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A new continuous model for flexural vibration analysis of a cracked beam

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ABSTRACT

In this paper a new continuous model for vibration analysis of a beam with an open edge crack is presented. A quasi-linear displacement filed is suggested for the beam and the strain and stress fields are calculated. The equation of motion of the beam is calculated using the Hamilton principle. The calculated equation of motion is solved with a modified weighted residual method and the natural frequencies and mode shapes are obtained. The results are compared with those obtained by finite element method and an excellent agreement has been observed. The presented model is a simple and accurate method for analysis of the cracked beam behavior near or far from the crack tip.

Keywords: vibration, crack, beam, natural frequency, mode shape, weighted residual

INTRODUCTION

Fatigue and cracks are important subjects in industrial machineries which can lead to catastrophic failures in certain conditions. The importance of the early detection of the cracks takes researchers to study various aspects of the behavior of a structure defected by cracks. One of these aspects is the vibration of cracked structures. Crack development in a system changes the vibration behavior. With measurement and analysis of these vibrations the cracks can be identified well in advance and appropriate actions can be taken to prevent more damage to the system.

The vibration behavior of cracked structures has been investigated by many researchers. Dimaragonas presented a review on the topic of vibration of cracked structures [1]. His review contains vibration of cracked rotors, bars, beams, plates, pipes, blades and shells. Two more literature reviews are also available on the dynamic behavior of cracked rotors by Wauer and Gasch [2, 3].

Cracked beam is one of the structural elements which has been studied by researchers. There exist three methods for vibration modeling of cracked beams: discrete models with local flexibility model for crack, continuous models with local flexibility model for crack and continuous models with continues model for crack.

For the first time, Dimaragonas suggested the local flexibility method for modeling the crack [4]. He assumed the crack to be a rotational spring between two healthy parts of the beam. The stiffness of this spring obtained from the concept of J-integral in fracture mechanics. This local flexibility idea has been followed by several researchers till now. Some researchers modeled two healthy half beams discretely and added the flexibility of the rotational spring to the flexibility matrix of the system [5,6]. While others modeled two healthy half beams continuously and used appropriate boundary conditions for each part to link them through the rotational spring [7,8]. Some other researchers are tried to modify and improve the local flexibility model of the crack by adding one or two linear springs beside the rotational one [9]. These methods have also been extended for beams with more than one crack [10-12]. The local flexibility model for the crack is a simple approach and has a relative good result in finding fundamental natural frequency of a cracked beam. However this method offers no solutions for finding the stress at the crack area under the dynamic loads, mode shapes in free vibrations and operational deformed shape in forced vibrations.

Another approach to vibration analysis of cracked beams is continuous modeling of the crack. Christides and Barr developed a continuous theory for vibration of a uniform Euler-Bernoulli beam containing one or more pairs of symmetric cracks [13]. They suggested some modifications on the familiar stress field of a normal Euler-Bernoulli beam in order to consider the crack effect. The differential equation of motion and corresponding boundary conditions are given as the results. However in their model two different and incompatible assumptions have been made for displacement and strain fields. Although the accuracy of the results in finding the natural frequencies is acceptable for some applications their model is not still reliable for more accurate analyses such as stress analysis near the crack tip under dynamic loading and mode shape analysis. In addition the resulted partial differential equation is complex and dependent on some constants which are unknown and must be calculated by correlating the analytically obtained results with those calculated by finite element in each case. Several researchers followed the Christides and Barr approach by modifying their method and gained some improvements [14-18]. However there still exists the inconsistency between strain and displacement fields which causes inaccuracy of the results especially in mode shapes and stress analysis.

In this paper a new continuous approach for vibration analysis of cracked beams has been presented. The crack is assumed to be an open edge crack. A bilinear displacement field has been suggested for the cracked beam and the strain and stress fields have been calculated. The differential equation of motion of the cracked beam has been obtained using the Hamilton principle. This partial differential equation has been solved with special numerical algorithm based on Galerkin projection method. The required constant needed in this model can be obtained using fracture mechanics. The results of this study are compared with the finite element results for verification.

MAIN IDEA AND ASSUMPTIONS

The basic assumption in the Euler-Bernoulli bending theory for beams is that the plane sections of beam which are perpendicular to the neutral axis remain plane and perpendicular to the neutral axis after deformation. In the presence of an edge crack, the planes will not remain plane after deformation particularly at the vicinity of the crack due to a shear stress near the crack tip which leads to warping in plane sections. Thus at the vicinity of the crack the displacement field is completely nonlinear. For the planes far from the crack tip, the warping will be smaller and the displacement filed can be assumed linear. In order to have a better sense of the bending in a cracked beam, a real model has been produced in this research and the mid span crack behavior under a pure bending moment can be seen in Fig. 1. The beam is made from a linear elastic material with low modulus of elasticity and a U-shape notch at the mid-span as a crack.

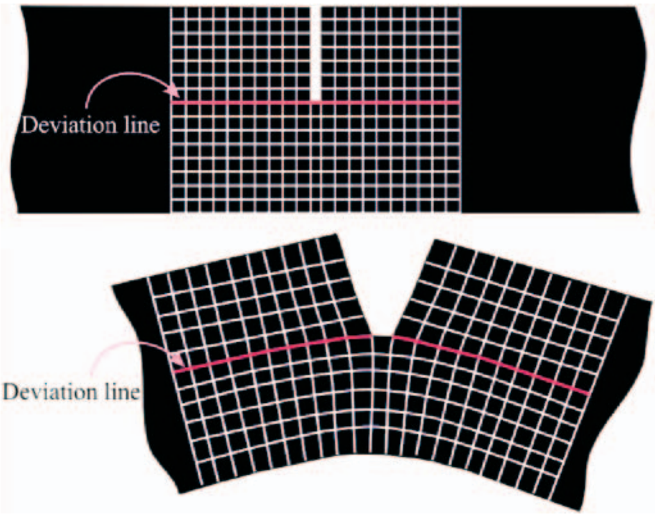

***Fig. 1.** A linear elastic cracked beam subjected to pure bending*

Near the crack area the plane sections will no longer remain plane. With a good approximation it can be supposed that each plane section turns into two straight planes after deformation. The horizontal line passing through the crack tip is called "deviation line" in this research which is shown in Fig. 1. Each straight plane section turns into two planes with different slopes one beneath and the other above the deviation line. The slope difference between these two planes decreases while the distance from the crack increases. These two straight planes connect to each other through a nonlinear part near the deviation line.

In order to find the stress, strain and deformation functions for a cracked beam in flexural vibrations a bilinear displacement field for the beam has been suggested in this research. In fact it is assumed that each plane section turns into two straight planes after deformation. The essential assumptions used in this research can be listed as follows:

★ the beam is slender and prismatic
★ the crack is considered to be an open edge notch
★ the deformations are supposed to be small
★ the plane strain assumption has been used in this research. Consequently the displacements along y-axis have been neglected
★ the stresses are small enough and the crack does not grow.

DISPLACEMENT FIELD

On the base of the above assumptions and explanations, the following displacement field is introduced for a cracked beam in flexural vibrations:

$$\begin{cases} w = w(x,t) \\ v = 0 \\ u(x,z,t) = u_0(x,t) - z\psi(x,t) + \Delta(x,z,t)h(z) \end{cases} \tag{1}$$

In which u, v, w are the displacement components along x, y and z axes. $u_0(x, t)$ is the longitudinal displacement of the deviation line along the x-axis and $\psi(x, t)$ is the slope of the plane sections below the deviation line. In an Euler-Bernoulli beam theory by neglecting the shear stress effect one has $\psi(x, t) = w_{,x}(x, t)$. In a cracked beam the shear stress near the crack tip cannot be ignored thus $\psi(x, t)$ is different from $w_{,x}(x, t)$. However far from the crack the shearing stress decreases gradually and $\psi(x, t)$ tends to be equal to $w_{,x}(x, t)h(z)$ is the unit step function which is equal to zero for $z \le 0$ and 1 for $z > 0$. Accordingly the term $w_{,x}(x, t)h(z)$ can be considered as the extra displacement of the plane sections above the deviation line. Fig. 2 shows these parameters graphically.

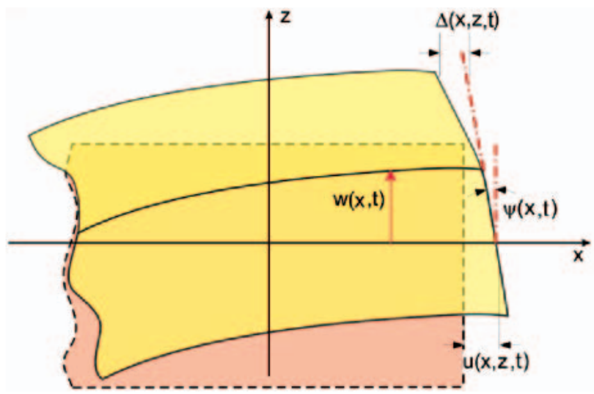

***Fig. 2.** Deformation filed definition of a cracked beam*

The additional displacement of the plane section above the deviation line has its maximum value at the crack faces and decreases gradually with distance from the crack tip. This additional displacement is a nonlinear and complex variable with respect to x. Here in this research an exponential regime has been assumed for the function $\Delta(x, z, t)$ along the x-axis as follows:

$$\Delta(x,z,t)=\varphi(z,t)\cdot e^{-\alpha\frac{|x-x_c|}{d}}\ \mathrm{sgn}(x-x_c) \tag{2}$$

In equation (2) α is a dimensionless exponential decay rate which will be obtained later in this paper, x_c is the crack position, d is the depth of the beam and sgn($x - x_c$) is the sign function which is -1 for $x < x_c$ and +1 for $x > x_c$. The application of sign function is due to the fact that the additional displacement function has a discontinuity at the position of the crack and the sign of its value changes when passing through the crack tip.

In order to find φ(x, t), zero normal stress condition at the crack faces can be used. The normal strain function can be found using equation (2):

$$\varepsilon_x=u_{,x}=u_{0,x}-z\psi_{,x}-\frac{\alpha}{d}\varphi(z,t)e^{-\alpha\frac{|x-x_c|}{d}}h(z) \tag{3}$$

In which the subscript ,x denotes the partial derivative with respect to x. The normal stress at the crack faces should be zero so one has:

$$\varphi(z,t)=\frac{d}{\alpha}\left(u_{0,x}(x_c,t)-z\psi_{,x}(x_c,t)\right) \tag{4}$$

To avoid discontinuity at the crack tip and considering the nonlinearity at the crack tip, the function φ(x, t) is modified in this paper as follows:

$$\varphi(z,t)=\frac{d}{\alpha}\left(u_{0,x}(x_c,t)-u_{0,x}(x_c,t)e^{-\beta\frac{z}{d}}-z\psi_{,x}(x_c,t)\right) \tag{5}$$

In equation (5) β is a dimensionless parameter and will be discussed later in this paper.

EQUATION OF MOTION

Now the strain field can be extracted from the displacement field. The only nonzero components of the stress field are ε_x and y_{xy} as follows:

$$\begin{cases}\varepsilon_x=u_{,x}=u_{0,x}-z\psi_{,x}-\left(u_{0,x}(x_c,t)-u_{0,x}(x_c,t)e^{-\beta\frac{z}{d}}-z\psi_{,x}(x_c,t)\right)h(z)e^{-\alpha\frac{|x-x_c|}{d}}\\ \gamma_{xy}=\frac{1}{2}\left(w_{,x}+u_{,z}\right)=\frac{1}{2}\left(w_{,x}-\psi+\left(\frac{\beta}{\alpha}u_{0,x}(x_c,t)e^{-\beta\frac{z}{d}}-\frac{d}{\alpha}\psi_{,x}(x_c,t)\right)e^{-\alpha\frac{|x-x_c|}{d}}\mathrm{sgn}(x-x_c)\right)\end{cases} \tag{6}$$

The normal stress energy of the beam can be obtained using the following relation:

$$V=\frac{1}{2}\int_V\sigma_{xx}\varepsilon_{xx}dV=\frac{1}{2}\mathrm{E}\int_V\varepsilon_{xx}{}^2dV \tag{7}$$

In which V is the normal strain energy function, V is the volume of the beam and E is the modulus of elasticity.

In this research the cracked beam is assumed to be slender. So the Euler-Bernoulli assumption can be used and one can neglect the shear strain energy in compare with the normal strain energy. Similar to a normal Euler-Bernoulli beam the average shear strain in each cross section can be assumed to be zero. So the following relation can be hold:

$$\int_A\gamma_{xy}dA=0 \tag{8}$$

The kinetic energy of the cracked beam can be also calculated as follows:

$$T=\frac{1}{2}\int_V\rho w_{,t}{}^2dV \tag{9}$$

In equation (9) similar to the Euler-Bernoulli beam theory the rotational moment of inertia has been neglected.

Using the Hamilton principle one has:

$$\delta\int_{t_0}^{t_1}(T-V)dt=0 \tag{10}$$

Now using equations (6) to (10) and performing appropriate calculations the following equations can be obtained:

$$\begin{cases}u_{0,x}=\bar{z}\psi_{,x}+k_3\bar{z}_h\psi_{,x}(x_c,t)e^{-\alpha\frac{|x-x_c|}{d}}\\ \psi_{,x}=w_{,xx}+k_5w_{,xx}(x_c,t)e^{-\alpha\frac{|x-x_c|}{d}}\end{cases} \tag{11}$$

In equation (11) $\bar{z}$ is the vertical coordinate of the centroid of the cross section and $\bar{z}_h$ is the vertical coordinate of the centroid of the healthy part of the cross section as shown in Fig. 3. The parameters k_3 and k_5 are geometrical dimensionless constants defined in equation (13).

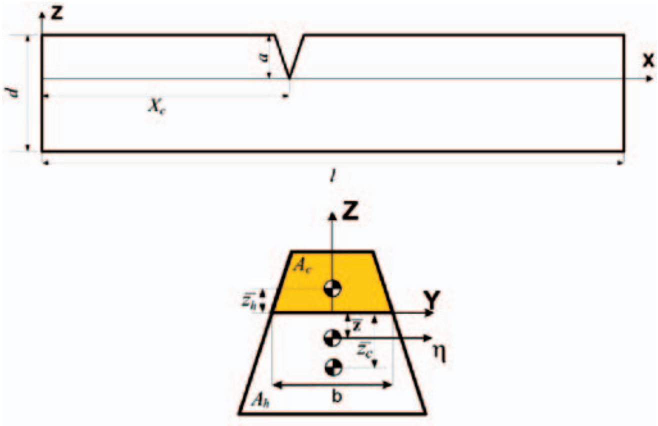

***Fig. 3.** A cracked beam parameter definition*

And finally the equation of motion for free vibrations of a cracked slender beam can be obtained as follows:

$$\frac{\partial^2}{\partial x^2} EI_\eta \left(w_{,xx} + \kappa w_{,xx}(x_c,t) e^{-\alpha\frac{|x-x_c|}{d}} \right) + \rho A w_{,tt} = 0 \quad (12)$$

In equations (11) and (12) the parameters $k_{1\text{-}6}$ and κ are geometrical dimensionless constants which can be defined as follows:

$$\begin{cases} k_1 = \dfrac{1}{A_c} \displaystyle\int_{A_c} e^{-\beta\frac{z}{d}} dA \\ k_2 = \dfrac{A_h}{A_h + k_1 A_c} \\ k_3 = \dfrac{A_c}{A}\left(k_2 - k_1 k_2 - \dfrac{\bar{z}_c}{\bar{z}_h} \right) \\ k_4 = \dfrac{Ad}{A_h d + \beta k_1 k_2 \bar{z}_h A_c} \\ k_5 = \dfrac{A_c}{A} k_4 \left(1 - \dfrac{\beta}{d} k_1 k_2 \bar{z}_h \right) \\ k_6 = \dfrac{1}{A_c \bar{z}_c} \displaystyle\int_{A_c} z e^{-\beta\frac{z}{d}} dA \\ \kappa = k_5 - \dfrac{k_3 k_4 A \bar{z}\, \bar{z}_h}{I_\eta} + \dfrac{k_4}{I_\eta}\left(k_2 A_c \bar{z}_c\, \bar{z}_h (1-k_6) - I_{c_y} \right) \end{cases} \quad (13)$$

Where A is the cross section area of the beam, A_c is the area of the crack face, I_{cy} is the moment of inertia of the crack face about y-axis and I_η is the moment of inertia of the cross section about the horizontal axis passing through the centroid of the cross section η.

The equation (12) is the main result of this investigation. In a normal beam the geometrical parameter κ is zero and hence equation (12) results in the Euler-Bernoulli beam equation for slender beams. The dimensionless exponential decay rates (α, β) are the only factors which has not been discussed yet. In the next section the parameters α and β are calculated.

EXPONENTIAL DECAY RATES α AND β CALCULATION

When a pair of bending moments M are applied to the cracked beam an additional relative rotation θ^* will exist between two ends of the beam due to the crack as shown in Fig. 4.

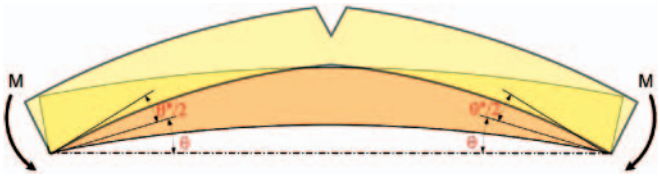

***Fig. 4.** Additional remote point rotation of a cracked beam*

It can be shown that for a cracked beam under pure bending equation (12) will turn into the following form:

$$\frac{d^2 w}{dx^2} = \frac{M}{EI_\eta}\left(1 - \frac{\kappa}{1+\kappa} e^{-\alpha\frac{|x-x_c|}{d}} \right) \quad (14)$$

Solving equation (14) will result in the load-deflection relation of a cracked beam under static pure bending. The results are as follows:

$$w = \begin{cases} \dfrac{M}{EI_\eta}\left(\dfrac{x^2}{2} + c_1 x + c_2 - \dfrac{d^2}{\alpha^2} \cdot \dfrac{\kappa}{1+\kappa} e^{\alpha\frac{x-x_c}{d}} \right) & x \le x_c \\ \dfrac{M}{EI_\eta}\left(\dfrac{x^2}{2} + c_3 x + c_4 - \dfrac{d^2}{\alpha^2} \cdot \dfrac{\kappa}{1+\kappa} e^{-\alpha\frac{x-x_c}{d}} \right) & x > x_c \end{cases} \quad (15)$$

In which the constants c_1, c_2, c_3 and c_4 will be as follows:

$$\begin{cases} c_2 = \dfrac{d^2}{\alpha^2} \cdot \dfrac{\kappa}{1+\kappa} e^{-\alpha\frac{x_c}{d}} \\ c_4 = c_2 + 2x_c \dfrac{d}{\alpha} \cdot \dfrac{\kappa}{1+\kappa} \\ c_3 = -\dfrac{l}{2} - \dfrac{c_4}{l} + \dfrac{1}{l} \cdot \dfrac{d^2}{\alpha^2} \cdot \dfrac{\kappa}{1+\kappa} e^{-\alpha\frac{l-x_c}{d}} \\ c_1 = c_3 + 2\dfrac{d}{\alpha} \cdot \dfrac{\kappa}{1+\kappa} \end{cases} \quad (16)$$

Now using equation (15) one can obtain the additional remote point rotation θ^* as follows:

$$\theta^* = \left(\theta_{cracked}(0) - \theta(0) \right) - \left(\theta_{cracked}(l) - \theta(l) \right) = = -\frac{2M}{EI_\eta} \frac{d}{\alpha} \cdot \frac{\kappa}{1+\kappa} \quad (17)$$

In equation (17) the parameter κ is a function of β. However comparing the finite element results with those obtained by this model shows that the parameter β has a very large value and accordingly it can be assumed that the parameter β is infinity. So in equation (17) one can substitute κ with $\lim_{\beta\to\infty} \kappa$.

On the other hand the additional remote point rotation θ^* has been obtained by empirical methods as follows [19]:

$$\theta^* = \frac{2Md}{E(1-\nu^2)I_\eta}\left(\frac{\frac{a}{d}}{1-\frac{a}{d}}\right)^2\left(5.93-19.69\left(\frac{a}{d}\right)+37.1\left(\frac{a}{d}\right)^2-35.8\left(\frac{a}{d}\right)^3+13.1\left(\frac{a}{d}\right)^4\right) \tag{18}$$

Equating the right hand sides of (17) and (18) will result in parameter á values as presented in Fig. 5.

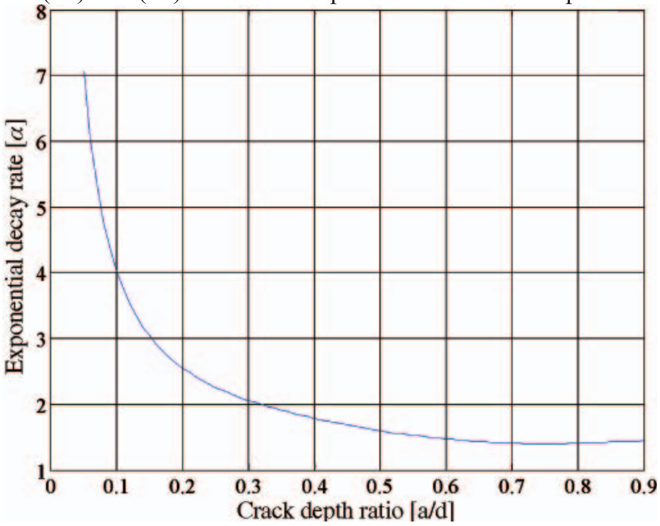

Fig. 5. *Exponential decay rate á versus crack depth ratio a/d*

In the next section the partial differential equation (12) has been solved and the natural frequencies and mode shapes have been calculated.

EIGEN SOLUTION

In order to find the natural frequencies and mode shapes of a cracked beam, the equation of motion presented in equation (12) must be solved. However this equation cannot be solved analytically and a numerical method must be used. It can be assumed that the solution is a harmonic function so one has:

$$w(x,t) = X(x)e^{i\omega t} \tag{19}$$

In which ω is the natural frequency of the beam. Substituting equation (19) into (12) and assuming EI_η to be constant along the beam the following eigenvalue problem will be resulted:

$$\frac{d^2}{dx^2}\left(X'' + \kappa X''(x_c)e^{-\alpha\frac{|x-x_c|}{d}}\right) - \frac{\rho A}{EI_\eta}\omega^2 X = 0 \tag{20}$$

Equation (20) has a special form and contains a singular function and depends on the value of the solution at the crack position. Theses anomalies prevent one to use the normal weighted residual solution for this Strum-Liouville problem. In a normal Strum-Liouville problem one can easily consider the function X to be in the form of $\Sigma c_i S_i(x)$ in which $S_i(x)$are the shape functions which satisfy the physical boundary conditions. However in this research the results show that such an approach will lead to divergence of the results. Since the function $e^{-\alpha\frac{|x-x_c|}{d}}$ in equation (20) is not a smooth function it seems that the solution specially for larger crack depth ratio tends to have large derivatives near the crack tip. Accordingly extracting the value of $X''(x_c)$ from X by derivation can lead to large fluctuations in the results and divergence. In order to avoid the divergence problem the function X″ and the value of $X''(x_c)$ is not extracted from Xby direct derivation. Instead the X″ is discretised independently from X and then a constraint equation provided to link X″ to X.

Considering the above discussion the following relations can be written:

$$\begin{cases} X'' + \kappa X''(x_c)e^{-\alpha\frac{|x-x_c|}{d}} = \sum_{i=1}^{N} c_i S_i(x) \\ X = \sum_{i=1}^{N} c_i' S_i(x) \end{cases} \tag{21}$$

In which c_i and c_i' are two independent set of constants and functions $S_i(x)$ are the shape functions which must satisfy the physical boundary conditions. Substituting equation (21) into (20) and multiplying two sides of the equation by $S_j(x)$ then integrating along the length of the beam one has:

$$\sum_{i=1}^{N} c_i \int_0^l S_i''(x)S_j(x)dx - \frac{\rho A}{EI_\eta}\omega^2 \sum_{i=1}^{N} c_i' \int_0^l S_i(x)S_j(x)dx = 0 \quad j = 1,2,\ldots,N \tag{22}$$

Or in the matrix form:

$$\left[K_{ij}\right]\left[c_i\right]-\frac{\rho A}{EI_\eta}\omega^2\left[P_{ij}\right]\left[c_i'\right]=0\ ,\ K_{ij}=\int_0^l S_i''(x)S_j(x)dx,\ P_{ij}=\int_0^l S_i(x)S_j(x)dx \tag{23}$$

On the other hand from equations (23) the following relation can be obtained:

$$\sum_{i=1}^{N} c_i' S_i''(x)=\frac{1}{1+\kappa}\sum_{i=1}^{N} c_i S_i(x) \tag{24}$$

Multiplying two sides of equation (24) by $S_j(x)$, integrating along the length of the beam and writing the equations in the matrix form one has:

$$\left[c_i'\right]=\left[Q_{ij}\right]^{-1}\left[R_{ij}\right]\left[c_i\right]\ ,\ Q_{ij}=\int_0^l S_i''(x)S_j(x)dx\ ,\ R_{ij}=\frac{1}{1+\kappa}\int_0^l S_i(x)S_j(x)dx \tag{25}$$

Substituting equation (25) into (23) the following equation will be resulted:

$$\left(\left[K_{ij}\right]-\frac{\rho A}{EI_\eta}\omega^2\left[M_{ij}\right]\right)\left[c_i\right]=0\ ,\left[M_{ij}\right]=\left[P_{ij}\right]\left[Q_{ij}\right]^{-1}\left[R_{ij}\right] \tag{26}$$

The natural frequencies and corresponding mode shapes for the cracked beam can be calculated solving the matrix eigenvalue problem of equation (26). In the next section the results are presented for a simply supported beam with rectangular cross-section.

RESULTS FOR A SIMPLY SUPPORTED BEAM WITH RECTANGULAR CROSS-SECTION

In this section the new approach has been applied for free vibration analysis of a simply supported slender prismatic cracked beam with rectangular cross-section. In such a beam the exponential decay rate β can be assumed to be infinity and the exponential decay rate α can be calculated from equations (17) and (18). The geometrical factor κ and the exponential decay rate α are as follows:

$$\kappa=\left(\frac{a}{d}\right)\left(\frac{a}{d}-2\right),\alpha=\frac{-91\left(\frac{a}{d}-2\right)}{593\left(\frac{a}{d}\right)-1969\left(\frac{a}{d}\right)^2+3714\left(\frac{a}{d}\right)^3-3584\left(\frac{a}{d}\right)^4+1312\left(\frac{a}{d}\right)^5} \tag{27}$$

In a simply supported cracked beam the shape functions $S_i(x)$ can be assumed to be in the form of sin(iđx/l) which satisfy the physical boundary conditions. The natural frequency and mode shapes can be calculated using eigenvalue problem of (26). In this research the number of shape functions N is set to be 100. In order to generalize the results the natural frequencies of the cracked beam have been divided to the corresponding values for a normal beam. Figures (6), (7) and (8) show the fundamental, second and third natural frequency ratios of the cracked beam respectively. In figures (6) to (8) the natural frequency ratios have been plotted versus the crack depth ratio (a/d) for several crack positions.

In figures (6) to (8) the results of finite element (FE) analysis are also presented for verification. The finite element results have been obtained using ANSYS software. In order to have an accurate and reliable model the PLANE183 singular element has been used in the cracked area [20]. This element is an 8-node quadratic solid singular element which specially designed for crack analysis. In this research a fine mesh has been used at the vicinity of the crack and dependency of the results to the mesh size has been checked. In all of the results there is a good agreement between analytical results and those obtained by FE analysis.

As it can be seen in figure (6) the reduction rate of the fundamental natural frequency has a direct relation with the position of the crack. This rate reduces for cracks which have more distance from the mid span of the beam. For the cracks at $x_c/l = 0.1$ the fundamental natural frequency only drops nearly 1 percent when the crack reaches to the half of the beam depth while for the cracks at the mid span this value is about 15 percent.

The dependency of the reduction of the natural frequency to the crack position is also seen in the first few natural frequencies. For the cracks at the mid span the second natural frequency remains nearly constant with the crack depth because this point coincides with the node of the second vibration mode of the beam.

Figures (9) to (11) show the first three normalized mode shapes for a cracked beam with a/d = 0.5 and $x_c/l = 0.1, 0.3, 0.5$. Comparison of the analytic and finite element results in this set of figures shows the efficiency of the model presented in this research.

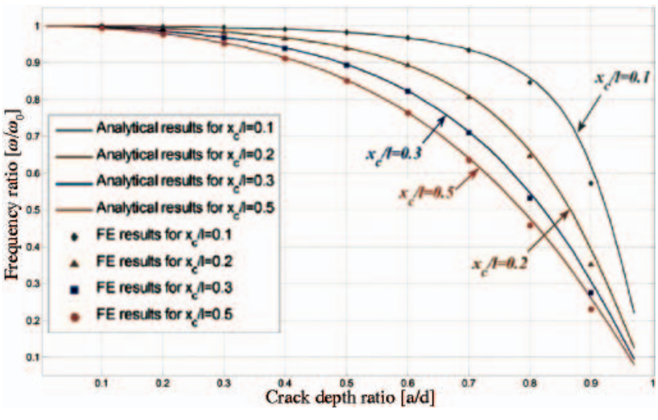

Fig. 6. Fundamental natural frequency ratio of a cracked beam versus crack depth ratio (a/d)

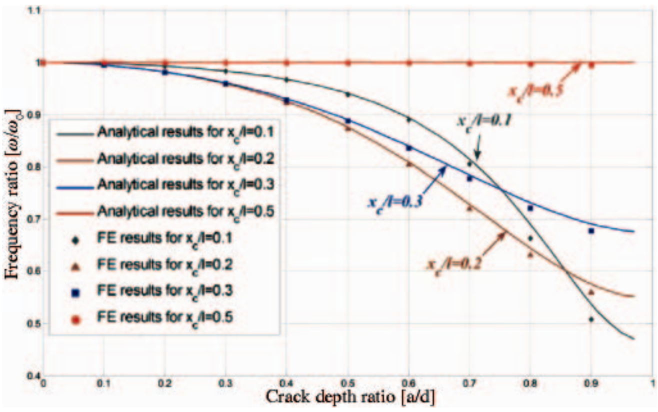

Fig. 7. Second natural frequency ratio of a cracked beam versus crack depth ratio (a/d)

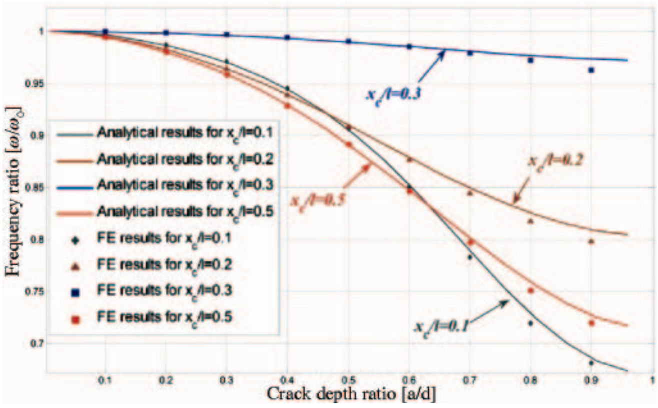

Fig. 8. Third natural frequency ratio of a cracked beam versus crack depth ratio (a/d)

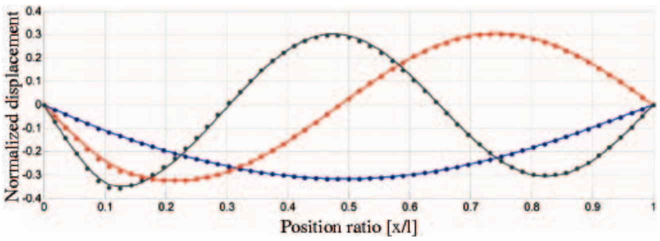

Fig. 9. First three normalized mode shapes of a cracked beam with $x_c/l=0.1$ and $a/d=0.5$ (——): Analytical results; (••••): Finite element results

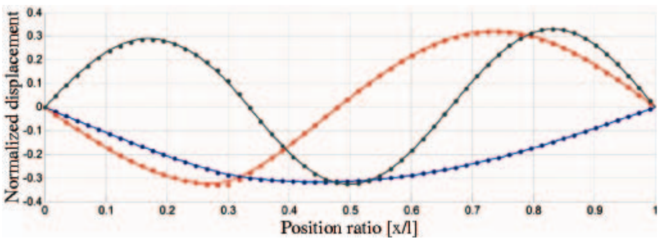

Fig. 10. First three normalized mode shapes of a cracked beam with $x_c/l=0.3$ and $a/d=0.5$ (——): Analytical results; (••••): Finite element results

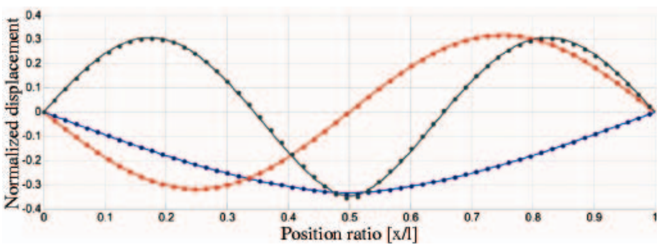

Fig. 11. First three normalized mode shapes of a cracked beam with $x_c/l=0.5$ and $a/d=0.5$ (——): Analytical results; (••••): Finite element results

CONCLUSIONS

- A new continuous theory for flexural vibration analysis of a beam with an edge crack has been introduced in this paper. The crack is assumed to be an open edge crack. A bilinear displacement filed has been suggested for the beam and the strain and stress fields have been calculated. The extraction of the strain field from the displacement field is based on elasticity rules and hence the displacement and strain fields are completely compatible. The governing equation of motion has been obtained using the Hamilton principle. The required constants in this model are calculated from empirical formula in fracture mechanics. The equation of motion has been solved for natural frequencies and mode shapes using a special numerical algorithm presented in this paper.

- The analytical results have been compared with finite element results and an excellent agreement has been observed. The model is accurate for both natural frequencies and mode shapes calculations. Results show that for a simply supported beam the reduction of natural frequency is a function of crack depth ratio as well as the crack position.

- The presented model is a simple and accurate model which predicts the behavior of the cracked beam and its results are reliable near the crack tip and far from it. This model can be used for dynamic stress and strain calculations near the crack tip.

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