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On the behaviour of viscoelastic solids under multiaxial loads

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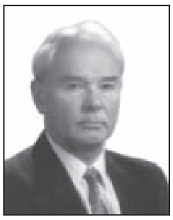

ABSTRACT

On the basis of modified Hooke's law for multiaxial stress in viscoelastic solids, three-dimensional constitutive equations for strains have been derived. It is shown that after application or removal of triaxial static load, normal and shear strain components vary in course of time proportionally to each other and that in-phase stress components produce in-phase strain components. Harmonic out-of-phase stress as well as multiaxial periodic and stationary random stresses are also considered. The matrix of dynamical flexibility of viscoelastic materials is determined which depends on three material constants (Young modulus, Poisson's ratio and coefficient of viscous damping of normal strain) and load circular frequency.

Keywords: viscoelastic material, multiaxial stress, constitutive equations, static load, vibratory load

INTRODUCTION

Deformation of solids is accompanied by internal friction [1]. Consequently, even in the region below the proportionality limit metals are not perfectly elastic [2]. If tensile strain has occurred in an anelastic rod, the complete removal of the load will be followed by a gradual decrease in length. When the length does decrease with time after unloading, the shortening is called "creep recovery". The anelastic strain, which occurs during long-time creep, is basically the same as that which occurs during vibratory load, and in both cases the anelastic behaviour can be expressed in the same terms [2].

There are various models of anelastic materials and damping mechanisms in use [1-3]. When the Kelvin-Voigt's model (spring and dashpot in parallel) of viscoelastic material is applied to the rod, its behaviour is easy to deduce. In particular, it is obvious that:

- creep recovery at a given time, following partial or complete unloading from a given prior load of a given duration, is proportional to the stress decrement
- after creep recovery, deformation should cease and the dimensions should remain constant.

The governing differential equation that relates stress σ and strain ε , the creep compliance I(t) and the relaxation modulus G(t) read [3]:

$$\sigma = q_0\varepsilon + q_1\dot{\varepsilon} \tag{1}$$

$$I(t) = \frac{1}{q_0}\left(1 - e^{-\frac{q_0}{q_1}t}\right) \tag{2}$$

$$G(t) = q_0 + q_1\delta(t) \tag{3}$$

where:

q_0, q_1 – aterial constants in the Kelvin-Voigt's model
I(t) – esponse of strain due to a unit step input of stress
G(t) – response of stress due to a unit step input of strain
δ(t) – Kronecker delta.

As the relaxation modulus is known, the stress response can be determined under any strain loading condition through a convolution integral. These equations are:

$$\sigma(t) = \varepsilon(0)G(t) + \int_0^t G(t-\tau)\frac{d\varepsilon(\tau)}{d\tau}d\tau \tag{4}$$

or

$$\sigma(t) = \varepsilon(t)G(0) + \int_0^t \varepsilon(\tau)\frac{dG(t-\tau)}{d\tau}d\tau \tag{5}$$

with a similar equation for the strain response due to arbitrary stress input:

$$\varepsilon(t) = \sigma(0)I(t) + \int_0^t I(t-\tau)\frac{d\sigma(\tau)}{d\tau}d\tau \tag{6}$$

or

$$\varepsilon(t) = \sigma(t)I(0) + \int_0^t \sigma(\tau)\frac{dI(t-\tau)}{d\tau}d\tau \tag{7}$$

The one-dimensional constitutive equations can be extended to three-dimensional constitutive equations. A general and most common form of these are [3]:

$$\sigma_{ij}(t) = G_{ijkl}(t)\varepsilon_{kl}(0) + \int_0^t G_{ijkl}(t-\tau)\frac{d\varepsilon_{kl}(\tau)}{d\tau}d\tau \quad (8)$$

$$\sigma_{ij}(t) = G_{ijkl}(0)\varepsilon_{kl}(t) + \int_0^t \varepsilon_{kl}(\tau)\frac{dG_{ijkl}(t-\tau)}{d\tau}d\tau \quad (9)$$

The stress σ_{ij} and the strain ε_{kl} are second-order tensor quantities, and G_{ijkl} are the four-order relaxation modulus. A set of similar constitutive equations for strains, when the stress history is known, can be obtained.

The aim of the present paper is to derive three-dimensional constitutive equations for strains by means of the modified Hooke's law for multiaxial stress in viscoelastic solids [4] because its simplicity in the case of homogeneous isotropic materials may be advantageous.

STRAIN RESPONSE OF VISCOELASTIC SOLIDS TO REMOVAL AND APPLICATION OF MULTIAXIAL STATIC LOAD

After a long period of static load resulting in normal and shear components σ_{j0} and τ_{k0} (j = x, y, z; k = xy, yz, zx), the normal and shear strain components ε_{j0} and γ_{k0} in a viscoelastic material can be calculated as those in elastic materials [3,5]:

$$\begin{aligned}
\varepsilon_{x0} &= \frac{1}{E}\left[\sigma_{x0} - \upsilon(\sigma_{y0} + \sigma_{z0})\right] \\
\varepsilon_{y0} &= \frac{1}{E}\left[\sigma_{y0} - \upsilon(\sigma_{x0} + \sigma_{z0})\right] \\
\varepsilon_{z0} &= \frac{1}{E}\left[\sigma_{z0} - \upsilon(\sigma_{x0} + \sigma_{y0})\right] \\
\gamma_{k0} &= \frac{1}{G}\tau_{k0}
\end{aligned} \quad (10)$$

where E is the Young modulus, υ is the Poisson's ratio and

$$G = \frac{E}{2(1+\upsilon)} \quad (11)$$

is the shear modulus. If at t = 0 the load is removed, in accordance with the modified Hooke's law [4], the following equations can be applied:

$$\begin{aligned}
E\varepsilon_j + \eta\dot{\varepsilon}_j &= 0 \\
G\gamma_k + \lambda\dot{\gamma}_k &= 0
\end{aligned} \quad (12)$$

and solved with the initial conditions:

$$\varepsilon_j(0) = \varepsilon_{j0} \quad , \quad \gamma_k(0) = \gamma_{k0} \quad (13)$$

In Eqs (12), η is the coefficient of viscous damping of normal strain and [4]:

$$\lambda = \frac{\eta}{2(1+\upsilon)} \quad (14)$$

is the coefficient of viscous damping of shear strain.

The solutions of Eqs (12) have the form:

$$\varepsilon_j(t) = A_j e^{-rt} \quad , \quad \gamma_k(t) = B_k e^{-st} \quad (15)$$

where A_j, B_k , r and s are constants. Substitution of Eqs (15) into Eqs (12) yields:

$$E - r\eta = 0 \quad , \quad G - s\lambda = 0 \quad (16)$$

so that:

$$r = \frac{E}{\eta}, \quad s = \frac{G}{\lambda} = \frac{E}{\eta} = r \quad (17)$$

From Eqs (13) and (15) one obtains:

$$A_j = \varepsilon_{j0} \ , \quad B_k = \gamma_{k0} \quad (18)$$

Hence:

$$\begin{aligned}
\varepsilon_j(t) &= \varepsilon_{j0} e^{-\frac{E}{\eta}t} \\
\gamma_k(t) &= \gamma_{k0} e^{-\frac{E}{\eta}t}
\end{aligned} \quad (19)$$

It is also easy to prove that sudden application of a multiaxial static load producing stress components σ_{j0} and τ_{k0} to viscoelastic solids evokes their dimensional changes and distortions described by equations:

$$\begin{aligned}
\varepsilon_j(t) &= \varepsilon_{j0}\left(1 - e^{-\frac{E}{\eta}t}\right) \\
\gamma_k(t) &= \gamma_{k0}\left(1 - e^{-\frac{E}{\eta}t}\right)
\end{aligned} \quad (20)$$

It is noteworthy that in conformity with Eqs (19) and (20), after removal or application of multiaxial static loads, the strain components in viscoelastic solids vary in course of time proportionally to each other.

Eqs (19) and (20) apply to creep recovery following a long period at constant stress, or to creep at a particular stress level following a long period of zero stress. These, of course, are very special cases, but with the modified Hooke's law we are able to handle effectively also other load patterns. Some of them are considered below.

THE CASE OF HARMONIC IN-PHASE STRESS

The relations between stress and strain in viscoelastic materials subjected below the yield point to time-dependent loads are governed by the modified Hooke's law [4]:

$$\begin{aligned}
E\varepsilon_x + \eta\dot{\varepsilon}_x &= \sigma_x - \upsilon(\sigma_y + \sigma_z) \\
E\varepsilon_y + \eta\dot{\varepsilon}_y &= \sigma_y - \upsilon(\sigma_x + \sigma_z) \\
E\varepsilon_z + \eta\dot{\varepsilon}_z &= \sigma_z - \upsilon(\sigma_x + \sigma_y) \\
G\gamma_k + \lambda\dot{\gamma}_k &= \tau_k \quad ; \quad k = xy, yz, zx
\end{aligned} \quad (21)$$

Under in-phase stress components:

$$\begin{aligned}
\sigma_j &= \sigma_{ja}\sin\omega t; \quad j = x, y, z \\
\tau_k &= \tau_{ka}\sin\omega t
\end{aligned} \quad (22)$$

the strain components take the form:

$$\begin{aligned}
\varepsilon_j &= \varepsilon_{j1}\sin\omega t + \varepsilon_{j2}\cos\omega t \\
\gamma_k &= \gamma_{k1}\sin\omega t + \gamma_{k2}\cos\omega t
\end{aligned} \quad (23)$$

In Eqs (22), σ_{ja} and τ_{ka} are the amplitudes of stress components and ω is their circular frequency. With Eqs (22) and (23), Eqs (21) become:

$$\begin{aligned}
&E(\varepsilon_{x1}\sin\omega t+\varepsilon_{x2}\cos\omega t)+\eta\omega(\varepsilon_{x1}\cos\omega t-\varepsilon_{x2}\sin\omega t)=[\sigma_{xa}-\upsilon(\sigma_{ya}+\sigma_{za})]\sin\omega t\\
&E(\varepsilon_{y1}\sin\omega t+\varepsilon_{y2}\cos\omega t)+\eta\omega(\varepsilon_{y1}\cos\omega t-\varepsilon_{y2}\sin\omega t)=[\sigma_{ya}-\upsilon(\sigma_{xa}+\sigma_{za})]\sin\omega t\\
&E(\varepsilon_{z1}\sin\omega t+\varepsilon_{z2}\cos\omega t)+\eta\omega(\varepsilon_{z1}\cos\omega t-\varepsilon_{z2}\sin\omega t)=[\sigma_{za}-\upsilon(\sigma_{xa}+\sigma_{ya})]\sin\omega t\\
&G(\gamma_{k1}\sin\omega t+\gamma_{k2}\cos\omega t)+\lambda\omega(\gamma_{k1}\cos\omega t-\gamma_{k2}\sin\omega t)=\tau_{ka}\sin\omega t
\end{aligned}\tag{24}$$

Eqs (24) are satisfied if:

$$\begin{gathered}
E\varepsilon_{x1}-\eta\omega\varepsilon_{x2}=\sigma_{xa}-\upsilon(\sigma_{ya}+\sigma_{za})\\
E\varepsilon_{y1}-\eta\omega\varepsilon_{y2}=\sigma_{ya}-\upsilon(\sigma_{xa}+\sigma_{za})\\
E\varepsilon_{z1}-\eta\omega\varepsilon_{z2}=\sigma_{za}-\upsilon(\sigma_{xa}+\sigma_{ya})\\
\eta\omega\varepsilon_{j1}+E\varepsilon_{j2}=0\ ,\ G\gamma_{k1}-\lambda\omega\gamma_{k2}=\tau_{ka}\ ,\ \lambda\omega\gamma_{k1}+G\gamma_{k2}=0
\end{gathered}\tag{25}$$

Eqs (23) and (25) lead to the constitutive equations for strains as follows:

$$\begin{aligned}
\varepsilon_x&=\frac{\sigma_{xa}-\upsilon(\sigma_{ya}+\sigma_{za})}{\sqrt{E^2+(\eta\omega)^2}}\sin(\omega t-\alpha),\quad \alpha=\operatorname{arctg}\frac{\eta\omega}{E}\\
\varepsilon_y&=\frac{\sigma_{ya}-\upsilon(\sigma_{xa}+\sigma_{za})}{\sqrt{E^2+(\eta\omega)^2}}\sin(\omega t-\alpha)\\
\varepsilon_z&=\frac{\sigma_{za}-\upsilon(\sigma_{xa}+\sigma_{ya})}{\sqrt{E^2+(\eta\omega)^2}}\sin(\omega t-\alpha)
\end{aligned}\tag{26}$$

and

$$\gamma_k=\frac{\tau_{ka}}{\sqrt{G^2+(\lambda\omega)^2}}\sin(\omega t-\beta),\quad \beta=\operatorname{arctg}\frac{\lambda\omega}{G}\tag{27}$$

Through Eqs (11) and (14), Eqs (27) become:

$$\gamma_k=\frac{2(1+\upsilon)\tau_{ka}}{\sqrt{E^2+(\eta\omega)^2}}\sin(\omega t-\alpha),\quad \beta=\alpha\tag{28}$$

It means that in-phase stress components produce in homogeneous, isotropic viscoelastic materials in-phase strain components.
Introducing the strain vector:

$$\boldsymbol{\varepsilon}=[\varepsilon_x\varepsilon_y\varepsilon_z\gamma_{xy}\gamma_{yz}\gamma_{zx}]^T\tag{29}$$

and the vector of amplitudes of the stress components:

$$\boldsymbol{\sigma}_a=[\sigma_{xa}\sigma_{ya}\sigma_{za}\tau_{xya}\tau_{yza}\tau_{zxa}]^T\tag{30}$$

Eqs (26) and (28) can be rewritten in a matrix form:

$$\boldsymbol{\varepsilon}=\mathbf{H}\boldsymbol{\sigma}_a\sin(\omega t-\alpha)\tag{31}$$

where:

$$H=H(\omega)=\frac{1}{\sqrt{E^2+(\eta\omega)^2}}\begin{bmatrix}1&-\upsilon&-\upsilon&0&0&0\\-\upsilon&1&-\upsilon&0&0&0\\-\upsilon&-\upsilon&1&0&0&0\\0&0&0&2(1+\upsilon)&0&0\\0&0&0&0&2(1+\upsilon)&0\\0&0&0&0&0&2(1+\upsilon)\end{bmatrix}\tag{32}$$

is the matrix of dynamical flexibility of the viscoelastic material at the load circular frequency ω.

THE CASE OF HARMONIC OUT-OF-PHASE STRESS

When the stress components are given by:

$$\begin{aligned}\sigma_j &= \sigma_{ja} \sin(\omega t + \varphi_j)\\ \tau_k &= \tau_{ka} \sin(\omega t + \varphi_k)\end{aligned} \tag{33}$$

where φ_j and φ_k are the phase angles, it is convenient to introduce the complex stress components:

$$\begin{aligned}\overline{\sigma}_j &= \sigma_{ja} e^{i(\omega t+\varphi_j)} = \overline{\sigma}_{ja} e^{i\omega t}\\ \overline{\tau}_k &= \tau_{ka} e^{i(\omega t+\varphi_k)} = \overline{\tau}_{ka} e^{i\omega t}\end{aligned} \tag{34}$$

Here i is the imaginary unity and

$$\overline{\sigma}_{ja} = \sigma_{ja} e^{i\varphi_j} \quad , \quad \overline{\tau}_{ka} = \tau_{ka} e^{i\varphi_k} \tag{35}$$

are the complex amplitudes of the stress components. Then the real stress components are represented by the imaginary parts of the complex stress components:

$$\sigma_j = \mathrm{Im}\overline{\sigma}_j \quad , \quad \tau_k = \mathrm{Im}\overline{\tau}_k \tag{36}$$

Consequently, in view of Eq. (31) the strain response of the viscoelastic material to the stress (33) can be calculated as:

$$\boldsymbol{\varepsilon} = \mathbf{H}\ \mathrm{Im}\left[\overline{\sigma}_a e^{i(\omega t-\alpha)}\right] \tag{37}$$

where:

$$\overline{\sigma}_a = \left[\overline{\sigma}_{xa}\overline{\sigma}_{ya}\overline{\sigma}_{za}\overline{\tau}_{xya}\overline{\tau}_{yza}\overline{\tau}_{zxa}\right]^T \tag{38}$$

is the vector of complex amplitudes of the stress components.

STRAIN RESPONSE OF VISCOELASTIC SOLIDS TO MULTIAXIAL PERIODIC AND STATIONARY RANDOM LOADS

The solution (37) can be utilized for determination of behaviour of viscoelastic materials under multiaxial periodic loads. The resulting stress components can be expanded in Fourier series:

$$\begin{aligned}\sigma_j &= \sigma_{j0} + \sum_n \sigma_{jn} \sin(n\omega t + \varphi_{jn})\\ \tau_k &= \tau_{k0} + \sum_n \tau_{kn} \sin(n\omega t + \varphi_{kn})\end{aligned} \tag{39}$$

where:

σ_{j0} , τ_{k0} – mean stress components
σ_{jn} , φ_{jn} – amplitude and phase angle of n-th term in Fourier expansion of j-th stress component
τ_{kn} , φ_{kn} – amplitude and phase angle of n-th term in Fourier expansion of k-th stress component
ω – fundamental circular frequency.

Since Eqs (21) are linear, the principle of superposition can be applied. For this purpose we define:

- the vector of mean stress components:

$$\sigma_0 = \left[\sigma_{x0}\sigma_{y0}\sigma_{z0}\tau_{xy0}\tau_{yz0}\tau_{zx0}\right]^T \tag{40}$$

- the vector of mean strain components:

$$\boldsymbol{\varepsilon}_0 = \left[\varepsilon_{x0}\varepsilon_{y0}\varepsilon_{z0}\gamma_{xy0}\gamma_{yz0}\gamma_{zx0}\right]^T \tag{41}$$

- the vector of complex amplitudes of n-th terms of the stress components:

$$\overline{\sigma}_n = \left[\overline{\sigma}_{xn}\overline{\sigma}_{yn}\overline{\sigma}_{zn}\overline{\tau}_{xyn}\overline{\tau}_{yzn}\overline{\tau}_{zxn}\right]^T \tag{42}$$

with

$$\overline{\sigma}_{jn} = \sigma_{jn} e^{i\varphi_{jn}} \quad , \quad \overline{\tau}_{kn} = \tau_{kn} e^{i\varphi_{kn}} \tag{43}$$

- the matrix of dynamical flexibility of the viscoelastic material at the load circular frequency nω:

$$H_n = H(n\omega) \tag{44}$$

- the phase angle of n-th terms of the strain components:

$$\alpha_n = \mathrm{arctg}\frac{\eta n\omega}{E} \tag{45}$$

Under assumption that the material remains viscoelastic, its strain response to the stress (39) is described by the following equation:

$$\varepsilon = \boldsymbol{\varepsilon}_0 + \sum_n H_n \mathrm{Im}\left[\overline{\sigma}_n e^{i(n\omega t-\alpha_n)}\right] \tag{46}$$

where the elements of the vector $\boldsymbol{\varepsilon}_0$ are given in Eqs (10).

As the stress is increased above the yield point, the linear behaviour of viscoelastic material expressed by the modified Hooke's law (21) is terminated by the onset of plastic flow. In the case of uniaxial static tension, the part will yield if the uniaxial stress equals the yield strength of the material. For biaxial or triaxial static stress, various theories of failure by yielding have been developed, for example the distortion-energy strength theory [3, 5]. This theory is an important one because it comes closest of all to verifying experimental results [3]. Therefore in [6] an attempt was made to extend its use also to the case of multiaxial periodic stress and to model the stress components (39) by the reduced uniaxial stress:

$$\sigma_e(t) = \sigma_{e0} + \sigma_{ea} \sin\omega_e t \tag{47}$$

The mean value σ_{e0} amplitude σ_{ea} and circular frequency ω_e of the reduced stress are determined in [6]. Such an approach suggests that under multiaxial periodic stress the yield strength is not exceeded at a given point if the following condition is met:

$$\sigma_{e0} + \sigma_{ea} < R_e \tag{48}$$

where R_e is the tensile yield strength of the material.

Similar condition of avoiding plastic flow in viscoelastic solids can be postulated in the cases of multiaxial random loads if a uniaxial reduced random stress [6, 7] is taken under consideration.

As far as the strain response of viscoelastic solids to random loads is concerned, we shall confine ourselves to the solution in frequency domain [8, 9]. When the stress components represent zero mean stochastic processes that are stationary and stationary correlated with each other, and when their power spectral densities are given, the power spectral densities of the strain components can be calculated from the following equation:

$$S_\varepsilon = HS_\sigma H \tag{49}$$

where **H** is the matrix (32) of dynamical flexibility of the material, and:

$$S_\varepsilon = \begin{bmatrix} S_{\varepsilon_x\varepsilon_x} & S_{\varepsilon_x\varepsilon_y} & S_{\varepsilon_x\varepsilon_z} & S_{\varepsilon_x\gamma_{xy}} & S_{\varepsilon_x\gamma_{yz}} & S_{\varepsilon_x\gamma_{zx}}\\ S_{\varepsilon_y\varepsilon_x} & S_{\varepsilon_y\varepsilon_y} & S_{\varepsilon_y\varepsilon_z} & S_{\varepsilon_y\gamma_{xy}} & S_{\varepsilon_y\gamma_{yz}} & S_{\varepsilon_y\gamma_{zx}}\\ \hdashline \\ S_{\gamma_{zx}\varepsilon_x} & S_{\gamma_{zx}\varepsilon_y} & S_{\gamma_{zx}\varepsilon_z} & S_{\gamma_{zx}\gamma_{xy}} & S_{\gamma_{zx}\gamma_{yz}} & S_{\gamma_{zx}\gamma_{zx}} \end{bmatrix} \tag{50}$$

$$S_\sigma = \begin{bmatrix} S_{\sigma_x\sigma_x} & S_{\sigma_x\sigma_y} & S_{\sigma_x\sigma_z} & S_{\sigma_x\tau_{xy}} & S_{\sigma_x\tau_{yz}} & S_{\sigma_x\tau_{zx}} \\ S_{\sigma_y\sigma_x} & S_{\sigma_y\sigma_y} & S_{\sigma_y\sigma_z} & S_{\sigma_y\tau_{xy}} & S_{\sigma_y\tau_{yz}} & S_{\sigma_y\tau_{zx}} \\ \hdashline \\ S_{\tau_{zx}\sigma_x} & S_{\tau_{zx}\sigma_y} & S_{\tau_{zx}\sigma_z} & S_{\tau_{zx}\tau_{xy}} & S_{\tau_{zx}\tau_{yz}} & S_{\tau_{zx}\tau_{zx}} \end{bmatrix} \tag{51}$$

where:

$S_{\varepsilon x\varepsilon x}$, $S_{\varepsilon y\varepsilon y}$, S_{yzxyzx} – power spectral densities of the strain components ε_x , ε_y , ... , y_{zx}

$S_{\varepsilon x\varepsilon y}$, $S_{\varepsilon x\varepsilon z}$, S_{yzxyyz} – cross power spectral densities of the strain components ε_x and ε_y , ε_x and ε_z , ... , y_{zx} and y_{zx}

$S_{\sigma x\sigma x}$, $S_{\sigma y\sigma y}$, $S_{\tau zx\tau zx}$ – power spectral densities of the stress components σ_x , σ_y , ... , τ_{zx}

$S_{\sigma x\sigma y}$, $S_{\sigma x\sigma z}$, $S_{\tau zx\tau yz}$ – cross power spectral densities of the stress components σ_x and σ_y , σ_x and σ_z , ... , τ_{zx} and τ_{yz}.

Within the static, purely mechanical theory of continua there are two numbers which may be associated with the deformed state of a structure: its mass and its stored energy [10]. This association provides the criteria for the comparison of various designs. Apart from these numbers, in dynamic problems the load frequency and in fatigue design the stress range and number of cycles also play an important role. In this context it is clear that the physical models of structural materials should incorporate their mass density. As a result of application of the two-parameter Kelvin-Voigt's model, the relationships derived in the foregoing ignore inertia forces which in many cases may not be negligible in comparison with external loads and internal forces due to the viscoelastic properties of the material. However, the problems dealt with in the present paper have been aimed at gaining additional information on the influence of dissipative properties on the behaviour of structural materials and mathematical solutions within the assumed simpler model. Three-parameter models and more comprehensive stress-strain relations in viscoelastic materials are discussed, e.g., in [1, 3, 11, 12].

As to the behaviour of engineering details under vibratory loads with inertia forces taken into account, this problem has been widely addressed in the literature on vibration of continuous systems by exact mathematical treatment and numerical methods (see, e.g., [1, 13-15]) and will not be considered here.

CONCLUSIONS

❍ Three-dimensional constitutive equations for strains in homogeneous, isotropic viscoelastic solids have been derived by means of the modified Hooke's law.

❍ Owing to the fact that for homogeneous, isotropic viscoelastic materials the ratio of moduli E and G is equal to the ratio of damping coefficients η and λ , after sudden change of multiaxial static loads the strain components vary in time proportionally to each other because the time function of normal and shear creep is the same.

❍ If the stress components in a homogeneous, isotropic viscoelastic material are in phase, there are no phase shifts between the strain components.

❍ The matrix of dynamical flexibility of the homogeneous, isotropic viscoelastic material has been determined which depends on three material constants (Young modulus, Poisson's ratio, coefficient of viscous damping of normal strain) and load circular frequency.

NOMECLATURE

E	– oung modulus
G	– shear modulus, relaxation modulus
H	– matrix of dynamical flexibility of the viscoelastic material at the load circular frequency ω
H_n	– matrix of dynamical flexibility of the viscoelastic material at the load circular frequency nω
i	– imaginary unity
I	– creep compliance
Im	– imaginary part
n	– natural number
$q_0 = E$, $q_1 = \eta$	– material constants in the Kelvin-Voigt's model of the viscoelastic material
R_e	– tensile yield strength
S_ε	– matrix of power spectral densities of the strain components
S_σ	– matrix of power spectral densities of the stress components
t	– time
α	– phase angle of the strain components
α_n	– phase angle of n-th terms of the strain components
γ	– shear strain
γ_k	– k-th strain component (k = xy, yz, zx)
γ_{k0}	– k-th strain component at the static load, mean value of k-th strain component
ε	– normal strain
$\boldsymbol{\varepsilon}$	– vector of the strain components
ε_j	– j-th strain component (j = x, y, z)
ε_{j0}	– j-th strain component at the static load, mean value of j-th strain component
$\boldsymbol{\varepsilon}_0$	– vector of mean values of the strain components
η	– coefficient of viscous damping of normal strain
λ	– coefficient of viscous damping of shear strain
υ	– Poisson's ratio
σ	– normal stress
$\boldsymbol{\sigma}_a$	– vector of the amplitudes of stress components
σ_{e0}	– mean value of the reduced stress
σ_{ea}	– amplitude of the reduced stress
σ_j	– j-th stress component
σ_{ja}	– amplitude of j-th stress component
σ_{jn}	– amplitude of n-th term in Fourier expansion of σ_j
σ_{j0}	– j-th stress component at the static load, mean value of j-th stress component
$\overline{\sigma_n}$	– vector of complex amplitudes of n-th terms in Fourier expansions of the stress components
$\boldsymbol{\sigma}_0$	– vector of mean values of the stress components
τ	– shear stress
τ_k	– k-th stress component
τ_{ka}	– amplitude of k-th stress component
τ_{kn}	– amplitude of n-th term in Fourier expansion of τ_k
τ_{k0}	– k-th stress component at the static load, mean value of k-th stress component
φ_j	– phase angle of j-th stress component
φ_{jn}	– phase angle of n-th term in Fourier expansion of σ_j
φ_k	– phase angle of k-th stress component
φ_{kn}	– phase angle of n-th term in Fourier expansion of τ_k
ω	– circular frequency, fundamental circular frequency
ω_e	– circular frequency of the reduced stress
$(\bar{\cdot})$	– complex quantity

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