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A discrete model of the plate heat exchanger

Andrzej Mielewczyk, Ph.D.
Gdynia Maritime University

ABSTRACT

Developing numerical control methods requires precise models of technical machines. The article presents a discrete model of a plate heat exchanger. The model was worked out based on differential equations to secure high accuracy. Correctness of the model was verified using the data for the M10 – MFM cooler produced by Alfa-Laval.

Keywords: simulation, dynamic model, Z transform, plate cooler, ship facilities

INTRODUCTION

Heat exchangers are used in all types of power plants for heating or cooling the working medium having the liquid or gaseous form. The operation of heat exchangers is crucial for keeping accurate temperature of the process.

In marine power plants three-step main engine cooling systems have been introduced, which increased the number of coolers and control systems. Plate heat exchangers are in common use here. Moreover, the plate construction is also used in the designs of condensers, evaporators, and fuel heaters.

Marine power plant simulators require fast algorithms simulating the operation of plate heat exchangers to determine new control points for numerous installations. At the same time, the simulation is expected to model properly static and dynamic properties of the heat transfer process [7, 13, 15].

The principles applicable when working out the simulation algorithm have been formulated in [10]. The there presented procedure is used for working out equations for the fresh water/sea water heat exchanger model.

MODEL OF THE PLATE HEAT EXCHANGER

The plate heat exchanger consists of plates arranged parallel to each other, between which the first and second working medium flows alternately. The heat transfer process is repeated between successive plates. Each plate takes the working medium from the main collector and generates repeatable heat transfer conditions, including constant medium flow velocity and heat transfer conditions represented by the overall heat transfer coefficient. In order to develop a mathematical model of the plate heat exchanger, a two-plate system with co-current or counter-current flow is to be analysed, Fig. 1. When modelling the heat transfer, the following simplifying assumptions were adopted [2, 3, 4, 12, 14]:

- ✰ physical parameters of the working media do not depend on temperature within the range of temperature changes taking place in the exchanger,
- ✰ the averaged profile of the working medium flow velocity is characteristic for the plug flow (turbulent flow),
- ✰ temperature gradients in y and z directions are equal to zero,
- ✰ thermal capacity of the plate membrane is neglected due to its small thickness (g = 0.4 mm).

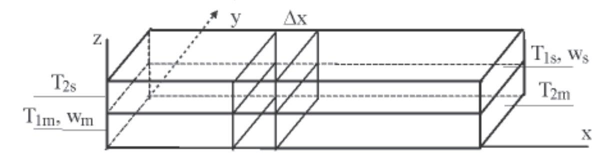

Fig. 1. *Scheme of the plate heat exchanger segment for the counter-current flow*

Taking into account the above assumptions we can write the energy balance for an element having length Δx [11, 13]. The heat transfer process in the plate heat exchanger is described by a system of differential equations (1) and (2):

$$\begin{cases} \dfrac{\partial T_s(x,\tau)}{\partial \tau} + w_s \dfrac{\partial T_s(x,\tau)}{\partial x} = -a\left[T_s(x,\tau) - T_m(x,\tau)\right] & (1) \\ \dfrac{\partial T_m(x,\tau)}{\partial \tau} + w_m \dfrac{\partial T_m(x,\tau)}{\partial x} = b\left[T_s(x,\tau) - T_m(x,\tau)\right] & (2) \end{cases}$$

where:

$$a = \frac{2k}{z c_s \rho_s} \tag{3}$$

$$b = \frac{2k}{z c_m \rho_m} \tag{4}$$

The basic variable is the temperature of the liquid T_s and T_m with respect to plate length x and time τ. The system of equations (1) and (2) was solved using the **Z** transform, and its solution was:

$$\begin{cases} \overline{T}_s(x,z)\left(a + \frac{z-1}{T}\right) + w_s \frac{d\overline{T}_s(x,z)}{dx} - \frac{z}{T}\overline{T}_s(x,0) = a \cdot T_m(x,z) & (5) \\ \overline{T}_m(x,z)\left(b + \frac{z-1}{T}\right) + w_m \frac{d\overline{T}_m(x,z)}{dx} - \frac{z}{T}\overline{T}_m(x,0) = b \cdot \overline{T}_s(x,z) & (6) \end{cases}$$

The process is nonlinear due to the flow velocity w and heat transfer conditions k. However, their changes are slower than temperature changes in a given discretisation step, and are assumed constant in a single iteration step. This way we arrive at a system of first-order differential equations (5) and (6) with constant coefficients.

To solve the system of equations (5) and (6) we assume a constant non-zero value of the temperature along plate length in time τ = 0:

$$T_s(x,0) = T_{s0}(x) = \text{const} \qquad T_m(x,0) = T_{m0}(x) = \text{const} \tag{7}$$

The general solution of the system of equations (5) and (6) for non-zero initial conditions (7) is a composition of functions:

$$\begin{cases} \overline{T}_s(x,z) = Ae^{r_1 x} + Be^{r_2 x} + \overline{T}_{ss}(x,z) & (8) \\ \overline{T}_m(x,z) = \left(1 + \frac{z-1}{aT} + \frac{w_s}{a} r_1\right)Ae^{r_1 x} + \left(1 + \frac{z-1}{aT} + \frac{w_s}{a} r_2\right)Be^{r_2 x} + \overline{T}_{mm}(x,z) & (9) \end{cases}$$

where:

$$r_1 = \alpha\left[z + \beta - \sqrt{(z+\beta)^2 + \gamma}\right] - \frac{z-1}{Tw_s}x - \frac{a}{w_s}x \tag{10}$$

$$r_2 = -\alpha\left[z + \beta - \sqrt{(z+\beta)^2 + \gamma}\right] - \frac{z-1}{Tw_m}x - \frac{b}{w_m}x \tag{11}$$

$$\overline{T}_{ss}(x,z) = \frac{z\frac{z-1}{T} + bz}{Tw_s w_m r_1 r_2} T_{s0} + \frac{az}{Tw_s w_m r_1 r_2} T_{m0} = \frac{z(z-1)T_{s0} + zT(bT_{s0} + aT_{m0})}{(z-1)^2 + (z-1)T(a+b)} \tag{12}$$

$$\overline{T}_{mm}(x,z) = \frac{bz}{Tw_s w_m r_1 r_2} T_{s0} + \frac{z\frac{z-1}{T} + az}{Tw_s w_m r_1 r_2} T_{m0} = \frac{z(z-1)T_{m0} + zT(bT_{s0} + aT_{m0})}{(z-1)^2 + (z-1)T(a+b)} \tag{13}$$

and

$$\alpha = \frac{w_m - w_s}{2Tw_m w_s} x \tag{14}$$

$$\beta = T\frac{aw_m - bw_s}{w_m - w_s} - 1 \tag{15}$$

$$\gamma = 4abT^2 \frac{w_m w_s}{(w_m - w_s)^2} \tag{16}$$

The complete solution will be obtained for known initial conditions. The most typical flow arrangement used in plate coolers bases on the counter-flow. The assumed initial conditions include a step temperature change at plate cooler inlet, recorded simultaneously on the cooled and cooling medium sides, Fig. 1:

$$\overline{T}_s(x=l,z) = Ae^{r_1 l} + Be^{r_2 l} + \overline{T}_{ss}(l,\ z) = \overline{T}_{s1}(l,\ z) \tag{17}$$

$$\overline{T}_m(x=0,z) = \left(1+\frac{z-1}{aT}+\frac{w_s}{a}r_1\right)A + \left(1+\frac{z-1}{aT}+\frac{w_s}{a}r_2\right)B + \overline{T}_{mm}(0,z) = \overline{T}_{m1}(0,z) \tag{18}$$

After calculating the integration constants A and B from equations (17) and (18), and placing them to the system of equations (8) and (9) we get the solution:

$$\overline{T}_{s2}(0,z) = \left[\overline{T}_{s1}(l,z) - \overline{T}_{ss}(l,z)\right] \frac{\left(1+\dfrac{\gamma}{\left(z+\beta+\sqrt{(z+\beta)^2+\gamma}\right)^2}\right) e^{-\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]} e^{\frac{z-1}{Tw_s}l} e^{\frac{a}{w_s}l}}{1+\dfrac{\gamma}{\left(z+\beta+\sqrt{(z+\beta)^2+\gamma}\right)^2} e^{-2\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]} e^{(z-1)\frac{w_m-w_s}{Tw_m w_s}l} e^{\frac{aw_m-bw_s}{w_m w_s}l}} +$$

$$(19)$$

$$+\left[\overline{T}_{m1}(0,z) - \overline{T}_{mm}(0,z)\right] \frac{2Taw_m}{(w_m - w_s)} \frac{\dfrac{1}{z+\beta+\sqrt{(z+\beta)^2+\gamma}} - \dfrac{e^{-2\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]} e^{(z-1)\frac{w_m-w_s}{Tw_m w_s}l} e^{\frac{aw_m-bw_s}{w_m w_s}l}}{z+\beta+\sqrt{(z+\beta)^2+\gamma}}}{1+\dfrac{\gamma}{\left(z+\beta+\sqrt{(z+\beta)^2+\gamma}\right)^2} e^{-2\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]} e^{(z-1)\frac{w_m-w_s}{Tw_m w_s}l} e^{\frac{aw_m-bw_s}{w_m w_s}l}} +$$

$$+\overline{T}_{ss}(0,z)$$

— — — — — — — — — —

$$\overline{T}_{m2}(l,z) =$$

$$= \left[\overline{T}_{s1}(l,z) - \overline{T}_{ss}(l,z)\right] \frac{2bTw_s}{(w_m - w_s)} \frac{-\dfrac{1}{z+\beta+\sqrt{(z+\beta)^2+\gamma}} + \dfrac{e^{-2\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]} e^{(z-1)\frac{w_m-w_s}{Tw_m w_s}l} e^{\frac{aw_m-bw_s}{w_m w_s}l}}{z+\beta+\sqrt{(z+\beta)^2+\gamma}}}{1+\dfrac{\gamma}{\left(z+\beta+\sqrt{(z+\beta)^2+\gamma}\right)^2} e^{-2\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]} e^{(z-1)\frac{w_m-w_s}{Tw_m w_s}l} e^{\frac{aw_m-bw_s}{w_m w_s}l}} +$$

$$(20)$$

$$+\left[\overline{T}_{m1}(0,z) - \overline{T}_{mm}(0,z)\right] \frac{\left(1+\dfrac{\gamma}{\left(z+\beta+\sqrt{(z+\beta)^2+\gamma}\right)^2}\right) e^{-\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]} e^{-\frac{z-1}{Tw_m}l} e^{-\frac{b}{w_m}l}}{1+\dfrac{\gamma}{\left(z+\beta+\sqrt{(z+\beta)^2+\gamma}\right)^2} e^{-2\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]} e^{(z-1)\frac{w_m-w_s}{Tw_m w_s}l} e^{\frac{aw_m-bw_s}{w_m w_s}l}} +$$

$$+\overline{T}_{mm}(l,z)$$

The original form of functions T_{s2} T_{m2} is obtained after applying the definition [6]:

$$f_n = \lim_{p\to 0} \frac{1}{n!} \frac{d^n F\left(z=\dfrac{1}{p}\right)}{dp^n} \tag{21}$$

$$F_a(z) = \frac{1}{z+\beta+\sqrt{(z+\beta)^2+\gamma}}$$

$$f_a(nT) = 0, \sum_{m=0}^{S} (-1)^{n-1}(-1)^m \frac{(n-1)!}{2^{2m+1}m!(m+1)!(n-1-2m)!}\beta^{n-1-2m}\gamma^m \tag{22}$$

where:

$$S = \begin{cases} \frac{n-1}{2} & \text{for odd } n \\ \frac{n-2}{2} & \text{for even } n \end{cases}$$

$$F_b(z) = \frac{1}{z+\beta+\sqrt{(z+\beta)^2+\gamma}} e^{-2\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]}$$

$$f_b(nT) = 0, \sum_{k=0}^{n-1}\sum_{m=0}^{S} (-1)^{n-1}(-1)^k(-1)^m \frac{(k+1)(n-1)!}{2^{2m+1}k!m!(m+k+1)!(n-k-1-2m)!}\beta^{n-1-k-2m}\gamma^m(\alpha\gamma)^k \tag{23}$$

where:

$$S = \begin{cases} \frac{n-k-1}{2} & \text{for odd } n \text{ and even } k \text{ or even } n \text{ and odd } k \\ \frac{n-k-2}{2} & \text{for even } n \text{ and even } k \text{ or odd } n \text{ and odd } k \end{cases}$$

$$F_c(z) = \frac{\gamma}{\left(z+\beta+\sqrt{(z+\beta)^2+\gamma}\right)^2} e^{-\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]}$$

$$f_c(nT) = 0,0, \sum_{k=0}^{n-2}\sum_{m=0}^{S} (-1)^{n}(-1)^k(-1)^m \frac{(k+2)(n-1)!}{2^{2m+k+2}k!m!(m+k+2)!(n-k-2-2m)!}\beta^{n-2-k-2m}\gamma^{m+1}(\alpha\gamma)^k \tag{24}$$

$$S = \begin{cases} \frac{n-k-3}{2} & \text{for odd } n \text{ and even } k \text{ or even } n \text{ and odd } k \\ \frac{n-k-2}{2} & \text{for even } n \text{ and even } k \text{ or odd } n \text{ and odd } k \end{cases}$$

$$F_d(z) = \frac{\gamma}{\left(z+\beta+\sqrt{(z+\beta)^2+\gamma}\right)^2} e^{-2\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]}$$

$$f_d(nT) = 0,0, \sum_{k=0}^{n-2}\sum_{m=0}^{S} (-1)^{n}(-1)^k(-1)^m \frac{(k+2)(n-1)!}{2^{2m+2}k!m!(m+k+2)!(n-k-2-2m)!}\beta^{n-2-k-2m}\gamma^{m+1}(\alpha\gamma)^k \tag{25}$$

where:

$$S = \begin{cases} \frac{n-k-3}{2} & \text{for odd } n \text{ and even } k \text{ or even } n \text{ and odd } k \\ \frac{n-k-2}{2} & \text{for even } n \text{ and even } k \text{ or odd } n \text{ and odd } k \end{cases}$$

$$F_e(z) = e^{-\alpha\left[z+\beta-\sqrt{(z+\beta)^2+\gamma}\right]}$$

$$f_e(nT) = 1, \sum_{k=0}^{n-1}\sum_{m=0}^{S} (-1)^{n}(-1)^k(-1)^m \frac{(n-1)!}{2^{2m+k+1}k!m!(m+k+1)!(n-k-1-2m)!}\beta^{n-1-k-2m}\gamma^m(\alpha\gamma)^{k+1} \tag{26}$$

where:

$$S = \begin{cases} \frac{n-k-1}{2} & \text{for odd } n \text{ and even } k \text{ or even } n \text{ and odd } k \\ \frac{n-k-2}{2} & \text{for even } n \text{ and even } k \text{ or odd } n \text{ and odd } k \end{cases}$$

After dividing numerically by series we arrive at the response in discrete time instants:

$$
\begin{aligned}
T_{m2}(l,nT) = {} & \sum_n \left[T_{s1}(nT) - T_{ss}(l,nT)\right] \frac{2bTw_s}{(w_m - w_s)} \frac{f_b\left[nT - \frac{1}{w_m} + \frac{1}{w_s}\right] e^{\frac{aw_m - bw_s}{w_m w_s} l} - f_a(nT)}{1 + f_d\left[nT - \frac{1}{w_m} + \frac{1}{w_s}\right] e^{\frac{aw_m - bw_s}{w_m w_s} l}} + \\
& + \sum_n \left[T_{m1}(nT) - T_{mm}(0,nT)\right] \frac{f_e\left[nT - \frac{1}{w_m}\right] e^{\frac{b}{w_m} l} + f_c\left[nT + \frac{1}{w_m}\right] e^{\frac{b}{w_m} l}}{1 + f_d\left[nT - \frac{1}{w_m} + \frac{1}{w_s}\right] e^{\frac{aw_m - bw_s}{w_m w_s} l}} + T_{mm}(l,nT)
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
T_{s2}(0,nT) = {} & \sum_n \left[T_{s1}(nT) - T_{ss}(l,nT)\right] \frac{f_e\left[nT + \frac{1}{w_s}\right] e^{\frac{a}{w_s} l} + f_c\left[nT + \frac{1}{w_s}\right] e^{\frac{a}{w_s} l}}{1 + f_d\left[nT - \frac{1}{w_m} + \frac{1}{w_s}\right] e^{\frac{aw_m - bw_s}{w_m w_s} l}} + \\
& + \sum_n \left[T_{m1}(nT) - T_{mm}(0,nT)\right] \frac{2Taw_m}{(w_m - w_s)} \frac{f_a(nT) - f_b\left[nT - \frac{1}{w_m} + \frac{1}{w_s}\right] e^{\frac{aw_m - bw_s}{w_m w_s} l}}{1 + f_d\left[nT - \frac{1}{w_m} + \frac{1}{w_s}\right] e^{\frac{aw_m - bw_s}{w_m w_s} l}} + T_{ss}(0,nT)
\end{aligned}
\tag{28}
$$

The excitation functions T_{s1} and T_{m1} can be arbitrary, but this requires the use of the function convolution operation. For the step excitation function, its response is the sum of terms of a given series. In case of another function, its approximation by a staircase function is recommended. Such approximation is used in numerical control algorithms.

The final solution includes all process variables and the time in a discreet form. In one discretisation step the process variables take constant values and can be only changed in the next discretisation step. Thus we reduce the model to a linear form. By modifying parameters in successive steps we arrive as a quasi-linear solution to a nonlinear system. If the parameters do not have to be changed, a series of consecutive process values is calculated.

Sample results of calculations performed using the above-described method are shown in Fig. 2.

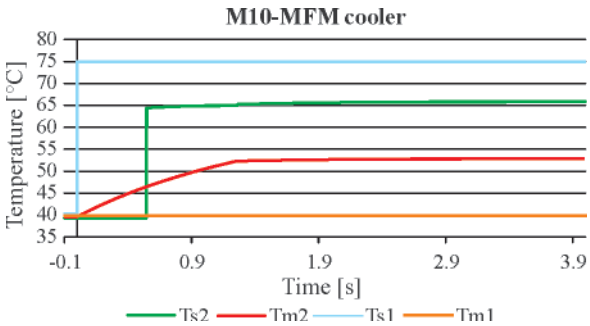

Fig. 2. Interval of the transformed function discretisation

The simulation of cooling plate operation made use of the following input data:

M10 – MFM cooler, type: fresh water - sea water

$l = 0.675$ m

$a = 0.75$ [1/s] $\quad$ $b = 0.74$ [1/s]

$w_s = 1.23$ [m/s] $\quad$ $w_m = 0.944$ [m/s]

The initial temperature distribution along the plate length:

$T_{0s}(x) = 39.3°C \qquad T_{0m}(x) = 39.3°C$

The signal assumed at cooler input:

$T_{1s}(\tau) = 75.0°C \qquad T_{1m}(\tau) = 39.3°C$

CONCLUSIONS

The presented model of the plate heat exchanger reveals good agreement with the real results. It can be used in numerical simulations, depending on the available simulation time.

The performed cooler model analyses make the basis for formulating the following conclusions:

- The obtained static and dynamic characteristics are fully compatible within the entire range of loads taking place in operating practice.
- The model correctly reacts to changes of working medium flow velocities and other parameters.
- Detailed analysis of the simulated phenomena and processes is possible. It can refer to both the control processes, and diagnostics of the technical state of the examined cooler.
- Comparing counter-current and co-current flows produces better results, but makes the time in which the steady state is reached longer.

NOMENCLATURE

A, B – integration constants
a, b – heat transfer constant for plate cooler cooled and cooling side, respectively [s^{-1}]
c – specific heat of the liquid [J/kgK]
k – plate cooler heat transfer coefficient [W/m^2K]
l – plate length [m]
ρ – liquid density [kg/m^3]
T_m – sea water temperature in the plate cooler [K, °C]
T_s – fresh water temperature in the plate cooler [K, °C]
T – interval of the transformed function discretisation [s],

$$Z(f) = \overline{f}(z) = \sum_{n=0}^{\infty} f(nT) z^{-n}$$

τ – time [s]
x – temperature distribution along plate length
w_m – sea water flow velocity [m/s]
w_s – fresh water flow velocity [m/s]
z – distance between plates [m]

Indices

0 – initial value,
1 – input boundary value,
2 – output boundary value,
m – sea water, cooling medium
s – fresh water, cooled medium

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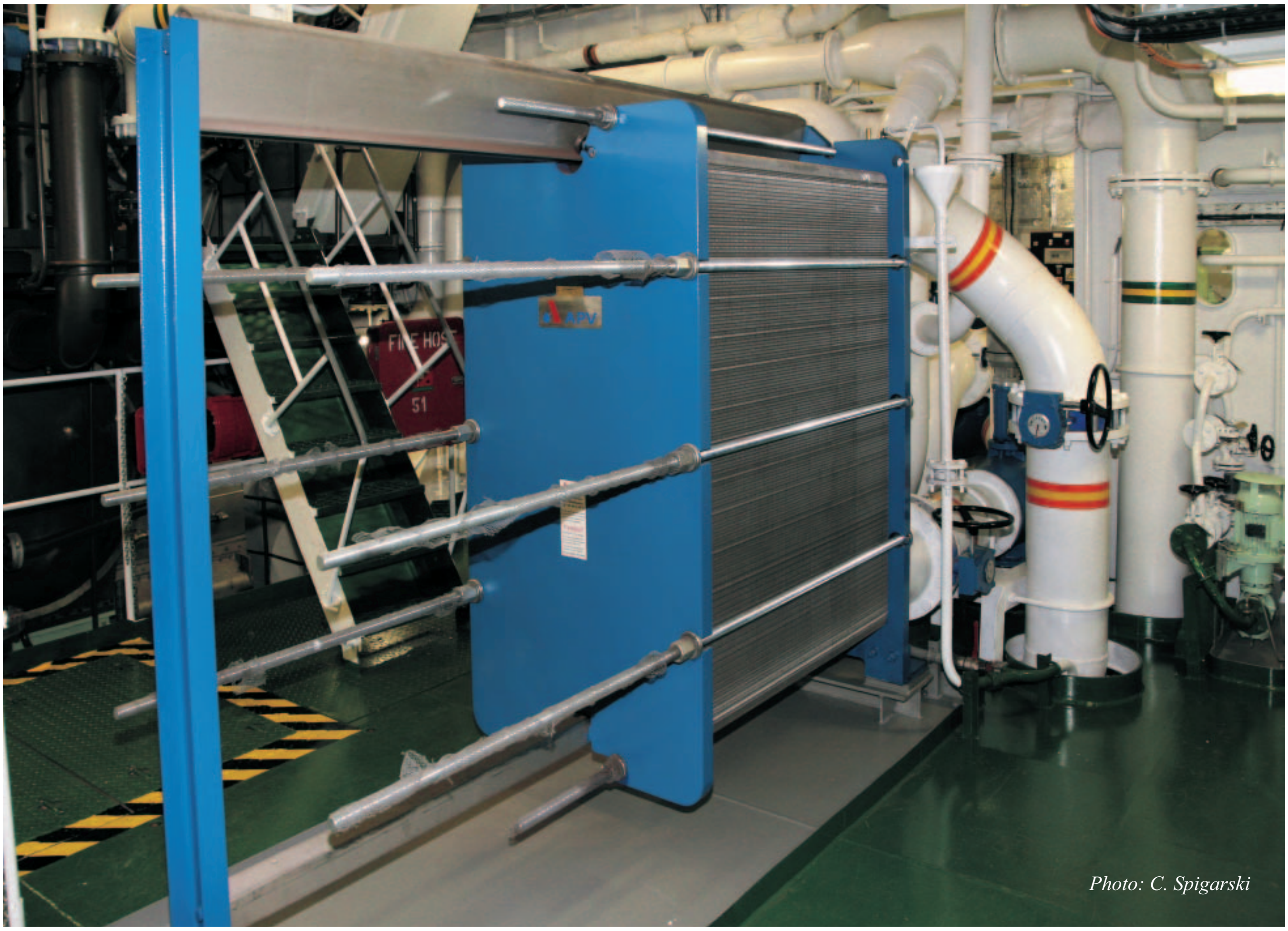
CONTACT WITH THE AUTHOR

Andrzej Mielewczyk, Ph. D.
Department of Basic Engineering Sciences,
Gdynia Maritime University
Morska 81-87
81-225 Gdynia, POLAND
e-mail: mieczyk@am.gdynia.pl



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