

# Stability of free-floating ship

## Part II

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### ABSTRACT



*This is the second part of the paper published in Polish Maritime Research no. 2/2005, dealing with the calculation problem of righting arms of the free-floating ship, i.e. longitudinally balanced at any heel angle. In such case the righting arms are ambiguous as they depend on a way the heeling moment acts. Two cases were considered : when the heeling moment is parallel to the ship plane of symmetry, and the case when it performs the least work, i.e. when the moment is parallel to the main axis of ship waterline. It was demonstrated that angular translations (heel and trim) are then the Euler angles associated with a relevant reference axis. Some cases of the incorrect defining and using of those angles in today design practice were indicated. The most important features of the curve of righting arms of free-floating ship were demonstrated.*

**Keywords :** stability, free floating ship

### FEATURES OF RIGHTING ARM CURVE

Knowing metacentric radii of a free-floating ship one can easily find the remaining characteristics of its curve of righting arms. They are analogous to those known from the classical theory of ships.

Hence, the metacentric height  $h$  is equal to :

$$h = \frac{d}{d\phi} GZ = BM - BZ \quad (25)$$

where :

- BM** - metacentric height according to Eq. (12)
- BZ = -r·n** - height of ship centre of gravity over its centre of buoyancy (Fig. 2)
- r = GB** - radius-vector of ship centre of buoyancy relative to its centre of gravity, and
- n** - versor normal to ship waterline according to Eq.(6) or (20).

Eq. (25) can be obtained immediately by considering the buoyancy action line in the rotation plane (Fig.6) for the heel angle increased by  $d\phi$ . The metacentric height can be also obtained by differentiating the righting arm  $GZ$ , acc. to Eq.(10), respective to the heel angle in the ship-fixed reference system.

The derivative is given by the formula :

$$GZ' = e' \cdot (r \times n) + e \cdot (r' \times n) + e \cdot (r \times n') = BM + r \cdot n$$

where the sign [ ' ] stands for differentiating respective to  $\phi$ .

It can be demonstrated that the first term equals zero, the second gives the metacentric height  $BM$ , and the third is equal to  $r \cdot n$ . Hence the formula is identical with Eq. (25).

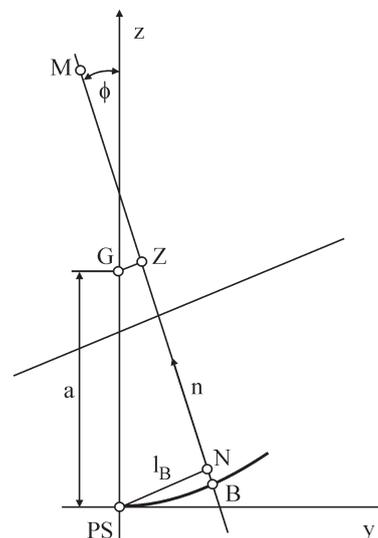


Fig. 6. Ship rotation plane

Work done by the heeling moment  $M$  is expressed by the formula :

$$L = \int_0^{\phi} M d\phi = \Delta \int_0^{\phi} GZ d\phi = \Delta I_d \quad (26)$$

where :

- $\Delta$  - ship buoyancy
- $I_d$  - dynamic arm.

It can be observed that the area under the curve of righting arms, called the *dynamic arm*, is proportional to the work of heeling moment.

Considering rotation of the rotation plane (Fig.6) by the angle  $d\phi$ , one can easily demonstrate that the differential  $GZ d\phi = d(BZ)$  is an increment of the segment BZ due to vertical translation of the point Z, as the buoyancy centre B translates horizontally relative to ship's waterline. Hence the known formula for the dynamic arm  $l_d = BZ - a$ , is obtained, where  $a = B_0G$  - the height of the gravity centre G over the buoyancy centre of ship in the upright position (i.e. for  $\phi = 0$ ). The equation has simple physical interpretation – the dynamic arm is equal to the vertical increment of the distance between the centre of gravity and centre of buoyancy. It can be useful in checking the calculation accuracy of the righting arm curve.

Worth mentioning that the curve of righting arms of free-floating ship complies with the theorem of minimum potential energy, i.e. ship's heel (understood as a rotation of the plane of rotation) by a given angle demands work done to be minimum. This is an important feature of the curve. To prove it let's observe that to incline the ship from its position of longitudinal equilibrium is not possible without applying a trimming moment and doing an additional work that could increase its potential energy. Hence it results that righting arms of a free-floating ship are at the most equal to or smaller than those of a fixed-trim ship, that is clearly illustrated in Fig.1. If it is not the case, it means that some errors appear in the calculation algorithm.

### Cross-curves of stability

The hull form arm, i.e. the arm of buoyancy force relative to the initial location of the centre of buoyancy, shown in Fig.6, is given as follows :

$$l_B = GZ + a \cdot \sin\phi \quad (27)$$

For a free-floating ship which changes its trim during heeling, the hull form arms depend on the height of centre of gravity over BP. Because of trim changes the idea of cross-curves of stability does not apply, strictly speaking, to a free-floating ship. Let's observe that z - axis fixed to the ship does not lie in the rotation plane which is inclined to it by the angle  $\theta$ . Projection of the axis z to the rotation plane agrees with the axis  $B_0Z$  (Fig.6)<sup>4</sup>. Therefore, by shifting the centre of gravity along Oz-axis by the quantity  $\Delta z_G$  it moves before the rotation plane by the distance  $\Delta x_2 = \Delta z_G \sin\theta$ . As a result the ship becomes unbalanced and it must trim in the vertical plane by the angle  $d\alpha_2 = \Delta x_2 / BM_L$ , where  $BM_L$  is the longitudinal metacentric radius at a given heel angle  $\phi$ . The ship can be balanced only by changing its trim, without changing its heel angle. In this case the relationship between both trim angles is the same as in Eq. (23) but without minus sign, i.e.  $d\alpha_2 = d\theta \cos\phi$ . Hence the trim correction is  $d\theta = d\alpha_2 / \cos\phi$ . From Fig.6 it can be stated that the new righting arm is :

$$GZ_1 = GZ - \Delta z_G \cdot \cos\theta \cdot \sin\phi + (D/V) d\alpha_2 \quad (28)$$

where : D - moment of deviation  
of ship waterline in  $\xi\eta$  - coordinate system (Fig.4).

For normally occurring trim values the function  $\cos\theta$  can be omitted as practically being equal to 1. Two first terms of Eq. (28) are the same as for fixed-trim ship. The last term,  $(D/V) d\alpha_2$ , denoted now by  $\Delta l$ , accounts for the effect of trim on the righting arm curve of a free-floating ship. Taking into account that  $d\alpha_2 = \Delta x_2 / BM_L$ , one obtains :

$$\Delta l = \Delta z_G \cdot \sin\theta \cdot \text{tg}\chi \quad (29)$$

Hence it can be observed that it is possible to find the righting arm curve for a new location of the centre of gravity by means of cross - curves of stability for free-floating ship provided additional information on the run of the angles  $\theta$  and  $\chi$  in function of the heel angle  $\phi$  is available, which makes it possible to calculate the correction  $\Delta l$ . Let's observe that the correction  $\Delta l$  disappears not only when  $\theta = 0$ , as it could be expected, but also when  $\chi = 0$ , i.e. when the ship's floatation axis is parallel to the rotation axis. Moreover from Eq. (29) it results that the quotient  $\Delta l / \Delta z_G$  is not dependent on a value of changes of the height of centre of gravity over BP; therefore in order to calculate the correction  $\Delta l$  it is enough to know a course of the quotient  $\Delta l / \Delta z_G = \sin\theta \text{tg}\chi$  in function of the heel angle  $\phi$ . With the use of the quotient, Eq. (28) for the new curve of righting arms obtains the following form :

$$GZ_1 = GZ - \Delta z_G (\sin\phi - \Delta l / \Delta z_G) \quad (30)$$

To improve calculation accuracy it is advisable that cross-curves of stability are calculated in the form of the righting arm curves GZ for a selected, typical location of ship's centre of gravity. In such case the correction  $\Delta l$  is small, hence it can be often neglected. Cross-curves of stability are usually presented in the form of the diagram :  $GZ = GZ(\nabla, \phi = \text{const})$ .

The diagram showing the differential quotient  $\Delta l / \Delta z_G = \sin\theta \text{tg}\chi$  should be presented in the similar way, in function of the buoyancy  $\nabla$ , for a fixed  $\phi$ .

### OTHER DEGREES OF FREEDOM

In ship hydrostatics in order to describe ship inclination relative to water, are used two mutually independent angles which can be interpreted as degrees of freedom, e.g. the analytical angles  $\phi$  and  $\theta$  which describe orientation of waterline relative to ship. By using them all other angles between various planes and axes can be expressed.

However the analytical angles are not convenient to describe angular translations of ship's hull. To this end three natural angles (in dynamics called the *Euler angles*) are used. In static problems two angles are sufficient to describe angular translations as any third angle is not necessary due to lack of yaw which means that nodal line orientation is constant in the sea plane. The above discussed angles  $\phi$  and  $\theta$  associated with Oy-axis and PS (playing the role of plane of nodes) exemplify the Euler angles. As there are two other axes, one can choose degrees of freedom associated with the remaining axes.

For instance, in NAPA software as well as many other computer programs<sup>5</sup> the heel angle is defined as the slope angle of trace of water in stations, relative to Oy - axis, hence it is the angle  $\phi$ , whereas the trim angle is rotation of the ship around this trace, denoted by  $\theta_x$ . Hence they are Euler angles associated with x-axis, commonly used in ship dynamics. The nodal line is the trace of water on midship section, fixed in space. The angle  $\theta_x$  is equal to that of slope of Ox-axis relative to water-level. As  $\cos(90^\circ + \theta_x) = \mathbf{i} \cdot \mathbf{n}$ , then :  $\sin\theta_x = \cos\alpha \text{tg}\theta$ , or simpler :

$$\text{tg}\theta_x = \cos\phi \cdot \text{tg}\theta \quad (31)$$

The versor  $\mathbf{n}$  can be directly expressed by means of the Euler angles. Dividing  $\sin\theta_x$  by  $\text{tg}\theta_x$  one obtains :  $\cos\theta_x = \cos\alpha / \cos\phi$ , hence :  $n_z = \cos\alpha = \cos\theta_x \cos\phi$ . By substituting them in Eq.(6), the components of the vector  $\mathbf{n}$  take the following form :

$$\mathbf{n} = (-\sin\theta_x, -\cos\theta_x \cdot \sin\phi, \cos\theta_x \cdot \cos\phi) \quad (32)$$

As  $n_y = -\sin\phi$ , hence :  $\sin\phi = \cos\theta_x \sin\phi$ , from which it results that :  $\phi \leq \theta$ , which has been already obtained from Eq.(7); both angles are equal to each other in case of no trim. From the

formula for  $\cos\alpha$  it results that :  $\varphi \leq \alpha$ , hence :  $\phi \leq \alpha$ . The angle  $\varphi = \alpha$ , in case of no trim or when the heel angle equals  $90^\circ$ .

In the case of the reference axis  $Oz$ , the Euler angles are :

- the inclination angle  $\alpha$  by which the axis  $z$  is deflected from the vertical, given by Eq.(5); it is identical with the slope angle of BP relative to waterline
- as well as the trim angle  $\psi$ , i.e. the ship rotation angle around  $Oz$  - axis .

The edge of intersection of BP and water level forms a nodal line as its orientation is fixed in space. Both the angles and the nodal line is shown in Fig.3. The angle  $\psi$  is equal to the slope angle of trace of water in the plane  $z = const$  (i.e. nodal line) relative to  $Ox$ -axis<sup>6</sup>.

This way one obtains as follows :

$$tg\psi = - tg\theta/tg\varphi \quad (33)$$

From Eqs.(5) and (33) it results that  $tg\alpha$  can be taken as a vector composed of  $tg\varphi$  and  $-tg\theta$ . Hence :  $tg\varphi = tg\alpha\cos\psi$ , and :  $tg\theta = -tg\alpha\sin\psi$ . By substituting them in Eq.(6) the components of the vector  $\mathbf{n}$  take the following form :

$$\mathbf{n} = (\sin\psi \cdot \sin\alpha, -\cos\psi \cdot \sin\alpha, \cos\alpha) \quad (34)$$

As :  $n_y = -\sin\phi$ , hence :  $\sin\phi = \cos\psi\sin\alpha$ , from which it results that:  $\phi \leq \alpha$ ; both the angles are equal to each other only in case of no trim.

It is not indifferent which reference axis (and the Euler angles associated with it) is chosen to calculate righting arm curve because location of the rotation plane in which the ship has to be balanced depends on the axis chosen. During balancing the ship, a change of trim angle does not influence the rotation angle.

In the case of  $Ox$  reference axis the rotation plane is a vertical section perpendicular to water-level and parallel to water-trace on frame sections (Fig.7); in the case of  $Oy$ -axis – it is perpendicular to water-trace on PS (Fig.2), and in the case of  $Oz$ -axis – perpendicular to water-trace on BP (Fig.8). In other words, in the case of the reference axis  $x$  the rotation plane is *parallel* to nodal line, and in the two latter cases – *perpendicular* to nodal line. The three planes of rotation are not identical which makes that for a free-floating ship righting arms depend on the choice of reference axis. In all the cases the rotation plane is fixed in space, perpendicular to heeling moment vector, vertical, and crossing ship's centre of gravity. The permanent orientation in space is associated with the permanent di-

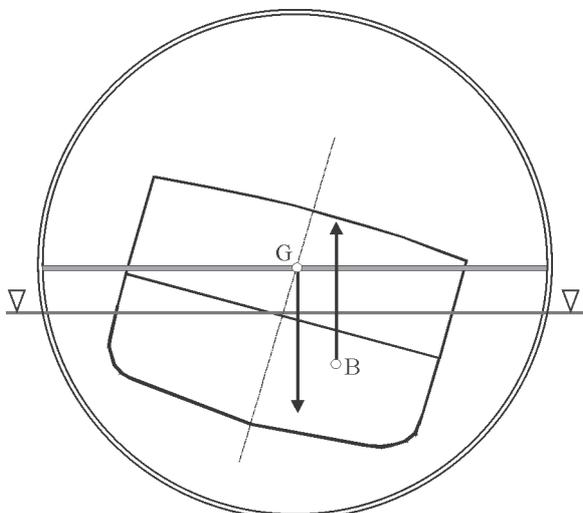


Fig. 7. Rotation plane of free-floating ship which trims around trace of water on midship section

rection of nodal line in sea-level, due to the lack of yaw. During heeling the buoyancy centre moves along the curve of buoyancy centres located on the rotation plane whose features have been already discussed. For this reason the righting arm curve is a function of the rotation angle of rotation plane, further marked  $\eta$ . In other words, the heel angle of the ship is identical with the rotation angle of the rotation plane.

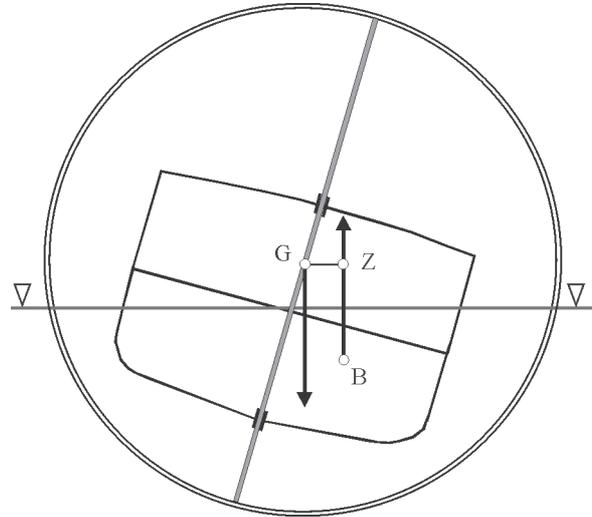


Fig. 8. Rotation plane of free-floating ship which trims around  $Oz$ - axis

In the case of  $Ox$ -axis the rotation angle  $\eta$  cannot be interpreted simply in geometrical terms. It is also difficult to realize such heeling model physically, that can be observed in Fig.7.

As a result the reference axis is not located in the rotation plane. The rotation angle and heel angle are mutually related by the formula :  $d\eta = d\varphi\cos\theta_x$ , hence the rotation angle  $\eta = \int \cos\theta_x d\varphi$ , from which it results that :  $\eta < \varphi$ , and that for  $\varphi = 90^\circ$  the rotation angle of rotation plane is a little smaller than  $90^\circ$  ( $\eta < 90^\circ$ ).

Generally, differences between the angles  $\eta$  and  $\varphi$  are marginal as  $\cos\theta_x$  is practically equal to 1, nevertheless both the angles are not identical. For  $Oy$ -axis the rotation angle  $\eta$  is identical with the angle  $\phi$ , and for  $Oz$ -axis – with the angle  $\alpha$ . In two latter cases shown in Fig.2 and 8, the reference axis is parallel to the rotation plane and it rotates together with the plane.

It can be mathematically demonstrated that the rotation planes associated with particular reference axes are differently situated relative to the ship. To this end it is sufficient to demonstrate that versors normal to the planes, called rotation axes, are different.

When  $x$ -axis is a reference axis the vector normal to rotation plane is given by the formula :  $\mathbf{e} = \mathbf{e}_2 \times \mathbf{n}$ , where  $\mathbf{e}_2$  – versor of trace of water on frame sections. The versor  $\mathbf{e}$  differs from the versor  $\mathbf{e}_1 = (\cos\theta, 0, \sin\theta)$  as for an arbitrarily inclined ship the water-traces on PS and midship section are not mutually normal. As  $\mathbf{e}_2 = (0, \cos\varphi, \sin\varphi)$ , hence the versor of rotation axis is as follows :

$$\mathbf{e} = (\cos\theta_x, -\sin\theta_x \cdot \sin\varphi, \sin\theta_x \cdot \cos\varphi) \quad (35)$$

When  $Oz$ - axis is a reference one the versor  $\mathbf{e}$  normal to rotation plane is parallel to the vector :  $\mathbf{k} \times \mathbf{n}$ . Accounting for that the length of the vector  $|\mathbf{k} \times \mathbf{n}| = \sin\alpha$ , one obtains as follows :

$$\mathbf{e} = (\cos\psi, \sin\psi, 0) \quad (36)$$

which can be directly observed from Fig.3. The versor differs from the two remaining by that it is parallel to BP and waterline – therefore parallel to common edge of both planes, i.e the

trace of BP on waterline. Let's observe that for  $\alpha = 0$  the rotation axis  $\mathbf{e}$  is undefined as the above mentioned planes are parallel and their common edge does not exist; the product  $\mathbf{k} \times \mathbf{n}$  is also undefined as both versors are parallel.

As rotation axis in ship's upright position is undefined its choice is arbitrary. It is usually assumed that for  $\alpha = 0$  the angle  $\psi = 0$ , i.e. that in the initial position the rotation axis coincides with the trace of PS on waterline, however not necessarily. As the rotation axis an arbitrary axis on waterline can be chosen, especially its main inertia axis. From the previous considerations on the floatation axis it follows that in the latter case it must permanently coincide with the rotation axis (the angle  $\chi = 0$ , i.e.  $\mathbf{f} = \mathbf{e}$ ) for every angle of heel Eq. (36) is valid regardless of the choice of rotation axis for  $\alpha = 0$  (an axis fixed in space).

The rotation axis is determined by a direction of heeling moment action in a ship-fixed reference system. As for the same analytical angles  $\phi$  and  $\theta$  the rotation axes are differently located relative to ship depending on an assumed reference axis, the rotation planes are also different, and hence the righting arms are different. This can be clearly stated from Eq.(10) where the righting arm GZ depends on  $\mathbf{r}$ ,  $\mathbf{e}$  and  $\mathbf{n}$ . For the same angles  $\phi$  and  $\theta$ , the versor  $\mathbf{n}$  is the same but the rotation planes have different  $\mathbf{e}$  and  $\mathbf{r}$ , which makes GZ arms different. Moreover if in physical space the rotation planes are the same as in Fig.2, 7 and 8, rotated by the same angle  $\eta$ , then the analytical angles are different, which gives different curves of righting arms in function of the rotation angle  $\eta$  taken as ship's heel angle.

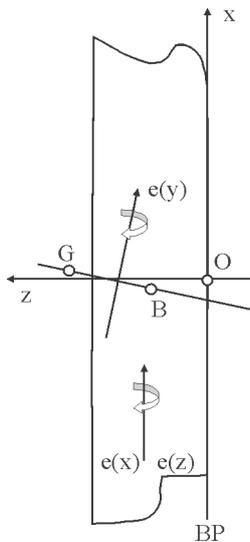


Fig. 9. Top view of the ship heeled by the angle  $\phi = 90^\circ$

The case of the ship heeled by  $90^\circ$  is interesting as differences between various reference axes can be then distinctly observed. In the case of the reference axis  $Oy$  at the angle  $\phi = 90^\circ$  PS is horizontal, and the rotation plane crosses ship's gravity centre and buoyancy centre (Fig.9), which means that the ship is longitudinally balanced. As the plane is motionless in space the entire figure should be horizontally rotated around the point G (in this case to the left) by the angle of inclination relative to the axis z. The righting arm is negative and equal to the horizontal distance between the points G and B.

For the axes x and z the rotation planes in Fig.9 cover each other – as they cross the point G parallel to the axis z. As the buoyancy centre is not located on the plane, the ship would so trim (in this case by bow) as to make the centre to be located on the rotation plane.

In both cases the righting arm would be the same, equal to the horizontal distance between the gravity centre and transla-

ted buoyancy centre. However the heel angle  $\eta$ , in the sense of the rotation angle of the plane of rotation, would be different. For the axis x the angle  $\eta$  will be less than  $90^\circ$ , even by a few degrees, if trim at large heel angles is significant, which is possible especially in the case of a damaged ship; and for the axis z the angle  $\eta = \alpha = 90^\circ$ .

The previous considerations dealing with the metacentric radii, floatation axes and curve of righting arms do not lose anything in value if to replace the differential  $d\phi$  with the differential  $d\eta$ , and to substitute  $\eta$  for  $\phi$ . The directed angle  $\mathbf{f}d\alpha_1$  is inclined by the angle  $\chi$  to the appropriate rotation axis  $\mathbf{e}$  defining the axis  $\xi$  in Fig.4. The axial component of the angle  $d\eta = d\alpha_1 \cos \chi$  is the rotation angle of the rotation plane. The component  $d\alpha_2$  normal to the rotation axis is given by the formula analogous to Eq. (23) :

$$d\alpha_2 = d\alpha_1 \cdot \sin \chi = d\eta \cdot \operatorname{tg} \chi = \begin{cases} -d\theta_x, & \text{for the reference axis } Ox \\ -d\psi \cdot \sin \alpha, & \text{for the reference axis } Oz \end{cases} \quad (37)$$

From the above given equation two conclusions analogous to the previous ones, result :

- the more deflected the floatation axis from the rotation axis the greater trim changes during heeling, and
- when  $\chi = 0$ , i.e. when the floatation axis is parallel to the rotation axis, the ship trim does not change due to rotation of the waterline itself. As opposed to the reference axis  $Oy$ , there is no problem with determining the rotation axis at  $90^\circ$  heel for the remaining reference axes.

In the case of the reference axis  $Ox$  the rotation axis of the midship section around the normal  $\mathbf{id}\phi$  has the horizontal component  $d\eta = d\phi \cos \theta_x$  which is axial one. The angle  $\mathbf{id}\phi$  has also the vertical component equal to  $-d\phi \sin \theta_x = -d\eta \operatorname{tg} \theta_x$ .

In the case of the reference axis  $Oz$  the vertical component of the rotation angle of BP around the normal  $\mathbf{kd}\psi$ , equals  $d\psi \cos \alpha$ . The vertical component makes the angle  $\chi$  changing by  $d\chi_0$  as a result of the change of orientation of rotation axis (relative to the hull), caused by the rotation of the waterline alone. The angle  $d\chi_0$  is equal to the vertical component of the rotation. Hence one obtains :

$$d\chi_0 = \begin{cases} -d\eta \operatorname{tg} \theta_x, & \text{for the reference axis } Ox \\ -d\alpha \operatorname{tg} \chi / \operatorname{tg} \alpha, & \text{for the reference axis } Oz \end{cases} \quad (38)$$

In the second part of Eq.(38) the identity appearing in Eq. (37) was utilized. The remaining comments concerning equi-volume waterlines are still valid also for other reference axes. In considerations dealing with cross-curves of stability the angle  $\phi$  should be obviously replaced with a relevant angle  $\eta$ . The quantity  $\Delta x_2 = \Delta z_G e_z$ , where  $e_z$  is z-component of the relevant rotation axis  $\mathbf{e}$ . The differential quotient amounts to :

$$\Delta l / \Delta z_G = e_z \operatorname{tg} \chi.$$

## RIGHTING ARM CURVE OF MINIMUM STABILITY

As previously mentioned, most heeling moments affecting the ship is parallel to PS, therefore a free - floating ship takes such position relative to its rotation plane as to make trace of water on PS normal to it. Therefore the question arises which position the ship takes when direction of the moment is not associated with its orientation relative to ship's hull. In other words, which reference axis should be then chosen. In order to unambiguously answer this question it is necessary to know the mechanism of taking position by the ship relative to its rotation plane in the case of a free-heeling moment.

In the case of a heeling moment not associated with the hull, such, for instance, as that resulting from shifting a weight on ship, or taking a weight to any place on ship, the ship in question rotates in such a way as to make potential energy of the heeled ship minimum [13]. In other words, the ship heels in such a way as the work associated with its heel is minimum. As moment work is proportional to the dynamic arm the minimum value of potential energy corresponds with minimum of the dynamic arm. From the classical ship theory it is known that dynamic arm depends on a course of metacentric radii in function of the heel angle, which – for a free-floating ship – means : in function of the rotation angle of the rotation plane.

Hence, in general case :

$$l_d = \int_0^{\eta} BM \cdot \sin(\eta - \nu) d\nu - a(1 - \cos\eta) \quad (39)$$

where :

- $\nu$  - dummy variable of integration, varying from 0 to  $\eta$  (the assumed rotation angle of rotation plane)
- BM - metacentric radius given by Eq.(12)
- $a = B_0G$  - constant value.

Obviously the integrand in Eq. (39) decides on minimum dynamic arms, which takes its minimum value for minimum metacentric radii in function of rotation angle. And these appear when instantaneous floatation axis is permanently parallel to the rotation axis ( $\mathbf{f} = \mathbf{e}$ ), i.e. when the angle  $\chi = 0$ . Hence a free-floating ship under influence of a free-heeling moment takes such position relative to its rotation plane as to make the instantaneous axis of floatation normal to the rotation plane.

A few conclusions yield from that :

- ❖ From Eq.(15) it results that the rotation axis coincides – then and only then – with the floatation axis when the deviation moment  $D = 0$ . The floatation axis is then the main inertia axis of the waterplane, for which the transverse inertia moment is minimum. When the ship heel is being changed a new floatation axis rotates relative to the previous one by the angle  $d\chi_f = d\chi$ , which results only from the change of orientation of the main inertia axis of waterline since  $d\chi_0 = 0$  due to zero value of the angle  $\chi$ , which can be deduced from Eq.(38).
- ❖ As the floatation axis is permanently parallel to the rotation axis Eq.(11) is reduced to the form :  $d\eta = d\alpha_1 = d\alpha$ . And, this in turn means that the rotation angle of rotation plane is equal to the angle contained between the actual waterplane and the initial one. Hence the curve of righting arms is a function of the angle  $\alpha$ , and Oz - axis is the appropriate reference axis.
- ❖ As the angle  $\chi$  is equal to zero for any angle  $\alpha$  the correction  $\Delta l$  takes zero value, which can be seen from Eq. (29). Therefore righting arm curve can be found on the basis of cross-curves of stability by neglecting the correction as in the case of even-keel ship.
- ❖ For a given angle  $\alpha$  it is necessary to find such value of the angle of rotation around the versor  $\mathbf{n}_0$ ,  $\psi$ , as to obtain the centre of buoyancy located in the rotation plane containing the ship centre of gravity, and perpendicular to the longitudinal axis of inertia of waterline.
- ❖ Righting arms are then as small as possible and such righting arm curve is called *the curve of minimum stability*. Any other curve can have the same or greater arms.

Worth mentioning that NAPA software as well as many other computer programs used in West countries for ship stability

calculations have no possibility to calculate the righting arm curve of minimum stability as such mode of calculations is there unknown at all.

## CONCLUSIONS

This paper presents the theoretical background for determining the righting arm curve for a free-floating ship, under assumption that the righting moment has constant direction in space. It is also highlighted what is meant by the heel angle and how ship's longitudinal balance is understood. Results of the performed considerations can be summarized as follows :

- Mode of calculation and choice of a reference axis ( Euler angles) considerably influence the results of calculation of the righting arm curve, hence they should be obligatorily defined. Such recommendations are not included in the current rules, which sometimes leads to great discrepancies between calculation results obtained from different sources.
- Righting arm curves should be calculated – as a rule – for a free-floating ship balanced in an appropriate rotation plane fixed in space and passing through the ship centre of gravity. The righting arm curve is then a function of the rotation angle of the rotation plane around the normal axis (rotation axis).
- The rotation plane of an *intact* ship is perpendicular to trace of water on PS, being a nodal line, due to the assumed character of heeling moments resulting mainly from horizontal, wind-generated forces. A rotation angle is the slope angle of y-axis relative to water-level, and a trim angle is the slope angle of trace of water on PS relative to BP. They are Euler angles associated with y-axis.
- For a *damaged* ship which mainly heels under influence of vertical forces the rotation plane is perpendicular to the main inertia axis of waterline, being a nodal line. A rotation angle is the angle  $\alpha$  contained between the initial waterline and water-level, and a trim angle is the angle contained between trace of water-level and trace of PS on the initial waterline. They are Euler angles associated with the ship-fixed axis normal to the initial waterline. The calculations yield the righting arm curve of minimum stability.
- The notion of cross-curves of stability is valid also for a free-floating ship heeled around the main inertia axis of instantaneous waterline. In other cases the calculated righting arm curve should be corrected by introducing the correction which accounts for an influence of change of trim, resulting from change of height of ship's centre of gravity.
- It is advisable to perform calculations of the righting arm curve by means of the equi-volume waterplane method (Krilov-Dargnies's) which in natural way tracks translation of buoyancy centre during heeling [4]. The method has not been used so far in calculation practice, though it appears not only more accurate and well numerically conditioned, but also it significantly shortens time of calculations (about 20 times) in comparison with buoyancy methods. Therefore it is worth implementing into common practice.

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## NOMENCLATURE

a	– height of gravity centre over buoyancy centre in upright position of ship
B	– buoyancy centre
BM	– transverse metacentric radius
BM <sub>L</sub>	– longitudinal metacentric radius
BZ	– height of gravity centre over buoyancy centre
e	– direction of rotation axis (unit vector normal to rotation plane)
e <sub>1</sub> , e <sub>2</sub>	– versors (unit vectors) of trace of water on PS and on midship plane, respectively
f	– versor (unit vector) of floatation axis
F	– freeboard
g	– gravity acceleration
G	– ship gravity centre
GM	– metacentric height
GZ	– righting arm
i, j, k	– versors (unit vectors) of the ship-fixed reference system whose origin is in the point K (intersection point of the plane of symmetry, PS, midship plane and base plane BP)
l, l <sub>d</sub>	– righting arm and dynamic arm, respectively
L, B, T	– length, breadth and mean draught of ship, respectively
n	– upward pointing versor (unit vector) normal to waterline
∇	– volumetric displacement of ship
α	– angle between initial waterline and water-level
β	– angle between trace of water on PS and midship plane
Δ	– ship buoyancy (weight of displaced water)
Δl	– correction of righting arm obtained by means of cross-curves of stability, accounting for oblique translation of gravity centre relative to rotation plane, due to changing the height of ship gravity centre over BP
η	– rotation angle of plane of rotation
θ	– slope angle of trace of water on PS relative to x-axis of ship
ρ	– water density
φ	– slope angle of trace of water on the stations relative to y-axis of ship
φ	– angle of PS inclination from the vertical
χ	– angle between floatation axis and rotation axis
ψ	– angle between water-level trace line and PS trace line on initial waterline.

## Footnotes

- <sup>4)</sup> B<sub>0</sub>YZ reference system is fixed to the rotation plane and its origin is located in the initial position of the buoyancy centre B<sub>0</sub>.
- <sup>5)</sup> e.g. such as STATAW and SEA software systems used by CTO and PRS.
- <sup>6)</sup> In general case the nodal plane is a plane parallel to waterline of ship in the upright position, which rotates together with it. Its normal versor **n**<sub>0</sub> is the same as that of initial waterline. The angle α is determined by :  $\cos\alpha = \mathbf{n}_0 \cdot \mathbf{n}$ , and ψ is the angle between trace of water and trace of PS in nodal plane.

## Acronyms

BP	– ship's base plane
PS	– ship's plane of symmetry
CTO	– Ship Design & Research Centre, Gdańsk
PRS	– Polish Register of Shipping

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Model test of ship stability (photo: Miroslaw Grygorowicz)