POLISH MARITIME RESEARCH 3(61) 2009 Vol 16; pp. 62-69
10.2478/v10012-008-0035-3

Occurrence probability of maximum sea levels in Polish ports of Baltic Sea coast

Bernard Wisniewski, Prof.
Tomasz Wolski, Ph. D.
University of Szczecin

ABSTRACT

In this work long-term probability of occurrence of maximum sea levels in some points of Polish Baltic Sea coast, was determined. Use was made of multi-year series of measurement data on maximum yearly sea levels, and their probability distributions were determined. To the analysis Gumbel's distribution and Pearson distribution of 3rd type as well as quantile methods and the highest credibility method, were applied. Kolmogorov test was used to examine conformity of the theoretical distributions with real random variable distribution. As results from the analysis, the highest sea levels of 1000- year return period can be expected in Polish ports of the west part of the coast , i.e. Kolobrzeg (750, 2 cm , i.e. 2,5 m above the average sea level) and Swinoujscie (723,6 cm) . Lower sea levels of the same return period can be expected in Ustka (720,2 cm), Wladyslawowo (709,7 cm) and Gdansk (716, 7 cm), respectively.

Keywords: sea levels; probability; Polish Baltic Sea coast

INTRODUCTION

This work is aimed at determination of long-term probabilistic prediction of maximum yearly sea levels in five Polish Baltic Sea coast observation stations: Swinoujscie, Kolobrzeg, Ustka, Wladyslawowo and Gdansk. Knowledge of sea levels of a given occurrence probability is necessary in designing the marine hydro-engineering buildings as well as in determining the characteristics of storm swellings.

The most important is to know high water level stages which can happen once a determined number of years, e.g. once 50, 100, 150 or 200 years. This period is called the return period **T.** It is expressed as follows:

$$T = \frac{100}{p\%} \ [\text{years}] \tag{1}$$

where:
T – mean return period
p – probability [%.].

Empirical probability is defined by Weibull's function [1]:

$$p(m, N)\% = \frac{100m}{N+1} \tag{2}$$

or Czegodajew formula used in hydro-engineering [7]:

$$p(m, N)\% = \frac{m - 0.25}{N+0.50} * 100\% \tag{3}$$

where:
m – successive term of distribution series
N – size of series.

To estimate a probable water level, available measurement data series from previous years should be used. Credibility of such estimation depends both on length of water level observation series and representativeness of a chosen theoretical probability distribution function. In determining probability of the yearly highest sea levels a random sample covering a few dozen years of measurements is usually at one's disposal. Hence for Kolobrzeg the observed water levels from the years of 1867 ÷ 2006 were at one's disposal, for Swinoujscie – from the years of 1901 ÷ 2006, for Gdansk – from the years of 1886 ÷ 2006, for Ustka – from the years of 1946 ÷ 2006 and for Wladyslawowo – from the years of 1947 ÷ 2006.

Calculations of exceedance probability of water levels consist in appropriate selection of theoretical probability distributions and next in assumption of estimation methods of parameters of a selected distribution, with the use of statistical data. Confirmation of the fact whether the distribution has been properly selected can be obtained by using the tests of goodness of fit, proposed in the literature. Calculation results are given together with a confidence interval within which real variable value can be found with a given probability.

In probability theory the random variable X is such quantity which - due to random coincidence of various factors – can achieve different numerical values with a given probability. In this work the series of yearly maximum water levels observed at the stations of Swinoujscie, Kolobrzeg, Ustka, Wladyslawowo and Gdansk, are taken as the random variable X. A function which determines how large is occurrence probability of the event that the random variable X takes one of the values contained within the numerical interval S on the x - variable axis is called the probability distribution of the random variable X.

In hydrology and oceanology the distributions:
- log-normal distribution (Gauss – Laplace)
- Pearson's distribution of type III
- Debski's distribution
- Gumbel's distribution of extreme values (Fisher-Tippett, type I) are most often used [1, 6, 8].

Every theoretical distribution density function has a number of parameters $g_1,...,g_k$, on which its graphical form depends. One of the crucial tasks is, on the basis of random sample, to estimate values of the paramters $g_1,...,g_k$, in such a way as random features of general population to be represented as best as possible by the function $f(x, g_1,...,g_k)$. In statistics to this end serve the methods of estimation of probability distribution parameters. Usefulness of particular estimation methods is dependent first of all on random sample size as a well as on a required degree of approximation of estimated values to theoretical ones. To large random samples simpler methods which provide lower estimation accuracy, can be applied [7].

In practice it is not possible to fully confirm how far a theoretical distribution fits actual distribution of random variable, i.e. that empirically elaborated on the basis of statistical observation series. In such situation consistency of theoretical distribution and observation results should be examined. To this end serve statistical consistency tests which determine occurrence probability of differences between theoretical distribution and empirical one. The λ point test of Kolmogorow and the χ^2 linear test of Pearson belong to the most often used [1, 8].

A few calculation methods of water levels with a given exceedance probability can be distinguished depending on an assumed type of probability distribution curve as well as estimation method of parameters. Among the most frequently applied the following methods can be numbered:
- quantile method (of position characteristics)
- maximum likelihood method and
- method of statitstical moments.

In this work Pearson's distribution and Gumbel's one was used and from among the applied methods the quantile one and that of highest likelihood was selected as recommended for the determining of occurrence probability of extreme hydrological phenomena such as water flow rates and water levels [6], [7].

Quantile method

The method is based on determined specific values of the variable X called quantiles. The p – order quantile is called such value of the variable x_p for which the exceedance probability is equal to p. Distribution quantiles are compared to sample quantiles. To determine quantiles from random sample its elements should be ordered according to their values to form the so called distribution series. The following quantiles are usually determined:
- $x_{50\%}$ – a value corresponding to the middle of series, equivalent of median
- x_p, $x_{100\%-p}$ – two symmetrical quantiles, equally distant from the middle of series, where: p = 10% or p = 5% is selected most frequently
- $x_{100\%}$, $x_{0\%}$ – two quantiles equivalent to lower and upper limit values of random variable.

If distribution series is long nad regular then values of symmetrical and middle quantiles can be determined by using interpolation between terms of the series. In the case of short and irregular series the searched - for quantiles are read from the equalized diagram of summed-up frequencies. However the values $x_{100\%}$ and $x_{0\%}$ are usually determined subjectively as they are located beyond the range of values of sample elements [1, 8].

Maximum likelihood method

The basic notion of the method is the likelihood function:

$$L = f(x_1, g_1, ..., g_n) * f(x_2, g_1, ..., g_n) * ... f(x_N, g_1, ..., g_n) = = \prod_1^N f(x_i, g_1, ..., g_n) \tag{4}$$

where:
$f(x_i, g_1,...g_n)$ – probability density function of the random variable x_i in a given mathematical form and having unknown distribution parameters g_i,$(g_1,g_2...g_n)$

The function can be approximately interpreted as probability of obtaining the random sample $Z_N(x_i)$ exactly the same as that really observed. For calculation reasons to use the logarithm of likelihood function is more convenient:

$$\ln L = \ln f(x_1, g_1, ..., g_n) + ... \ln f(x_N, g_1, ..., g_n) = = \sum_i^N \ln f(x_i, g_1, ..., g_n) \tag{5}$$

Estimation of distribution parameters consits in finding such system of the values g_1, g_2,...g_n at which probability of observing the given random sample $Z_N(xi)$ is the greatest. The discussed method is equivalent to searching for maximum value of the function L respective to $g_1$value, i.e. to solving the set of equations:

$$\frac{\partial L}{\partial g_i} = 0 \quad \text{or} \quad \frac{\partial \ln L}{\partial g_i} = 0 \tag{6}$$

where:
i = 1,2,...g_n.

The maximum likelihood method involves great troubles in the case when one of the estimated parameters is lower or upper variability range limitation of the random variable X. In such cases to use other methods is proposed for estimating the limitations [1, 8].

APPLIED METHODS

Determination of theoretical probable maximum sea levels by means of the quantile method with the use of Gumbel's distribution

The Gumbel's distribution density function is based on statistical distributions of extreme values which occur in certain larger sets of values. For instance it can be maximum values of sea levels considered in this paper. The Gumbel's distribution density function is double exponential and described by the formula [21:

$$f(x) = \frac{1}{\hat{a}} e \left[-\frac{x-\hat{b}}{\hat{a}} - e\left(-\frac{x-\hat{b}}{a}\right) \right] \tag{7}$$

where:

$\hat{\alpha}$ – scale parameter (it determines dispersion of the distribution along x-axis)

$\hat{u}$ – location parameter (it determines location of the distribution on x-axis)

e – Napierian base.

Essence of estimation of the assumed distribution against given measurement data is to determine estimators of the distribution parameters $\hat{\alpha}$ and $\hat{u}$.

After finding logarithms and simplifying the above given formula the following was obtained:

$$X_{p\%} = \hat{u} - 1/\hat{\alpha} * \ln[-\ln(1-f(x))] \tag{8}$$

To estimate the parameters $\hat{\alpha}$ and $\hat{u}$, formulas for the three quantiles: upper one X_p, middle one $X_{50\%}$ and lower one $X_{1-p,}$ were preliminarily determined. Here the following values were assumed: p = 5% and 1-p = 95%.

In compliance with the formula (8) for $X_{p\%}$ the following was achieved [7]:

$$X_{5\%} = \hat{u} - 1/\hat{\alpha}(-2.9702) \tag{9}$$

$$X_{50\%} = \hat{u} - 1/\hat{\alpha}(-0.36651) \tag{10}$$

$$X_{95\%} = \hat{u} - 1/\hat{\alpha}(1.097189) \tag{11}$$

Next the variability measure was determined form the formula:

$$V = (X_p - X_{1-p})/2 \tag{12}$$

By substituting relevant expressions from Eqs (9) and (11) for X_p and X_{1-p}, respectively, the following was obtained:

$$V = 4.0672/2\hat{\alpha} \rightarrow \hat{\alpha} = 2.0336/V \tag{13}$$

Then, on the basis of distribution series of maximum water levels for each of the considered stations the water levels were read or interpolated by using the empirical probability from the formula (2), for X_p and X_{1-p} at p = 5%. The obtained values were used for determining the distribution parameters $\hat{\alpha}$ and $\hat{u}$ from Eqs. (12), (13) and (10), respectively. The so achieved distribution parameters $\hat{\alpha}$ and $\hat{u}$ were introduced to Eq. (8) for $X_{p\%}$ and the theoretical sea levels for selected probability quantiles were calculated for each of the observation stations (Tab. 1, 2, 3, 4 and 5)

Tab. 1. Theoretical maximum sea levels and probability of their occurrence at Swinoujscie observation station in the period of 1901÷2006 years

T (years)	F(X)	Sea level [cm] Gumbel's distribution		Person's distribution	
		Quantile method	Maximum likelihood method	Quantile method	Maximum likelihood method
1000	0.1%	717.9	723.6	695.1	715.0
200	0.5%	686.3	690.9	671.4	679.1
100	1%	672.6	676.8	661.2	674.7
50	2%	658.9	662.6	650.6	662.0
20	5%	640.6	643.7	635.8	644.3
10	10%	626.5	629.1	623.8	630.1
5	20%	611.7	613.9	610.6	614.7
2	50%	589.5	590.9	589.5	590.5
1.33	75%	575.9	576.7	576.1	575.7
1.11	90%	565.9	566.5	566.7	565.6
1.05	95%	560.8	560.2	562.1	560.8
1.01	99%	552.3	555.3	555.0	554.2
1.00	100%	544.3	544.3	544.3	544.3

Tab. 2. Theoretical maximum sea levels and probability of their occurrence at Kolobrzeg observation station in the period of 1867÷2006 years

T (years)	F(X)	Sea level [cm] Gumbel's distribution		Person's distribution	
		Quantile method	Maximum likelihood method	Quantile method	Maximum likelihood method
1000	0.1%	740.1	750.2	749.8	739.0
200	0.5%	704.1	711.5	713.8	708.1
100	1%	688.5	694.8	698.8	692.0
50	2%	672.9	678.0	683.0	677.0
20	5%	652.1	655.7	660.7	656.3
10	10%	636.1	638.4	643.3	639.6
5	20%	619.3	620.4	623.9	621.6
2	50%	594.0	593.1	593.5	593.1
1.33	75%	578.5	576.4	575.7	575.5
1.11	90%	567.2	564.2	562.3	563.5
1.05	95%	561.3	555.0	555.8	557.8
1.01	99%	551.7	546.1	547.3	549.2
1.00	100%	537.6	537.6	537.6	537.6

Tab. 3. Theoretical maximum sea levels and probability of their occurrence at Ustka observation station in the period of 1946÷2006 years

T (years)	F(X)	Sea level [cm] Gumbel's distribution		Person's distribution	
		Quantile method	Maximum likelihood method	Quantile method	Maximum likelihood method
1000	0.1%	725.7	720.0	698.3	696.1
200	0.5%	692.3	687.5	673.5	675.1
100	1%	677.8	673.5	664.3	662.3
50	2%	663.4	659.4	654.0	651.4
20	5%	644.0	640.5	638.9	636.2
10	10%	629.1	626.0	626.2	623.8
5	20%	613.5	610.9	612.6	610.2
2	50%	590.0	588.0	590.0	588.0
1.33	75%	575.6	573.9	575.5	573.8
1.11	90%	565.1	563.7	564.3	563.6
1.05	95%	559.6	556.0	559.1	558.4
1.01	99%	550.7	550.0	550.4	550.7
1.00	100%	535.3	535.3	535.3	535.3

Tab. 4. Theoretical maximum sea levels and probability of their occurrence at Wladyslawowo observation station in the period of 1947÷2006 years

T (years)	F(X)	Sea level [cm]			
		Gumbel's distribution		Person's distribution	
		Quantile method	Maximum likelihood method	Quantile method	Maximum likelihood method
1000	0.1%	709.7	709.7	681.0	682.9
200	0.5%	679.4	679.1	659.4	669.4
100	1%	666.3	665.9	651.3	652.6
50	2%	653.1	652.5	642.4	642.8
20	5%	635.6	634.7	629.2	629.1
10	10%	622.0	621.0	618.1	617.9
5	20%	607.9	606.6	606.2	605.5
2	50%	586.5	585.0	586.5	585.2
1.33	75%	573.4	571.6	573.8	571.9
1.11	90%	563.9	561.9	564.1	562.2
1.05	95%	558.9	554.6	559.5	557.3
1.01	99%	550.8	549.3	551.9	549.6
1.00	100%	532.8	532.8	532.8	532.8

Tab. 5. Theoretical maximum sea levels and probability of their occurrence at Gdansk observation station in the period of 1886÷2006 years

T (years)	F(X)	Sea level [cm]			
		Gumbel's distribution		Person's distribution	
		Quantile method	Maximum likelihood method	Quantile method	Maximum likelihood method
1000	0.1%	716.9	716.7	695.5	695.3
200	0.5%	684.9	683.9	670.7	670.8
100	1%	671.1	669.8	661.4	660.2
50	2%	657.2	655.6	651.2	648.9
20	5%	638.7	636.6	636.0	633.2
10	10%	624.4	622.0	623.3	620.4
5	20%	609.5	606.7	609.6	606.4
2	50%	587.0	583.7	587.0	583.8
1.33	75%	573.2	569.5	572.4	569.3
1.11	90%	563.2	559.2	561.3	559.1
1.05	95%	557.9	550.6	556.0	557.5
1.01	99%	549.4	544.9	547.3	549.1
1.00	100%	532.3	532.3	532.3	532.3

Determination of theoretical probable maximum sea levels by means of the quantile method with the use of Pearson's distribution of type III

In hydrology Pearson's distribution is used most often. Its type III has found the widest application to equalization of series of numerical features of hydrological phenomena. The density function of Pearson's 3rd type distribution is of the following form [5, 6]:

$$f(x) = \frac{\alpha^{\lambda}}{\Gamma(\lambda)} e^{-\alpha(x-\varepsilon)} (x-\varepsilon)^{\lambda-1} \tag{14}$$

where:

$\alpha, \varepsilon, \lambda$ – distribution parameters which should satisfy the following conditions: $x \geq \varepsilon$ (lower limitation of the distribution), $\alpha > 0$, $\lambda > 0$

$\Gamma(\lambda)$ – gamma function of the variable λ.

In this work the simplified form of the function which determines Pearson's 3rd type probability distribution, was used [5, 6]:

$$X_{p\%} = X_{50\%} * [1 + \Phi(p, s) * C_v] \tag{15}$$

where:

$X_{50\%}$ – value of the middle quantile,

C_v – variability coefficient,

$\Phi(p,s)$ – function of exceedance probability and skewness (asymmetry).

The following are parameters of the distribution:

– the variability measure

$$V = (X_{10\%} - X_{90\%})/2$$

– the variability coefficient

$$C_v = V/X_{50\%} \tag{16}$$

– the skewness coefficient *s* determined from an auxiliary table depending on value of the expression:

$$C_p = (C_v * X_{50\%})/(X_{50\%} - X_{100\%}) \tag{17}$$

Like in the Gumbel's distribution, on the basis of the distribution series of maximum water levels for each of the observation stations the water levels were read or interpolated by using the empirical probability from Weibull's formula (2), for X_p and X_{1-p} at p = 10% (the quantiles: $X_{10\%}$, $X_{50\%}$, $X_{90\%}$, $X_{100\%}$). The obtained values of quantiles were used for determining the distribution parameters: V, C_v and *s* from Eqs. (12) and (16) as well as (17) and the auxiliary table, respectively [5]. The calculated distribution parameters and the function $\Phi(p,s)$ read from the above mentioned auxiliary table, were introduced to the successive formula, (15), for X_p, and the theoretical sea levels for the selected probability quantiles for each of the observation stations were calculated (Tabs 1, 2, 3, 4 and 5).

Determination of theoretical probable maximum sea levels by means of the maximum likelihood method with the use of Pearson's distribution of type III

In this method the form of Pearson's distribution function, proposed by Foster, was applied [6, 1]:

$$X_p = \varepsilon + 1/\alpha * tp \tag{18}$$

where:

ε – lower limitation of the distribution, equivalent of the quantile $X_{100\%}$

t_p – value of exceedance probability function and parameter λ, read from the auxiliary tables [6, 9]

α – the second parameter of the distribution, determined from the formula:

$$\alpha = \lambda/x_{sr} \tag{19}$$

where:

x_{sr} – arithmetic mean of the values x_i, where x_i - the substitute variable determined from the formula:

$$x_i = x\ max - \varepsilon \text{ for } i = 1, 2, 3, \ldots N \tag{20}$$

where:

x_i – maximum sea levels in distribution series as well as the parameter λ determined from the formula:

$$\lambda = 1/(4*A) * [1 + (1 + 4/3 * A)^{\wedge}0.5] \quad (21)$$

where:

A – function defined by the formula:

$$A = \ln(x_{sr}) - (\ln x)_{sr} \quad (22)$$

where:

$\ln x_{sr}$ – logarithm taken from the arithmetic mean of the values x_i

$(\ln x)_{sr}$ – arithmetic mean of the values lnx.

In the maximum likelihood method, on the basis of the distribution series of maximum water levels, for each of the observation stations the substitute variable x_i, acc. Eq. (20), and its arithmetic mean x_{sr}, as well as the logarithm of the mean, $\ln x_{sr}$, and the arithmetic mean of the logarithm values, (lnx)sr, then also the functions *A*, acc. Eq. (22), and the parameter ë, acc. Eq. (21), which was used to calculate the second parameter of Pearson's distribution, α, acc. Eq. (19), were determined. The lower limitation of the distribution, ĺ, was determined from the equation of the trend-line of maximum sea levels, depicted on the quantile - quantile diagram (by using *Statistica* software).

The functions t_p were read from the auxiliary tables [6, 9]. Having all the Pearson's distribution parameters one made use of the Foster's formula, acc. Eq. (18), and calculated theoretical sea levels for selected probability quantiles for each of the observation stations (Tab. 1, 2, 3, 4 and 5).

Test of consistency of the assumed theoretical probability distribution with empirical distribution

In this work consistency of the assumed theoretical distribution with empirical one (observation series of sea levels) was examined with the use of Kolmogorow consistency test. The testing consists in checking the following condition [1, 8]:

$$D_{max}\,[p/m,\ N/\% - p\%] < \lambda_{kr}/\sqrt{N} \quad (23)$$

where:

p/m, N/ % – empirical occurrence probability of *m*-th term of distributive series

p % – theoretical occurrence probability of sea level value corresponding with *m*-th term of distributive series

λ_{kr} – critical value of the Kolmogorow distribution (at α = 5% → λ_{kr} = 136, at α = 1% → λ_{kr} = 163)

N – size of distributive series.

The condition of the Kolmogorow test was checked for all the methods of determination of theoretical probable maximum sea levels for each of the observation stations. Results of the test are given in Tab. 6.

Calculation of estimation error and confidence interval

Confidence interval is a range within which real value of quantile will occur with the assumed probability P_α (confidence level). In this work the confidence interval limit x_p was determined on the level P_α = 68.3% as well as P_α = 95% by means of the following formula [1, 4]:

for the upper confidence interval:

$$x_p^\alpha = x_p + t_\alpha * \sigma(x_p) \quad (24)$$

for the lower confidence interval:

$$x_p^\alpha = x_p - t_\alpha * \sigma(x_p) \quad (25)$$

where:

$t_\alpha = 1$ – for the confidence level = 68.3%

$t_\alpha = 2$ – for the confidence level = 95%

$\sigma_{(xp)}$ – estimation error determined according to the following formula:

$$\sigma_{x_p} = F(s,p) * C_v * x_{50\%}/\sqrt{N} \quad (26)$$

where:

F(s,p) – function of probability and assymetry measure, read from the auxiliary tables [4]

The determined confidence interval limits for the confidence levels P_α = 68.3% and P_α = 95% are given in Tab. 7.

ESTIMATION RESULTS OF THEORETICAL MAXIMUM WATER LEVELS FOR PARTICULAR OBSERVATION STATIONS AND OCCURRENCE PROBABILITY OF THE LEVELS

In this work the sea levels with assumed occurrence probability were determined on the basis of Gumbel's distribution and Pearson's distribution by applying both the quantile and maximum likelihood methods.To check consistency of the assumed theoretical distributions with empirical ones the Kolmogorow test was used (Tab. 6). Results of the test calculations do not lead to rejection of the in-advance-assumed hypothesis on consistency of the distributions.

The best adjusted to long-term observation series are the theoretical maximum sea levels and their occurence probability determined by means of the Gumbel's distribution and maximum likelihood method, that is confirmed by description given in the chapter „Applied methods" and the diagrams presented in Fig. 1 through 5. The so-calculated highest theoretical sea

Tab. 6. *Results of Kolmogorow test for various methods of determination of theoretical sea levels at particular observation stations*

Observation stations	$\lambda_{kr}/\sqrt{N}$ for α = 1%	$\lambda_{kr}/\sqrt{N}$ for α = 5%	D_{max}			
			Gumbel's distr., quantile method	Gumbel's distr., method of maximum likelihood	Pearson's distr., quantile method	Pearson's distr., method of maximum likelihood
Swinoujscie (1901÷2006)	15.83	13.21	0.30	0.04	0.73	0.43
Kolobrzeg (1867÷2006)	13.88	11.58	-0.26	0.03	0.12	0.22
Ustka (1946÷2006)	20.87	17.41	-3.78	0.07	-2.89	-2.39
Wladyslawowo (1947÷2006)	21.04	17.56	-1.26	0.08	-0.16	-0.36
Gdansk (1886÷2006)	14.82	12.36	-1.38	0.06	-0.68	-0.28

Tab. 7. Limits of confidence intervals, [cm], of maximum yearly sea levels at Swinoujscie, Kolobrzeg, Ustka, Wladyslawowo and Gdansk for the confidence levels: Pá = 68.3% and Pá = 95.0%

P (%)		99.9-50	10	5	1	0.1	0.01
Swinoujscie	$P_\alpha = 68.3\%$	± 2.7	± 5.3	± 7.6	± 14.3	± 25.3	± 37.2
	$P_\alpha = 95.0\%$	± 5.4	± 10.6	± 15.3	± 28.6	± 50.6	± 74.3
Kolobrzeg	$P_\alpha = 68.3\%$	± 3.4	± 6.8	± 9.7	± 18.4	± 34.5	± 47.6
	$P_\alpha = 95.0\%$	± 6.7	± 13.7	± 19.3	± 36.9	± 69.0	± 95.2
Ustka	$P_\alpha = 68.3\%$	± 3.9	± 7.2	± 10.5	± 19.7	± 35.0	± 51.3
	$P_\alpha = 95.0\%$	± 7.7	± 14.5	± 20.9	± 39.5	± 70.0	± 102.7
Wladyslawowo	$P_\alpha = 68.3\%$	± 3.4	± 6.4	± 9.2	± 17.4	± 30.8	± 45.2
	$P_\alpha = 95.0\%$	± 6.8	± 12.8	± 18.4	± 34.7	± 61.6	± 90.3
Gdansk	$P_\alpha = 68.3\%$	± 2.8	± 5.2	± 7.4	± 14.0	± 24.9	±36.5
	$P_\alpha = 95.0\%$	± 5.5	± 10.3	± 14.9	± 28.1	± 49.8	± 73.0

levels which can occur once a 1000 years, 200 years, 100 years and 50 years are given in detail in Tab. 1 through 5. From the results it can be concluded that the highest of them will occur and did really occur in Polish west - coast ports, i.e. Kolobrzeg and Swinoujscie. The lowest of them will occur in the port of Wladyslawowo which belongs to the Polish middle coast of Baltic Sea and is directly exposed to sea. The remaining ports are located in river estuaries and water level indicators installed in them are about 1 km distant from sea coast line. For instance in the port of Kolobrzeg the water level of 750.2 cm, i.e. 2.5 m above the average mean sea level, should be expected once a 1000 years (zero points of Polish water level indicators are based on the Amsterdam's zero-level equal to -500 N.N.). Swinoujscie (with 723.6 cm), Ustka (with 720.2 cm),

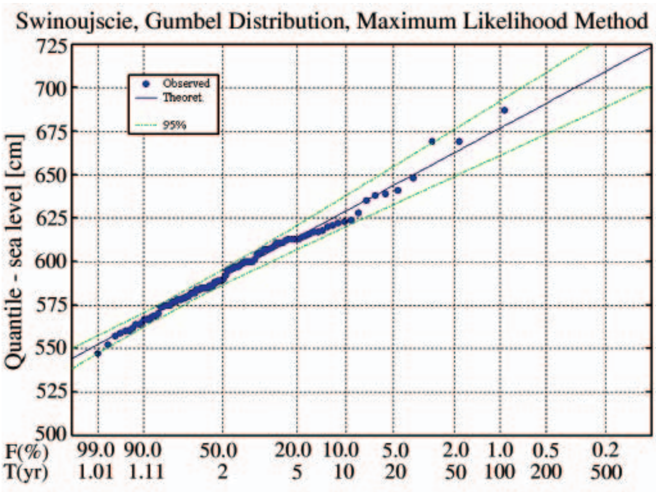

Fig. 1. Occurrence probability of maximum yearly sea levels at Swinoujscie in the period of 1901÷2006 years (Gumbel's distribution, maximum likelihood method)

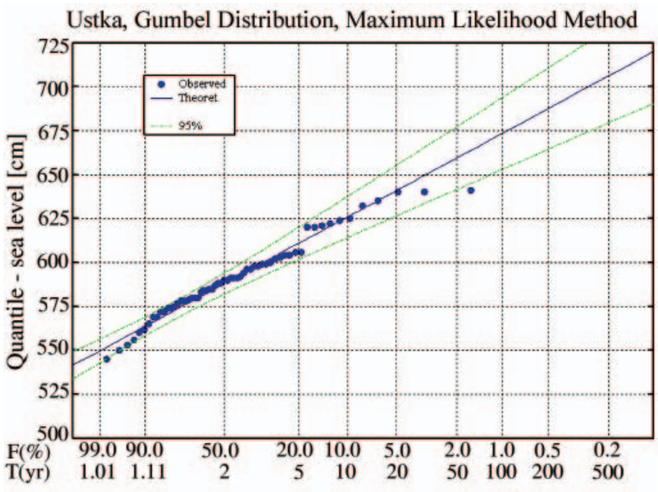

Fig. 3. Occurrence probability of maximum yearly sea levels at Ustka in the period of 1946÷2006 years (Gumbel's distribution, maximum likelihood method)

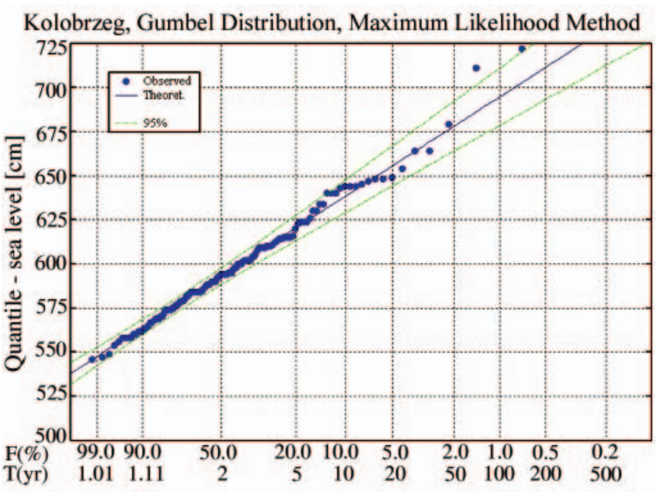

Fig. 2. Occurrence probability of maximum yearly sea levels at Kolobrzeg in the period of 1867÷2006 years (Gumbel's distribution, maximum likelihood method)

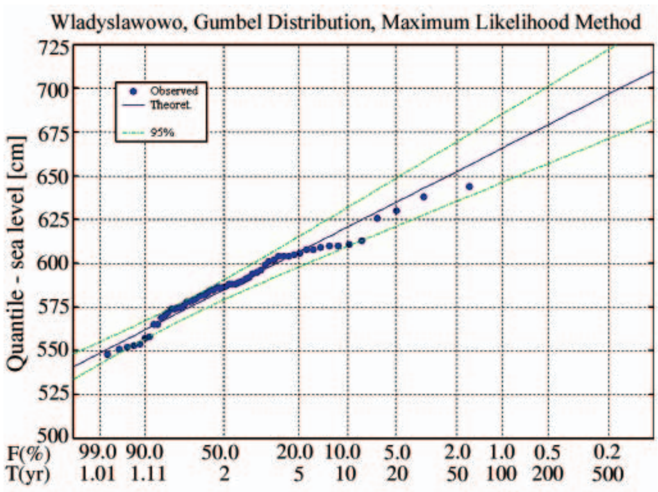

Fig. 4. Occurrence probability of maximum yearly sea levels at Wladyslawowo in the period of 1947÷2006 years (Gumbel's distribution, maximum likelihood method)

Gdansk (with 716.7 cm) and Wladyslawowo (with 709.7 cm) are the successive ports in which the maximum thousand-year levels can be expected. Other occurrence probabilities can be also considered. Graphical representation of the relations is presented in Fig. 1 through 5 for particular Polish ports of Baltic Sea coast.

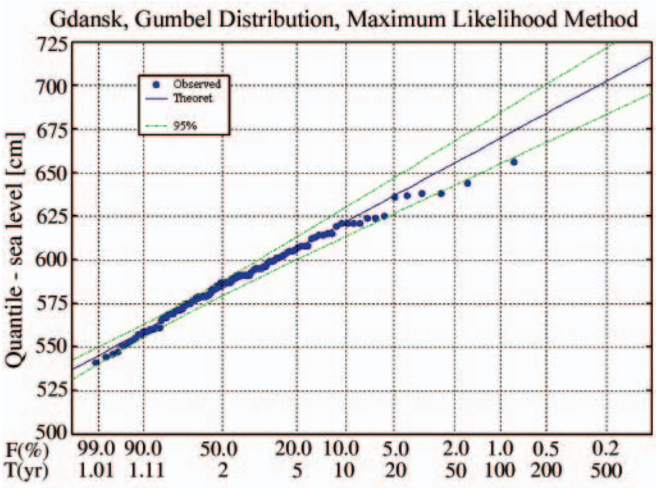

Fig. 5. Occurrence probability of maximum yearly sea levels at Gdansk in the period of 1886÷2006 years (Gumbel's distribution, maximum likelihood method)

DISCUSSION OF THE RESULTS

As already mentioned, due to its characteristics Gumbel's distribution has been considered the closest to empirical one. Its advantage results from lack of lower limitation of variability range of the random variable X, among estimated parameters. Estimation of lower or upper limitation of Pearson's distribution ($x_{100\%}$, or $x_{0\%}$) faces serious difficulty due to a subjective way of its determination [1], [8].

On the basis of comparison of both methods of estimation and determination of distribution parameters it can be concluded that the maximum likelihood method makes it possible to obtain more precise results as in this method - in contrast to the quantile method – to use interpolation of distribution parameters is not necessary.

In Polish subject-matter literature the methods of determination of probability of extreme sea levels were described in the publications of Wroblewski, A. [10, 11, 12, 13, 14] and Massel, S. [7]. Prediction of Baltic Sea extreme levels was also considered by Jednoral, T., Sztobryn, M. and Milkowska, M. [3]. This work presents the methods in question in the most complete way as it takes into account calculation approaches used by different authors. In this work, the characteristics of theoretical maximum sea levels and their occurrence probability in five reperesentative Polish ports of Baltic Sea coast, were considered. The obtained results are based on the longest observation series. From the point of view of the applied methods the most important have been so far two publications by Wroblewski [11, 13] with which to compare results of this work is deemed purposeful. In Tab. 8 can be found the comparison of results of the theoretical sea levels, determined by Wroblewski for selected quantiles, with those obtained by these authors for Gdansk observation station.

From the comparison of the results of the theoretical sea levels for Gdansk observation station, obtained by these authors by using the maximum likelihood method and Gumbel's and Pearson's distributions, with those determined by Wroblewski [11, 13], the difference from several up to a dozen or so centimeters for different quantiles, can be observed. The differences are mainly caused by a different way of determination of the lower limitation of the distribution, l. Wroblewski, determining the lower limitation of the distribution, made use of the maximum likelihood method as well as the tables of modal values. In this work the equation of the trend-line of maximum sea levels, depicted on the quantile-quantile diagram obtained from *Statistica* software, was applied. The other cause of the differences in determining the theoretical sea levels is length of observation series. In this work the series for Swinoujscie, Kolobrzeg and Gdansk are by 37 years longer than that used by Wroblewski [11, 13]

Tab. 8. Theoretical maximum sea levels and probability of their occurrence at Gdansk observation station, determined by means of both Gumbel's and Pearson's distributions with the use of the maximum likelihood method, according to Wroblewski [11, 13] and these authors, respectively

T (years)	F(X) (%)	Sea levels [cm]			
		Gumbel distr.	Pearson distr.	Gumbel distr.	Pearson distr.
		Results by these authors		Results by Wroblewski	
1000	0.1%	716.7	695.3	694	678
200	0.5%	683.9	670.8	664	-
100	1%	669.8	660.2	651	647
50	2%	655.6	648.9	639	637
20	5%	636.6	633.2	621	623
10	10%	622.0	620.4	608	612
5	20%	606.7	606.4	594	599
2	50%	583.7	583.8	573	578
1.33	75%	569.5	569.3	-	-
1.11	90%	559.2	559.1	551	555
1.05	95%	550.6	557.5	-	549
1.01	99%	544.9	549.1	538	541
1.00	100%	532.3	532.3	-	523

CONCLUSIONS

- In this work sea levels for a given occurrence probability, based both on Gumbel's and Pearson's distributions, were obtained with the use of both quantile method and maximum likewlihood method. The achieved results have been presented in Tab. 1 through 5 and partially in Fig. 1 through 5.
- For the determined theoretical sea levels the confidence intervals at the confidence levels: Pá = 68,3% i Pá = 95,0%, were calculated (Tab. 7).
- Consistency of the assumed theoretical distributions and empirical one was checked by means of the Kolmogorow test (Tab. 6). Results of the test calcultations do not lead to rejection of the assumed hypothesis on consistency of the distributions. In this work the Gumbel's distribution was considered the closest to the empirical one. Its advantage is lack of lower limitation of variability range of the random variable X, among estimated parameters.
- Among the methods applied in this work the maximum likelihood method has appeared the most accurate because of lack of necessity of interpolating the distribution parameters (in contrast to the case of using the quantile method).

– The obtained results can be considered reliable because of the long observation series of sea levels, especially those for Gdansk (1887 ÷ 2006), Swinoujscie (1901 ÷ 2006) and Kolobrzeg (1867 ÷ 2006). Large (upper) values of the observed sea level series are due to storm swellings and their impact on sea coast. The occurrence probability of high sea levels determined in this work for Swinoujscie, Kolobrzeg, Ustka, Wladyslawowo and Gdansk can be used in designing the coastal hydro-engineering buildings as well as in managing the costal zone and inudation areas during storm and flood phenomena.

BIBLIOGRAPHY

1. Byczkowski A.: *Hydrology* (in Polish), Vol. I and II, Publishing House of Warsaw Agriculture University (Wydawnictwo SGGW), Warszawa 1996
2. Gumbel, E.J.: *Statistics of Extremes*. Columbia University Press 1958
3. Jednoral T., Sztobryn M., Milkowska M.: *Application of position statistics for prediction of extreme levels of Baltic Sea in Polish coastal zone* (in Polish), Inżynieria Morska i Geotechnika, no.5/2008
4. Kaczmarek Z.: *Effectiveness of estimation of maximum flow rates of a given return period* (in Polish), Archiwum Hydrotechniki, vol. 4, no. 3/1957
5. Kaczmarek Z., Trykozko E.: *Application of the method of quantiles to estimation of the Pearson distribution.* Acta Geophysica Polonica, vol. 12, no. 1/1964
6. Kaczmarek Z.: *Meteorolgical and statistical methods in hydrology and meteorology* (in Polish). Transport and Telecommunication Publishers (Wyd. Komunikacji i Łącznosci), Warszawa 1970
7. Massel S. (ed.): *Handbook on hydro-engineering. Marine environmental loads on hydro-engineering buildings* (in Polish). Maritime Publishing House (Wyd. Morskie), Gdansk 1992
8. Ozga-Zielinska M., Brzezinski J.: *Applied hydrology* (in Polish). Scientific Publishing House (Wyd. PWN), Warszawa 1997
9. Weglarczyk S.: *Statistical methods* (in Polish). Printed series of course lectures for students of Krakow University of Technology, Krakow 1993
10. Wroblewski A.: *Occurrence of minimum yearly water states in South Baltic Sea (Based on observation results in Nowy Port of Gdansk in 1886÷1939 and 1946÷1966)* (in Polish), Przegląd Geofizyczny, vol. XV, no. 1/1970
11. Wroblewski A.: *Occurrence probability of maximum yearly levels of Baltic Sea in Gdansk Nowy Port, Kolobrzeg and Swinoujscie* (in Polish). Oceanology, no. 6/1975
12. Wroblewski A.: *Occurrence probability of minimum yearly levels of Baltic Sea in Gdansk Nowy Port, Kolobrzeg and Swinoujscie* (in Polish). Archiwum Hydrotechniki, vol. 29, no. 4/1982
13. Wroblewski A.: *Analysis and forecast of long-term sea level changes along the Polish Baltic Sea coast*, Part I. Annual sea level maxima, Oceanologia, 33, 1992
14. Wroblewski A.: *Long-term prediction of sea level rise* (in Polish). Inżynieria Morska i Geotechnika, no.3/2001

CONTACT WITH THE AUTHORS

Bernard Wisniewski, Prof.
Tomasz Wolski, Ph. D.
University of Szczecin
Wąska 13
71-415 Szczecin, POLAND
e-mail: natal@univ.szczecin.pl