Some aspects of vibration control

Part II: An optimal active controller

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ABSTRACT

The paper presents a theoretical method for determining the optimal correction to be introduced in a mechanical system. The active control of harmonic vibrations may be achieved by applying a control unit which ought to reduce the vibration amplitude of the selected elements of the system. The proposed method makes it possible to determine the controller parameters which provide an optimum value of the chosen quality index. This criterion includes the reduction of weighted amplitudes of the elements on the one hand, and minimizes energy of the control signal on the other. The described method is suitable for determination of an optimum controller of turbine rotor vibrations caused by bearing oil whip, bearing oil whirl or aerodynamic forces. For the case of rotor self-excited vibrations of aerodynamic type the linear model of excitations was compared with the neural network method.

Key words : active control, mechanical vibrations, optimal controller

PROBLEM DESCRIPTION

Let us consider a mechanical system with n degrees of freedom. Its steady harmonic vibrations at the frequency ω_i are described by the following equation:

$$\mathbf{q}_i = \mathbf{G}_i \cdot \mathbf{f}_i \qquad (28)$$

where :

- $\mathbf{f}_i$ - vector of amplitudes of harmonic forces (or moments) acting upon the inertial elements of system
- $\mathbf{q}_i$ - vector of displacement amplitude of the inertial elements of system
- $\mathbf{G}_i$ - system dynamic flexibility matrix.

When taking into consideration vibrations with k different values of the frequency ω one can write :

$$\mathbf{q} = \mathbf{G} \cdot \mathbf{f} \qquad (29)$$

The active control of the system vibrations is achieved by applying a controller. The block diagram of the system and its control unit are presented in Fig.11 (for details see Part I of the paper).

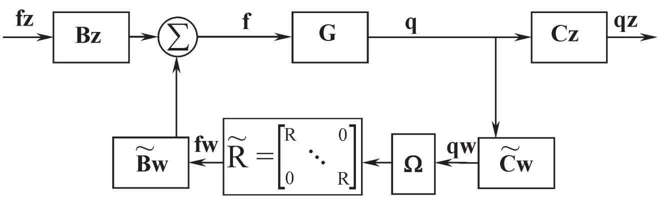

Fig 11. *Block diagram of active control of mechanical vibrations*

All of the vectors **f**, **fz**, **fw**, **q**, **qz**, **qw** in the generalized scheme are column vectors composed of the corresponding vectors for a single frequency:

$$\mathbf{f} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_k \end{bmatrix} \quad \mathbf{fz} = \begin{bmatrix} \mathbf{fz}_1 \\ \mathbf{fz}_2 \\ \vdots \\ \mathbf{fz}_k \end{bmatrix} \quad \mathbf{fw} = \begin{bmatrix} \mathbf{fw}_1 \\ \mathbf{fw}_2 \\ \vdots \\ \mathbf{fw}_k \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \vdots \\ \mathbf{q}_k \end{bmatrix} \quad \mathbf{qz} = \begin{bmatrix} \mathbf{qz}_1 \\ \mathbf{qz}_2 \\ \vdots \\ \mathbf{qz}_k \end{bmatrix} \quad \mathbf{qw} = \begin{bmatrix} \mathbf{qw}_1 \\ \mathbf{qw}_2 \\ \vdots \\ \mathbf{qw}_k \end{bmatrix}$$

The matrices **G**, Ω, $\widetilde{\mathbf{R}}$, $\widetilde{\mathbf{Bw}}$, $\widetilde{\mathbf{Cw}}$, **Bz**, **Cz** are described by the following formulas :

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{G}_k \end{bmatrix} \quad \Omega = \begin{bmatrix} \Omega_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \Omega_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \Omega_k \end{bmatrix}$$

$$\widetilde{\mathbf{R}} = \begin{bmatrix} \mathbf{R} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R} \end{bmatrix} \quad \widetilde{\mathbf{Bw}} = \begin{bmatrix} \mathbf{Bw} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{Bw} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{Bw} \end{bmatrix}$$

$$\widetilde{\mathbf{Cw}} = \begin{bmatrix} \mathbf{Cw} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{Cw} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{Cw} \end{bmatrix} \quad \mathbf{Bz} = \begin{bmatrix} \mathbf{Bz}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{Bz}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{Bz}_k \end{bmatrix}$$

$$\mathbf{Cz} = \begin{bmatrix} \mathbf{Cz}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{Cz}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{Cz}_k \end{bmatrix}$$

It is assumed that external forces **fz** act on selected inertial elements in a way which can be described by the binary matrix **Bz**. The vector **qz** represents the vibration amplitudes whose value has to be minimized. The selection of these amplitudes from the vector **q** is performed by means of the binary matrix **Cz**. In real mechanical systems it is possible to measure the amplitudes **qw** of vibrations of certain elements only. The vector **qw** can be obtained from vector **q** by multiplying it by the binary matrix **Cw**. Based on the vector **qw**, the controller (characterized by the parameters matrix **R**) creates the output signal **fw**, by using the transfer matrix $\tilde{\mathbf{R}} \cdot \mathbf{\Omega}$. This signal is introduced to some possible elements of the system, selected by the binary matrix **Bw**.

The system presented in Fig.11
can be described by the following set of equations :

$$\begin{cases} \mathbf{q} = \mathbf{G} \cdot \mathbf{f} & (30) \\ \mathbf{f} = \mathbf{Bz} \cdot \mathbf{fz} + \mathbf{Bw} \cdot \mathbf{fw} & (31) \\ \mathbf{qz} = \mathbf{Cz} \cdot \mathbf{q} & (32) \\ \mathbf{qw} = \mathbf{Cw} \cdot \mathbf{q} & (33) \\ \mathbf{fw} = \tilde{\mathbf{R}} \cdot \mathbf{\Omega} \cdot \mathbf{qw} & (34) \\ \tilde{\mathbf{R}} = \sum_{i=1}^{k} \left[(e_i \times \mathbf{I}_s) \mathbf{R} (e_i^T \times \mathbf{I}_{3r}) \right] & (35) \end{cases}$$

In the case of a PID controller,
the matrix **R** takes the following form :

$$\mathbf{R} = [\mathbf{K}_P \,|\, \mathbf{K}_I \,|\, \mathbf{K}_D] \qquad (36)$$

where :

$\mathbf{K}_P$, $\mathbf{K}_I$ and $\mathbf{K}_D$ are matrices of the proportional (P), integrating (I) and differentiating (D) action, respectively.

The optimization problem is to determine a controller matrix **R** which minimizes the amplitudes **qz** of chosen elements, taking into account the role of particular elements. This is obtained by assigning, to each of them, an appropriate coefficient from the weight matrix **Φ**. On the other hand, the energy of the control signals **fw** must be taken into account by applying the weight matrix **Λ**. Thus, the optimization index E may be written as follows :

$$\mathrm{E} = \mathrm{tr}(\mathbf{q}\overset{*}{\mathbf{z}} \cdot \mathbf{\Phi} \cdot \mathbf{qz} + \mathbf{f}\overset{*}{\mathbf{w}} \cdot \mathbf{\Lambda} \cdot \mathbf{fw}) \qquad (37)$$

where :

tr denotes the trace of a matrix, while the superscript asterisk of a matrix symbol stands for a transposed conjugate matrix.

From the set of equations (30÷35)
the following relation can be derived :

$$\mathbf{fw} = \mathbf{Z} \cdot \mathbf{Cw} \cdot \mathbf{G} \cdot \mathbf{Bz} \cdot \mathbf{fz} \qquad (38)$$

where the matrix **Z** is defined by the formula :

$$\mathbf{Z} := \tilde{\mathbf{R}} \cdot \mathbf{\Omega} (\mathbf{I} - \mathbf{Cw} \cdot \mathbf{G} \cdot \mathbf{Bw} \cdot \tilde{\mathbf{R}} \cdot \mathbf{\Omega})^{-1} = \\ = \sum_{i=1}^{k} \left[(e_i \times \mathbf{I}_s) \mathbf{R} (e_i^T \times \mathbf{I}_{3r}) \right] \mathbf{\Omega} \bullet \qquad (39) \\ \bullet \left\{ \mathbf{I} - \mathbf{Cw} \cdot \mathbf{G} \cdot \mathbf{Bw} \sum_{i=1}^{k} \left[(e_i \times \mathbf{I}_s) \mathbf{R} \cdot \mathbf{\Omega} (e_i^T \times \mathbf{I}_{3r}) \right] \right\}^{-1}$$

Furthermore, from the equations (30÷35,38)
it is possible to show that :

$$\mathbf{q} = \mathbf{G}(\mathbf{I} + \mathbf{Bw} \cdot \mathbf{Z} \cdot \mathbf{Cw} \cdot \mathbf{G}) \mathbf{Bz} \cdot \mathbf{fz} \qquad (40)$$

The equations (32) and (39) lead directly to the relation :

$$\mathbf{qz} = \mathbf{Cz} \cdot \mathbf{G}(\mathbf{I} + \mathbf{Bw} \cdot \mathbf{Z} \cdot \mathbf{Cw} \cdot \mathbf{G}) \mathbf{Bz} \cdot \mathbf{fz} \qquad (41)$$

By substituting **fw** and **qz** in the equation (37) for the right sides of the equations (38) and (41) respectively, the quality index E may be expressed directly as a function of the parameters of the mechanical system and the controller, the system input **fz** and the weight matrices **Φ** and **Λ** :

$$\mathrm{E} = \mathrm{tr}\Big\{ \mathbf{f}\overset{*}{\mathbf{z}} \cdot \mathbf{B}\overset{*}{\mathbf{z}} \, [(\mathbf{I} + \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{w}} \cdot \mathbf{Z}^* \cdot \mathbf{B}\overset{*}{\mathbf{w}}) \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{z}} \cdot \mathbf{\Phi} \cdot \mathbf{Cz} \cdot \mathbf{G} \bullet \\ \bullet (\mathbf{I} + \mathbf{Bw} \cdot \mathbf{Z} \cdot \mathbf{Cw} \cdot \mathbf{G}) + \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{w}} \cdot \mathbf{Z}^* \cdot \mathbf{\Lambda} \cdot \mathbf{Z} \cdot \mathbf{Cw} \cdot \mathbf{G}] \mathbf{Bz} \cdot \mathbf{fz} \Big\} \qquad (42)$$

The optimal value of the index E for a controller matrix **R** must occur in a stationary point of the function E(**R**), i.e. for a matrix **R**, which fulfills the relation :

$$\frac{\partial \mathrm{E}}{\partial \mathbf{R}} = 0 \qquad (43)$$

The condition for all partial derivatives with respect to single coefficients $\partial\mathbf{R}[i, j]$ of the matrix **R** may be written in the form :

$$\frac{\partial \mathrm{E}}{\partial \mathbf{R}[i,j]} = \sum_{k} \sum_{l} \frac{\partial \mathrm{E}}{\partial \mathbf{Z}[k,l]} \cdot \frac{\partial \mathbf{Z}[k,l]}{\partial \mathbf{R}[i,j]} = 0 \qquad (44)$$

It is possible to conclude from the relations (39) and (44), that the condition (43) will be always fulfilled for k=1 (which refers to the case in which a single frequency is considered) as long as :

$$\frac{\partial \mathrm{E}}{\partial \mathbf{Z}} = 0 \qquad (45)$$

The formula of the derivative $\frac{\partial \mathrm{E}}{\partial \mathbf{Z}}$ can be obtained from the equation (42), with the aid of formulas given in [1,3]. The result may be written in the following form :

$$\frac{\partial \mathrm{E}}{\partial \mathbf{Z}} = 2 \begin{bmatrix} \mathbf{B}\overset{*}{\mathbf{w}} \cdot \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{z}} \cdot \mathbf{\Phi} \cdot \mathbf{Cz} + \\ + (\mathbf{B}\overset{*}{\mathbf{w}} \cdot \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{z}} \cdot \mathbf{\Phi} \cdot \mathbf{Cz} \cdot \mathbf{G} \cdot \mathbf{Bw} + \mathbf{\Lambda}) \mathbf{Z} \cdot \mathbf{Cw} \end{bmatrix} \bullet \\ \bullet \mathbf{G} \cdot \mathbf{Bz} \cdot \mathbf{fz} \cdot \mathbf{f}\overset{*}{\mathbf{z}} \cdot \mathbf{B}\overset{*}{\mathbf{z}} \cdot \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{w}} \qquad (46)$$

From the condition (45) for the extremum of function E(**Z**), the matrix $\mathbf{Z}_{opt}$ matching the optimal controller matrix $\mathbf{R}_{opt}$, can be determined :

$$\mathbf{Z}_{opt} = -(\mathbf{B}\overset{*}{\mathbf{w}} \cdot \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{z}} \cdot \mathbf{\Phi} \cdot \mathbf{Cz} \cdot \mathbf{G} \cdot \mathbf{Bw} + \mathbf{\Lambda})^{-1} \bullet \\ \bullet \mathbf{B}\overset{*}{\mathbf{w}} \cdot \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{z}} \cdot \mathbf{\Phi} \cdot \mathbf{Cz} \cdot \mathbf{G} \cdot \mathbf{Bz} \cdot \mathbf{fz} \cdot \mathbf{f}\overset{*}{\mathbf{z}} \cdot \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{w}} \bullet \\ \bullet (\mathbf{Cw} \cdot \mathbf{G} \cdot \mathbf{Bz} \cdot \mathbf{fz} \cdot \mathbf{f}\overset{*}{\mathbf{z}} \cdot \mathbf{B}\overset{*}{\mathbf{z}} \cdot \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{w}})^{-1} \qquad (47)$$

The matrix $\mathbf{Z}_{opt}$ exists if the two matrices inverted in the above given equation, i.e.: $(\mathbf{B}\overset{*}{\mathbf{w}} \cdot \mathbf{G}^* \cdot \mathbf{C}\overset{*}{\mathbf{z}} \cdot \mathbf{\Phi} \cdot \mathbf{Cz} \cdot \mathbf{G} \cdot \mathbf{Bw} + \mathbf{\Lambda})$

and $(\mathbf{Cw} \cdot \mathbf{G} \cdot \mathbf{Bz} \cdot \mathbf{fz} \cdot \mathbf{fz}^* \cdot \mathbf{Bz}^* \cdot \mathbf{G}^* \cdot \mathbf{Cw}^*)$ are nonsingular. If the conditions are fulfilled, the optimal matrix $\mathbf{R}_{opt}$ exists and may be easily calculated from the relations (39) and (47) :

$$\mathbf{R_{opt}} = (\mathbf{I} + \mathbf{Z_{opt}} \cdot \mathbf{Cw} \cdot \mathbf{G} \cdot \mathbf{Bw})^{-1} \mathbf{Z_{opt}} \cdot \mathbf{\Omega}^{-1} \qquad (48)$$

The final values of the controller matrix **R** should be determined by taking into account some additional limits or criteria, for example stability requirements.

EXAMPLES OF APPLICATION

A ship propulsion unit

An example of a ship propulsion unit equipped with flexible couplings, shown in Fig.12, was described in Part I of this paper. It was shown that the controller using the angular velocity of the generator as its correction signal offers the largest possibilities of reducing torsional vibration amplitude in the main coupling, Fig.13.

The linear dynamic model of this power plant was compared with experimental data, followed by an analysis of the vibrations due to operation of the engine with one misfiring cylinder [2]. By using the method presented in this paper, the optimal matrix **R** of the parameters of the additional controller was estimated, under the assumption that the values of parameters should be contained within a range of practical applications. The optimum parameters of the matrix **R** were determined as those fulfilling the criterion (43) for minimizing the quality index E in the range of vibration frequencies 12÷26 Hz.

The comparison of the amplitude of the fundamental harmonic frequency of the torque acting on the main flexible coupling is presented in function of engine speed in Fig.13 for the cases with and without correction. By using the method described in Part I and II of the paper, it turned out possible to change the structure and parameters of the main engine control system and to remarkably decrease torque amplitudes (much below the hazardous limit).

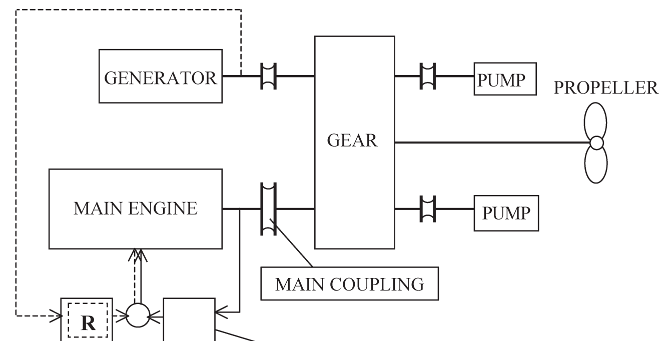

Fig 12. Ship propulsion unit with the correction loop for reduction of main coupling vibrations

Turbine rotor self-excited vibrations

Currently conducted work (see Part I of this paper) concentrates on active control of rotor vibrations of a 200 MW steam turbine by means of pressurized bearings. The aim is to investigate the possibilities of applying the pressurized bearings to the large output steam turbine rotor to reduce self-excited vibration caused by oil whip or oil whirl, as well as aerodynamic excitations.

The proposed method is used for :

- determining the optimum construction of the applied pressurized bearings (i.e. location of the external oil supply, Fig.14a)

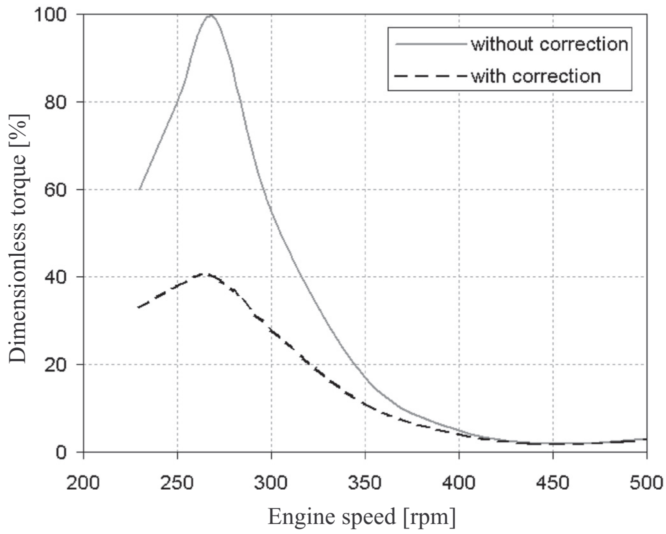

Fig 13. Comparison of the main coupling torque (current torque related to the maximum torque for the case without correction)

- finding the optimal controller for active control of the construction parameters of the bearing supports, Fig.14b
- determining the optimum structure of the active control system and the optimum controller parameters for governing the rotor vibrations by means of pressurized bearings, Fig.15.

After research is completed, the results will be presented in a separate paper.

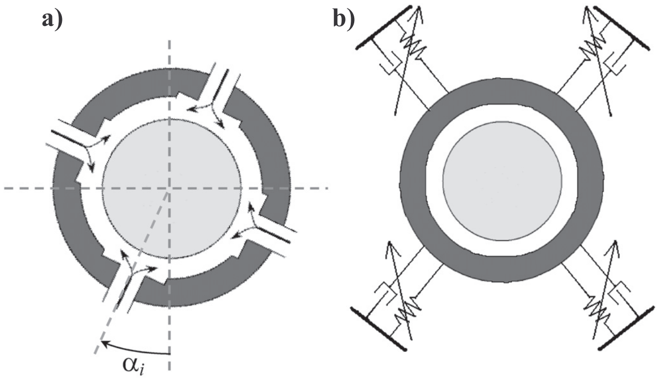

*Fig 14. Diagrams for determining optimum location of external oil supplying (**a**) and for determining optimum controller for active control of bearing supports (**b**)*

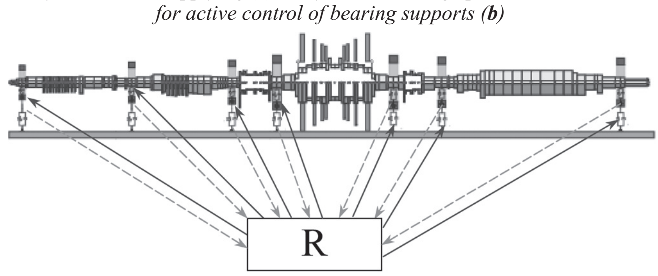

Fig 15. Diagram for determining the optimum structure of the active control system for governing the turbine rotor vibrations

CONCLUSIONS

- A theoretical approach for determining the optimal correction loop for improving the dynamic behaviour of a linear mechanical system was described in the paper. The active control of harmonic vibrations was achieved by applying a control unit which reduces the vibration amplitude of selected elements of the system. This method was applied to a ship propulsion unit in order to change the structure and parameters of the main engine control system, and by use of it a remarkable decrease of torque amplitudes was achieved.

- The presented method makes it possible to determine the controller parameters which provide the optimum value of the chosen quality index. This criterion includes the reduction of weighted amplitudes of the elements on the one hand, and minimizes energy of the control signal on the other.
- The proposed method is suitable for the determination of optimum controller of turbine rotor vibrations due to bearing oil whip, bearing oil whirl or aerodynamic forces.

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NOMENCTLATURE

Bw, **Bz**, **Cw**, **Cz** - binary matrices
e_i - i-th versor of dimension k
E - optimization index
f - vector of force amplitudes
fw - controller output vector
fz - vector of external forces
G - dynamic flexibility matrix
I - unitary matrix
j, k, l, n, r, s - dimensions of matrices and vectors
K - matrix of stiffness coefficients
q - vector of displacement amplitudes
qw - vector of measured amplitudes
qz - vector of amplitudes of controlled elements
R - controller matrix

Φ, **Λ** - weight matrices
ω - frequency
Ω - frequency multiplier matrix

Indices

P - proportional controller
I - integrating controller
D - differentiating controller

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Conference

21st Scientific Session on Marine Technology

On 27-28 September 2004 Polish shipbuilders met in Gdańsk to held their 21^{st} scientific conference which was organized by the Polish Society of Naval Architects and Marine Engineers KORAB, Faculty of Ocean Engineering and Ship Technology of Gdańsk University of Technology (GUT), (which hosted the conference), and North Shipyard Co.

The Conference started with the session on *History of Shipbuilding* which contained six papers prepared by scientific workers of the Faculty of Ocean Engineering and Ship Technology, GUT, namely :

- *Main Directions of the Development of Ships over the Centuries* – by K. Rosochowicz, D. Duda
- *A short Pictorial History of Orthodox and Unorthodox Hydrodynamic Marine Propulsors* – by J.A. Szantyr
- *Outline of Marine Piston Engines' Technology Development* – by J. Girtler
- *Steam Turbine as the Main Ship Propulsion. More than a Hundred Years at Sea* – by K. Kosowski
- *Gas Turbine : Advanced Marine Propulsion* by K. Kosowski
- *Marine Turbines : Cogeneration and Combined Propulsion Systems* – by K. Kosowski

The interesting essence of the papers was enriched by presentation of many illustrations and drawings.

39 papers were read during the technical part of the Conference, which was divided into 7 topical groups :

- Shipbuilding Techniques (7 papers)
- Ship Design and Hydromechanics (6 papers)
- Ship Hydromechanics and Structural Mechanics (7 papers)
- Safety of Ships and Shipping (4 papers)
- Ship Power Plants (6 papers)
- Ship Power Plants and Ship Equipment (4 papers)
- Ship Equipment and Production Management (5 papers).

Representatives of Gdańsk University of Technology and Technical University of Szczecin prepared the greatest number of papers (13 and 11, respectively). Authors and co-authors of the15 remaining papers came from Gdynia Maritime University, Polish Register of Shipping, Naval University of Gdynia, Ship Design and Research Centre – Gdańsk, Wrocław University of Technology, Foundation for Safety of Navigation and Environment Protection, Gdańsk Shiprepair Yard.

Photo: C. Spigarski