

# Processing methods of measured runs of torque and angular speed of combustion engine propulsion system

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## ABSTRACT



*In this paper four processing methods of cyclic data runs, namely : Fourier interpolation and transformation, least-squares approximation by means of trigonometric polynomials , synchronous averaging and movable approximation , were presented by using, as an example, discretely measured runs of torque and angular speed. It was demonstrated that for extracting a useful signal out of disturbance background and decomposing the disturbances the appropriate methods are the synchronous averaging and multiple movable approximation.*

*To emerge significant spectral lines from a spectrum the least-squares approximation by using trigonometric polynomials can be applied. The method contains a criterion for signal filtration and is insensitive to disturbances, run truncation and sampling irregularity. Fast Fourier Transform (FFT) may have an auxiliary significance for preliminary spectral analysis. Its main disadvantage is that in order to build a smoothing-out filter its parameters should be determined in advance, e.g. by applying one of the remaining methods.*

**Keywords :** Ship piston engine, torque, angular velocity, data processing, Fast Fourier Transform (FFT), least-squares approximation by using trigonometric polynomials (TLSA), movable approximation, synchronous averaging.

## INTRODUCTION

A convenient source of diagnostic information about work of ship combustion engine cylinders may be the torque measured on shafts of propulsion systems and angular speed measured by using proximity detectors placed over flywheels and free ends of shafts or other elements kinematically connected with engine crankshaft. In runs of the above mentioned quantities an information about work of all cylinders and entire propulsion system is aggregated that is why they are interesting from the point of view of possible gaining a diagnostic information from them. On the torque and angular speed runs natural vibrations of propulsion system and its elements, variations of consumer's power demand and technical state changes of propulsion system are superimposed. Runs of the above specified quantities are characterized by periodicity which results from cyclic work of piston engines. Sampling is timed by pulses coming from angular position of crankshaft.

Searching for different symptoms in measurement series including those of technical state changes requires applying mathematical tools for making it possible to extract useful signals out of the background of other signals and disturbances.

## INTERPOLATION AND APPROXIMATION OF RUNS BY MEANS OF TRIGONOMETRIC POLYNOMIALS

### *Essence of Fourier interpolation and transformation*

In order to highlight differences in results of data processing realized by using other methods it seems reasonable to remind in this work that the crucial element of Fourier interpolation is expansion of the periodic function with the period  $T$  :

$$f(\tau) = f(\tau + kT) \quad k = 1, 2, 3, \dots$$

into the infinite trigonometric series of the form :

$$f(\tau) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos 2\pi v_k \tau + b_k \sin 2\pi v_k \tau) \quad (1)$$

where :  $v_k = k/T$  – frequency.

The constant  $a_0$  and coefficients  $a_k$  and  $b_k$  are determined by means of commonly known integral formulae. Dimension of the argument  $\tau$  results from physical phenomena represented

by a measured run and is usually considered identical with time or axis of natural numbers ordering the set of samples of measurement series.

Eq. (1) can be expressed in the following form :

$$f(\tau) = A_0 + \sum_{k=1}^{\infty} A_k \cos(2\pi v_k \tau - \Phi_k) \quad (2)$$

$$A_0 = \frac{a_0}{2} ; A_k = \sqrt{a_k^2 + b_k^2} ; \Phi_k = \text{arctg}\left(\frac{b_k}{a_k}\right) \quad (3)$$

where :

$A_0$  – constant (mean value of series)

$A_k, \Phi_k$  – amplitude and phase of  $n$ -th spectral line, respectively, [2].

Ordered runs of the quantities  $A_n$  and  $\Phi_n$  constitute spectra (images) of amplitudes and phases of a considered run in frequency domain and are Fourier transformation of the analyzed run. Initial run can be reconstructed by means of Fourier inverse transformation. Filtration in frequency domain is realized by means of relevant operations on spectra e.g. by multiplying a given spectrum by suitably selected weighting functions prior to making the inverse transformation. In particular if some spectral lines or their groups are removed it will constitute a frequency filtration.

Generally the function  $f(\tau)$  and its image are considered in complex domain. Transition to the complex form is based on the known Euler formula [2]. In this work to apply the complex description was not necessary. However its knowledge is necessary in the case of making use of a software for determining transforms where results are given only in the complex form, e.g. as in Excel software.

The Discrete Fourier Transform (DFT) concerns the transformation of discrete data series described with the finite constant sampling period  $\Delta\tau$  where form of the sampled function is no longer known, that means that to shorten sampling period, if required, is not possible. If a series is finite (i.e. it contains  $N$  samples) then the series corresponds with the measurement series.

The idea of transformation maintains unchanged and in this case it consists in interpolation of series by means of trigonometric polynomials, as shown above.

In connection with the transition from continuous function to its discrete representation, two problems arose already in theoretical phase: of Nyquist frequency and of aliasing [1, 2].

In 1918 H. Nyquist announced that  $2v_g T$  samples are sufficient for approximate representing the signal whose spectrum is limited by the frequency  $v_g$ , and duration time is  $T$ . In 1949 K. C. E. Shannon published the proof of the theorem on run sampling and that on run reconstruction [2]. The Shannon theorem on run sampling cannot be expressed in a simple way and it is better to make use of the Shannon theorem on run reconstruction, which states that if for the sampling frequency  $v$  the relationship  $v \geq v_g$  is fulfilled then the function  $f(\tau)$  is unambiguously reproducible on the basis of its discrete values [2]. In practice are applied greater sampling frequencies depending on sampling purpose [2] since for reproduction of a given function a sufficiently complex Shannon's interpolation polynomial should be used and that the sampling process parameters do not satisfy the assumptions of Dirac's pulse comb function.

When a run whose spectrum contains spectral lines of frequencies greater than  $v_g$  is sampled with the frequency  $v_g$  then the aliasing phenomenon will occur [1, 2]. In order to be protected against this phenomenon, during measurements low-pass filters or, if possible, a sampling frequency increased

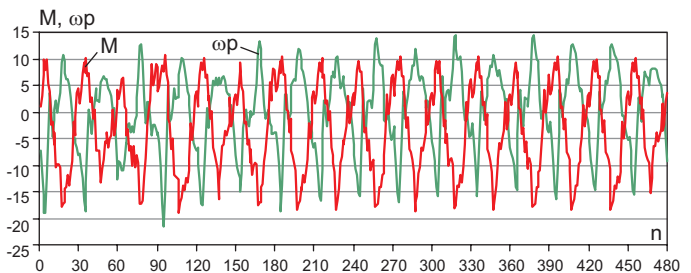
up to a safe value, is applied. As possible recording is limited only to finite representations of the investigated measurement runs the next problem arises, namely that of leak which consists in appearing, in a given spectrum, not existing spectral lines resulting from cutting-off sinusoidal components. Therefore it follows that the interval of measured series does not constitute an integral multiple of every sinusoidal component. An entire branch of mathematics and measurement engineering has been developed to elaborate suitable time filters (windows) to mitigate influence of the phenomenon on derived frequency spectrum [1, 2].

In the domain of practical computer-based spectral analyses the basic notion is Fast Fourier Transform (FFT). From the point of view of transformation theory the essence the FFT does not differ from that of the DFT. All the above given comments dealing with the DFT remain valid. In some elaborations any distinction between the DFT and FFT is not introduced [2]. The FFT problem consists in the computation algorithms owing to which it has been possible to greatly decrease number of main computational operations. Many kinds of FFT software have been developed. Cooley and Tukey, who presented their algorithm [6] in 1965, are regarded as the authors of the computer-based FFT.

The FFT software is offered in many recognized program packets such as e.g. Matlab, Mathematica, Excel. They may differ from each other in different algorithms for determining the FFT, and results obtained by means of different programs may appear not identical for some parameters of the same data sets.

### *Analysis of results of measurements of torque and angular speed runs with the use of the FFT*

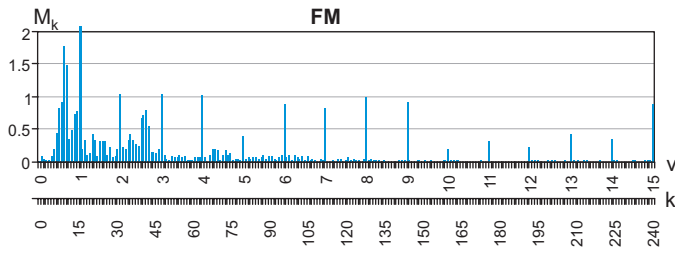
Transforms of example measurement series and several its testing modifications were determined in order to compare features of the analysis of measured runs made by using the FFT and least-squares approximation with application of trigonometric polynomials. To this end was used the torque and angular speed runs (shown in Fig. 1) measured on a real object.



**Fig. 1.** Measured runs obtained by means of a photo-electric speed indicator :  $M$  – torque (dimensionless),  $\omega_p$  [rad/s  $\times$  20] – angular speed pulses related to its mean value.

The above presented measurement results were obtained by means of the experimental unit of photo-electric disc torque meter of ENAMOR [4], on the test bed of one-cylinder two-stroke combustion engine [3]. From every shaft rotation 30 samples of both torque and angular speed were derived in the domain of shaft rotation angle, that results from the torque meter construction. Values of the torque  $M$  were left dimensionless as it does not matter for discussion on run processing. This example is interesting since characteristic symptoms and disturbances are distinctly present in the measured runs, especially those of the torque  $M$ .

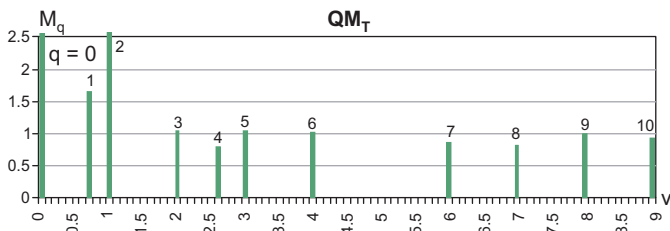
The amplitude spectrum of the run of the torque  $M$ , determined by means of the FFT nested in Matlab, is shown in Fig. 2.



**Fig. 2.** The amplitude spectrum (FM) of the run of the torque  $M$  (Fig.1) derived by using Matlab software;  $M_k$  – amplitude of spectral lines,  $v$  – frequency;  $k$  – FFT natural axis (numbers of spectral lines).

Majority of FFT programs (among them also Matlab and Excel) take measurement input data as complex numbers. In such case the number of symmetrical spectral lines  $N = 2K$  where  $N$  – number of samples of measurement series, and a constant is calculated. In the considered case where  $N = 480$ , apart from the constant, just 480 values of amplitudes and phases, 240 out of which constitute real values, are calculated. There are programs where only real values are determined. Phase spectra are not presented in this work as they are deemed useless for presentation and discussion of the problems in question.

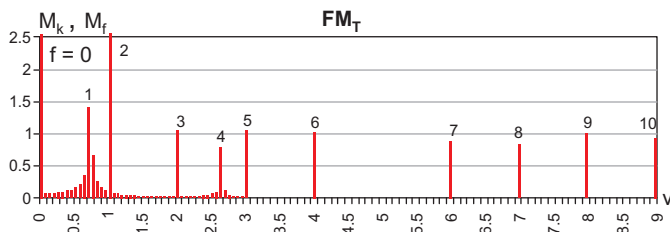
To illustrate the symptoms which appear in results of real measurement data processing with the use of the FFT, was determined the test run  $M_T$  whose amplitude spectrum  $QM_T$  is shown in Fig.3.



**Fig. 3.** The amplitude spectrum  $QM_T$  of the test run  $M_T$ , determined by applying the least-squares approximation method to the run  $M$  (Tab.1) :  $M_0 = 3.1$  ;  $M_2 = 10.46$ .

The spectral lines of the spectrum (Fig.1) were obtained by using the least-squares approximation of the run  $M$  (Tab.1). In the figures spectral lines for  $v > 9$  as well as those of small amplitude values were omitted in order to obtain better quality of imaging the spectra. Only 10 spectral lines were left due to which it was possible to better demonstrate influence of some factors on transformation results.

The amplitude FFT of the  $M_T$  test run, achieved by using the Matlab software is shown in Fig.4.

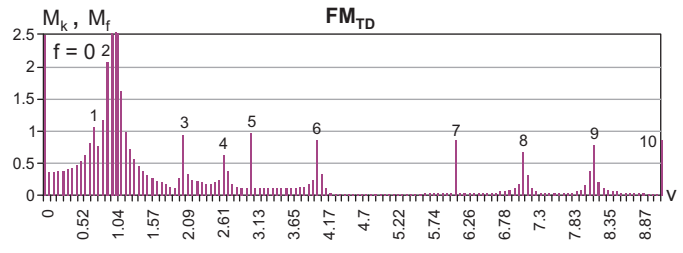


**Fig. 4.** The amplitude spectrum  $FM_T$  (FFT) of the  $M_T$  test run, obtained by means of the Matlab software.

As compared with the spectrum  $QM_T$  (Fig.3) in the spectrum  $FM_T$  (Fig.4) characteristic side spectral lines (stripes) appeared at non-integral frequencies.

The next test run,  $M_{TD}$ , was formed by removing 10 samples from each its end. The  $M_{TD}$  run contains  $N = 460$  samples.

The FFT (obtained by using Matlab) is characterized by a much greater number of side spectral lines (stripes) especially in the range of low frequencies (Fig.5).

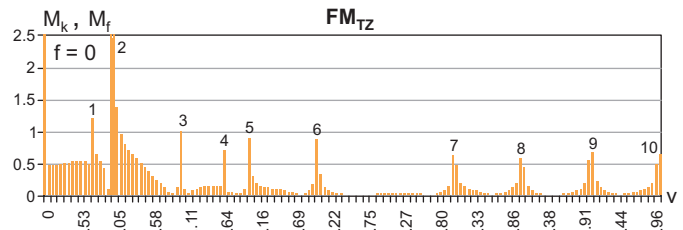


**Fig. 5.** The amplitude spectrum  $FM_{TD}$  of the  $M_{TD}$  test run, obtained by means of Matlab software;  $M_{TD}$  – the run formed from  $M_T$  run by removing 10 samples from each its end.

The phenomenon may constitute an obstacle in searching for characteristic spectral lines of lower amplitudes. Such cases can be met in measuring practice. Runs recorded without synchronizing and controlling pulses of starting-up and stopping the measurements will be generally truncated and even in the case of synchronization the truncation effect may happen for some sinusoids.

In the case of application of some programs, removal of a part of samples or extension of measurement series by adding samples of zero values may be necessary. For instance the FFT program nested in Excel software demands sets of samples of size  $N = 2^l$ , where  $l$  - natural number.

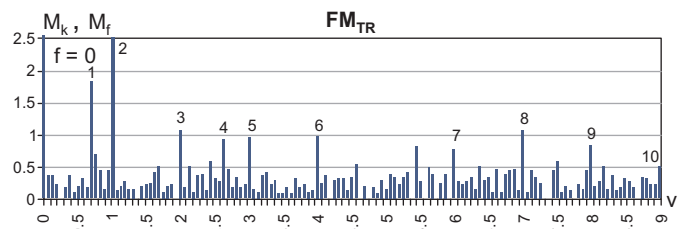
In Fig.6 is shown the transform  $FM_{TZ}$  of the test set  $M_{TZ}$  obtained from the set  $M_T$  of  $N = 480$  samples by complementing it with zeros up to  $N = 512$  samples.



**Fig. 6.** The amplitude spectrum  $FM_{TZ}$  of the test run  $M_{TZ}$  formed by applying Excel software:  $M_{TZ}$  – test run obtained from the set  $M_T$  of  $N = 480$  samples by complementing it with zeros up to  $N = 512$  samples.

In the spectrum  $FM_{TZ}$  (Fig.6) the side stripes of significant values can be observed. The disturbances are analogous to those as in the case of truncation of the run (Fig.5).

The successive test run,  $M_{TR}$ , was obtained by adding, to the run  $M_T$ , pseudo-random disturbances of sampling frequencies generated by computer from the interval  $\langle -5, 5 \rangle$  (of uniform distribution). As observed in Fig.7 the spectrum  $FM_{TR}$  contains a significant number of spectral disturbing lines.



**Fig. 7.** The amplitude spectrum  $FM_{TR}$  of the test run  $M_{TR}$  formed by applying Matlab software:  $M_{TR}$  – test run obtained from the run  $M_T$  by adding pseudo-random disturbances from the interval  $\langle -5, 5 \rangle$ .

In the example shown in Fig.7 the spectral lines resulting from disturbances can be considered significant.

To prevent occurring such symptoms during measurements, as mentioned above, low-pass filters, synchronized recording and various time windows are applied [1, 2].



## Approximation of runs by means of trigonometric polynomials and least-squares method (TLSA)

It should be stressed once more that any transform obtained by using the FFT contains complete measured information, i.e. together with disturbances of analyzed time series, hence this is time series transformation into domain of frequency and phase, performed in the way of interpolation of measurement data by means of trigonometric polynomials of a given uniform (natural) frequency scale.

Determination of spectral lines having their parameters close to real ones, which is connected with elimination of disturbances, can be realized by using the least-squares method, in the simplest case by assuming a power polynomial, as the orthogonal base, and sum of pairs of terms of a trigonometric polynomial :

$$y(t) = \sum_{u=0}^U c_u t^u + \sum_{k=1}^K (a_k \sin v_k t + b_k \cos v_k t) \quad (4)$$

$$t = 2\pi \frac{\tau}{T} ; \quad U + 2K < N$$

where :

- $c_u$  – power polynomial coefficients
- $a_k, b_k$  – trigonometric polynomial coefficients
- $k$  – spectral line number
- $t$  – angular axis, rad
- $v_k$  – dimensionless spectral line frequency,  $v_k > 0$
- $\tau$  – measurement axis (time, rotation angle, measurement point's number)
- $T$  – period of main measurement axis
- $U$  – number of power polynomial coefficients
- $K$  – number of trigonometric polynomial coefficients
- $N$  – size of set of measurement samples.

The power polynomial has been introduced to approximate the constant and trends. In the Matlab software linear trend of data can be removed by using the command „detrnd”. The trend is subtracted off the run by means of least-squares linear approximation. In the model (4) the power polynomial can be replaced by other functions  $\tau$ , and varying frequency can be introduced, that may be reasonable in the case of analyzing some runs of specific models. However, level of computation outlays may be limiting in such case.

The coefficients  $c_u$  and  $a_k, b_k$  are determined from the condition for minimum of sum of squares of deviations, and this sum is used to assess approximation quality and extract significant spectral lines. Values of amplitudes and phases are determined by using the formulae (3). The model (4) differs from the interpolation formula (2) by that values of frequencies are not to be multiples of the fundamental period and total number of determined coefficients and spectral lines must be smaller than that of samples,  $N$ , to ensure some number of degrees of freedom necessary to make it possible to perform statistical significance assessment of determined coefficients and spectral lines.

The significant spectral lines are included one-by-one or collectively into the model with taking into account a degree of decreasing the sum of squares of deviations after their inclusion. If the successively included spectral lines do not make the sum of squares of deviations to decrease significantly then the approximation process will be stopped.

In this work to analyze the torque and angular speed runs (Fig.1) the least-squares approximation algorithm using

trigonometric polynomials (the TLSA method) in accordance with the author's elaboration based on the formulae (4) was applied.

However in this case the Nyquist's frequency limitation is also present. The problem is known in the approximation theory of discrete sets by using a set of selected functions where the functions are required not only to be mutually orthogonal in general, but also to be orthogonal on the approximated data set. It concerns arbitrary functions, also trigonometric ones.

The idea of the method of including - to approximation model - the function which most contributes to decreasing the sum of squares of deviations, is known in a general form, and as in the case of invention of the FFT it is connected with development and applications of computers [6]. Sometimes more firm results may be obtained by using the method of rejecting low-significant functions (terms). In such case the approximation is performed in successive steps with the use of a model which contains a redundant number of terms. Other strategies are also known [6]. In the considered examples of run analysis both approaches were applied. For relatively small sets of spectral lines (up to 50) a kind of algorithm and processing program applied to determining significant spectral lines is rather unimportant if confronted with vast computational capability of contemporary laptop computers.

Results of the determination of significant spectral lines in the spectrum of the measurement series  $M$  with the use of the least-squares approximation are presented in Tab.1.

**Tab. 1.** Results of the determination of significant spectral lines for the run  $M$  with the use of TLSA method :  $q$  – number of spectral line,  $v_q$  – frequency of spectral line,  $M_q$  – amplitude,  $\Phi_q$  – phase,  $\delta_q$  – degree of decrease of the sum of squares of deviations after including the spectral line  $q$  into the base of approximating functions .

<b>q</b>	1	2	3	4	5	6	7	8	9	10
<b>v<sub>q</sub></b>	1	0.7	2	3	4	8	9	6	7	2.6
<b>M<sub>q</sub></b>	10.46	1.66	1.03	1.04	1.01	0.99	0.92	0.87	0.82	0.80
<b>Φ<sub>q</sub></b>	0.86	0.66	0.97	-1.31	1.76	-1.33	-0.01	1.22	-0.34	-1.48
<b>δ<sub>q</sub> [%]</b>	83.7	13.3	5.8	6.1	6.4	6.5	5.9	5.8	5.4	5.4
<b>q</b>	11	12	13	14	15	16	17	18	19	
<b>v<sub>q</sub></b>	15	2.3	1.3	5	13	14	11	12	10	
<b>M<sub>q</sub></b>	0.89	0.54	0.42	0.40	0.41	0.36	0.31	0.22	0.21	
<b>Φ<sub>q</sub></b>	1.57	1.35	-1.43	1.33	-0.76	0.97	1.25	-0.63	-1.19	
<b>δ<sub>q</sub> [%]</b>	5.9	2.4	1.5	1.4	1.4	1.1	0.8	0.4	0.4	

In the above presented example the searching for significant spectral lines was realized by hand without using the FFT in advance. The values  $v$  given in Tab.1 were rounded to one digit after the point. Some of them were determined with a greater accuracy (up to 3 significant digits). Sequence of including the spectral lines is of no influence on final result of approximation and precise positioning the spectral lines regarding their share in decreasing the minimum of sum of squares of deviations is not performed in practice but some influence levels are assumed instead, which makes the process of searching for spectral lines much faster and its automation possible.

On the basis of the constant and first 10 spectral lines, was built the test run  $M_1$  (Fig.2) which constitutes the least-squares approximation of the run  $M$  containing the constant and strictly the first 10 amplitude spectral lines (Fig.2) and 10 respective phases.

To check the method of searching for significant spectral lines the test runs  $M_T$ ,  $M_{TD}$  and  $M_{TR}$  were analyzed also by using the TLSA method together with an algorithm for including and excluding the spectral lines (Tab.2).

**Tab. 2.** Results of the determination of main spectral lines in spectra of the test runs with the use of TLSA method :  $QM_T$  – spectrum of the run  $M_T$ ,  $QM_{TD}$  – spectrum of the run  $M_{TD}$ ,  $QM_{TR}$  – spectrum of the run  $M_{TR}$ .

	q	1	2	3	4	5	6	7	8	9	10
$QM_T$	$v_q$	1	0.7	2	3	4	8	9	6	7	2.6
	$\delta_q$ [%]	91.7	28.4	15.1	17.5	20.8	25.0	28.6	36.6	51.4	100
$QM_{TD}$	$v_q$	1	0.7	2	3	4	8	9	6	7	2.6
	$\delta_q$ [%]	91.5	27.5	14.9	17.0	20.6	24.6	29.0	37.0	51.2	100
$QM_{TR}$	$v_q$	1	0.7	2	3	4	8	9	6	7	2.7
	$\delta_q$ [%]	80.1	15.0	4.6	3.4	4.4	3.4	1.2	3.0	5.8	4.6

From Tab.2 it results that among the first 10 spectral lines for each of the analyzed runs the spectral lines of the same frequencies can be found. The obtained result of 100% decrease of the sum of squares for 10th spectral line in the spectra  $QM_T$  and  $QM_{TD}$  is obvious in this case.

Also, in the case of the run  $M_{TR}$  (Tab.2) within the set of significant spectral lines 10 of them have the same frequencies, that confirms effectiveness of the method. The level of disturbances introduced into the run  $M_{TR}$  was significant as the maximum amplitudes of the disturbances reached 30% of the maximum values of amplitudes of the run  $M_T$ .

The absence of correct order of some spectral lines, observed in Tab.2, as regards the decreasing of the sum of squares of deviations is not a mistake but manifestation of features of orthogonalization process after inclusion of successive spectral lines. The analogous result is obtained in every attempt to changing this sequence of the spectral lines. Such phenomena can happen when values of amplitudes of spectral lines are close to each other and their share in approximation of run is similar and concerns groups of a few spectral lines (2 or 3) that makes determining the spectral lines only a little more difficult.

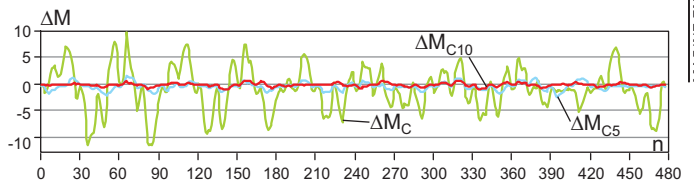
## SYNCHRONOUS AVERAGING AND MOVABLE APPROXIMATION OF RUNS

### Run synchronous averaging

Sampling the torque run  $M$  was timed by pulses from shaft angle position and for successive rotations of the shaft the angular positions of sampling points could be assumed identical. For each of the shaft rotations 30 measurements initiated by shaft angular position, were made.

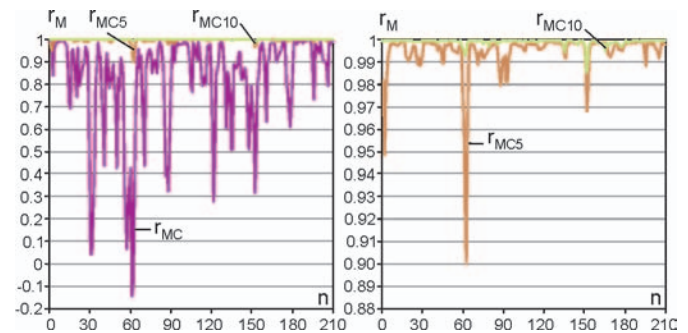
Therefore for steady loading conditions of propulsion system the measurements performed within a given shaft rotation constitute successive realizations of the same process and the deviations from mean values in successive points can be considered random ones which have the sum of deviations from the mean tending to zero along with increasing number of samples. Hence it is reasonable to perform the synchronous averaging of run (called also : coherent, linear, prior-detecting or vectorial averaging) [2]. It should be mentioned that the averaging is a point-wise least-squares approximation procedure.

Fig.8 shows which way the differences between the measured run and its averaged values go depending on a number of synchronous averaging cycles.



**Fig. 8.** Runs of the cyclic differences  $\Delta M = \Delta M_{(n+30)} - \Delta M_n$  for some selected numbers of synchronous averaging cycles :  $\Delta M_{C1}$  – without averaging,  $\Delta M_{C5}$  – for 5-cycle averaging,  $\Delta M_{C10}$  – for 10-cycle averaging.

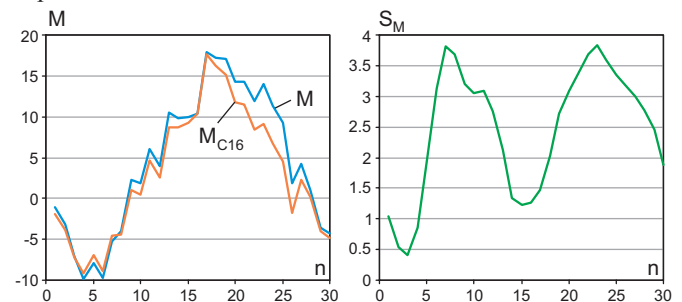
The runs  $\Delta M$  for the whole range of  $n$  were achieved by extending the run  $M$  by the successive run  $M$ . To assess effectiveness of the averaging procedure also the runs of Pearson's correlation coefficient values synchronically calculated can be applied (Fig.9).



**Fig. 9.** Runs of the Pearson's correlation coefficient  $r_M$  determined for the neighbouring cycles of the run  $M_M$ :  $r_{MC}$  – without averaging,  $r_{MC5}$  – for averaging after 5 cycles,  $r_{MC10}$  – for averaging after 10 cycles.

The runs  $\Delta M$  are characterized by the deviations between the points distant by one cycle, but the runs  $r_M$  are characterized by their trends. To determine  $r_M$  values for each of them 5 successive samples were taken into consideration.

In the case of the analyzed data set the maximum possible number of averaging was 16. In Fig.10 a fragment of the measured run  $M$  and the averaged run  $M_{C16}$  (for 16-cycle averaging) is compared, as well as the run of the standard deviations  $S_M$  is presented.



**Fig. 10.** Comparison of a fragment of the measured run  $M$  and the averaged run  $M_{C16}$  (for 16-cycle averaging):  $S_M$  – standard deviation of the run  $M$ .

It is easy to observe that the waving period of the standard deviations  $S_M$  (Fig.10) amounts to about 0.7 of the shaft rotation period, hence it can be concluded that this is the large amplitude of torsional vibrations of the propulsion system (i.e that fitted with one-cylinder combustion engine) which constitutes the main cause of torque measurement disturbances. The smoothness disturbances observed in the run  $M_{C16}$  may result from manufacturing and assembling inaccuracies of the discs.

The FFT of the single averaged run obtained contains spectral lines solely for integral values of  $v$  (Fig.11).

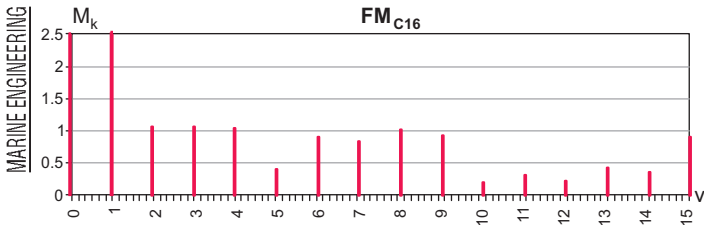


Fig. 11. The amplitude spectrum  $FM_{C16}$  of the run  $M_{C16}$ , obtained with the use of Matlab software.

In Fig.12 a fragment of the averaged run  $M_{C16}$  was compared with its two trigonometric approximations for an assumed number of spectral lines, obtained by means of the least squares method.

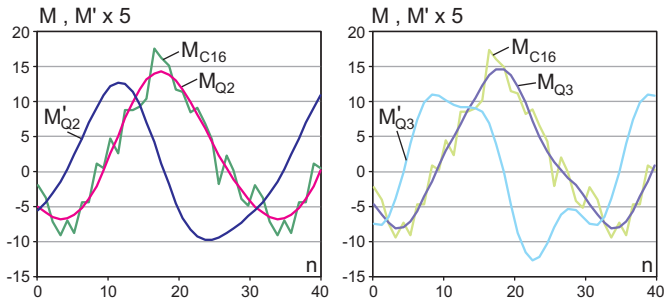


Fig. 12. Comparison of the averaged run  $M_{C16}$  and its approximations:  $M_{Q2}$  – that based on the spectral lines  $q = 1, 2$ ;  $M_{Q3}$  – that based on the spectral lines  $q = 1, 2, 3$  (Tab.4);  $M'_{Q2}$ ,  $M'_{Q3}$  – their first derivatives.

In order to better demonstrate differences between the runs  $M_{Q2}$  and  $M_{Q3}$  the runs of their first derivatives  $M'_{Q2}$  and  $M'_{Q3}$  are also presented (Fig.12). However in both the cases shown in Fig.12 one may doubt, making only the use of general knowledge on physical nature of the run  $M$ , whether the approximation models are correct. Possible adding successive spectral lines would decrease quality of the approximation. It will be further demonstrated that the problem of smoothing the run does not consists in adding or removing the spectral lines which appeared in the spectrum of the run  $M$ .

The information removed from the run  $M$  as a result of averaging has been now stored in the run of deviations,  $\Delta M_{C16}$  whose FFT obtained by means of the Matlab, is shown in Fig.13.

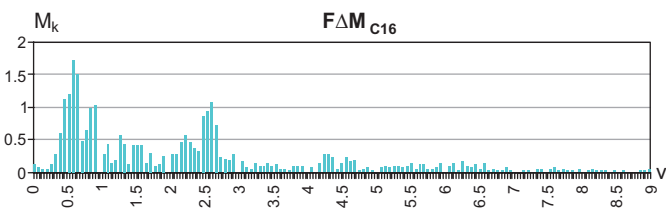


Fig. 13. A fragment of the amplitude spectrum  $F\Delta M_{C16}$  obtained by means of the Matlab for the run of deviations,  $\Delta M_{C16} = M - M_{C16}$ .

In Tab.3 are presented results of searching for the most significant spectral lines in the run  $\Delta M_{C16}$ , by using the least squares method.

Tab. 3. Results of determining the most significant spectral lines for the run of deviations,  $\Delta M_{C16}$ , by using the TLSA method.

q	1	2	3	4	5	6	7	8	9	10
$v_q$	1	0.6	0.7	0.5	0.9	2.6	2.3	1.3	1.4	2.8
$\delta_q$ [%]	0.0...	24.4	30.1	9.7	5.7	10.8	5.8	4.1	2.8	2.7
$\Delta M_{C16q}$	0.11	1.77	1.69	0.81	0.6	0.81	0.53	0.44	0.35	0.34

In Tab.3 the spectral line of  $v_1 = 1$  was left in order to demonstrate effectiveness of the performed averaging. The value of the spectral line amplitude  $\Delta M_1 = 0.17$  appeared significantly smaller as compared with that of the averaged run  $M_2 = 10.46$  (Fig.2, that confirms good effectiveness of the performed averaging.

In addition, the spectrum  $Q\Delta M_{C16}$  is graphically presented (Fig.14) to facilitate making comparisons with the spectrum  $F\Delta M_{C16}$  (Fig.13).

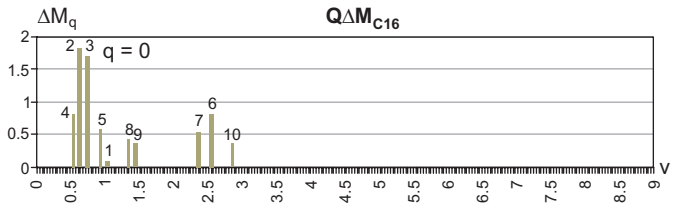


Fig. 14. Main spectral lines of the amplitude spectrum  $Q\Delta M_{C16}$  obtained by using the TLSA method for the run of deviations,  $\Delta M_{C16} = M - M_{C16}$ .

The run of deviations,  $\Delta M_{C16}$ , and runs of its selected approximations are shown in Fig.15.

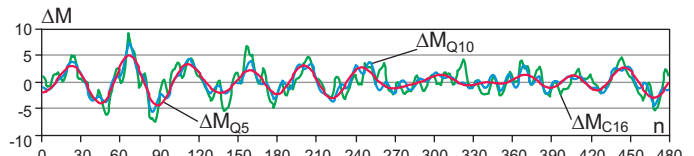


Fig. 15. The run of deviations,  $\Delta M_{C16} = M - M_{C16}$ , and its selected LSA approximations:  $\Delta M_{Q5}$  – that containing the spectral lines for  $q = 2 - 5$ ,  $\Delta M_{Q10}$  – that containing all 10 spectral lines (Fig.14).

From the comparison of the runs shown in Fig.15 it results that four neighbouring spectral lines of  $k = 2-5$  are mainly responsible for the observed „beating” (Fig.14).

The run of deviations,  $\Delta M_{C16}$ , can be also subjected to further synchronous averaging for selected non-integral frequencies  $v$  and such operation can be repeated in order to split the disturbances regarding their characteristic frequencies – in an analogous way as it takes place in the case of approximation itself.

Examples of synchronous averaging the run of deviations,  $\Delta M_{C16}$ , for two values of  $v$  are shown in Fig.16.

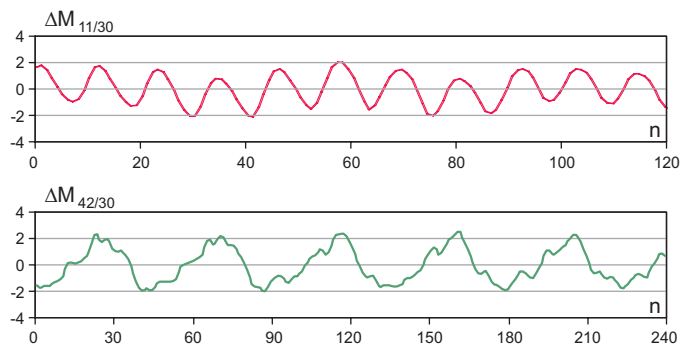


Fig. 16. Results of synchronous averaging of the run of deviations,  $\Delta M_{C16}$ , of 10 averaging operations, for the various values of  $v$ :  $M_{11/30}$  – a fragment of the averaged run for  $v = 11/30$ ,  $M_{42/30}$  – a fragment of the averaged run for  $v = 42/30$ .

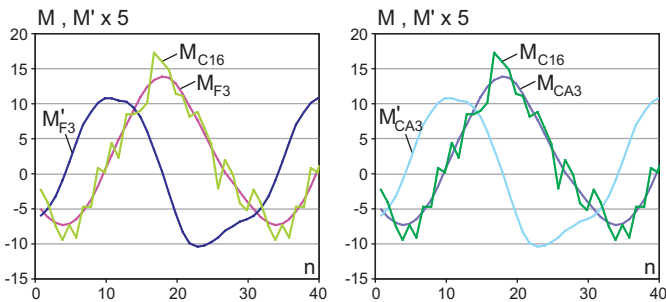
The above mentioned procedures could be applied again to the obtained runs or those of differences resulting from the averaging in order to finally extract interesting periodic components. Limited accuracy of  $v$  - values make a problem in their setting. In the case of the run in question they arise from the number of samples derived from one shaft revolution.



The synchronous averaging method should be considered as basic one for processing the data of periodical character, measured in steady operation conditions of propulsion system.

### Smoothing the runs by means of movable approximation method

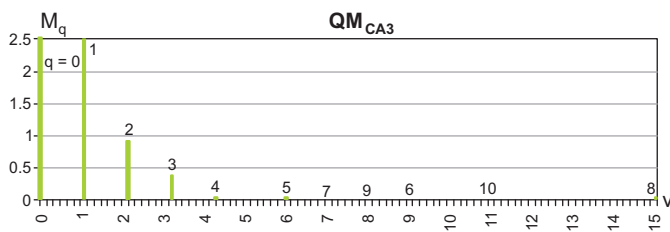
In the systems for measuring torque or angular speed the main aim of the processing of measurement results is to determine in real time the runs of the quantities, characterized by an appropriately low level of disturbances. Examples of smoothing the run  $M_{C16}$  with the use of two different approximating objects elaborated by this author [9], are shown in Fig.17.



**Fig. 17.** Comparison of the averaged torque run  $M_{C16}$  and its approximations obtained by using the movable approximation method:  $M_{F3}$  – the result of the triple approximation by using 3rd order power polynomial (Savitzky’s – Golay’s filter) within the interval of 11 samples;  $M_{CA3}$  – the result of the triple approximation by using 3rd order power polynomial within the interval of 21 samples and 2 nodes having free constraints (cut derivatives: 1<sup>st</sup>; 2<sup>nd</sup> and 3<sup>rd</sup>);  $M'_{F3}$ ,  $M'_{CA3}$  – 1<sup>st</sup> derivatives.

As results from the comparison of the runs  $M_{F3}$  and  $M_{CA3}$  as well as their derivatives  $M'_{F3}$  and  $M'_{CA3}$ , the achieved results are identical in the applied scale of imaging. The differences are negligible and they can be disclosed e.g. by comparing the higher order derivatives or remote significant places of values of spectral lines of Fourier transforms. As already mentioned there is a justified supposition that the approximations by using trigonometric polynomials (Fig.12) are loaded by large errors resulting from a trigonometric model inadequacy, that can be concluded from the comparison of the runs of 1<sup>st</sup> derivatives for both the approximation cases (Fig.12 and 17). The problem can be solved by analyzing the motion models of propulsion system, e.g. by taking into consideration a run of indicated pressure.

If the Fourier transform for the runs  $M_{F3}$  and  $M_{CA3}$  (Fig.17) is elaborated it will have the form shown in Fig.18, whereas the FFT of the run  $M_{O2}$  consists 2 spectral lines, and that of the run  $M_{O3}$  - 3 ones (plus constant) taken from the transform  $FM_{C16}$  (Fig.11).



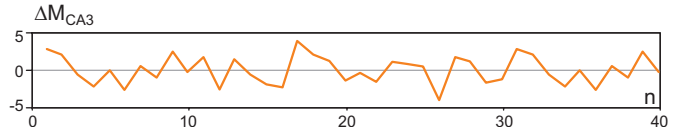
**Fig. 18.** Ten main spectral lines of the amplitude spectrum  $QM_{CA3}$ , determined by using the least squares method for the smoothed run  $M_{CA3}$ .

The transform  $QM_{CA3}$  (Fig.18) is characteristic by having a dozen or so spectral lines of similarly small values of amplitudes but significantly important for approximation quality (smoothing) of the run, which is manifested by a large degree of decreasing the sum of squares of deviations after adding successive spectral lines (Tab.4).

**Tab. 4.** Values of input data to determination process of main spectral lines and their parameters for the spectrum  $QM_{CA3}$  (Fig. 18).

q	1	2	3	4	5	6	7	8	9	10
$v_q$	1	2	3	4	6	9	7	15	8	11
$\delta_q$ [%]	99.3	84.6	98.9	71.9	83.3	56.1	77.2	94.2	70.7	54.0
$M_{CA3q}$	10.23	0.82	0.34	0.031	0.018	0.006	0.005	0.002	0.0005	0.0002

In order to fully recognize differences between the runs their phase spectra should be also compared. In Fig.19 is shown the run of the deviations,  $\Delta M_{CA3}$ , between the run  $\Delta M_{C16}$  and the smoothed run  $M_{CA3}$ .



**Fig. 19.** The run of deviations :  $\Delta M_{CA3} = M_{C16} - M_{CA3}$

The transform of the above mentioned deviations amounts approximately to the difference of the spectrum  $FM_{C16}$  (Fig.11) and  $QM_{CA3}$  (Fig.18).

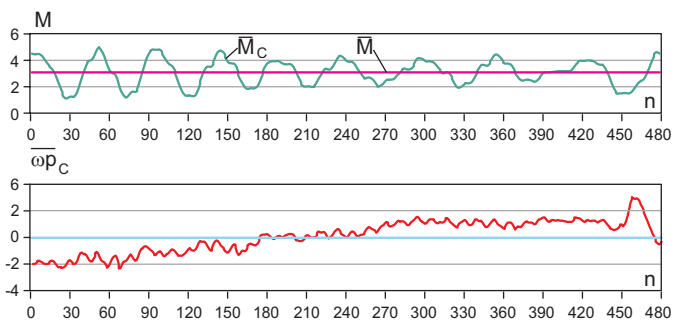
Smoothing by means of the movable approximation method can be also applied to the runs of deviations  $\Delta M_{C16}$ .

Smoothing by means of the movable approximation method and decomposition of disturbances can be applied both before and after synchronous averaging. A very important advantage of the movable approximation is its possible application to smoothing and decomposing the disturbances measured in unsteady conditions : starting-up, stopping, variable loading. The disturbances extracted as a result of decomposition can be next subjected to frequency analysis, if necessary.

Alternative smoothing the runs by applying the Fourier inverse transform would require prior multiplication of the transforms by appropriate weighting functions. In order to find such weighting functions it would be necessary to smooth the runs in advance e.g. by applying the movable approximation.

### Interim averaging per cycle

The interim averaging per cycle is a peculiar case of the movable approximation. It is aimed at imaging the variability of mean values usually determined as a function of time or shaft rotation angle. Such characteristics may be important for diagnostics of propulsion systems. In Fig.20 are presented results of interim averaging the runs of torque and angular speed oscillations, shown in Fig.1.



**Fig. 20.** Results of interim averaging the runs  $M$  and  $\omega_p$  (Fig. 1):  $\overline{M}_c$ ,  $\overline{\omega_p}_c$  – results of averaging per one cycle (rotation),  $\overline{M}$  – mean value of the set.

The achieved results can be used in successive steps of their analyzing with the use of the above presented methods of data processing.

## CONCLUSIONS

- The methods of synchronous averaging and movable approximation should be deemed basic for processing the discretely measured runs of torque and angular speed.
- The spectral analysis with the use of least-squares approximation of random disturbances and uneven sampling makes it possible to extract significant spectral lines in the situation of truncated run, random disturbances and non-uniform sampling. It also makes it possible to precisely determine characteristic frequencies as well as extract significant spectral lines of neighbouring frequencies.
- The FFT can be used for preliminary recognizing the spectrum if there is no computer program available for automatic determining the significant spectral lines by means of least-squares approximation.

## NOMENCLATURE

- $a_k, b_k$  – trigonometric polynomial coefficients  
 $a_0$  – trigonometric polynomial constant  
 $c_u$  – power polynomial coefficients  
 $F...$  – Fourier transform of a given run  
 $k$  – spectral line number  
 $K$  – number of trigonometric polynomial coefficients  
 $M$  – torque, measured torque  
 $M_{C16}$  – torque run synchronically averaged per 16 cycles  
 $M_T$  – torque test run containing 10 spectral lines  
 $M_{TD}$  – test run obtained from the run  $M_T$  by removing 10 samples from each end of set of samples  
 $M_{TR}$  – test run obtained from the run  $M_T$  by adding to it random disturbances from the interval  $\langle -5, 5 \rangle$   
 $M_{TZ}$  – test run obtained from the run  $M_T$  by its extending by zeros as to reach  $N = 512$  samples  
 $n$  – number of a sample in measurement set : 1, ...,  $N$   
 $N$  – number of samples in measurement set  
 $Q...$  – spectrum of a given run, obtained by using LSA method  
 $t$  – angular axis, rad  
 $T$  – period  
 $U$  – number of power polynomial coefficients  
 $\Delta M$  – run of deviations of the run  $M$  (a kind of deviations is indicated by its indices)  
 $\Delta M_{C16}$  – run of deviations,  $\Delta M_{C16} = M - M_{C16}$   
 $\delta_q$  – degree of decreasing the sum of squares of deviations after including the component  $q$  to the basis of approximating functions  
 $v$  – frequency  
 $v_k$  – dimensionless spectral line frequency,  $v_k > 0$   
 $\tau$  – time  
 $\Phi$  – phase.

## Indices

- $f$  – number of a distinguished (significant) component of a spectrum (spectral line) in Fourier transform  
 $g$  – limit value  
 $k$  – successive number of spectral line in Fourier transform  
 $q$  – number of spectral line in the transform obtained by means of TLSA method  
 $'$  – symbol of 1<sup>st</sup> derivative.

## Abbreviations

- DFT – Discrete Fourier Transform  
 FFT – Fast Fourier Transform  
 TLSA – approximation by means of trigonometric polynomials and least squares method.

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## Conference

# REGIONAL GROUP of the Section on Exploitation Foundations

On 23 March 2006 Faculty of Ocean Engineering and Ship Technology, Gdańsk University of Technology, hosted the members of the Regional Group of the Section on Exploitation Foundations, Machine Building Committee, Polish Academy of Sciences ( PAS), who took part in their successive scientific seminar.

The seminar's scientific program consisted of four papers presented by the scientific workers of the Faculty :

- *Comparative analysis of main engine loads and output of multi-bucket dredgers by using probabilistic approach* – by D. Bocheński
- *Diagnostics of the energy transformation process of steam turbine power plant of large output* – by J. Gluch
- *Research on water-lubricated short-sleeve bearings of propeller shaft* – by W. Litwin
- *A concept of application of artificial neural networks to represent reliability probabilistic models* – by J. Rudnicki

After interesting discussion on the presented topics the seminar participants were acquainted with the Faculty's scientific and didactic laboratories.