{d^2 v_{\varphi 1\Sigma}}{d\chi^2} + 2\chi \frac{dv_{\varphi 1\Sigma}}{d\chi} + 4v_{\varphi 1\Sigma} = \frac{1}{N^2}\frac{\partial p_{11}}{\partial \varphi} + \frac{d^2 v_{\varphi 0\Sigma}}{d\chi^2} + \frac{1}{2}\chi \frac{d^2 v_{\varphi 0\Sigma}}{d\chi^2} \tag{20}$$

$$\frac{d^2 v_{z1\Sigma}}{d\chi^2} + 2\chi \frac{dv_{z1\Sigma}}{d\chi} + 4v_{z1\Sigma} = \frac{1}{N^2}\frac{\partial p_{11}}{\partial z_1} + \frac{d^2 v_{z0\Sigma}}{d\chi^2} + \frac{1}{2}\chi \frac{d^2 v_{z0\Sigma}}{d\chi^2} \tag{21}$$

$$\frac{d^2 v_{\varphi 2\Sigma}}{d\chi^2} + 2\chi \frac{dv_{\varphi 2\Sigma}}{d\chi} + 8v_{\varphi 2\Sigma} = \frac{1}{N^2}\frac{\partial p_{12}}{\partial \varphi} + 2\frac{d^2 v_{\varphi 1\Sigma}}{d\chi^2} + \frac{1}{2}\chi \frac{d^3 v_{\varphi 1\Sigma}}{d\chi^3} \tag{22}$$

$$\frac{d^2 v_{z2\Sigma}}{d\chi^2} + 2\chi \frac{dv_{z2\Sigma}}{d\chi} + 8v_{z2\Sigma} = \frac{1}{N^2}\frac{\partial p_{12}}{\partial z_1} + 2\frac{d^2 v_{z1\Sigma}}{d\chi^2} + \frac{1}{2}\chi \frac{d^3 v_{z1\Sigma}}{d\chi^3} \tag{23}$$

and so on.

The general solutions of the differential equations (18), (19) are of the following form :

$$v_{\varphi 0\Sigma}(\chi,\varphi,z_1) = C_{\varphi 1} v_{01}(\chi) + C_{\varphi 2} v_{02}(\chi) + v_{\varphi 03}(\chi,\varphi,z_1) \tag{24}$$

$$v_{z0\Sigma}(\chi,\varphi,z_1) = C_{z1} v_{01}(\chi) + C_{z2} v_{02}(\chi) + v_{z03}(\chi,\varphi,z_1) \tag{25}$$

where: $C_{\varphi 1}$, $C_{\varphi 2}$, C_{z1} , C_{z2} are integration constants. The particular solutions of the uniform and non-uniform ordinary differential equation are as follows :

$$v_{01}(\chi) = \frac{\sqrt{\pi}}{2}\operatorname{erf}(\chi) \quad , \quad v_{02}(\chi) = 1 \quad , \quad \operatorname{erf}(\chi) \equiv \frac{2}{\sqrt{\pi}}\int_0^{\chi} e^{-\chi_1^2} d\chi_1 \tag{26}$$

$$v_{\varphi 03}(\chi,\varphi,z_1) = -\frac{\sqrt{\pi}}{2N^2}\frac{\partial p_{10}}{\partial \varphi}\Omega(\chi) \quad , \quad v_{z03}(\chi,\varphi,z_1) = -\frac{\sqrt{\pi}}{2N^2}\frac{\partial p_{10}}{\partial z_1}\Omega(\chi) \tag{27}$$

$$\Omega(\chi) \equiv \int_0^{\chi} e^{\chi_1^2} \operatorname{erf}\chi_1 d\chi_1 - \operatorname{erf}\chi \int_0^{\chi} e^{\chi_1^2} d\chi_1 \tag{28}$$

where : $0 \le \chi_1 \le \chi \equiv r_1 N$

For $t_1 \to 0$, one obtains $N \to \infty$, hence $\chi \to \infty$. If $t_1 \to \infty$, then $N \to 0$, as well as for $r_1 > 0$ one obtains $\chi \to 0$. If $t_1 > 0$ and $r_1 = 0$ then $\chi = 0$. The following limits are true :

$$\begin{aligned} &v_{01}(\chi) = \pi^{0.5}/2 && \text{for : } \chi\to\infty \ , \ t_1\to 0 \ , \ N\to\infty \\ &v_{01}(\chi) = 0 && \text{for : } \chi\to 0 \ , \ r_1 = 0 \ , \ 0 < t_1 < t_2 < \infty \ , \ N > 0 \\ &v_{i03}(\chi) = 0 && \text{for : } \chi\to 0 \ , \ r_1 = 0 \ , \ 0 < t_1 < t_2 < \infty \ , \ N > 0 \ ; \ \text{where : } i = \varphi, z \\ &v_{01}(\chi) = 0 && \text{for : } \chi\to 0 \ , \ r_1 > 0 \ , \ t_1\to\infty \ , \ N\to 0 \end{aligned} \tag{29}$$

$$\begin{aligned} &v_{\varphi 03} = -\frac{r_1^2}{2}\frac{\partial p_{10}}{\partial \varphi} && \text{for : } \chi\to 0 \ , \ r_1 > 0 \ , \ t_1\to\infty \ , \ N\to 0 \\ &v_{z03} = -\frac{r_1^2}{2}\frac{\partial p_{10}}{\partial z_1} && \text{for : } \chi\to 0 \ , \ r_1 > 0 \ , \ t_1\to\infty \ , \ N\to 0 \end{aligned} \tag{30}$$

The cylindrical journal executes only the rotational motion in the φ direction. Hence the oil velocity component on the journal surface in the tangential direction is equal to the velocity of the cylindrical surface of the journal.

The longitudinal component of oil velocity on the journal's cylindrical surface equals zero because the journal is motionless along the z-axis. However the shaft undergoes pulsatory changes of its location with time along the gap height direction. Hence the gap height is time dependent. Thus, the following boundary conditions appear :

$$v_{\varphi 0\Sigma}(\chi = 0) = 1 \ , \ v_{z0\Sigma}(\chi = 0) = 0 \ , \ v_{r0\Sigma}(\chi = 0) = Str \cdot (\partial h_1/\partial t_1) \quad \text{for : } r_1 = 0 \Leftrightarrow \chi = 0 \ , \ 0 < t_1 < t_2 < \infty \ , \ N > 0 \tag{31}$$

The cylindrical sleeve surface is motionless in the tangential, longitudinal (axial) and radial directions. Viscous oil flows around the sleeve. Hence the oil velocity on the sleeve surface equals zero in the tangential and longitudinal directions as well as in the gap height direction r. Thus the following boundary conditions are valid :

$$v_{\varphi 0\Sigma}(\chi = M) = 0 \ , \ v_{z0\Sigma}(\chi = M) = 0 \ , \ v_{r0\Sigma}(\chi = M) = 0 \quad \text{for : } r_1\to h_1 \Leftrightarrow \chi\to N h_1 \equiv M \ , \ 0 < t_1 < t_2 < \infty \ , \ N > 0 \tag{32}$$

where : $h = \varepsilon h_1$ - gap height , h_1- dimensionless gap height

Applying the conditions (31), (32) to the solutions (24), (25) one obtains :

$$\begin{aligned}
&C_{\varphi1}v_{01}(\chi=0)+C_{\varphi2}+v_{\varphi03}(\chi=0)=1 && \text{for}: r_1=0\\
&C_{\varphi1}v_{01}(\chi=M)+C_{\varphi2}+v_{\varphi03}(\chi=M)=0 && \text{for}: r_1=h_1\\
&C_{z1}v_{01}(\chi=0)+C_{z2}+v_{z03}(\chi=0)=0 && \text{for}: r_1=0\\
&C_{z1}v_{01}(\chi=M)+C_{z2}+v_{z03}(\chi=M)=0 && \text{for}: r_1=h_1
\end{aligned} \tag{33}$$

If the limits (29), (30) are taken into account the set of equations (33) yields the following solutions :

$$C_{\varphi1}=-\frac{1+v_{\varphi03}(M)}{v_{01}(M)}\ ,\ C_{z1}=-\frac{v_{z03}(M)}{v_{01}}\ ,\ C_{\varphi2}=1\ ,\ C_{z2}=0 \tag{34}$$

Now, to the right hand sides of the equations (20), (21) the solutions (24), (25), (26), (27), (28), (34) are substituted. Hence the general solution of the equations (20), (21) obtains the following form :

$$v_{\varphi1\Sigma}(\chi,\varphi,z_1)=C_{\varphi3}v_{11}(\chi)+C_{\varphi4}v_{12}(\chi)+v_{\varphi13}(\chi,\varphi,z_1) \tag{35}$$

$$v_{z1\Sigma}(\chi,\varphi,z_1)=C_{z3}v_{11}(\chi)+C_{z4}v_{12}(\chi)+v_{z13}(\chi,\varphi,z_1) \tag{36}$$

where : $C_{\varphi3}$, $C_{\varphi4}$, C_{z3} , C_{z4} - integration constants

The particular solutions are as follows :

$$v_{11}(\chi)=\chi e^{-\chi^2} \tag{37}$$

$$v_{12}(\chi)=\chi e^{-\chi^2}\int_{\delta}^{\chi}\frac{1}{\chi_1^2}e^{-\chi_1^2}d\chi_1 \tag{38}$$

$$\begin{aligned}
v_{\varphi13}(\chi,z_1,C_{\varphi1})&=v_{11}(\chi)\int_0^{\chi}\left\{C_{\varphi1}\chi_1(\chi_1+2)-\left(1+\frac{\chi_1}{2}\right)e^{\chi_1^2}\frac{d^2}{d\chi_1^2}\left[v_{\varphi03}(\chi_1)\right]+\frac{1}{N^2}\frac{\partial p_{11}}{\partial\varphi}\right\}v_{12}(\chi_1)d\chi_1+\\
&+v_{12}(\chi)\int_0^{\chi}\left\{\left(1+\frac{\chi_1}{2}\right)e^{\chi_1^2}\frac{d^2}{d\chi_1^2}\left[v_{\varphi03}(\chi_1)\right]+\frac{1}{N^2}\frac{\partial p_{11}}{\partial\varphi}-C_{\varphi1}\chi_1(\chi_1+2)\right\}v_{11}(\chi_1)d\chi_1\\
v_{z13}(\chi,z_1,C_{\varphi1})&=v_{11}(\chi)\int_0^{\chi}\left\{C_{z1}\chi_1(\chi_1+2)-\left(1+\frac{\chi_1}{2}\right)e^{\chi_1^2}\frac{d^2}{d\chi_1^2}\left[v_{z03}(\chi_1)\right]+\frac{1}{N^2}\frac{\partial p_{11}}{\partial z_1}\right\}v_{12}(\chi_1)d\chi_1+\\
&+v_{12}(\chi)\int_0^{\chi}\left\{\left(1+\frac{\chi_1}{2}\right)e^{\chi_1^2}\frac{d^2}{d\chi_1^2}\left[v_{z03}(\chi_1)\right]+\frac{1}{N^2}\frac{\partial p_{11}}{\partial z_1}-C_{z1}\chi_1(\chi_1+2)\right\}v_{11}(\chi_1)d\chi_1
\end{aligned} \tag{39}$$

for : $0<\delta\le\chi_1\le\chi$

The solutions (35), (36) represent corrections to the components of oil velocity because of its viscoelastic properties. On the basis of the solutions (37) ÷ (39) for $\chi\rightarrow0$, $r_1\rightarrow0$, $N>0$ the following limits are achieved :

$$\lim_{\chi\to0,N>0}v_{12}(\chi)=\lim_{\chi\to0,N>0}\chi e^{-\chi^2}\int_{\delta}^{\chi}\left(\frac{1}{\chi_1^2}e^{\chi_1^2}\right)d\chi_1=-1 \tag{40}$$

The following limits are also true :

$$\begin{aligned}
&v_{11}(\chi)=0 && \text{for}: \chi\rightarrow0\ ,\ r_1=0\ ,\ 0<t_1<t_2<\infty\ ,\ N>0\\
&v_{12}(\chi)=-1 && \text{for}: \chi\rightarrow0\ ,\ r_1=0\ ,\ 0<t_1<t_2<\infty\ ,\ N>0\\
&v_{i13}(\chi)=0 && \text{for}: \chi\rightarrow0\ ,\ r_1=0\ ,\ 0<t_1<t_2<\infty\ ,\ N>0\ ;\ \text{where } i=\varphi, z
\end{aligned} \tag{41}$$

The corrections to the oil velocity components cannot have the same boundary conditions as those previously assumed (31), (32) for the cylindrical journal and sleeve in the longitudinal and tangential directions. Therefore the corrections satisfy the following boundary conditions :

$$\begin{aligned}
&v_{\varphi1\Sigma}(\chi=0)=0\ ,\ v_{z1\Sigma}(\chi=0)=0 && \text{for}: r_1=0\Leftrightarrow\chi=0\ ,\ 0<t_1<t_2<\infty\ ,\ N>0\\
&v_{\varphi1\Sigma}(\chi=M)=0\ ,\ v_{z1\Sigma}(\chi=M)=0 && \text{for}: r_1\rightarrow h_1\Leftrightarrow\chi\rightarrow Nh_1\equiv M\ ,\ 0<t_1<t_2<\infty\ ,\ N>0
\end{aligned} \tag{42}$$

Applying the conditions (42) to the general solutions (35), (36) one obtains :

$$\begin{aligned} &C_{\varphi 3}v_{11}(\chi=0)+C_{\varphi 4}v_{12}(\chi=0)+v_{\varphi 13}(\chi=0)=0 && \text{for : } r_1=0\\ &C_{\varphi 3}v_{11}(\chi=M)+C_{\varphi 4}v_{12}(\chi=M)+v_{\varphi 13}(\chi=M)=0 && \text{for : } r_1=h_1\\ &C_{z3}v_{11}(\chi=0)+C_{z4}v_{12}(\chi=0)+v_{z13}(\chi=0)=0 && \text{for : } r_1=0\\ &C_{z3}v_{11}(\chi=M)+C_{z4}v_{12}(\chi=M)+v_{z13}(\chi=M)=0 && \text{for : } r_1=h_1 \end{aligned} \tag{43}$$

If the limits (41) are accounted for the set of equations (43) yields the following solutions :

$$C_{i3}=-\frac{v_{i13}(\chi=h_1N)}{v_{11}(\chi=h_1N)}\ ,\quad C_{i4}=0 \quad \text{for : } i=\varphi, z \tag{44}$$

The general solutions of the oil velocity components (15), (14) at making use of the solutions (25), (35), (36), can be presented in the following form :

$$v_{\varphi 1}=C_{\varphi 1}v_{01}(\chi)+C_{\varphi 2}+v_{\varphi 03}(\chi,\varphi,z_1)+\frac{Des}{t_1}\left[C_{\varphi 3}v_{11}(\chi)+C_{\varphi 4}v_{12}(\chi)+v_{\varphi 13}(\chi,\varphi,z_1)\right]+O\left(\frac{Des}{t_1}\right)^2 \tag{45}$$

$$v_{z1}=C_{z1}v_{01}(\chi)+C_{z2}+v_{z03}(\chi,\varphi,z_1)+\frac{Des}{t_1}\left[C_{z3}v_{11}(\chi)+C_{z4}v_{12}(\chi)+v_{z13}(\chi,\varphi,z_1)\right]+O\left(\frac{Des}{t_1}\right)^2 \tag{46}$$

If $t_1\to\infty$ then $N\to 0$, hence $\chi \equiv r_1N\to 0$.

For further analysis it is worthwhile to find the following limits :

$$\lim_{\chi\to 0, N\to 0} N^2 v_{11}(\chi)=\lim_{\chi\to 0} N^2\chi e^{-\chi^2}=0 \tag{47}$$

$$\lim_{\chi\to 0, N\to 0} N^2 v_{12}(\chi)=\lim_{\chi\to 0, N\to 0} N^2\chi e^{-\chi^2}\cdot\int_\delta^\chi\left(\frac{1}{\chi_1^2}e^{\chi_1^2}\right)d\chi_1=0 \tag{48}$$

for : $0\le\chi_1\le h_1N$, $0<t_1<\infty$, $0\le r_1\le h_1$, $-1\le z_1\le +1$, $0<\varphi<2\pi$

VALUES OF OIL VELOCITY AND PRESSURE AT NON-STATIONARY NEWTONIAN LUBRICATION

If viscoelastic properties of oil are neglected, then on the basis of the solutions (45), (46) the oil velocity components in the φ and z directions, at non-stationary flow, are of the following form :

$$v_{\varphi 0\Sigma}(\varphi,r_1,z_1,t_1)=+1-\left\{1-\frac{\sqrt{\pi}}{2N^2}\frac{\partial p_{10}}{\partial\varphi}\Omega(\chi=Nh_1)\right\}\frac{\operatorname{erf}(r_1N)}{\operatorname{erf}(h_1N)}-\frac{\sqrt{\pi}}{2N^2}\frac{\partial p_{10}}{\partial\varphi}\Omega(\chi=Nr_1) \tag{49}$$

$$v_{z0\Sigma}(\varphi,r_1,z_1,t_1)=\frac{\sqrt{\pi}}{2N^2}\frac{\partial p_{10}}{\partial z_1}\Omega(\chi=Nh_1)\frac{\operatorname{erf}(r_1N)}{\operatorname{erf}(h_1N)}-\frac{\sqrt{\pi}}{2N^2}\frac{\partial p_{10}}{\partial z_1}\Omega(\chi=Nr_1) \tag{50}$$

for : $0\le t_1<\infty$, $0\le r_1\le h_1$, $-1\le z_1\le 1$, $0<\varphi<2\pi$, $0\le\chi_2\le\chi_1\le\chi\equiv r_1N\le h_1N\equiv M$, $h_1=h_1(\varphi,z_1)$

The oil velocity components (49), (50) are now inserted to the continuity equation (12) and next the equation is integrated respective to the variable r_1.

The oil velocity component $v_{r0\Sigma}$ in the gap height direction is not equal to zero on the cylindrical journal surface due to impulse displacements of the shaft. Therefore by applying the condition $v_{r0\Sigma}=Str\,\partial h_1/\partial t_1$ for $r_1=0$, the following form of this oil velocity component is obtained :

$$v_{r0\Sigma}(\varphi,r_1,z_1,t_1)=-\int_0^{r_1}\frac{\partial v_{\varphi 0\Sigma}}{\partial\varphi}dr_2-\frac{1}{L_1^2}\int_0^{r_1}\frac{\partial v_{z0\Sigma}}{\partial z_1}dr_2+Str\frac{\partial h_1}{\partial t_1} \tag{51}$$

for : $0\le t_1<\infty$, $0\le r_2\le r_1\le h_1$, $-1\le z_1\le +1$, $0<\varphi<2\pi$, $0\le\chi_2\le\chi_1\le\chi\equiv r_1N\le h_1N\equiv M$

The oil velocity component $v_{r0\Sigma}$ equals zero on the sleeve surface. Integrating the continuity equation (12) along the direction r and applying the boundary condition (32) for $r_1=h_1$ to the oil velocity component in the gap height direction, and making use of the conditions (29) one obtains the equation :

$$\frac{\partial}{\partial\varphi}\int_0^{h_1}v_{\varphi 0\Sigma}dr_1+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\int_0^{h_1}v_{z0\Sigma}dr_1=Str\frac{\partial h_1}{\partial t_1} \tag{52}$$

The solutions (49) ÷ (50) are now inserted to the equation (52). One then obtains the following modified Reynolds equation :

$$\frac{\sqrt{\pi}}{2N^2}\frac{\partial}{\partial\varphi}\left\{\left[\frac{\int_0^{h_1}\mathrm{erf}(r_1N)dr_1}{\mathrm{erf}(h_1N)}\Omega(\chi=Nh_1)-\int_0^{h_1}\Omega(\chi=Nr_1)dr_1\right]\frac{\partial p_{10}}{\partial\varphi}\right\}+$$

$$+\frac{\sqrt{\pi}}{2N^2L_1^2}\frac{\partial}{\partial z_1}\left\{\left[\frac{\int_0^{h_1}\mathrm{erf}(r_1N)dr_1}{\mathrm{erf}(h_1N)}\Omega(\chi=Nh_1)-\int_0^{h_1}\Omega(\chi=Nr_1)dr_1\right]\frac{\partial p_{10}}{\partial z_1}\right\}= \quad (53)$$

$$=-\int_0^{h_1}\left[1-\frac{\mathrm{erf}(r_1N)}{\mathrm{erf}(h_1N)}\right]dr_1+\mathrm{Str}\frac{\partial h_1}{\partial t_1}$$

for : $0 \le r_2 \le r_1 \le h_1$, $0 \le \varphi < 2\pi$, $-1 \le z_1 < +1$, $0 \le t_1 < \infty$, $0 \le \chi_1 \le \chi \le h_1N$, $0 \le N(t_1) = 0.5(Res/t_1)^{0.5} < \infty$

The modified Reynolds equation (53) defines an unknown pressure function p_{10} (φ, z_1, t_1). If the dimensionless time t_1 approaches infinity, i.e. the coefficient N approaches zero, then the equation (53) approaches the classical Reynolds equation (see App. 2) :

$$\frac{\partial}{\partial\varphi}\left(h_1^3\frac{\partial p_{10}}{\partial\varphi}\right)+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left(h_1^3\frac{\partial p_{10}}{\partial z_1}\right)=6\frac{\partial h_1}{\partial\varphi}-12\,\mathrm{Str}\frac{\partial h_1}{\partial t_1} \quad (54)$$

for : $0 \le \varphi < 2\pi$, $-1 \le z_1 < +1$

The dimensionless time-dependent height of the bearing gap height, at accounting for its periodical disturbances, may be described by the following relationship :

$$h_1 = [1 + \lambda \cdot \cos\varphi + s \cdot z \cdot \cos(\varphi)] \cdot f(t_1) \;\; ; \;\; f(t_1) = [1 + c \cdot \exp(-t_o \cdot t_1 \cdot \omega_o)] \quad (55)$$

where : $s = \frac{L}{\psi}\tan(\gamma)$ - skew coefficient.

The symbol ω_o stands for an angular velocity given in $[s^{-1}]$ which describes the disturbances in the gap height direction for unsteady oil flow through the cylindrical bearing gap height, and "c" stands for a coefficient used to control values of gap height changes. If c- value is positive the bearing gap height is increased, if negative - the bearing gap height is decreased relative to the classical gap height. If t_1 approaches infinity then the gap height equation (55) approaches the classical gap height equation independent on time during stationary motion.

PRESSURE CORRECTIONS FOR VISCOELASTIC OIL PROPERTIES

Particular solutions of oil velocity components in the directions φ and z_1 due to viscoelastic properties in non-stationary motion are contained in the solutions (14) , (16) multiplied by the term DeStr/t_1. By making use of (38), (39), (44) and the boundary conditions (42), the corrections of the oil velocity components (35) , (36) can be transformed to the following forms :

$$\frac{Des}{t_1}v_{\varphi 1\Sigma}(\varphi,z_1,r_1,t_1)=\frac{4\beta}{\rho\varepsilon^2}Nr_1e^{-r_1^2N^2}\left\{\frac{\partial p_{11}}{\partial\varphi}\left[\int_{r_1N}^{h_1N}\chi\Omega_1(\chi)d\chi+\right.\right.$$

$$\left.+\frac{N^2}{2}\left(h_1^2-r_1^2\right)\Omega_1(\chi=h_1N)\right]+\frac{\partial p_{10}}{\partial\varphi}\left[\int_{r_1N}^{h_1N}\Omega_2(\chi)d\chi-\Omega_1(\chi=h_1N)\int_0^{h_1N}\chi_1e^{-\chi^2}\Omega_2(\chi)d\chi+\right.$$

$$\left.+\Omega_1(\chi=r_1N)\int_0^{r_1N}\chi_1e^{-\chi^2}\Omega_2(\chi)d\chi\right]-\frac{2}{\sqrt{\pi}\mathrm{erf}(h_1N)}\frac{\partial p_{10}}{\partial\varphi}\left[\int_{r_1N}^{h_1N}\Omega_1(\chi)\Omega_3(\chi)\Omega(\chi)d\chi+\right. \quad (56)$$

$$\left.\left.-\Omega_1(\chi=h_1N)\int_0^{h_1N}\Omega_3(\chi)\Omega(\chi)d\chi+\Omega_1(\chi=r_1N)\int_0^{r_1N}\Omega_3(\chi)\Omega(\chi)d\chi\right]\right\}+$$

$$-\frac{8\beta N^2r_1e^{-r_1^2N^2}}{\sqrt{\pi}\rho\varepsilon^2\mathrm{erf}(h_1N)}\left[\Omega_1(\chi=h_1N)\int_0^{h_1N}\Omega_3(\chi)d\chi-\Omega_1(\chi=r_1N)\int_0^{r_1N}\Omega_3(\chi)d\chi-\int_{r_1N}^{h_1N}\Omega_1(\chi)\Omega_3(\chi)d\chi\right]$$

$$\frac{\mathrm{Des}}{t_1} v_{z1\Sigma}(\varphi, z_1, r_1, t_1) = \frac{4\beta}{\rho\varepsilon^2}\frac{1}{L_1^2} N r_1 e^{-r_1^2 N^2}\left\{\frac{\partial p_{11}}{\partial z_1}\left[\int_{r_1 N}^{h_1 N}\chi\Omega_1(\chi)d\chi + \right.\right.$$

$$+\frac{N^2}{2}\left(h_1^2 - r_1^2\right)\Omega_1(\chi = h_1 N)\Bigg] + \frac{\partial p_{10}}{\partial z_1}\left[\int_{r_1 N}^{h_1 N}\Omega_2(\chi)d\chi - \Omega_1(\chi = h_1 N)\int_0^{h_1 N}\chi e^{-\chi^2}\Omega_2(\chi)d\chi + \right.$$

$$+\Omega_1(\chi = r_1 N)\int_0^{r_1 N}\chi e^{-\chi^2}\Omega_2(\chi)d\chi\Bigg] - \frac{2}{\sqrt{\pi}\,\mathrm{erf}(h_1 N)}\frac{\partial p_{10}}{\partial z_1}\left[\int_{r_1 N}^{h_1 N}\Omega_1(\chi)\Omega_3(\chi)\Omega(\chi)d\chi + \right. \tag{57}$$

$$\left.\left. - \Omega_1(\chi = h_1 N)\int_0^{h_1 N}\Omega_3(\chi)\Omega(\chi)d\chi + \Omega_1(\chi = r_1 N)\int_0^{r_1 N}\Omega_3(\chi)\Omega(\chi)d\chi\right]\right\}$$

where :

$$\Omega_1(\chi) \equiv \int_\delta^\chi \frac{1}{\chi_1^2} e^{\chi_1^2} d\chi_1 \ , \ \ \Omega_2(\chi) \equiv \left(\chi + \frac{\chi^2}{2}\right)\left(2\chi e^{-\chi^2}\int_0^\chi e^{\chi_1^2} d\chi_1 - 1\right) \ , \ \ \Omega_3(\chi) \equiv \chi^2(\chi + 2)e^{-\chi^2} \tag{58}$$

and : $0 \le t_1 < \infty$, $0 \le r_2 \le r_1 \le h_1$, $-1 \le z_1 \le 1$, $0 < \varphi < 2\pi$, $0 \le \chi_1 \le \chi \equiv r_1 N \le h_1 N \equiv M$

The corrections (56) , (57) are now inserted to the continuity equation (12) and then both its sides are integrated respective to the variable r.

From the viscoelastic oil properties it results that the corrections of the oil velocity component along the gap height must equal zero on the journal surface at $r_1 = 0$. Hence the correction of the oil velocity component along the gap height is as follows :

$$v_{r1\Sigma}(\varphi, z_1, r_1, t_1) = \frac{\partial}{\partial\varphi}\left(\int_0^{r_1} v_{\varphi 1\Sigma}(\varphi, z_1, r_1, t_1)\,dr_1\right) + \frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left(\int_0^{r_1} v_{z1\Sigma}(\varphi, z_1, r_1, t_1)\,dr_1\right) \tag{59}$$

The velocity vector corrections cannot change the boundary conditions (31) , (32) which are assumed on the journal and sleeve surfaces in the direction of the bearing gap height. Therefore the oil velocity vector corrections in this direction are equal to zero on the sleeve surface at $r_1=h_1$. By applying this condition to the solution (59) the modified Reynolds equation can be obtained :

$$\frac{\partial}{\partial\varphi}\left\{\frac{\partial p_{11}}{\partial\varphi}\left[\int_0^{h_1} r_1 e^{-r_1^2 N^2}\left(\int_{r_1 N}^{h_1 N}\chi\Omega_1(\chi)\,d\chi - r_1^2\Omega_1(\chi = r_1 N)\right)dr_1 + \frac{1}{4}\left(1 - e^{-h_1^2 N^2}\right)h_1^2\Omega_1(\chi = h_1 N)\right]\right\} +$$

$$+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left\{\frac{\partial p_{11}}{\partial z_1}\left[\int_0^{h_1} r_1 e^{-r_1^2 N^2}\left(\int_{r_1 N}^{h_1 N}\chi\Omega_1(\chi)\,d\chi - r_1^2\Omega_1(\chi = r_1 N)\right)dr_1 + \frac{1}{4}\left(1 - e^{-h_1^2 N^2}\right)h_1^2\Omega_1(\chi = h_1 N)\right]\right\} =$$

$$= \frac{2N}{\mathrm{erf}\left(Nh_1\right)}\int_0^{h_1} r_1 e^{-r_1^2 N^2}\left[\Omega_1(\chi = r_1 N)\int_0^{h_1 N}\Omega_3(\chi)d\chi - \Omega_1(\chi = r_1 N)\int_0^{r_1 N}\Omega_3(\chi)d\chi - \int_{r_1 N}^{h_1 N}\Omega_1(\chi)\Omega_3(\chi)d\chi\right]dr_1 +$$

$$+\frac{2}{\sqrt{\pi}\,\mathrm{erf}(h_1 N)}\frac{\partial}{\partial\varphi}\left\{\frac{\partial p_{10}}{\partial\varphi}\left[\int_0^{h_1} r_1 e^{-r_1^2 N^2}\left(\int_{r_1 N}^{h_1 N}\Omega_1(\chi)\Omega_3(\chi)\Omega(\chi)\,d\chi + \right.\right.\right. \tag{60}$$

$$\left.\left.\left. +\Omega_1(\chi = r_1 N)\int_0^{r_1 N}\Omega_1(\chi)\Omega_3(\chi)\Omega(\chi)\,d\chi\right)dr_1 - \frac{1 - e^{-h_1^2 N^2}}{2N^2}\Omega_1(\chi = h_1 N)\int_0^{h_1 N}\Omega_1(\chi)\Omega_3(\chi)\Omega(\chi)\,d\chi\right]\right\} +$$

$$+\frac{2}{\sqrt{\pi}\,\mathrm{erf}(h_1 N)}\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left\{\frac{\partial p_{10}}{\partial z_1}\left[\int_0^{h_1} r_1 e^{-r_1^2 N^2}\left(\int_{r_1 N}^{h_1 N}\Omega_1(\chi)\Omega_3(\chi)\Omega(\chi)\,d\chi + \right.\right.\right.$$

$$\left.\left.\left. +\Omega_1(\chi = r_1 N)\int_0^{r_1 N}\Omega_1(\chi)\Omega_3(\chi)\Omega(\chi)\,d\chi\right)dr_1 - \frac{1 - e^{-h_1^2 N^2}}{2N^2}\Omega_1(\chi = h_1 N)\int_0^{h_1 N}\Omega_1(\chi)\Omega_3(\chi)\Omega(\chi)\,d\chi\right]\right\} +$$

$$-\frac{\partial}{\partial\varphi}\left\{\frac{\partial p_{10}}{\partial\varphi}\left[\int_0^{h_1} r_1 e^{-r_1^2N^2}\left(\int_{r_1N}^{h_1N}\Omega_2(\chi)\,d\chi+\Omega_1(\chi=r_1N)\int_0^{h_1N}\Omega_2(\chi)\,\chi e^{-\chi^2}d\chi\right)dr_1+\right.\right.$$

$$\left.\left.-\frac{1-e^{-h_1^2N^2}}{2N^2}\Omega_1(\chi=h_1N)\int_0^{h_1N}\Omega_2(\chi)\,\chi e^{-\chi^2}d\chi\right]\right\}+ \qquad (60)$$

$$-\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left\{\frac{\partial p_{10}}{\partial z_1}\left[\int_0^{h_1} r_1 e^{-r_1^2N^2}\left(\int_{r_1N}^{h_1N}\Omega_2(\chi)\,d\chi+\Omega_1(\chi=r_1N)\int_0^{h_1N}\Omega_2(\chi)\,\chi e^{-\chi^2}d\chi\right)dr_1+\right.\right.$$

$$\left.\left.-\frac{1-e^{-h_1^2N^2}}{2N^2}\Omega_1(\chi=h_1N)\int_0^{h_1N}\Omega_2(\chi)\,\chi e^{-\chi^2}d\chi\right]\right\}$$

for : $0 \le r_2 \le r_1 \le h_1$, $0 \le \varphi < 2\pi$, $-1 \le z_1 < +1$, $0 \le t_1 < \infty$, $0 \le \chi_1 \le \chi \le h_1N$, $0 \le N(t_1) = 0.5(Res/t_1)^{0.5} < \infty$

The modified Reynolds equation (60) determines unknown functions $p_{11}(\varphi, z_1, t_1)$ of the corrections of pressure values, resulting from viscoelastic oil properties during non-stationary motion.

NUMERICAL CALCULATIONS

The dimensionless distributions of values of the pressure p_{10} and its corrections p_{11}, p_{12}, ... in the lubrication area are determined by means of the Reynolds equations (53) , (60) with using the gap height (55). On the boundary of the area, dimensional values of the pressure and its corrections obtain the value of the atmospheric pressure p_{at}. The lubrication area and the gap height are shown in Fig.2. In order to determine a dimensional value of the gap height the dimensionless gap height indicated in Fig.2 should be multiply by the radial clearance ε. The lubrication area is defined by the following inequalities : $0 \le \varphi \le \pi$, $-1 \le z_1 \le 1$.

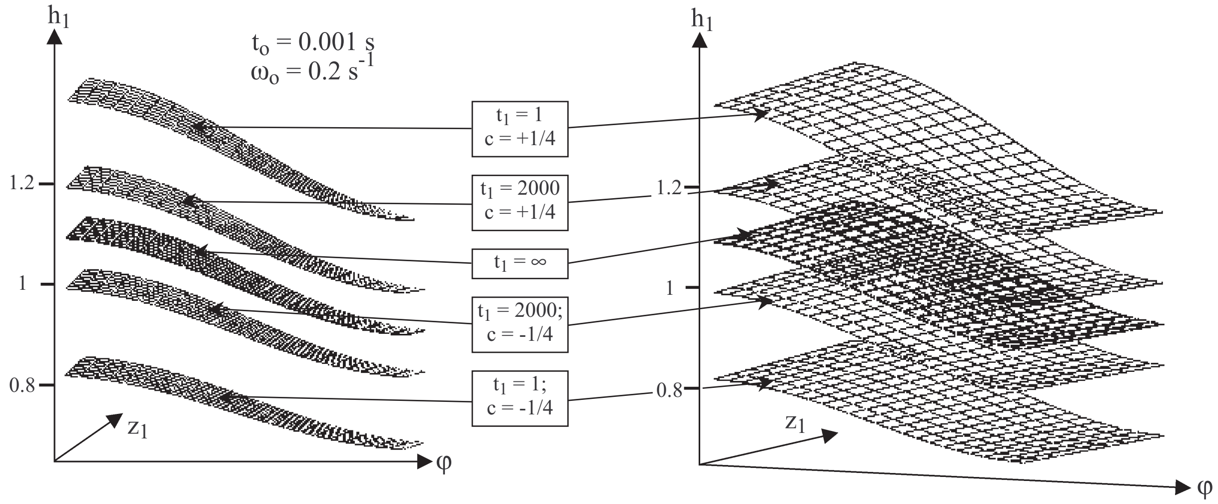

***Fig. 2.** The example lubrication area with the time-variable height of lubrication bearing gap*

The values of the dimensionless time t_1 increasing from 1 to 2000 and even more express the time of departing from the instant of impulse occurrence of the force acting on friction unit. The impulse which causes the gap height decreasing was denoted with the negative value of the gap height change coefficient c = -1/4, and that causing the gap height increasing – with the positive value of that coefficient c = 1/4.

In order to calculate definite hydrodynamic pressure values the following input data were assumed :

- relative radial clearance : $\psi = 10^{-3}$
- oil viscosity : $\eta_o = 0.03$ [Pa·s]
- oil pseudo-viscosity coefficient : $\beta = 6\cdot10^{-4}$ [Pa·s^2]
- oil density : $\rho = 900$ [kg/m^3]
- shaft angular speed : n = 1500 [rpm]
- relative excentricity : $\lambda = 0.5$
- shaft radius : R = 0.08 [m]
- dimensionless bearing length : $L_1 = 1$
- skewness coefficient : s = 0.05
- angular speed of sleeve perturbation : $\omega_o = 0.2$ [s^{-1}]
- characteristic time : $t_o = 0.001$ [s]
- time intervals : t_1 =1 ; 100 ; 10000 ; ∞

For the so assumed data and on the basis of the Reynolds equation (53) the dimensionless values of pressure distribution were numerically determined for the gap height defined by the equation (55) with the use of the method of finite differences and the software Mathcad 11 (Fig. 3, 4, 5, 6). In order to obtain real dimensional distributions of pressure values, the calculated dimensionless values of pressure distributions should be multiplied by the dimensional coefficient $UR\eta_o/\varepsilon^2$. The distributions of dimensionless hydrodynamic pressure values were presented for the dimensionless time intervals $t_1 = 1$; $t_1 = 100$; $t_1 = 10000$; $t_1 = \infty$ and the bearing gap height change coefficient c = ± 1/4.

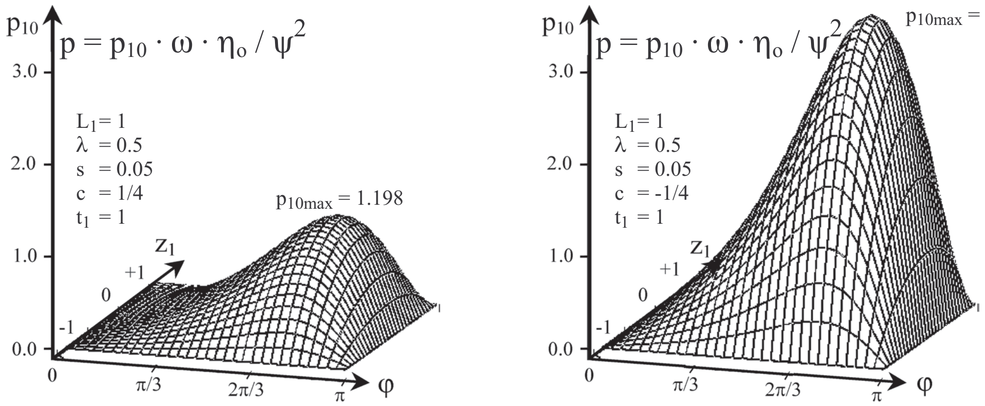

***Fig. 3.** Distributions of the dimensionless hydrodynamic pressure in the cylindrical bearing gap for the dimensionless time t_1=1 counted from the impulse instant and for the gap height change coefficient c = +1/4 (lubrication gap height increased due to impulse load), and for c = -1/4 (lubrication gap height decreased due to impulse load) at accounting for a skew of the journal*

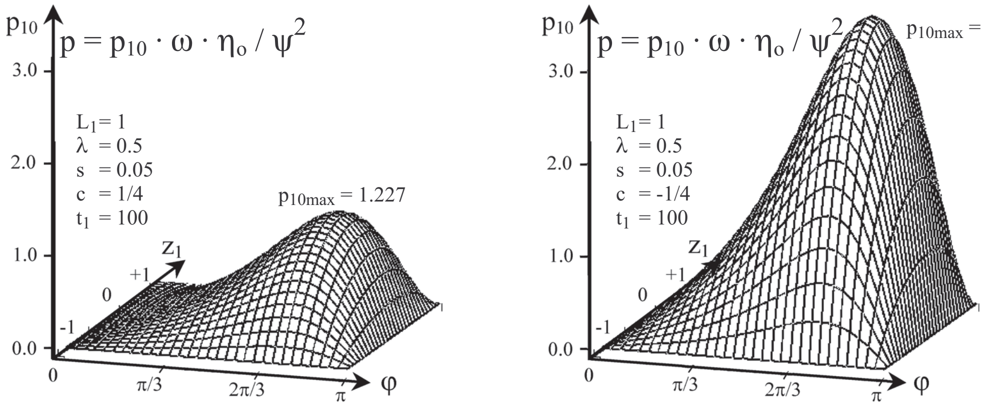

***Fig. 4.** Distributions of the dimensionless hydrodynamic pressure in the cylindrical bearing gap for the dimensionless time t_1=100 counted from the impulse instant and for the gap height change coefficient c = +1/4 (lubrication gap height increased due to impulse load), and for c = -1/4 (lubrication gap height decreased due to impulse load) at accounting for a skew of the journal*

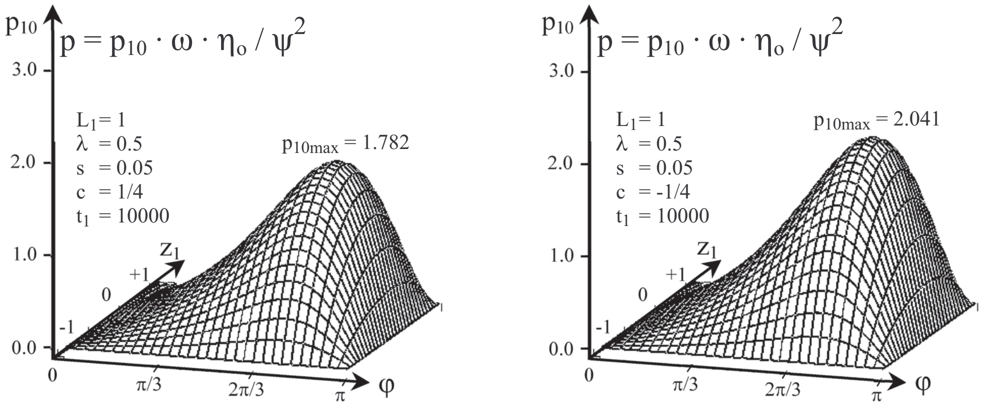

***Fig. 5.** Distributions of the dimensionless hydrodynamic pressure in the cylindrical bearing gap for the dimensionless time t_1=10000 counted from the impulse instant and for the gap height change coefficient c = +1/4 (lubrication gap height increased due to impulse load), and for c = -1/4 (lubrication gap height decreased due to impulse load) at accounting for a skew of the journal*

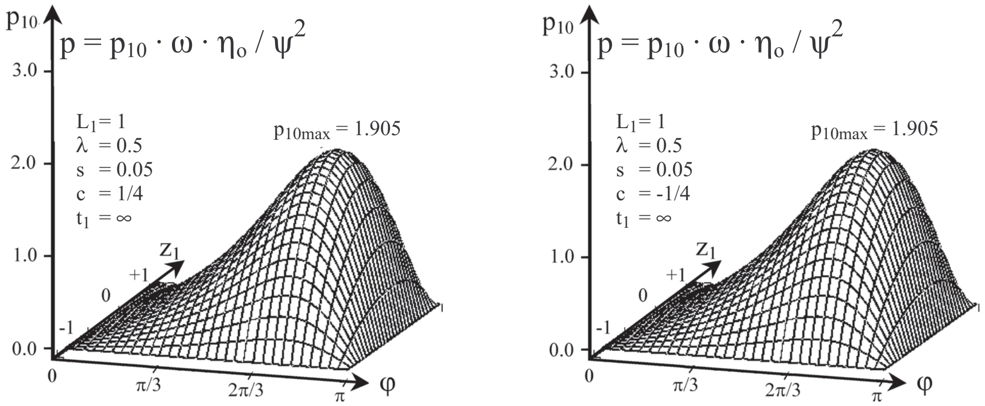

***Fig. 6.** Distributions of the dimensionless hydrodynamic pressure in the cylindrical bearing gap for the dimensionless time $t_1 = \infty$ counted from the impulse instant and for the gap height change coefficient c = +1/4 (lubrication gap height increased due to impulse load), and for c = -1/4 (lubrication gap height decreased due to impulse load) at accounting for a skew of the journal*

For the same data on the basis of the equation (60) the numerical calculations of the dimensionless values of hydrodynamic pressure corrections which result from oil viscoelastic properties, were performed. Their results are presented in Fig. 7, 8, 9 for the dimensionless time intervals $t_1 = 1, 2, 10$. For $t_1 = 100$, $t_1 = 10000$ and $t_1 = \infty$, the calculations of dimensionless values of the hydrodynamic pressure corrections were also performed, but they have not been attached here as being negligible. To obtain dimensional values of the pressure corrections the dimensionless values shown in Fig. 7, 8, 9, should be multiplied by the dimensional coefficient $UR\eta_o/\varepsilon^2$.

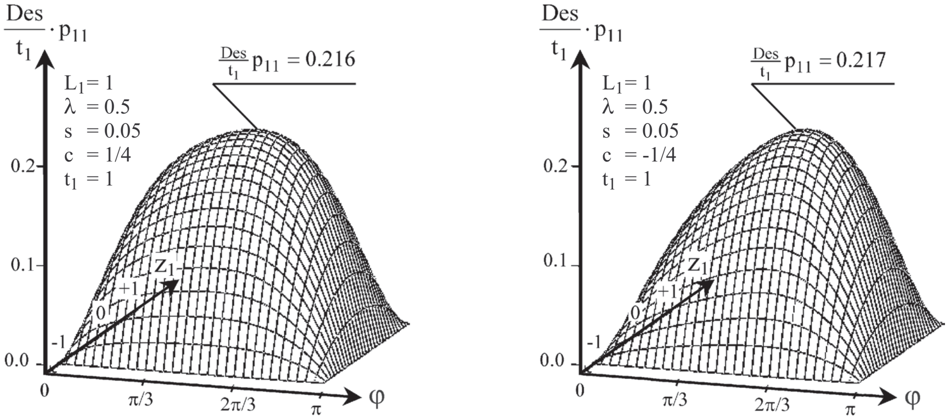

Fig.7. *Distributions of the dimensionless values of the hydrodynamic pressure corrections resulting from viscoelastic properties of oil in the cylindrical bearing gap for the dimensionless time $t_1 = 1$ counted from the impulse occurrence instant and for the gap height change coefficient c = +1/4 (lubrication gap height increased due to impulse load), and for c = -1/4 (lubrication gap height decreased due to impulse load) at accounting for a skew of the journal*

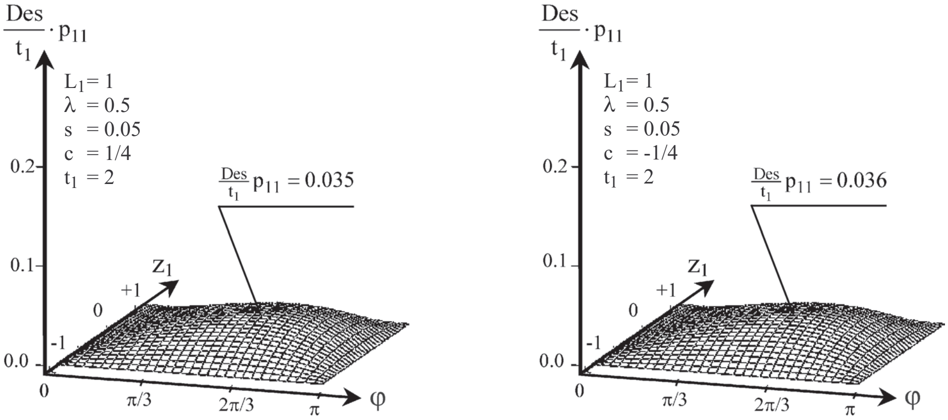

Fig.8. *Distributions of the dimensionless values of the hydrodynamic pressure corrections resulting from viscoelastic properties of oil in the cylindrical bearing gap for the dimensionless time $t_1 = 2$ counted from the impulse occurrence instant and for the gap height change coefficient c = +1/4 (lubrication gap height increased due to impulse load), and for c = -1/4 (lubrication gap height decreased due to impulse load) at accounting for a skew of the journal*

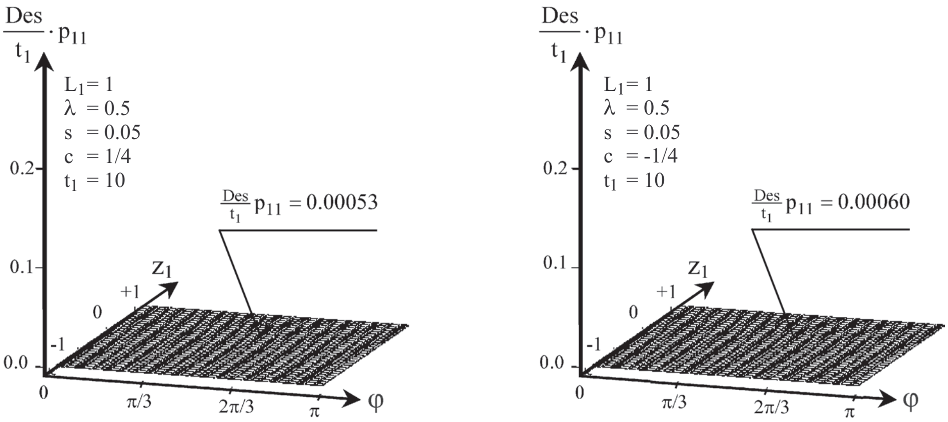

Fig.9. *Distributions of the dimensionless values of the hydrodynamic pressure corrections resulting from viscoelastic properties of oil in the cylindrical bearing gap for the dimensionless time $t_1 = 10$ counted from the impulse occurrence instant and for the gap height change coefficient c = +1/4 (lubrication gap height increased due to impulse load), and for c = -1/4 (lubrication gap height decreased due to impulse load) at accounting for a skew of the journal*

In Fig. 6 one can observe that when a load impulse occurs sufficiently far in time from the impulse occurrence instant, i.e. when $t_1 \to \infty$, then the distributions of pressure values approach the pressure distribution identical as regards its values and shape, both at the impulse increasing the gap height $c > 0$ and that decreasing the gap height $c < 0$, which can be also achieved from the classical Reynolds equation (54).

From the analysis of the pressure corrections (due to viscoelastic oil properties) it results that only in the initial instant $t_1 = 1$ after impulse occurrence (Fig.7) the corrections really influence the total pressure value. At the so assumed time instant $t_1 = 1$ the share of the corrections of pressure p_{11} in the value of the basic pressure p_{10} amounts to about 6% (at the gap height decreased due to impulse load, c = -1/4) and to about 18% (at that gap height increased due to impulse load, c = +1/4). The values were calculated for the relevant maximum values shown in Fig. 3 and 7.

DISCUSSION OF RESULTS AND CONCLUSIONS

- Numerical analysis of hydrodynamic pressure values during the unsteady impulse loading of radial slide cylindrical bearings reveals that in the first time interval counted from the instant of impulse load occurrence very high changes of the hydrodynamic pressure may appear, and also very high changes of bearing loads in comparison with their load carrying capacities which shall occur at no impulse load.
- In the case if due to an impulse load the journal changes its location relative to the sleeve in such a way that the lubrication gap height increases (left column of Fig. 3, 4, 5 and 6) then the bearing will suffer sudden drop of its load carrying capacity by a few dozen percent. As time runs after the impulse load occurrence the hydrodynamic pressure in the bearing gap height increases up to its value occurring under regular load (without any impulse).
- In the case when an impulse load results in decreasing the gap height (right column of Fig.3, 4, 5, 6) then an increase of pressure values appears in the initial phase of impulse loading and next, as time runs, the pressure decreases down to the hydrodynamic pressure value relevant for the bearing under regular load (without any impulse).
- The mixed case may also happen when the journal displaces itself due to simultaneous occurrence of two impulses of opposite tendencies leading to the decreasing and increasing of the gap height relative to its initial location. Then, rises and drops of hydrodynamic pressure in comparison to its initial value, may happen. Such hydrodynamic pressure changes may lead to an accelerated wear of elements of the cylindrical slide friction units in question.
- The accounting for the impulse-load-induced pressure changes in designing the cylindrical slide friction units, would contribute to elimination of engine failures resulting from seizure of cylindrical slide bearings in the service conditions in which impulse loads often occur. Transport safety would be this way improved.

Appendix 1

$$\frac{\partial}{\partial t_1}=\frac{\partial}{\partial\chi}\frac{\partial\chi}{\partial t_1}=-\frac{1}{4}\sqrt{Res}\,\frac{r_1}{t_1\sqrt{t_1}}\frac{\partial}{\partial\chi}=-\frac{\chi}{2t_1}\frac{\partial}{\partial\chi}$$
$$\frac{\partial^2}{\partial r_1^2}=\frac{\partial}{\partial r_1}\left(\frac{\partial}{\partial r_1}\right)=\frac{\partial}{\partial\chi}\left(\frac{\partial}{\partial\chi}\frac{\partial\chi}{\partial r_1}\right)\frac{\partial\chi}{\partial r_1}=\frac{Res}{4t_1}\frac{\partial^2}{\partial\chi^2} \tag{A1.1}$$

$$\frac{\partial^3}{\partial t_1\partial r_1^2}=\frac{\partial}{\partial t_1}\left(\frac{Res}{4t_1}\frac{\partial^2}{\partial\chi^2}\right)=-\frac{Res}{4t_1^2}\frac{\partial^2}{\partial\chi^2}+\frac{Res}{4t_1}\frac{\partial}{\partial\chi}\left(\frac{\partial^2}{\partial\chi^2}\right)\frac{\partial\chi}{\partial t_1}=-\frac{Res}{4t_1^2}\left(\frac{\partial^2}{\partial\chi^2}+\frac{\chi}{2}\frac{\partial^3}{\partial\chi^3}\right) \tag{A1.2}$$

Appendix 2

$$\lim_{N\to0}\frac{\sqrt{\pi}}{2N^2}\Omega(\chi=h_1N)\equiv\lim_{N\to0}\frac{\sqrt{\pi}}{2N^2}\left[\int_0^{h_1N}\exp(\chi^2)\mathrm{erf}(\chi)d\chi-\mathrm{erf}(h_1N)\int_0^{h_1N}\exp(\chi^2)d\chi\right]=$$
$$=\lim_{N\to0}\frac{1}{N^2}\left\{\int_0^{h_1N}\left[\exp(\chi^2)\int_0^{\chi}\exp(-\chi_1^2)d\chi_1\right]d\chi-\left(\int_0^{h_1N}\exp(-\chi^2)d\chi_2\right)\left(\int_0^{h_1N}\exp(\chi^2)d\chi\right)\right\}= \tag{A2.1}$$
$$\overset{H}{=}-\lim_{N\to0}\frac{h_1\int_0^{Nh_1}\exp(\chi^2)d\chi}{2N\exp(h_1^2N^2)}\overset{H}{=}-\frac{h_1^2}{2}\lim_{N\to0}\frac{\exp(h_1^2N^2)}{\exp(h_1^2N^2)+2h_1^2N^2\exp(h_1^2N^2)}=-\frac{h_1^2}{2}$$

Analogically :

$$\lim_{N\to0}\frac{\sqrt{\pi}}{2N^2}\left[\int_0^{r_1N}\exp(\chi^2)\mathrm{erf}(\chi)d\chi-\mathrm{erf}(r_1N)\int_0^{r_1N}\exp(\chi^2)d\chi\right]=-\frac{r_1^2}{2} \tag{A2.2}$$

$$\text{and: }\lim_{N\to0}\frac{\mathrm{erf}(r_1N)}{\mathrm{erf}(h_1N)}=\frac{r_1}{h_1} \tag{A2.3}$$

The equation (53) at N→0 approaches the following form :

$$\frac{\partial}{\partial\varphi}\left\{\left[\left(-\frac{h_1^2}{2}\right)\int_0^{h_1}\frac{r_1}{h_1}dr_1-\int_0^{h_1}\left(-\frac{r_1^2}{2}\right)dr_1\right]\frac{\partial p_{10}}{\partial\varphi}\right\}+$$
$$+\frac{1}{L_1^2}\frac{\partial}{\partial z_1}\left\{\left[\left(-\frac{h_1^2}{2}\right)\int_0^{h_1}\frac{r_1}{h_1}dr_1-\int_0^{h_1}\left(-\frac{r_1^2}{2}\right)dr_1\right]\frac{\partial p_{10}}{\partial z_1}\right\}=-\frac{\partial}{\partial\varphi}\left[\int_0^{h_1}\left(1-\frac{r_1}{h_1}\right)dr_1\right] \tag{A2.4}$$

After realisation of the calculations the classical Reynolds equation is obtained in the form of (54) for a cylindrical system.

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NOMENCLATURE

a - acceleration vector [m/s^2]
$\mathbf{A}_1$ - Rivlin-Ericksen strain tensors [s^{-1}]
$\mathbf{A}_2$ - Rivlin-Ericksen strain tensors [s^{-2}]
b - a half of bearing length [m]
c - coefficient for controlllling gap height changes
C_{z1} , C_{z2} , C_{z3} , C_{z4} , $C_{\varphi 1}$, $C_{\varphi 2}$, $C_{\varphi 3}$, $C_{\varphi 4}$ - integration constants
De - Deborah number
h - gap height in the cylindrical bearing [m]
h_1 - dimensionless gap height
I - unit tensor
L - tensor of oil velocity vector gradient [s^{-1}]
$\mathbf{L}^T$ - tensor with matrix transpose [s^{-1}]
$\mathbf{L}_1$ - dimensionless bearing length
N - dimensionless number
$O\left(\frac{Des}{t_1}\right)^2$ - estimate of all remaining corrections of velocity and pressure components
p - pressure [Pa]
p_o - characteristic value of hydrodynamic pressure [Pa]
p_1 - total dimensionless hydrodynamic pressure
p_{10} , p_{11} , p_{12} - dimensionless corrections of hydrodynamic pressure
P - load [N]
r - radial coordinate [m]
r_1 , r_2 - dimensionless radial coordinate
R - radius of cylindrical journal [m]
Re - Reynold's number
s - skew coefficient
S - stress tensor [Pa]
Str - Strouhal number
t - time [s]
t_o - characteristic time [s]
t_1 , t_2 - dimensionless time
U - tangential journal velocity [m/s]
v - velocity vector [m/s]
v_φ , v_r , v_z - dimensional values of tangential, radial and axial components of velocity vector, respectively [m/s]
$v_{\varphi 1}$, v_{r1} , v_{z1} - dimensionless values of tangential, radial and axial components of velocity vector, respectively
$v_{\varphi 0\Sigma}$, $v_{r0\Sigma}$, $v_{z0\Sigma}$ - dimensionless components of oil velocity vector, without accounting for changes due to disturbing impulse
$v_{\varphi 1\Sigma}$, $v_{r1\Sigma}$, $v_{z1\Sigma}$, $v_{\varphi 2\Sigma}$, $v_{r2\Sigma}$, $v_{z2\Sigma}$ - dimensionless corrections of oil velocity vector components, resulting from disturbing impulse impact on a bearing at sufficiently close instant from the impulse occurrence
v_{10} , v_{20} - parts of dimensionless velocity vector components, dependent on shaft rotation, without accounting for disturbing impulse action
v_{11} , v_{12} - parts of dimensionless velocity vector components, dependent on shaft rotation, with accounting for disturbing impulse action
$v_{\varphi 03}$, v_{z03} - parts of dimensionless velocity vector components, resulting from pressure gradient influence, without accounting for disturbing impulse action
$v_{\varphi 13}$, v_{z13} - parts of dimensionless velocity vector components, resulting from pressure gradient influence, with accounting for disturbing impulse action
z - longitudinal coordinate [m]
z_1 - dimensionless longitudinal coordinate
α , β - pseudo-viscosity constants of oil [Pa·s^2]
γ - skew angle
δ - a value close zero
ε - radial clearance
η_o - characteristic value of oil dynamic viscosity [Pa·s]
λ - relative eccentricity
ρ - oil density [kg/m3]
φ - tangential coordinate
χ , χ_1 , χ_2 - dimensionless coordinates
ψ - relative radial clearance
ω - angular journal velocity [s^{-1}]
ω_o - angular speed of sleeve perturbation [s^{-1}]
Ω , Ω_1 , Ω_2 , Ω_3 - auxiliary functions

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