Problems of welding in shipbuilding - an analytic-numerical assessment of the thermal cycle in HAZ with three dimensional heat source models in agreement with modelling rules

Part III
Non-linear analytic-numerical assessment of thermal cycle - examples

Eugeniusz Ranatowski, Prof
University of Technology and Life Science, Bydgoszcz

ABSTRACT

This part is continuation of PART II. Analytic solutions for the temperature distribution in HAZ – presented in the previous part of this article are transformed for computer calculation with used Mathcad programme. There are established algorithms in moving and stationary systems for thermal cycle calculating. Finally, a few analytical examples with use of C-I-N and D-E models are demonstrated.

Keywords: welding; shipbuilding; welding in shipbuilding; thermal cycle; heat affected zone; heat source model

ADAPTATION OF THE ANALYTICAL SOLUTIONS FOR NON-LINEAR COMPUTER CALCULATIONS

The equations (37)÷(42) of PART II of this article are algebraic form of linear heat flow solutions.

In order to execute computer calculations with temperature dependent physical parameters: λ, c_p, ρ the above algebraic expressions must be transformed. For this purpose we will use calculations in Mathcad programme [1]. This programme is very useful for modelling and simulation of welding thermal process [2, 3].

Therefore the following assumptions were done:
- heat source energy is being input to the metal during time Δt, not impulsively $\Delta t \rightarrow 0$. HS inputs are being summed up in points in distance $\Delta x = v \, \Delta t$. Considering this t' = (j-1) Δt, (j = 1, 2, 3 ..n).
- integrals were replaced by finished sums assuring sufficient exactness. Finally, the following computing expressions for linear heat flow solutions are obtained [4]:

A. from Cylindrical-Involution-Normal heat source model

- Stationary co-ordinates system :

$$T(x_0, y_0, z_0, t) = \sum_{j=1}^{n} \text{if} \left\{ t < (j-1) \cdot \Delta t, 0, \right.$$

$$\frac{q \cdot k \cdot K_z}{\pi \cdot c_\gamma \cdot (1 - \exp(-K \cdot s))} \cdot \frac{1}{4 \cdot a \cdot k \cdot (t - (j-1))\Delta t + 1} \cdot$$

$$\cdot \exp\left[\frac{-k \cdot ((x_0 - (j-1)v \cdot \Delta t)^2 + y_0^2)}{4 \cdot a \cdot k(t - (j-1)\Delta t) + 1} \right] \cdot \quad (1a)$$

$$\left. \cdot \sum_{j=1}^{last} B_i \cdot C_i \cdot D_i \cdot \exp[-a \cdot r_l^2 (t - (j-1)\Delta t)] \right\}$$

- Moving co-ordinates system:

$$T(x, y, z, t) = \sum_{j=1}^{n} \text{if} \left\{ t < (j-1) \cdot \Delta t, 0, \right.$$

$$\frac{q \cdot k \cdot K_z}{\pi \cdot c_\gamma \cdot (1 - \exp(-K \cdot s))} \cdot \frac{1}{4 \cdot a \cdot k \cdot (t - (j-1))\Delta t + 1} \cdot$$

$$\cdot \exp\left[\frac{-k \cdot ((x + v \, t - (j-1)v \cdot \Delta t)^2 + y^2)}{4 \cdot a \cdot k(t - (j-1)\Delta t) + 1} \right] \cdot \quad (1b)$$

$$\left. \cdot \sum_{j=1}^{last} B_i \cdot C_i \cdot D_i \cdot \exp[-a \cdot r_l^2 (t - (j-1)\Delta t)] \right\}$$

B. from Double Ellipsoidal configuration of source

- Stationary co-ordinates system

$$T(x_0, y_0, z_0, t) = \sum_{j=1}^{n} if\left\{ t < (j-1)\cdot \Delta t, 0, \right.$$

$$\frac{q\cdot f_f \cdot 3\cdot \sqrt{3}\cdot \Delta t}{\pi \cdot \sqrt{\pi}\cdot \dfrac{\lambda}{a}\cdot c_f \cdot \sqrt{\begin{pmatrix}(12\cdot a\cdot (t-(j-1)\Delta t)+a_f^2)\cdot \\ \cdot (12\cdot a\cdot (t-(j-1)\Delta t)+b_f^2)\end{pmatrix}}}\cdot$$

$$\exp\left(-\left(\frac{(x_0 - v\cdot (j-1)\Delta t)^2}{4\cdot a\cdot (t-(j-1)\Delta t)+\dfrac{1}{3}\cdot a_f^2} + \frac{y_0^2}{4\cdot a\cdot (t-(j-1)\Delta t)+\dfrac{1}{3}\cdot b_f^2}\right)\right) + \tag{2a}$$

$$+\frac{q\cdot f_r \cdot 3\cdot \sqrt{3}\cdot \Delta t'}{\pi \cdot \sqrt{\pi}\cdot \dfrac{\lambda}{a}\cdot c_f \cdot \sqrt{\begin{pmatrix}(12\cdot a\cdot (t-(j-1)\Delta t)+a_r^2)\cdot \\ \cdot (12\cdot a\cdot (t-(j-1)\Delta t)+b_r^2)\end{pmatrix}}}\cdot$$

$$\cdot \exp\left(-\left(\frac{(x_0 - v\cdot (j-1)\Delta t)^2}{4\cdot a\cdot (t-(j-1)\Delta t)+\dfrac{1}{3}\cdot a_r^2} + \frac{y_0^2}{4\cdot a\cdot (t-(j-1)\Delta t)+\dfrac{1}{3}\cdot b_r^2}\right)\right)\cdot$$

$$\cdot \sum_{j=1}^{last} B_i \cdot C_i \cdot D_i \cdot \exp[-a\cdot r_i^2(t-(j-1)\Delta t)]\Big\}$$

- Moving co-ordinates system

$$T(x, y, z, t) = \sum_{j=1}^{n} if\left\{ t < (j-1)\cdot \Delta t, 0, \right.$$

$$\frac{q\cdot f_f \cdot 3\cdot \sqrt{3}\cdot \Delta t}{\pi \cdot \sqrt{\pi}\cdot \dfrac{\lambda}{a}\cdot c_f \cdot \sqrt{\begin{pmatrix}(12\cdot a\cdot (t-(j-1)\Delta t)+a_f^2)\cdot \\ \cdot (12\cdot a\cdot (t-(j-1)\Delta t)+b_f^2)\end{pmatrix}}}\cdot$$

$$\cdot \exp\left(-\left(\frac{(x + v(t-(j-1)\Delta t))^2}{4\cdot a\cdot (t-(j-1)\Delta t)+\dfrac{1}{3}\cdot a_f^2} + \frac{y^2}{4\cdot a\cdot (t-(j-1)\Delta t)+\dfrac{1}{3}\cdot b_f^2}\right)\right) + \tag{2b}$$

$$+\frac{q\cdot f_r \cdot 3\cdot \sqrt{3}\cdot \Delta t'}{\pi \cdot \sqrt{\pi}\cdot \dfrac{\lambda}{a}\cdot c_f \cdot \sqrt{\begin{pmatrix}(12\cdot a\cdot (t-(j-1)\Delta t)+a_r^2)\cdot \\ \cdot (12\cdot a\cdot (t-(j-1)\Delta t)+b_r^2)\end{pmatrix}}}\cdot$$

$$\cdot \exp\left(-\left(\frac{(x + v(t-(j-1)\Delta t))^2}{4\cdot a\cdot (t-(j-1)\Delta t)+\dfrac{1}{3}\cdot a_r^2} + \frac{y^2}{4\cdot a\cdot (t-(j-1)\Delta t)+\dfrac{1}{3}\cdot b_r^2}\right)\right)\cdot$$

$$\cdot \sum_{j=1}^{last} B_i \cdot C_i \cdot D_i \cdot \exp[-a\cdot r_i^2(t-(j-1)\Delta t)]\Big\}$$

In order to execute computer calculations with temperature dependent physical parameters: λ, c_p, ρ the above algebraic expressions must be transformed.

Therefore the following assumptions were done:
- heat source energy is being input during time Δt, not impulsively $\Delta t \to 0$. Hs inputs are being summed up in points in distance $\Delta x = v \Delta t$. Considering this $t' = (j - 1) \Delta t$ ($j = 1, 2, 3,…S$),
- the integrals are changed to an algorithm executing proper summing with physical parameters upon temperature change control,
- as $\lambda(T)$, $c_p(T)$, $\rho(T)$, $\alpha(T)$ values in defined increments are known – like shown in table 1, the matrices containing T and corresponding $\lambda(T)$, $c_p(T)$, $\rho(T)$, $\alpha(T)$ values are defined.

With use of linear interpolation procedure, the continuous functions $\lambda(T)$, $c_p(T)$, $\rho(T)$, $\alpha(T)$ were created and built-in inside calculation sheet.

There are three main mini-procedures responsible for thermal cycle calculating. In the first of them initial values are presented (these values are specific for the given cycle)."Stab_ time" parameter is estimated time needed for stabilisation of thermal field in moving co-ordinates system, "Δt" is duration time of every heat impulse being input. Therefore "S" gives the total number of heat impulses to be generated in order to obtain the summary thermal field as a result (this parameter is being used finally in the third mini-procedure). The estimation of thermal fields from several impulses is running with changeable values of α and λ parameters according to Table 1. Parameter "Last" is used in the second mini-procedure which computes r_1, r_2, r_3,...r_n values – again with step by step $\lambda(T)$ values being modified. The final mini-procedure summarises thermal fields from several heat energy impulses using a proper formula specific for several HS model.

The following algorithms (3) and (4) with initial computing parameters are usually used to perform calculations for various heat sources in moving co-ordinates system:

A. from Cylindrical-Involution-Normal heat source model:

Stab_time = 19 sec Δt = 0.05 sec
S = Stab_time/ Δt = 380 last = 0

$$T_n(x, y, z, t) = \begin{vmatrix} T \leftarrow 0 \\ \text{for } j \in 1...S = \dfrac{Stab\_time}{\Delta t} \\ a \leftarrow a(T) \\ \lambda \leftarrow \lambda(T) \\ \text{for } i \in 1...last \end{vmatrix}$$

$$r_i = \begin{vmatrix} n \leftarrow threshold \\[4pt] root\left[\cot(n\cdot g) - \dfrac{\lambda^2\cdot\left[n+(i-1)\cdot\frac{\pi}{g}\right]^2 - \alpha_0\cdot\alpha_1}{(\alpha_0+\alpha_1)\cdot\left[\lambda\cdot\left[n+(i-1)\cdot\frac{\pi}{g}\right]\right]}\cdot n\right] + \\[4pt] \quad +(i-1)\cdot\dfrac{\pi}{g} \\[4pt] if\left[root\left[\cot(n\cdot g) - \dfrac{\lambda^2\cdot\left[n+(i-1)\cdot\frac{\pi}{g}\right]^2 - a_0\cdot a_1}{(a_0+a_1)\cdot\left[\lambda\cdot\left[n+(i-1)\cdot\frac{\pi}{g}\right]\right]}\cdot n\right]\right] > \\[4pt] \quad > threshold \\[4pt] threshold + (i-1)\cdot\dfrac{\pi}{g} \quad otherwise \end{vmatrix}$$

$$\begin{vmatrix} T \leftarrow T \ if \quad t < (j-1)\cdot\Delta t, 0, q\cdot \\[4pt] \cdot k\,\dfrac{K_z\Delta t}{\pi\cdot\frac{\lambda}{a}\cdot(1-\exp(-K_z\cdot s))}\cdot \\[6pt] \cdot \dfrac{1}{[1+4\cdot a\cdot k(t-(j-1)\cdot\Delta t)]} \\[6pt] \cdot \exp\left[-k\,\dfrac{[(x+v\cdot(t-(j-1)\Delta t))^2 + y^2]}{1+4\alpha\cdot k(t-(j-1)\Delta t)}\right]\cdot \\[6pt] \cdot \sum_{i=1}^{last} B_i\cdot C_i\cdot D_i\exp[a\cdot r_i^2\cdot(t-(j-1)\cdot\Delta t)] \\[6pt] T \end{vmatrix} \qquad (3)$$

B. from D-E configuration of source:

$$T_n(x,y,z,t) = \begin{vmatrix} T \leftarrow 0 \\[4pt] for\ j \in 1\ldots S = \dfrac{Stab\_time}{\Delta t} \\[4pt] a \leftarrow a(T) \\[4pt] \lambda \leftarrow \lambda(T) \\[4pt] for\ i \in 1\ldots last \end{vmatrix}$$

$$r_i = \begin{vmatrix} n \leftarrow threshold \\[4pt] root\left[\cot(n\cdot g) - \dfrac{\lambda^2\cdot\left[n+(i-1)\cdot\frac{\pi}{g}\right]^2 - \alpha_0\cdot\alpha_1}{(\alpha_0+\alpha_1)\cdot\left[\lambda\cdot\left[n+(i-1)\cdot\frac{\pi}{g}\right]\right]}\cdot n\right] + \\[4pt] \quad +(i-1)\cdot\dfrac{\pi}{g} \\[4pt] if\left[root\left[\cot(n\cdot g) - \dfrac{\lambda^2\cdot\left[n+(i-1)\cdot\frac{\pi}{g}\right]^2 - a_0\cdot a_1}{(a_0+a_1)\cdot\left[\lambda\cdot\left[n+(i-1)\cdot\frac{\pi}{g}\right]\right]}\cdot n\right]\right] > \\[4pt] \quad > threshold \\[4pt] threshold + (i-1)\cdot\dfrac{\pi}{g} \quad otherwise \end{vmatrix}$$

$$\begin{vmatrix} T \leftarrow T \ if \quad t < (j-1)\cdot\Delta t, 0, \\[6pt] \left(\dfrac{q\cdot f_f\cdot 3\cdot\sqrt{3}\cdot\Delta t}{\pi\cdot\sqrt{\pi}\cdot\frac{\lambda}{a}\cdot c_f\cdot\sqrt{\left((12\cdot a\cdot(t-(j-1)\Delta t)+a_f^{\,2})\cdot(12\cdot a\cdot(t-(j-1)\Delta t)+b_f^{\,2})\right)}}\cdot\right. \\[8pt] \left.\cdot\exp\left(-\left(\dfrac{(x+v\cdot(t-(j-1)\Delta t))^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\frac{1}{3}\cdot a_f^{\,2}} + \dfrac{y^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\frac{1}{3}\cdot b_f^{\,2}}\right)\right)\right) + \\[10pt] + \dfrac{q\cdot f_r\cdot 3\cdot\sqrt{3}\cdot\Delta t}{\pi\cdot\sqrt{\pi}\cdot\frac{\lambda}{a}\cdot c_r\cdot\sqrt{\left((12\cdot a\cdot(t-(j-1)\Delta t)+a_r^{\,2})\cdot(12\cdot a\cdot(t-(j-1)\Delta t)+b_r^{\,2})\right)}}\cdot \\[8pt] \cdot\exp\left(-\left(\dfrac{(x+v\cdot(t-(j-1)\Delta t))^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\frac{1}{3}\cdot a_r^{\,2}} + \dfrac{y^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\frac{1}{3}\cdot b_r^{\,2}}\right)\right)\cdot \\[10pt] \cdot\sum_{i=1}^{last} B_i\cdot C_i\cdot E_i\cdot\exp[a\cdot r_i^2\cdot(t-(j-1)\cdot\Delta t)] \end{vmatrix} \qquad (4)$$

For stationary system ($x_0 = x+vt$, $y_0 = y$, $z_0 = z$) the following algorithms (5) and () are presented:

C. from Cylindrical-Involution-Normal heat source model:

Stab_time = 19 sec $\Delta t = 0.05$ sec

$S = $ Stab_time$/ \Delta t = 380$ last $= 0$

$$T_n(x,y,z,t) = \begin{vmatrix} T \leftarrow 0 \\ \text{for } j \in 1...S = \dfrac{\text{Stab\_time}}{\Delta t} \\ a \leftarrow a(T) \\ \lambda \leftarrow \lambda(T) \\ \text{for } i \in 1...\text{last} \end{vmatrix}$$

$$r_i = \begin{vmatrix} n \leftarrow \text{threshold} \\ \text{root}\left[\dfrac{\cot(n \cdot g) - \dfrac{\lambda^2 \cdot \left[n+(i-1)\cdot\dfrac{\pi}{g}\right]^2 - \alpha_0 \cdot \alpha_1}{(\alpha_0+\alpha_1)\cdot\left[\lambda\cdot\left[n+(i-1)\cdot\dfrac{\pi}{g}\right]\right]}}{}\right] + \\ \quad + (i-1)\cdot\dfrac{\pi}{g} \\ \text{if}\left[\text{root}\left[\dfrac{\cot(n \cdot g) - \dfrac{\lambda^2 \cdot \left[n+(i-1)\cdot\dfrac{\pi}{g}\right]^2 - a_0 \cdot a_1}{(a_0+a_1)\cdot\left[\lambda\cdot\left[n+(i-1)\cdot\dfrac{\pi}{g}\right]\right]}}{}\right] \cdot n\right] > \\ \quad > \text{threshold} \\ \text{threshold} + (i-1)\cdot\dfrac{\pi}{g} \quad \text{otherwise} \end{vmatrix}$$

$$\begin{vmatrix} T \leftarrow T \quad \text{if} \\ t < (j-1)\cdot\Delta t, 0, q \cdot \\ \cdot k \dfrac{K_z \Delta t}{\pi \cdot \dfrac{\lambda}{a} \cdot (1-\exp(-K_z \cdot s))} \cdot \\ \cdot \dfrac{1}{[1+4\cdot a \cdot k(t-(j-1)\cdot\Delta t)]} \cdot \\ \cdot \exp\left[-k \dfrac{[(x+v\cdot(t-(j-1)\Delta t))^2 + y^2]}{1+4\alpha \cdot k(t-(j-1)\Delta t)}\right] \cdot \\ \cdot \sum_{i=1}^{\text{last}} B_i \cdot C_i \cdot D_i \cdot \\ \cdot \exp[a\cdot r_i^2 \cdot (t-(j-1)\cdot\Delta t)] \\ T \end{vmatrix} \quad (5)$$

D. from D-E configuration of source:

$$T_n(x,y,z,t) = \begin{vmatrix} T \leftarrow 0 \\ \text{for } j \in 1...S = \dfrac{\text{Stab\_time}}{\Delta t} \\ a \leftarrow a(T) \\ \lambda \leftarrow \lambda(T) \\ \text{for } i \in 1...\text{last} \end{vmatrix}$$

$$r_i = \begin{vmatrix} n \leftarrow \text{threshold} \\ \text{root}\left[\dfrac{\cot(n \cdot g) - \dfrac{\lambda^2 \cdot \left[n+(i-1)\cdot\dfrac{\pi}{g}\right]^2 - \alpha_0 \cdot \alpha_1}{(\alpha_0+\alpha_1)\cdot\left[\lambda\cdot\left[n+(i-1)\cdot\dfrac{\pi}{g}\right]\right]}}{}\right] + \\ \quad + (i-1)\cdot\dfrac{\pi}{g} \\ \text{if}\left[\text{root}\left[\dfrac{\cot(n \cdot g) - \dfrac{\lambda^2 \cdot \left[n+(i-1)\cdot\dfrac{\pi}{g}\right]^2 - a_0 \cdot a_1}{(a_0+a_1)\cdot\left[\lambda\cdot\left[n+(i-1)\cdot\dfrac{\pi}{g}\right]\right]}}{}\right] \cdot n\right] > \\ \quad > \text{threshold} \\ \text{threshold} + (i-1)\cdot\dfrac{\pi}{g} \quad \text{otherwise} \end{vmatrix}$$

$$\begin{vmatrix} T \leftarrow T \quad \text{if} \quad t < (j-1)\cdot\Delta t, 0, \\ \left(\dfrac{q\cdot f_f \cdot 3 \cdot\sqrt{3}\cdot\Delta t}{\pi\cdot\sqrt{\pi}\cdot\dfrac{\lambda}{a}\cdot c_f \cdot \sqrt{\left(12\cdot a\cdot(t-(j-1)\Delta t)+a_f^2\right)\cdot\left(12\cdot a\cdot(t-(j-1)\Delta t)+b_f^2\right)}} \cdot \right. \\ \left. \cdot \exp\left[-\left(\dfrac{(x_0-v\cdot(j-1)\Delta t))^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\dfrac{1}{3}\cdot a_f^2} + \dfrac{y_0^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\dfrac{1}{3}\cdot b_f^2}\right)\right]\right) + \\ + \dfrac{q\cdot f_r \cdot 3 \cdot\sqrt{3}\cdot\Delta t}{\pi\cdot\sqrt{\pi}\cdot\dfrac{\lambda}{a}\cdot c_r \cdot \sqrt{\left(12\cdot a\cdot(t-(j-1)\Delta t)+a_r^2\right)\cdot\left(12\cdot a\cdot(t-(j-1)\Delta t)+b_r^2\right)}} \cdot \\ \cdot \exp\left[-\left(\dfrac{(x_0-v\cdot(j-1)\Delta t))^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\dfrac{1}{3}\cdot a_r^2} + \dfrac{y_0^2}{4\cdot a\cdot(t-(j-1)\Delta t)+\dfrac{1}{3}\cdot b_r^2}\right)\right] \cdot \\ \cdot \sum_{i=1}^{\text{last}} B_i \cdot C_i \cdot E_i \exp[a\cdot r_i^2 \cdot(t-(j-1)\cdot\Delta t)] \end{vmatrix} \quad ()$$

The base procedure defined by algorithms (3) ÷ () requires cooperation with several sub-procedures such as *Stab_time*, last, calculation of roots r_i, $\lambda(T)$, $\alpha(T)$.

From essential point of view the sub-procedure *Stab_time* defines in time dimension the moment of time after which it estimated its value needed for stabilisation of thermal fields as follows:

- sub-procedure *Stab_time*

$$Stab\_time(x, y, z) =$$

$$
= \begin{bmatrix}
absolute \leftarrow 20 \\
settled \leftarrow 10 \\
stroke \leftarrow 1 \\
while \dfrac{T\_MOVE(x, y, z, absolute)}{T\_MOVE(x, y, z, settled)} < 1.02 \\
if \\
settled \leftarrow settled - stroke \\
while \dfrac{T\_MOVE(x, y, z, absolute)}{T\_MOVE(x, y, z, settled)} > 1.02 \\
otherwise \\
settled \leftarrow settled + stroke \\
\begin{bmatrix} \dfrac{(settled + stroke)}{T\_MOVE(x, y, z, absolute)} \\ \dfrac{}{T\_MOVE(x, y, z, settled + stroke)} \\ T\_MOVE(x, y, z, absolute) \end{bmatrix}
\end{bmatrix} \quad (7)
$$

Stab_time is compound sub-procedure realising calculation of wanted value of stabilised time for optional point in moving coordinates system. Parameters "absolute" and "settled" are preliminary set up values of start and end of partition of time, in which follows search of time stabilisation with "stroke" step.

The procedure is built in this way that last value of *Stab_time* does not depend on value "absolute" and "settled" but correct assumption of these values shorten time wanted for account of *Stab_time*.

Furthermore in the first main mini-procedure also cooperation is required with sub-procedures for inside calculation of $\lambda(T)$, $\alpha(T)$. The discrete values of $\lambda(T)$, $\alpha(T)$ are known and shown in Table 1 the matrices containing T and corresponding $\lambda(T)$, $\alpha(T)$ values are defined experimentally. Than with use of linear interpolation procedure, continuous functions were created and built in inside calculation sheet as follows:

- sub-procedure $\lambda(T)$

$$
A = \begin{array}{|c|c|}
\hline
\langle 1 \rangle & \langle 2 \rangle \\
\hline
0 & 0.6285 \\
100 & 0.5866 \\
200 & 0.5447 \\
300 & 0.5028 \\
400 & 0.4609 \\
500 & 0.419 \\
600 & 0.3771 \\
700 & 0.3487 \\
800 & 0.3268 \\
900 & 0.3226 \\
1000 & 0.3268 \\
1100 & 0.331 \\
1200 & 0.3352 \\
1300 & 0.3352 \\
1400 & 0.3352 \\
1500 & 0.3352 \\
\hline
\end{array}
\qquad
\lambda(T): = \begin{bmatrix}
i \leftarrow floor\left(\dfrac{T}{100}\right) \\
i \leftarrow 14 \text{ if } T \geq 1500 \\
\begin{bmatrix} \dfrac{(A_{i+2,2} - A_{i+1,2})}{(A_{i+2,1} - A_{i+1,1})} \cdot \\ \cdot(t - A_{i+1,1}) + A_{i+1,2} \end{bmatrix} \\
T := 1...1500
\end{bmatrix} \quad (8)
$$

- sub-procedure $\alpha(T)$

$$
C = \begin{array}{|c|c|}
\hline
\langle 1 & \langle 2 \rangle \\
\hline
0 & 0.19 \\
100 & 0.16 \\
200 & 0.13 \\
300 & 0.11 \\
400 & 0.093 \\
500 & 0.079 \\
600 & 0.062 \\
700 & 0.05 \\
768 & 0.034 \\
800 & 0.042 \\
900 & 0.055 \\
901 & 0.062 \\
1200 & 0.062 \\
1300 & 0.062 \\
1400 & 0.062 \\
1500 & 0.062 \\
\hline
\end{array}
=
\begin{bmatrix}
\alpha(T): = \\
i \leftarrow floor\left(\dfrac{T}{100}\right) \\
if \quad T < 768 \\
i \leftarrow 8 < if \ T \geq 768 \\
i \leftarrow 9 < if \ T \geq 800 \\
i \leftarrow 10 < if \ T \geq 900 \\
i \leftarrow 11 < if \ T \geq 901 \\
i \leftarrow 12 < if \ T \geq 1200 \\
\begin{bmatrix} \dfrac{(C_{i+2,2} - C_{i+1,2})}{(C_{i+2,1} - C_{i+1,1})} \cdot \\ \cdot(t - C_{i+1,1}) + C_{i+1,2} \end{bmatrix} \\
T := 0...1500
\end{bmatrix} \quad (9)
$$

On Fig. 1 are presented discrete values of $\lambda(T)$, $\alpha(T)$ and determined by continuous functions with used sub-procedures (8) and (9) for a. $\lambda(T)$, b. $\alpha(T)$.

We have high conformity of continuous functions and discrete value of $\lambda(T)$, $\alpha(T)$ from above-mentioned date on Fig. 1.
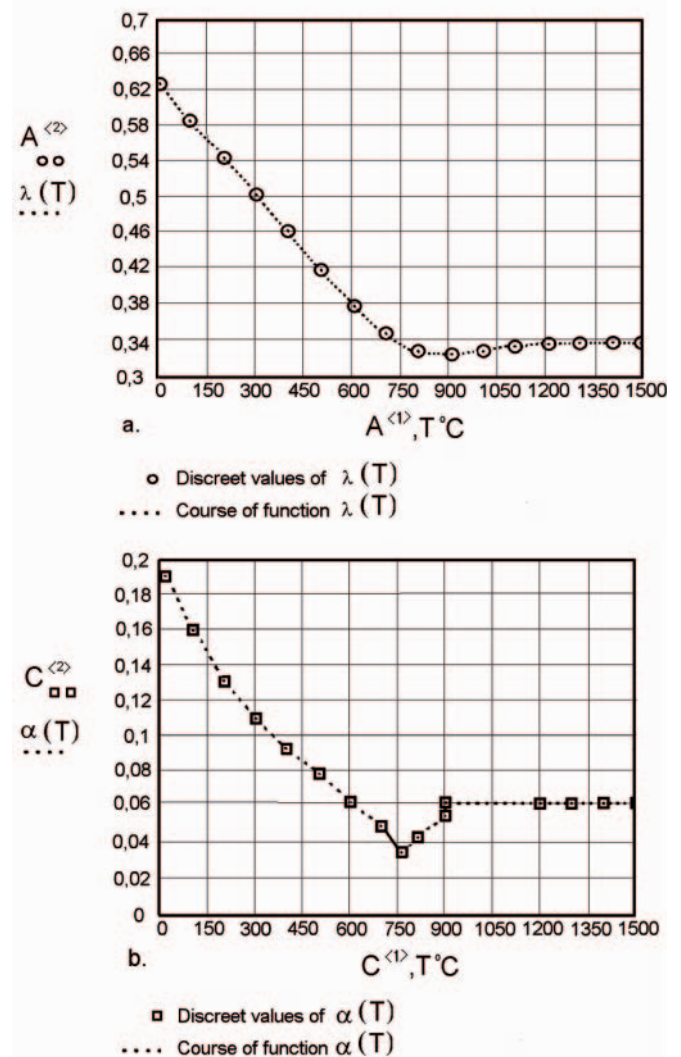

Fig. 1. *Values of **a.** λ(T) and **b.** α(T) in agreement with tab. 1 and continuous functions of for low carbon steel 0.1% C*

The sub-procedure "LAST" realises calculation of value of parameter "last" defining correctness of calculation in analytical meaning but it decides indirectly about amount of r_i roots which is taken into consideration in each computational step. It is possible to assure required accuracy of calculation when value of "last" is sufficiently large.

Sub-procedure "LAST" can find this value through analysis of moment of numeric convergence of neuralgic mathematical module in most adverse conditions at t = 0 and z = 0 as below:

- sub-procedure "LAST"

$$Last = \begin{bmatrix} i \rightarrow 1 \\ while \left[\left(\dfrac{Total_{i+1}}{Total_i} > 1.001 \right) + \\ + (Total_i < 0.96 \cdot max(W)) \right] \\ i \rightarrow i+1 \\ \begin{bmatrix} i \ if \ i > 15 \\ 15 \ otherwise \end{bmatrix} \end{bmatrix} \quad (10)$$

$$Pulse_i = \left[\frac{k\,K_z}{(1 - exp(-K \cdot s)) \cdot \dfrac{\lambda}{\alpha}} \right] cos(r_i \cdot z_0) + \\ + \frac{\alpha_0}{\lambda \cdot r_i} \cdot sin(r_i \cdot z_0) \cdot B_i C_i D_i \quad (11)$$

$$Total_i = \sum_{i=1}^{i} Pulse_i \quad (12)$$

Realisation of sub-procedure "LAST" relies on search of sufficiently large value "i". In order to set up condition of assumption converge of monotones growing series (from foundation for value 0.001) at necessity outreaching of 9% of maximum value "Total" in treated interval "i". It catches on that

minimal value of "i" and can't be smaller than 15. On Fig. 2 it is presented course of value "Pulse" and "Total".
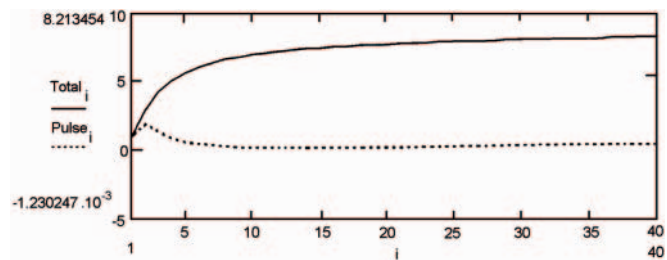

Fig. 2. Course of value "Pulse" and "Total" as function of growing value "i"

Furthermore let's notice that computational cycle is started with acceptance of value λ(T = 0), α(T = 0) but each next time step "Δt" change the values of λ(T), α(T) in agreement with current value of temperature T. Generation of value "r_i" is next computational step and for each time step, which is found in numeric way according to separate "sub-procedure r_i" which is directly insert in algorithms (3)÷() and simultaneously checks accuracy of calculation with used "threshold" parameter. The value of "threshold" parameter is established on very close zero but unzero.

For example in Tab. 2 some values are presented of roots r_i along with estimates of their accuracy for t = 0 and g = 1.2 cm, threshold = 0.0000001, α_0 = 0.02 W m^{-2}K^{-1}, α_1 = 0.01.

The final sub-procedure "Temperature – T" in algorithms (3) ÷ () summarises thermal fields from several heat energy impulses using a proper formula specific for appropriated H-S model.

EXAMPLES

A few examples of welding cycles in stationary co-ordinates system are shown. C-I-N and D-E configuration of source are used for analysis. The results from C-I-N and D-E models are compared with Rosenthal-Rykalin solution along with experimental results[5,] . Material parameters are accordance with Tab. 1 for low carbon steel.

Tab. 1. λ(T), c_p(T), ρ(T), and α(T) values in several temperatures for low carbon steel – 0.1%C

T °C	λ(T) Wcm^{-1}K^{-1}		T °C	ρ(T)·c_p(T) JK^{-1}c^{m-3}		T °C	α(T) cm^{2}s^{-1}
0	0.6285		0	3.307		0	0.190
100	0.5866		100	3.666		100	0.160
200	0.5447		200	4.190		200	0.130
300	0.5028		300	4.570		300	0.110
400	0.4609		400	4.950		400	0.093
500	0.4190		500	5.303		500	0.079
600	0.3771		600	6.082		600	0.062
700	0.3477		700	6.955		700	0.050
800	0.3268		768	9.809		768	0.034
900	0.3226		800	6.536		800	0.042
1000	0.3268		900	5.866		900	0.055
1100	0.3310		901	5.204		901	0.062
1200	0.3352		1200	5.406		1200	0.062
1300	0.3352		1300	5.406		1300	0.062
1400	0.3352		1400	5.406		1400	0.062
1500	0.3352		1500	5.406		1500	0.062

Tab. 2. *The assessment of a few r_i roots (11 of 60) for non-linear approach calculation at t = 0*

i	r_i values	r_i check
1	0.198811490	$5.153050313 \cdot 10^{-9}$
2	2.633099949	$3.597482134 \cdot 10^{-8}$
3	5.243573590	$6.028955113 \cdot 10^{-10}$
4	7.859042948	$6.232056649 \cdot 10^{-9}$
5	10.475772574	$5.70471002 \cdot 10^{-7}$
6	13.093007438	$5.118749868 \cdot 10^{-9}$
7	15.710495155	$3.640161481 \cdot 10^{-6}$
8	18.332817429	$2.0121055824 \cdot 10^{-7}$
9	20.945850074	$1.066098321 \cdot 10^{-8}$
10	23.563632979	$4.666276254 \cdot 10^{-10}$
11	26.1814807	$3.501158403 \cdot 10^{-6}$

Tab. 3. *Results of estimate temperature*

No of example	Temperature [°C]											
	1. experiment			2. C-I-N calcul.			3. D-E calcul.			4. R-R calcul.		
	T_{max}	T_{10s}	T_{20s}	T_{max}	T_{10s}	T_{20s}	T_{max}	T_{10s}	T_{20s}	T_{max}	T_{10s}	T_{20s}
1	1300	740	520	1410	660	420	1410	630	410	1610	360	250
2	1030	500	380	1110	470	330	1250	440	310	1560	380	265
3	1430	770	500	1470	730	470	1570	610	380	1820	470	340

Example 1.
Main parameters: q = 2400W; g = 0.4 cm; v = 0.75cm s^{-1}
C-I-N: s = 0.2 cm; Kz = 15 cm^{-1}; k = 12 cm^{-2}
D-E: f_f = 0.; a_f = 0.5 cm; b_f = 0.5 cm; c_f = 3.0 cm;
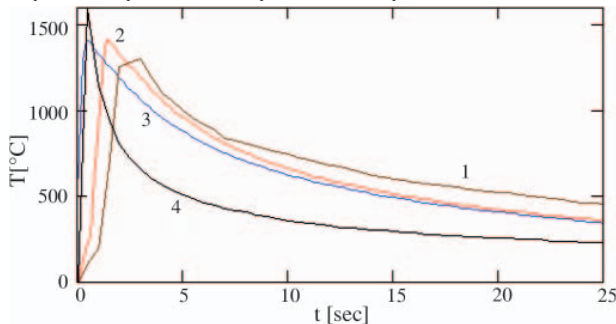f_r = 1.4; a_r = 1.0 cm; b_r = 0.5 cm; c_r = 3.0 cm – Fig.3.

Fig. 3. *Temperature change in points: x_0 = 0, y_0 = 0.1, z_0 = 0*
Symbols: 1 – experimental, 2 – C-I-N calculation, 3 – D-E calculation
4 – Rykalin-Rosenthal solution (plate model with the sector line source)

Example 2.
Main parameters: q = 3300 W; g = 0.8 cm; v = 0.5 cm s^{-1}
C-I-N: s = 0. cm; Kz = 5 cm^{-1}; k = 12 cm^{-2}
D-E: f_f = 0.; a_f = 0. cm; b_f = 0. cm; c_f = 4.2 cm;
f_r = 1.4; a_r = 1.2 cm; b_r = 0. cm; c_r = 4.2 cm – Fig. 4.
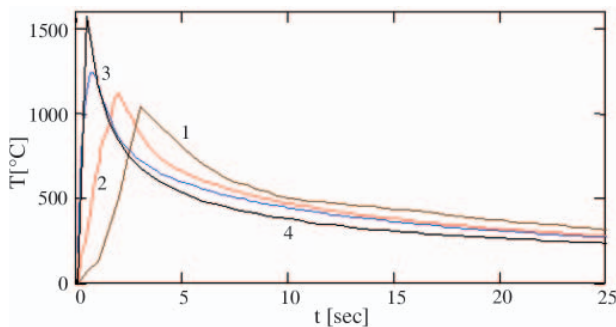

Fig. 4. *Temperature change in points: x_0 = 0, y_0 = 0.1, z_0 = 0*
Symbols: 1 – experimental, 2 – C-I-N calculation, 3 – D-E calculation
4 – Rykalin-Rosenthal solution (plate model with the sector line source)

Example 3.
Main parameters: q = 12000W; g = 0.8cm; v = 1.1 cm s^{-1}
C-I-N: s = 0. cm; Kz = 2 cm^{-1}; k = 8 cm^{-2}
D-E: f_f = 0.; a_f = 0. cm; b_f = 0.7 cm; c_f = .2 cm;
f_r = 1.4; a_r = 1.4 cm; b_r = 0.7 cm; c_r = .2 cm – Fig. 5.
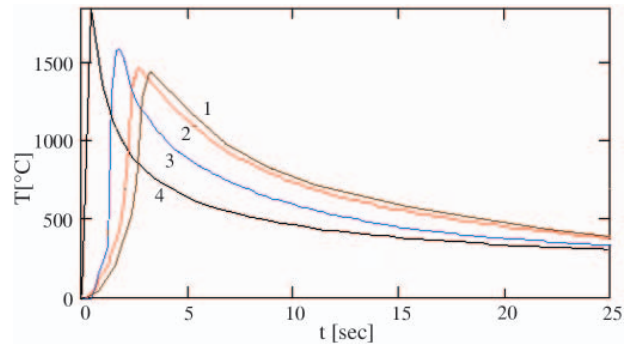

Fig. 5. *Temperature change in points: x_0 = 0, y_0 = 0.1, z_0 = 0*
Symbols: 1 – experimental, 2 – C-I-N calculation, 3 – D-E calculation
4 – Rykalin-Rosenthal solution (plate model with the sector line source)

For the comparison of account and experiment results, in Table 3, effects of estimated temperatures for examples 1 ÷ 3 are collected.

Temperature is defined for:
- T_{max} – maximum temperature,
- T_{10s} – temperature after 10 s,
- T_{20s} – temperature after 20 s.

In the first example we can see distinctly correspondence of assessed temperature and experiment when we used analytic-numerical method and equations (1a), (2a) with use C-I-N and D-E heat source models respectively. For above example it take a stand of difference of order 100 - 300°C for maximum temperature. Results got run away from these issues is analytical assessment of run of temperature by used of Rosenthal-Rykalin (R-R) solutions.

In the second example it takes a stand a similar situation but with certain difference in upper temperature. Highest

conformity with experiment in this range of temperature is for analytic-numerical solution and agreement with equation (1a) for C-I-N heat source model. Results of estimates temperature with used pure analytical R-R solutions definitely run away from analytic-numerical solution especially in upper temperatures where divergence in estimated T_{max} amount to 500°C. Results of third example are similar to second example in essential meaning.

The special feature of characteristic of above examples $1 \div 3$ depends on heat source power and line energy of welding ($1 - 3200$ J cm^{-1}, $2 - 00$ J cm^{-1} and $3 - 10909$ J cm^{-1}).

Furthermore these issues indicate on utility of analytic-numerical solution with adopted C-I-N heat source model rather for simulation of welding process of high concentrated energy used but may also be used for the simulation of arc welding process similarly as D-E heat source model. It confirms also laser welding process simulation [7].

CONCLUSIONS

In this work some extended consideration about analytic-numerical methods conforming has taken place.

It is obvious that:
- with an application of various heat source models one can obtain very effective temperature field solutions,
- with appropriate algorithms, calculations are very attractive, effective and can be quickly executed on PC computers,
- further impact should of course be put on still more detailed welding phenomenon analysis. The specificity of metal phase change and other complicated phenomena should be discovered and reflected in complex model in order to make more accurate and detailed analysis possible.

The results of proposed methods were compared with experimental data and Rosenthal-Rykalin solution. The accuracy of D-E and C-I-N results having in mind experimental data was discussed and there's no doubt that the accuracy of old solutions (R-R) seems to be out of date for these examples.

All this makes analytical solutions very competitive with numerical ones and makes them very useful in engineering practice.

REFERENCES

1. *Mathcad-User's Guide*. MathSoft, inc. 2001. Cambridge, MA02142. USA
2. B. Buchmayr: *Materials and engineering with Mathcad. Modelling and simulation in applications.* Springer-Verlag Berlin Heidelberg. 2002
3. H. Benker: *Mathematica with MathCAD*. Springer-Verlag Berlin Heidelberg. 1999
4. E. Ranatowski, A. Poćwiardowski: *Mathematical Modelling of Weld Phenomena 5*. (ed. H. Cejrak), pp. 725-742. 2000. IOM Communications
5. A. Marmołowski: *The optimal method of define of the welding temperature field in systems formulate*. PhD thesis. Technical University of Gdańsk. 2001
. A. Poćwiardowski: *The thermal identification of weld process with used analytical-numerical methods*. PhD thesis. Technical University of Bydgoszcz. 2002
7. E. Ranatowski: *Review of welding* No 8-10, pp. 152-155. 2002.

NOMENCLATURE

"absolute", s,	-	preliminary assumption of upper value of time stabilised
"settled", s,	-	preliminary assumption of lower value of time stabilised
"stroke" , s,	-	a step assumption on way of installation of time of stabilisation
T_MOVE (x, y, z ,t)	-	a temperature field in moving coordinate system, °C.

CONTACT WITH THE AUTHOR

Prof. Ranatowski Eugeniusz
Faculty of Mechanical Engineerig,
University of Technology and Life Science,
Prof. S. Kaliskiego 7
85-73 Bydgoszcz, POLAND
e-mail: ranatow@utp.edu.pl