Energy losses in the hydraulic rotational motor definitions and relations for evaluation of the efficiency of motor and hydrostatic drive

Zygmunt Paszota, Prof.
Gdansk University of Technology

Abstract

The evaluation methods of energy losses and efficiency of the hydraulic rotational motors for the hydrostatic drives, used so far in the scientific research and in the industrial practice, give wrong results because the parameters that the losses and efficiencies are a function of are themselves dependent on those losses.
The aim of the paper is to define the motor operating parameters, developed powers, energy losses and efficiencies and also to show the respective relations. Conclusions are drawn, based on the analyses of presented definitions and relations, on the motor energy investigations.

Keywords: hydrostatic drive; hydraulic motor; energy efficiency

INTRODUCTION

Evaluation of the energy behaviour of a hydraulic motor is an evaluation of its overall efficiency $\eta_M = f(n_M, M_M, \nu)$, i.e. evaluation of the overall efficiency η_M as a function of motor shaft speed n_M and load M_M and of the working fluid viscosity ν. This is also assessment of the value and proportions of the motor mechanical, volumetric and pressure losses deciding of the motor mechanical η_{Mm}, volumetric η_{Mv} and pressure η_{Mp} efficiency, where the product $\eta_M = \eta_{Mm} \, \eta_{Mv} \, \eta_{Mp}$ determines the motor overall efficiency η_M. The energy losses and the corresponding efficiencies η_{Mm}, η_{Mv} and η_{Mp} should be determined as a function of parameters having a direct impact on the particular losses and efficiencies.

Designers and makers of rotational hydraulic motors and hydrostatic systems have not had so far a tool to determine their energy behaviour in the $(0 \leq \overline{\omega}_M \langle \overline{\omega}_{M\max}, \; 0 \leq \overline{M}_M \langle \overline{M}_{M\max})$ field of change of the hydraulic motor shaft speed and load coefficients and in the $\nu_{\min} \leq \nu \leq \nu_{\max}$ field of change of the working fluid viscosity.

The rotational motor producers make erroneous routine evaluations of the following energy efficiencies and work parameters:
- the motor overall efficiency η_M as a function of the shaft speed n_M and motor pressure decrease Δp_M (e.g. [11÷14])
- the motor overall efficiency η_M as a product of the volumetric efficiency η_{Mv} and the so called „mechanical – hydraulic efficiency" η_{Mmh}, determined all the three as a function of the same parameters (e.g. [11, 12])
- motor shaft speed n_M as a function of the motor capacity Q_M and volumetric efficiency η_{Mv}, determined in turn as a function of the motor pressure decrease Δp_M (e.g. [11, 13])

- motor shaft torque M_M as a function of pressure decrease Δp_M and the so called „mechanical – hydraulic efficiency" η_{Mmh} of the motor (e.g. [11÷14])
- motor capacity Q_M as a function of the shaft speed n_M and volumetric efficiency η_{Mv}, determined in turn as a function of the motor pressure decrease Δp_M (e.g. [11÷13])
- motor shaft usefull power P_{Mu} as a function of the motor capacity Q_M and pressure decrease Δp_M and as a function of the motor overall efficiency η_M determined in turn as a function of the motor shaft speed n_M and pressure decrease Δp_M (e.g. [11÷13]).

The hydraulic motor researchers evaluate in a wrong way the losses arising in the motor:
- the motor torque M_{Mm} of mechanical losses as a function of the motor pressure decrease Δp_M and shaft speed n_M
- a sum of torque M_{Mm} of mechanical losses and the so called „torque of pressure losses" (resulting from the pressure losses Δp_M in the motor) – as a function of motor pressure decrease Δp_M and the shaft speed n_M
- the intensity Q_{Mv} of volumetric losses in the motor as a function of the motor pressure decrease Δp_M (or as a function of the motor shaft torque M_M) and as a function of the motor shaft speed n_M.

The evaluation methods of the energy losses and efficiency of the rotational hydraulic motors, used so far in the scientific research and in the industrial practice give wrong results because the parameters that the losses and efficiencies are a function of are themselves dependent of those losses.

There are very few informations of the motor makers presenting properly the motor overall efficiency $\eta_M = f(n_M,$

M_M) as a function of the motor shaft speed n_M and torque M_M at a specified fluid viscosity ν and presenting the impact of viscosity ν on the overall efficiency η_M (e.g. [10]).

It is a common deficiency that no information is given about the dependence of the motor mechanical, volumetric and pressure losses on the kinematic viscosity ν of the working fluid used in the hydrostatic drive system.

The fundamental reason of the erroneous evaluations are commonly accepted views on the research methodology and on the method of determining the energy losses in pumps and in hydraulic motors . That method is based, among others, on the traditional reading of the energy balance of a hydrostatic drive system from the Sankey diagram [1÷9]. The present unsatisfactory state is also effect of using simplified evaluations of the relations of particular losses to the motor or pump working parameters and to the working fluid viscosity.

Therefore, the aim of this paper is to define the work parameters, developed powers, losses and energy efficiency of a rotational hydraulic motor and also demonstrating their complex interdependence. The analysis of those definitions and relations will be a basis of conclusions regarding the investigations of motor energy characteristics.

ROTATIONAL HYDRAULIC MOTOR – WORK PARAMETERS, POWERS, ENERGY LOSSES, ENERGY EFFICIENCY – DEFINITIONS AND RELATIONS

– <u>Motor shaft rotational (angular) speed n_M (ω_M)</u> varies in the ($0 \leq \overline{\omega}_M \langle \overline{\omega}_{Mmax}$, $0 \leq \overline{M}_M \langle \overline{M}_{Mmax}$) field of the hydrostatic drive system operation. The instantaneous n_M (ω_M) value is required by the machine (device) driven by the motor. The instantaneous value of the n_M (ω_M) speed is independent of the instantaneous value of the M_M torque loading the motor shaft and also independent of the energy losses in the hydraulic motor and in the hydrostatic drive system.

– <u>M_M torque loading the motor shaft</u> varies in the ($0 \leq \overline{\omega}_M \langle \overline{\omega}_{Mmax}$, $0 \leq \overline{M}_M \langle \overline{M}_{Mmax}$) field of the hydrostatic drive system operation. The instantaneous value of M_M torque is required by the motor driven machine (device). The instantaneous value of M_M torque is independent of the instantaneous value of the required motor shaft speed n_M (ω_M) and also of the energy losses in a hydraulic motor and in the hydrostatic drive system.

– <u>Working fluid (hydraulic oil, oil-water emulsion) kinematic viscosity ν</u> changes in the $\nu_{min} \leq \nu \leq \nu_{max}$ range. The instantaneous value ν of the viscosity of fluid flux reaching the hydraulic motor is independent of the motor and of the energy losses in the motor.

– <u>Motor useful power P_{Mu}</u>, required on the motor shaft by the driven machine (device), is a product of the required M_M torque loading the motor shaft and the required shaft angular speed ω_M:

$$P_{Mu} = M_M \, \omega_M = 2\Pi \, M_M \, n_M \qquad (1)$$

The motor useful power P_{Mu} is independent of the energy losses in the hydraulic motor and the hydrostatic drive system.

– <u>M_{Mm} torque of mechanical losses in the motor</u>, occurring in the „shaft – working chambers" assembly, is a function of the required M_M torque loading the motor shaft and of the required shaft rotational speed n_M. The n_M speed influences the inertia forces of „shaft – working chambers" assembly elements and in effect the friction losses in the piston, satellite and vane motors. The M_{Mm} torque of losses is to a some extent also a function of the working fluid viscosity ν.

The impact of fluid viscosity on the mechanical losses in the „shaft – working chambers" assembly occurs mainly in the piston motors with fluid in the motor casing:

$$M_{Mm} = f (M_M, n_M, \nu) \qquad (2)$$

– <u>Power ΔP_{Mm} of mechanical losses in the motor</u>, occuring in the „shaft – working chambers" assembly, is a product of the M_{Mm} torque of mechanical losses and shaft angular speed ω_M:

$$\Delta P_{Mm} = M_{Mm} \, \omega_M = 2\Pi \, M_{Mm} \, n_M \qquad (3)$$

– <u>M_{Mi} indicated torque in the motor working chambers</u> (at the point of conversion of the working fluid pressure energy into mechanical energy of the „shaft – working chambers" assembly), required by the motor from the driving working fluid, must be greater than the M_M torque loading the motor shaft [required by the driven machine (device)] because of the necessity of balancing also the M_{Mm} torque of mechanical losses in the „shaft – working chambers" assembly. The M_{Mi} torque is equal to the sum of shaft torque M_M and M_{Mm} torque of mechanical losses. The indicated torque M_{Mi} requires a value of the product of decrease Δp_{Mi} of the indicated pressure in working chambers and the theoretical motor capacity q_{Mt} per one shaft revolution (theoretical motor working volume V_{Mt}) in accordance with the expression:

$$\frac{\Delta p_{Mi} \, q_{Mt}}{2\Pi} = M_{Mi} = M_M + M_{Mm} \qquad (4)$$

The M_{Mi} torque indicated in the motor working chambers is not a function of the decrease Δp_{Mi} and of theoretical motor capacity q_{Mt} per one shaft revolution.

For evaluation of the M_{Mi} torque indicated in the motor working chambers a formula can be used relating the M_M torque loading the motor shaft with the known motor mechanical efficiency η_{Mm} [formula (11)]:

$$\frac{\Delta p_{Mi} \, q_{Mt}}{2\Pi} = M_{Mi} = \frac{M_M}{\eta_{Mm}} \qquad (5)$$

with: $\eta_{Mm} = f (M_{Mm}, M_M) = f (M_M, n_M, \nu)$

i.e. a formula where mechanical efficiency η_{Mm} is defined as a function of parameters influencing the M_{Mm} torque of mechanical losses in the „shaft – working chambers" assembly and as a function of the M_M torque loading the motor shaft.

– <u>Pressure decrease Δp_{Mi} indicated in the motor working chambers</u> is a function of the required M_{Mi} torque indicated in the chambers and theoretical capacity q_{Mt} per one shaft revolution:

$$\Delta p_{Mi} = \frac{2\Pi \, M_{Mi}}{q_{Mt}} = \frac{2\Pi (M_M + M_{Mm})}{q_{Mt}} \qquad (6)$$

Therefore, the pressure decrease Δp_{Mi} indicated in the motor working chambers (with determined theoretical capacity q_{Mt} per one shaft revolution) is a function of the required M_M torque loading the motor shaft and the M_{Mm} torque of mechanical losses in the „shaft – working chambers" assembly. The pressure decrease Δp_{Mi} is indirectly a function of the shaft rotational speed n_M and a function of the working fluid viscosity ν, which have an impact (apart from M_M) on the M_{Mm} torque of mechanical losses:

$$\Delta p_{Mi} = f (M_M, M_{Mm}) = f (M_M, n_M, \nu) \qquad (7)$$

For evaluation of the decrease Δp_{Mi} of pressure indicated in the motor working chambers (with determined theoretical

capacity q_{Mt} per one shaft revolution) a formula can be used relating the M_M torque loading the motor shaft with the known motor mechanical efficiency η_{Mm} [formula (11)]:

$$\Delta p_{Mi} = \frac{2\Pi M_M}{q_{Mt}\, \eta_{Mm}} \qquad (8)$$

with: $\eta_{Mm} = f\,(M_{Mm}, M_M) = f\,(M_M, n_M, \nu)$

i.e. a formula where mechanical efficiency η_{Mm} is defined as a function of parameters influencing the M_{Mm} torque of mechanical losses in the „shaft – working chambers" assembly and as a function of the M_M torque loading the motor shaft.

– P_{Mi} indicated power in the motor working chambers is required by the motor from the driving working fluid at the point of conversion of the working fluid pressure energy into mechanical energy of the „shaft – working chambers" assembly. The P_{Mi} is equal to the product of the M_{Mi} torque indicated in the chambers and the shaft angular speed ω_M. The power P_{Mi} indicated in the working chamber is a sum of useful power P_{Mu} [required on the motor shaft by the driven machine (device)] and the power ΔP_{Mm} of mechanical losses in the „shaft – working chambers" assembly:

$$\frac{\Delta p_{Mi}\, q_{Mt}}{2\Pi}\, \omega_M = \Delta p_{Mi}\, q_{Mt}\, n_M = P_{Mi} =$$
$$= M_{Mi}\, \omega_M = (M_M + M_{Mm})\, \omega_M = P_{Mu} + \Delta P_{Mm} \qquad (9)$$

The P_{Mi} power indicated in the motor working chambers is not a function of the decrease Δp_{Mi} of pressure indicated in the chambers and of theoretical motor capacity q_{Mt} per one shaft revolution.

For evaluation of the P_{Mi} power indicated in the motor working chambers a formula can be used relating the motor shaft useful power P_{Mu} with the known motor mechanical efficiency η_{Mm} [formula (11)]:

$$P_{Mi} = \frac{P_{Mu}}{\eta_{Mm}} \qquad (10)$$

with: $\eta_{Mm} = f\,(M_{Mm}, M_M) = f\,(M_M, n_M, \nu)$

i.e. a formula, where mechanical efficiency η_{Mm} is defined as a function of parameters influencing the M_{Mm} torque of mechanical losses in the „shaft – working chambers" assembly and as a function of the M_M torque loading the motor shaft.

– The motor mechanical efficiency η_{Mm} is a ratio of useful power P_{Mu} on the shaft [required by the motor driven machine (device)] to the power P_{Mi} indicated in the motor working chambers (required by the motor of the driving fluid at the point of conversion (change) of the working fluid pressure energy into the mechanical energy of the „shaft – working chambers" assembly). The η_{Mm} efficiency can be also determined by the ratio of motor shaft torque M_M to the torque M_{Mi} indicated in the working chambers:

$$\eta_{Mm} = \frac{P_{Mu}}{P_{Mi}} = \frac{P_{Mu}}{P_{Mu} + \Delta P_{Mm}} =$$
$$= \frac{M_M\, \omega_M}{(M_M + M_{Mm})\, \omega_M} = \frac{M_M}{M_M + M_{Mm}} = \frac{M_M}{M_{Mi}} \qquad (11)$$

The motor mechanical efficiency η_{Mm} is a function of torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and the shaft loading torque M_M. Therefore, the η_{Mm} efficiency is a function of torque M_M and shaft rotational speed n_M and a function of working fluid viscosity ν, which influences (apart from M_M) the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly:

$$\eta_{Mm} = f\,(M_{Mm}, M_M) = f\,(M_M, n_M, \nu) \qquad (12)$$

because: $M_{Mm} = f\,(M_M, n_M, \nu)$.

The motor mechanical efficiency η_{Mm} can be also evaluated from the formula:

$$\eta_{Mm} = \frac{2\Pi M_M}{\Delta p_{Mi}\, q_{Mt}} \qquad (13)$$

However, mechanical efficiency η_{Mm} is not a function of the decrease Δp_{Mi} of pressure indicated in the motor working chambers and of the motor theoretical capacity q_{Mt} per one shaft revolution.

– Intensity Q_{Mv} of the motor volumetric losses in the working chambers takes into account internal volumetric losses (between the chamber inlet channel and chamber outlet channel) and external volumetric losses (between chambers and casing and then led out of the casing). The intensity Q_{Mv} of motor volumetric losses (with determined theoretical capacity q_{Mt} per one shaft revolution) is a function of the decrease Δp_{Mi} of pressure indicated in the chambers and, to some extent, of the shaft rotational speed n_M as well as working fluid viscosity ν:

$$Q_{Mv} = f\,(\Delta p_{Mi}, n_M, \nu) \qquad (14)$$

The intensity Q_{Mv} of volumetric losses in the motor working chambers is a complex function of torque M_M and motor shaft speed n_M and also working fluid viscosity ν, i.e. of parameters independent of the motor and motor losses. The decrease Δp_{Mi} of pressure indicated in the chambers, influencing directly Q_{Mv} [formula (14)], is a function [formula (6)] of the shaft torque M_M and of torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and, in turn, torque M_{Mm} of the losses [formula (2)] is a function of torque M_M and of motor shaft speed n_M and also of the working fluid viscosity ν. At the same time, the impact of shaft speed n_M and working fluid viscosity ν on the intensity Q_{Mv} of the volumetric losses in working chambers differs from the impact of n_M and ν on the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly. A direct evaluation of the relation of intensity Q_{Mv} of volumetric losses in the motor working chambers to the motor shaft torque M_M and speed n_M and also to the working fluid viscosity ν would be unjustified and wrong because it would be under a complex impact of the torque M_{Mm} of mechanical losses.

– Power ΔP_{Mv} of the motor volumetric losses, in the working chambers, is a product of decrease Δp_{Mi} of pressure indicated in the chambers and intensity Q_{Mv} of volumetric losses in the chambers (on the assumption that the external volumetric losses are small and negligible from the energy point of view):

$$\Delta P_{Mv} = \Delta p_{Mi}\, Q_{Mv} \qquad (15)$$

– Motor capacity Q_M required by the motor from the driving fluid must be greater than the product $q_{Mt}\, n_M$ [theoretical capacity q_{Mt} per one shaft resolution and motor shaft rotational speed n_M required by the motor driven machine (device)] because of the necessity of balancing also the intensity Q_{Mv} of volumetric losses in the motor working chambers. Capacity Q_M is equal to the sum of intensity $q_{Mt}\, n_M$ and intensity Q_{Mv}:

$$Q_M = q_{Mt}\, n_M + Q_{Mv} \qquad (16)$$

Evaluation of the motor capacity Q_M (with determined theoretical capacity q_{Mt} per one shaft revolution) can be performed with a formula including the motor shaft rotational speed n_M required by the motor driven machine and known motor volumetric efficiency η_{Mv} [formula (23)]:

$$Q_M = \frac{q_{Mt}\, n_M}{\eta_{Mv}} \qquad (17)$$

with: $\eta_{Mv} = f(Q_{Mv}, n_M) = f(\Delta p_{Mi}, n_M, \nu)$

i.e. a formula, where the volumetric efficiency η_{Mv} is determined as a function of parameters influencing the intensity Q_{Mv} of volumetric losses in the working chambers and as a function of the motor rotational speed n_M.

– Power P_{Mci} of the working fluid absorbed by the motor in working chambers is required by the motor from the driving fluid as a difference between power $p_{M1i} Q_M$ of the fluid inflowing in the working chambers from the inlet channel and power $p_{M2i} Q_M$ of the fluid outflowing from the chambers to the outlet channel. Assuming that the external volumetric losses are small and negligible from the energy point of view, it may be accepted that the intensity of the outflowing flux is equal to the intensity Q_M of the inflowing flux. Therefore, power P_{Mci} may be determined as a product of the decrease Δp_{Mi} of pressure indicated in working chambers and the motor capacity Q_M. Power P_{Mci} must be greater than power P_{Mi} indicated in the chambers (required by the motor from the driving fluid at the point of conversion (change) of pressure energy of working fluid into the mechanical energy of the „shaft – working chambers" assembly) because of the necessity of balancing also the power ΔP_{Mv} of volumetric losses in the chambers. Power P_{Mci} is equal to the sum of power P_{Mi} and power ΔP_{Mv}:

$$
\begin{aligned}
P_{Mci} &= p_{M1i}\, Q_M - p_{M2i}\, Q_M = \\
&= \Delta p_{Mi}\, Q_M = \Delta p_{Mi}\, (q_{Mt}\, n_M + Q_{Mv}) = \quad (18) \\
&= \Delta p_{Mi}\, q_{Mt}\, n_M + \Delta p_{Mi}\, Q_{Mv} = P_{Mi} + \Delta P_{Mv}
\end{aligned}
$$

Power P_{Mci} of the working fluid consumed by the motor in working chambers is a sum of useful power P_{Mu} [required on the motor shaft by driven machine (device)], power ΔP_{Mm} of mechanical losses in the „shaft – working chambers" assembly and power ΔP_{Mv} of volumetric losses in the motor working chambers:

$$P_{Mci} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} \qquad (19)$$

After replacing in the equation (19) the useful power P_{Mu}, power ΔP_{Mm} of mechanical losses and power ΔP_{Mv} of volumetric losses by the expressions relating those powers to the parameters and losses deciding of their values, a picture can be obtained of the impact of those parameters and losses on power P_{Mci} consumed in the working chambers:

$$
\begin{aligned}
P_{Mci} &= M_M\, \omega_M + M_{Mm}\, \omega_M + \Delta p_{Mi}\, Q_{Mv} = \\
&= M_M\, \omega_M + M_{Mm}\, \omega_M + \frac{2\Pi (M_M + M_{Mm})}{q_{Mt}}\, Q_{Mv} = \quad (20) \\
&= 2\Pi (M_M + M_{Mm}) \left(n_M + \frac{Q_{Mv}}{q_{Mt}} \right)
\end{aligned}
$$

Power P_{Mci} of working fluid consumed by the motor in working chambers can be evaluated by means of a formula

expressing the ratio of power P_{Mi} indicated in the working chambers to a known volumetric efficiency η_{Mv} of the motor [formula (23)]:

$$P_{Mci} = \frac{P_{Mi}}{\eta_{Mv}} \qquad (21)$$

with: $\eta_{Mv} = f(Q_{Mv}, n_M) = f(\Delta p_{Mi}, n_M, \nu)$

i.e. a formula where volumetric efficiency η_{Mv} is determined as a function of parameters influencing the intensity Q_{Mv} of volumetric losses in the working chambers and as a function of the motor rotational speed n_M.

Power P_{Mci} of the working fluid consumed by motor in the working chambers can be evaluated also from the known useful power P_{Mu} on the motor shaft, known mechanical efficiency η_{Mm} [formula (11)] and known volumetric efficiency η_{Mv} of the motor [formula (23)]:

$$P_{Mci} = \frac{P_{Mu}}{\eta_{Mm}\, \eta_{Mv}} \qquad (22)$$

with: $\eta_{Mm} = f(M_{Mm}, M_M) = f(M_M, n_M, \nu)$
and: $\eta_{Mv} = f(Q_{Mv}, n_M) = f(\Delta p_{Mi}, n_M, \nu)$

i.e. a formula where mechanical efficiency η_{Mm} is determined as a function of parameters influencing the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and as a function of torque M_M loading the motor shaft, and the volumetric efficiency η_{Mv} is determined as a function of parameters influencing the intensity Q_{Mv} of volumetric losses in the working chambers and of the motor rotational speed n_M.

– Volumetric efficiency η_{Mv} of the motor is a ratio of power P_{Mi} indicated in the motor working chambers to power P_{Mci} of working fluid consumed by the motor in the chambers:

$$
\begin{aligned}
\eta_{Mv} &= \frac{P_{Mi}}{P_{Mci}} = \frac{P_{Mi}}{P_{Mi} + \Delta P_{Mv}} = \\
&= \frac{\Delta p_{Mi}\, q_{Mt}\, n_M}{\Delta p_{Mi}\, q_{Mt}\, n_M + \Delta p_{Mi}\, Q_{Mv}} = \quad (23) \\
&= \frac{q_{Mt}\, n_M}{q_{Mt}\, n_M + Q_{Mv}} = \frac{q_{Mt}\, n_M}{Q_M}
\end{aligned}
$$

The motor volumetric efficiency η_{Mv} (with determined theoretical capacity q_{Mt} per one shaft revolution) is a function of intensity Q_{Mv} of volumetric losses in the motor and of the motor shaft rotational speed n_M. Therefore, efficiency η_{Mv} is a function of the decrease Δp_{Mi} of pressure indicated in working chambers and a function of the rotational speed n_M as well as a function of the working fluid viscosity ν (which have an impact on the intensity Q_{Mv} of volumetric losses) and also directly a function of rotational speed n_M:

$$\eta_{Mv} = f(Q_{Mv}, n_M) = f(\Delta p_{Mi}, n_M, \nu) \qquad (24)$$

because: $Q_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$.

– Losses Δp_{Mp} of working fluid pressure in the motor channels are a sum of two pressure losses i.e. loss Δp_{Mp1} of pressure in the inlet channel (between the motor inlet point and working chambers) and loss Δp_{Mp2} of pressure in the outlet channel (between the working chambers and the motor outlet point). Losses Δp_{Mp} are a function of motor capacity Q_M and of working fluid viscosity ν:

$$\Delta p_{Mp} = \Delta p_{Mp1} + \Delta p_{Mp2} = f(Q_M, \nu) \qquad (25)$$

Losses Δp_{Mp} of working fluid pressure in the motor channels

are a complex function of the motor shaft speed n_M and torque M_M and the working fluid viscosity ν, i.e. parameters independent of the motor and of losses in it. The motor capacity Q_M, having a direct impact on Δp_{Mp} [formula (14)], is a function [formula (16)] of the shaft rotational speed n_M and of intensity of volumetric losses Q_{Mv} in the working chambers. The decrease Δp_{Mi} of pressure indicated in the working chambers, having a direct impact on Q_{Mv} [formula (14)], is a function [formula (6)] of the shaft torque M_M and of torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly, and the torque M_{Mm} of mechanical losses [formula (2)] is in turn a function of the motor shaft torque M_M and speed n_M and of the viscosity ν of working fluid. At the same time, the impact of working fluid viscosity ν on the losses Δp_{Mp} of fluid pressure in the channels differs from the impact of viscosity ν on the intensity Q_{Mv} of volumetric losses in the working chambers and from the impact of ν on the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly. Also the impact of shaft speed n_M on the intensity Q_{Mv} of volumetric losses in the working chambers differs from the impact of n_M on the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly. Direct evaluation of the dependence of pressure losses Δp_{Mp} of the working fluid in the motor channels on the motor shaft speed n_M and torque M_M and on the viscosity ν of working fluid would be unjustified and wrong, because it would be under a complex impact of the intensity Q_{Mv} of volumetric losses in working chambers and of torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly.

– Power ΔP_{Mp} of pressure losses in the motor, in the motor channels, with the assumption that external volumetric losses are small and negligible from the energy point of view, is a product of the pressure losses Δp_{Mp} in the channels and the motor capacity Q_M:

$$\Delta P_{Mp} = \Delta p_{Mp}\, Q_M \qquad (26)$$

– Decrease Δp_M of pressure in the motor (with determined theoretical capacity q_{Mt} per one shaft revolution), required by the motor from the driving working fluid, must be greater than decrease Δp_{Mi} of pressure indicated in the working chambers (required by the torque M_{Mi} indicated in the chambers) because of the necessity of balancing also the losses Δp_{Mp} of pressure in the motor channels. Decrease Δp_M is equal to the sum of the indicated decrease Δp_{Mi} and losses Δp_{Mp}:

$$\Delta p_M = \Delta p_{Mi} + \Delta p_{Mp} \qquad (27)$$

Replacing in the equation (27) the decrease Δp_{Mi} of pressure indicated in the working chambers with expression (6), we obtain the dependence of decrease Δp_M of pressure in the motor on the required torque M_M loading the motor shaft and on the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and also on the pressure losses Δp_{Mp} in the motor channels:

$$\Delta p_M = \frac{2\Pi\,(M_M + M_{Mm})}{q_{Mt}} + \Delta p_{Mp} \qquad (28)$$

The decrease Δp_M of pressure in the motor can be evaluated by means of a formula expressing the ratio of decrease Δp_{Mi} of pressure indicated in the working chambers to the known pressure efficiency η_{Mp} of the motor [formula (37)]:

$$\Delta p_M = \frac{\Delta p_{Mi}}{\eta_{Mp}} \qquad (29)$$

with: $\eta_{Mp} = f(\Delta p_{Mp},\, \Delta p_{Mi}) = f(Q_M,\, \Delta p_{Mi},\, \nu)$

i.e. formula where the motor pressure efficiency η_{Mp} is defined as a function of parameters influencing the losses Δp_{Mp} of working fluid pressure in the channels and as a function of the decrease Δp_{Mi} of pressure indicated in the working chambers.

Decrease Δp_M of pressure in the motor can be evaluated also from a known torque M_M loading the motor shaft, from a known mechanical efficiency η_{Mm} [formula (11)] and a known motor pressure efficiency η_{Mp} [formula (37)]:

$$\Delta p_M = \frac{2\Pi\, M_M}{q_{Mt}\, \eta_{Mm}\, \eta_{Mp}} \qquad (30)$$

with: $\eta_{Mm} = f(M_{Mm},\, M_M) = f(M_M,\, n_M,\, \nu)$
and: $\eta_{Mp} = f(\Delta p_{Mp},\, \Delta p_{Mi}) = f(Q_M,\, \Delta p_{Mi},\, \nu)$

i.e. formula, where the mechanical efficiency η_{Mm} is defined as a function of parameters influencing the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and a function of torque M_M loading the motor shaft and the pressure efficiency η_{Mp} is defined as a function of parameters influencing the losses Δp_{Mp} of pressure in the channels and as a function of decrease Δp_{Mi} of pressure indicated in the motor working chambers.

– Working fluid power P_{Mc} consumed by the motor must be greater than power P_{Mci} (consumed by the motor in the working chambers) because of the necessity of balancing also the power ΔP_{Mp} of pressure losses in the motor channels. Power P_{Mc} is equal to the sum of power P_{Mci} and power ΔP_{Mp} of the losses. Power P_{Mc} is a product of the decrease Δp_M of pressure in the motor and motor capacity Q_M:

$$\begin{aligned} P_{Mc} &= \Delta p_M\, Q_M = (\Delta p_{Mi} + \Delta p_{Mp})\, Q_M = \\ &= \Delta p_{Mi}\, Q_M + \Delta p_{Mp}\, Q_M = P_{Mci} + \Delta P_{Mp} \end{aligned} \qquad (31)$$

The working fluid power P_{Mc} consumed by the motor is a sum of useful power P_{Mu} (required on the motor shaft by the driven machine (device)], power ΔP_{Mm} of mechanical losses in the „shaft – working chambers" assembly, power ΔP_{Mv} of volumetric losses in the working chambers and power ΔP_{Mp} of pressure losses in the motor channels:

$$P_{Mc} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} + \Delta P_{Mp} \qquad (32)$$

After replacing in equation (32) the useful power P_{Mu} and power ΔP_{Mm} of mechanical losses, power ΔP_{Mv} of volumetric losses and power ΔP_{Mp} of pressure losses with formulae expressing dependence on parameters and losses deciding of the values of those powers, we can obtain a picture of the impact of parameters and losses on the consumed power P_{Mc}:

$$\begin{aligned} P_{Mc} &= M_M\, \omega_M + M_{Mm}\, \omega_M + \Delta p_{Mi}\, Q_{Mv} + \Delta p_{Mp}\, Q_M = \\ &= M_M\, \omega_M + M_{Mm}\, \omega_M + \frac{2\Pi(M_M + M_{Mm})}{q_{Mt}} Q_{Mv} + \\ &\quad + \Delta p_{Mp}\left(q_{Mt}\, n_M + Q_{Mv}\right) = \\ &= 2\Pi(M_M + M_{Mm})\left(n_M + \frac{Q_{Mv}}{q_{Mt}}\right) + \\ &\quad + \Delta p_{Mp}\left(q_{Mt}\, n_M + Q_{Mv}\right) \end{aligned} \qquad (33)$$

The expression describing the working fluid power P_{Mc} consumed by the motor can be also obtained from the

product of decrease Δp_M of pressure in the motor [formula (28)] and motor capacity Q_M [formula (16)]:

$$P_{Mc} = \Delta p_M Q_M =$$

$$= \left[\frac{2\Pi (M_M + M_{Mm})}{q_{Mt}} + \Delta p_{Mp} \right] (q_{Mt} n_M + Q_{Mv}) \quad (34)$$

Expressions (33) and (34) are equivalent.
Evaluation of the working fluid power P_{Mc} consumed by the motor can be performed with a formula expressing the ratio of working fluid power P_{Mci} consumed by the motor in the working chambers to the motor pressure efficiency η_{Mp} [formula (37)]:

$$P_{Mc} = \frac{P_{Mci}}{\eta_{Mp}} \quad (35)$$

with: $\eta_{Mp} = f(\Delta p_{Mp}, \Delta p_{Mi}) = f(Q_M, \Delta p_{Mi}, \nu)$

i.e. a formula where the pressure efficiency η_{Mp} is defined as a function of parameters influencing the losses Δp_{Mp} of working fluid pressure in the channels and as a function of decrease Δp_{Mi} of pressure indicated in the motor working chambers.
The working fluid power P_{Mc} consumed by the motor can be evaluated from the known useful power P_{Mu} on the motor shaft, a known mechanical efficiency η_{Mm} [formula (11)], known volumetric efficiency η_{Mv} [formula (23)] and known pressure efficiency η_{Mp} [formula (37)] of the motor:

$$P_{Mc} = \frac{P_{Mu}}{\eta_{Mm}\, \eta_{Mv}\, \eta_{Mp}} \quad (36)$$

with: $\eta_{Mm} = f(M_{Mm}, M_M) = f(M_M, n_M, \nu)$
$\quad\quad \eta_{Mv} = f(Q_{Mv}, n_M) = f(\Delta p_{Mi}, n_M, \nu)$
and: $\quad \eta_{Mp} = f(\Delta p_{Mp}, \Delta p_{Mi}), = f(Q_M, \Delta p_{Mi}, \nu)$.

In formula (36), the mechanical efficiency η_{Mm} is defined as a function of parameters influencing the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and as a function of torque M_M loading the motor shaft. The volumetric efficiency η_{Mv} is defined as a function of parameters influencing the intensity Q_{Mv} of volumetric losses in the working chambers and as a function of the motor shaft rotational speed n_M. The pressure efficiency η_{Mp} is defined as a function of parameters influencing the pressure losses Δp_{Mp} of working fluid in the channels and as a function of the decrease Δp_{Mi} of pressure indicated in the motor working chambers.

– The motor pressure efficiency η_{Mp} is a ratio of the working fluid power P_{Mci} consumed by the motor in the working chambers to the power P_{Mc} consumed by the motor:

$$\eta_{Mp} = \frac{P_{Mci}}{P_{Mc}} = \frac{P_{Mci}}{P_{Mci} + \Delta P_{Mp}} =$$

$$= \frac{\Delta p_{Mi}\, Q_M}{(\Delta p_{Mi} + \Delta p_{Mp})\, Q_M} = \frac{\Delta p_{Mi}}{\Delta p_{Mi} + \Delta p_{Mp}} = \frac{\Delta p_{Mi}}{\Delta p_M} \quad (37)$$

Therefore, the pressure efficiency can by presented as a ratio of the decrease Δp_{Mi} of pressure indicated in the working chambers to the decrease Δp_M of pressure in the motor. The motor pressure efficiency η_{Mp} is a function of losses Δp_{Mp} of the working fluid pressure in motor channel and decrease Δp_{Mi} of pressure indicated in the motor working chambers. Therefore, efficiency η_{Mp} is a function of the motor capacity Q_M and a function of the working fluid viscosity ν (which influence the losses Δp_{Mp} of working fluid pressure in the

channels) as well as a function of the decrease Δp_{Mi} of pressure in the motor working chambers:

$$\eta_{Mp} = f(\Delta p_{Mp}, \Delta p_{Mi}) = f(Q_M, \Delta p_{Mi}, \nu) \quad (38)$$

because: $\Delta p_{Mp} = f(Q_M, \nu)$.

– The motor overall efficiency η_M is a ratio of the useful power P_{Mu} on the motor shaft required by the driven machine (device) to the power P_{Mc} consumed by the motor:

$$\eta_M = \frac{P_{Mu}}{P_{Mc}} = \frac{M_M\, \omega_M}{\Delta p_M\, Q_M} = \frac{2\Pi\, M_M\, n_M}{\Delta p_M\, Q_M} \quad (39)$$

Replacing in formula (39) the power P_{Mc} consumed by the motor with equations describing its dependence on the useful power P_{Mu} and on the powers ΔP_{Mm}, ΔP_{Mv} and ΔP_{Mp} of the energy losses in the motor (with a determined theoretical capacity q_{Mt} per one shaft revolution), we obtain the expressions describing the motor overall efficiency η_M as a function of the losses:

with reference to equation (32):

$$\eta_M = \frac{P_{Mu}}{P_{Mc}} = \frac{P_{Mu}}{P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} + \Delta P_{Mp}} \quad (40)$$

with reference to equation (33):

$$\eta_M = \frac{2\Pi\, M_M\, n_M}{2\Pi (M_M + M_{Mm}) \left(n_M + \dfrac{Q_{Mv}}{q_{Mt}} \right) + \Delta p_{Mp} (q_{Mt}\, n_M + Q_{Mv})} \quad (41)$$

with reference to equation (34):

$$\eta_M = \frac{2\Pi\, M_M\, n_M}{\left[\dfrac{2\Pi (M_M + M_{Mm})}{q_{Mt}} + \Delta p_{Mp} \right] (q_{Mt}\, n_M + Q_{Mv})} \quad (42)$$

Expressions (41) and (42) are equivalent.

The motor overall efficiency η_M is therefore a function of torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly, intensity Q_{Mv} of volumetric losses in the working chambers and losses Δp_{Mp} of pressure in the motor channels. The η_M efficiency is also a function of the motor shaft torque M_M and speed n_M required by the driven machine (device):

$$\eta_M = f(M_{Mm}, Q_{Mv}, \Delta p_{Mp}, M_M, n_M) \quad (43)$$

Torque $M_{Mm} = f(M_M, n_M, \nu)$ of mechanical losses from friction of elements in the motor „shaft – working chambers" assembly is a function of the required torque M_M loading the motor shaft. In the piston, satellite and vane motors, torque M_{Mm} of the losses is also a function of the required speed n_M influencing the inertia forces in the „shaft – working chambers" assembly and in effect the forces of friction between those elements. In the piston motors in particular, with working fluid in the casing, the torque M_{Mm} of the losses is also a function of the fluid viscosity ν, which influences the friction between the „shaft – working chambers" assembly elements and the fluid.

Intensity $Q_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$ of volumetric losses in the motor working chambers is a function of the decrease Δp_{Mi} of pressure indicated in the chambers and, to some extent, a function of motor shaft rotational speed n_M and also a function of working fluid viscosity ν.

Losses $\Delta p_{Mp} = f(Q_M, \nu)$ of the working fluid pressure in the motor channels are a function of the motor capacity Q_M and of the working fluid viscosity ν.

In order to evaluate the dependence of overall efficiency η_M of the motor (with determined theoretical capacity q_{Mt} per one shaft revolution) on parameters independent of the losses in the motor, i.e. evaluate η_M as a function of the required motor shaft torque M_M and speed n_M and also as a function of the working fluid viscosity ν, a product of the motor mechanical efficiency η_{Mm}, volumetric efficiency η_{Mv} and pressure efficiency η_{Mp} must be used:

$$\eta_M = f(M_M, n_M, \nu) = \eta_{Mm}\,\eta_{Mv}\,\eta_{Mp} \qquad (44)$$

with: $\quad \eta_{Mm} = f(M_{Mm}, M_M) = f(M_M, n_M, \nu)$
because: $\quad M_{Mm} = f(M_M, n_M, \nu)$ [equation (12)]
$\qquad\quad \eta_{Mv} = f(Q_{Mv}, n_M) = f(\Delta p_{Mi}, n_M, \nu)$
because: $\quad Q_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$ [equation (24)]
and: $\qquad \eta_{Mp} = f(\Delta p_{Mp}, \Delta p_{Mi}) = f(Q_M, \Delta p_{Mi}, \nu)$
because $\quad \Delta p_{Mp} = f(Q_M, \nu)$ [equation (38)].

In the above equation describing the overall efficiency η_M, mechanical efficiency η_{Mm} is defined as a function of parameters influencing the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and as a function of torque M_M loading the motor shaft. Volumetric efficiency η_{Mv} is defined as a function of parameters influencing the intensity Q_{Mv} of volumetric losses in the working chambers and also as a function of the motor shaft rotational speed n_M. Pressure efficiency η_{Mp} is defined as a function of parameters influencing the losses Δp_{Mp} of working fluid pressure in the channels and a function of decrease Δp_{Mi} of pressure indicated in the motor working chambers.

After replacing in equation (44) η_{Mm} with expression (13), η_{Mv} with expression (23) and η_{Mp} with expression (37), we obtain a formula describing the motor efficiency η_M as a ratio of useful power P_{Mu} to power P_{Mc} consumed by the motor, i.e. a formula confirming correctness of the expressions describing η_{Mm}, η_{Mv} and η_{Mp}:

$$\eta_M = f(M_M, n_M, \nu) = \eta_{Mm}\eta_{Mv}\eta_{Mp} =$$
$$= \frac{2\Pi M_M}{\Delta p_{Mi} q_{Mt}}\,\frac{q_{Mt} n_M}{Q_M}\,\frac{\Delta p_{Mi}}{\Delta p_M} = \frac{2\Pi M_M n_M}{Q_M \Delta p_M} = \frac{P_{Mu}}{P_{Mc}} \qquad (45)$$

In equation (45), the required motor capacity Q_M [equation (17)] is a function:

$$Q_M = \frac{q_{Mt} n_M}{\eta_{Mv}}$$

with: $\quad \eta_{Mv} = f(Q_{Mv}, n_M) = f(\Delta p_{Mi}, n_M, \nu)$
because: $\quad Q_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$

and the required pressure decrease Δp_M (equation (30)] is a function:

$$\Delta p_M = \frac{2\Pi M_M}{q_{Mt}\,\eta_{Mm}\,\eta_{Mp}}$$

with: $\quad \eta_{Mm} = f(M_{Mm}, M_M) = f(M_M, n_M, \nu)$
because: $\quad M_{Mm} = f(M_M, n_M, \nu)$
and: $\qquad \eta_{Mp} = f(\Delta p_{Mp}, \Delta p_{Mi}) = f(Q_M, \Delta p_{Mi}, \nu)$
because: $\quad \Delta p_{Mp} = f(Q_M, \nu)$.

After replacing in equation (44) η_{Mm} with formula (11), η_{Mv} with formula (23) and η_{Mp} with formula (37), we obtain an expression describing the efficiency η_M as a product of individual efficiencies described by losses and parameters deciding of their values and where at the same time Δp_{Mi} and Q_M are functions of the losses.

$$\eta_M = f(M_M, n_M, \nu) = \eta_{Mm}\eta_{Mv}\eta_{Mp} =$$
$$= \frac{M_M}{M_M + M_{Mm}}\,\frac{q_{Mt} n_M}{q_{Mt} n_M + Q_{Mv}}\,\frac{\Delta p_{Mi}}{\Delta p_{Mi} + \Delta p_{Mp}} \qquad (46)$$

where: $\quad M_{Mm} = f(M_M, n_M, \nu)$ [equation (2)]
$\qquad\quad Q_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$ [equation (14)]
$\qquad\quad \Delta p_{Mp} = f(Q_M, \nu)$ [equation (25)]

with: $\qquad \Delta p_{Mi} = \dfrac{2\Pi(M_M + M_{Mm})}{q_{Mt}}$ [equation (6)]

and: $\qquad Q_M = q_{Mt} n_M + Q_{Mv}$ [equation (16)].

Decrease Δp_{Mi} of pressure indicated in the working chambers [equation (6)] is a function of the loading torque M_M and torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly. Capacity Q_M in the motor channels [equation (16)] is a function of motor shaft speed n_M and the intensity Q_{Mv} of volumetric losses in the working chambers.

Formula (46) shows a direct dependence of the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly on the torque M_M and on the motor shaft rotational speed n_M as well as on the working fluid viscosity ν.

Formula (46) presents a complex dependence of the intensity Q_{Mv} of volumetric losses in the working chambers on the shaft loading torque M_M and on the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly (decrease Δp_{Mi} of pressure indicated in the working chambers depends on M_M and M_{Mm} and has direct impact on Q_{Mv}) and also on the shaft speed n_M (influencing in diversified way the torque M_{Mm} of mechanical losses and intensity Q_{Mv} of volumetric losses). The intensity Q_{My} of volumetric losses depends on diversified impact of the working fluid viscosity ν: indirectly by impact of ν on the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and directly by impact of ν on intensity Q_{Mv} of losses in the chambers.

Formula (46) presents also a complex dependence of losses Δp_{Mp} of working fluid pressure in the channels on the shaft rotational speed n_M and on intensity Q_{Mv} of volumetric losses in the working chambers. The intensity Q_{Mv} of losses influences the motor capacity Q_M [equation (16)] and at the same time Q_{Mv} depends in a complex way on the shaft loading torque M_M and on the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly. Pressure losses Δp_{Mp} in the channel are also dependent on the diversified impact of the working fluid viscosity ν: indirectly by impact of ν on the torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly and by impact of ν on the intensity Q_{My} of volumetric losses in the working chambers and directly by impact of ν on the losses Δp_{Mp} of pressure in the channels.

ANALYSIS OF THE PRESENTED DEFINITIONS AND RELATIONS

1. The power P_{Mc} consumed by the motor is a sum of motor shaft useful power P_{Mu} and powers of three different energy losses in the motor. The losses occur in series increasing power stream in the opposite direction to the direction of power flow. In effect, the power stream in the motor increases from the shaft useful power P_{Mu} to the working fluid power P_{Mc} consumed by the motor:

$$P_{Mc} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} + \Delta P_{Mp}$$

Mechanical losses (and power ΔP_{Mm}) occur in the „shaft – working chambers" assembly, volumetric losses (and power ΔP_{Mv}) occur in the working chambers, pressure losses (and power ΔP_{Mp}) occur in the motor channels.

2. Figure 1 presents a diagram of the direction of increasing power stream in a hydraulic motor. Direction of the increase of power stream is opposite to the direction of power flow in the motor. The diagram replaces the Sankey diagram of distribution of power flowing in a power transmission system. The use of Sankey diagram for description of power stream in the power transmission systems is a basic cause of errors in evaluation of losses in the power flow. The Sankey diagram suggests determination of losses in a system as a function of input parameters of that system. The suggestion can be noticed in the method of hydraulic motor investigations and in the related evaluations of losses and the motor energy efficiency. But the input parameters of the system depend on the losses.

3. Torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly should be evaluated as a function $M_{Mm} = f(M_M, n_M, \nu)$, i.e. as a function of the motor shaft loading torque M_M and the shaft rotational speed n_M and also as a function of a working fluid viscosity ν.

   The picture of torque M_{Mm} of mechanical losses in the motor, presented in literature and in the industrial practice as a function $M_{Mm} = f(\Delta p_M, n_M, \nu)$, i.e. as a direct dependence on the decrease Δp_M of pressure in the motor, is incorrect because it bears the impact of the mechanical losses and also of volumetric losses in the working chambers and pressure losses in the motor channels.

4. Intensity Q_{Mv} of volumetric losses in the motor working chambers should be evaluated as a function $Q_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$, i.e. a function of decrease Δp_{Mi} of pressure indicated in the working chambers and of motor shaft rotational speed n_M and also as a function of the working fluid viscosity ν.

   The picture of intensity Q_{Mv} of volumetric losses, presented in literature and in industrial practice as a function $Q_{Mv} = f(\Delta p_M, n_M, \nu)$, i.e. as a direct dependence on the decrease Δp_M of pressure in the motor, is incorrect, because it bears the impact of the pressure losses in the motor channels.

   Similarly, the picture of intensity Q_{Mv} of volumetric losses as a function $Q_{Mv} = f(M_M, n_M, \nu)$, i.e. as a direct dependence on the motor shaft loading torque M_M, is also incorrect, because it bears the impact of the mechanical losses in the „shaft – working chambers" assembly.

5. Pressure losses Δp_{Mp} in the motor channels should be evaluated as a function $\Delta p_{Mp} = f(Q_M, \nu)$, i. e. a function of motor capacity Q_M and of working fluid viscosity ν.

   The picture of pressure losses Δp_{Mp}, presented sometimes in literature and in the industrial practice as a function $\Delta p_{Mp} = f(n_M, \nu)$, i.e. as a direct dependence on the motor shaft rotational speed n_M, is incorrect because it bears the impact of mechanical losses in the „shaft – working chambers" assembly and the impact of volumetric losses in the motor working chambers.

6. The torque M_{Mm} of mechanical losses in the „ shaft – working chambers" assembly and the so called „torque" of pressure losses Δp_{Mp} in the motor channels cannot make up a „sum" and also that „sum" cannot be evaluated as directly dependent on the same chosen parameters (which is practiced in literature and in industry), because those losses are of different character and are dependent on different parameters $[M_{Mm} = f(M_M, n_M, \nu), \Delta p_{Mp} = f(Q_M, \nu)]$.

7. The impact of working fluid viscosity ν on:
   – torque M_{Mm} of mechanical losses in the „shaft – working chambers" motor assembly
   – intensity Q_{Mv} of volumetric losses in the motor working chambers
   – pressure losses Δp_{Mp} in the motor channels
   is differentiated.

The dependence of particular kinds of losses on the working fluid viscosity ν should be presented in expressions describing dependence of those losses on other parameters which have a direct impact on them $[M_{Mm} = f(M_M, n_M, \nu), Q_{Mv} = f(\Delta p_{Mi}, n_M, \nu), \Delta p_{Mp} = f(Q_M, \nu)]$.

8. The overall efficiency of the motor (with determined theoretical capacity q_{Mt} per one shaft revolution), in the $(0 \leq \overline{\omega}_M \langle \overline{\omega}_{Mmax}, 0 \leq \overline{M}_M \langle \overline{M}_{Mmax})$ range of hydraulic motor shaft speed and torque coefficients and in the $\nu_{min} \leq \nu \leq \nu_{max}$ range of working fluid viscosity, must be evaluated only as a function $\eta_M = f(M_M, n_M, \nu)$, i.e. as a function of the required motor shaft torque M_M, required shaft rotational speed n_M and as a function of the working fluid viscosity ν. Torque M_M and speed n_M are parameters required by the motor driven machine (device), independent of the motor and of the motor losses. The working fluid viscosity ν is also independent of the motor and of the motor losses. At the same time those parameters (M_M, n_M, ν), in a direct or indirect way, have an impact on the motor mechanical, volumetric and pressure losses and also on the internal parameters deciding of the losses: on pressure decrease Δp_{Mi} indicated in the working chambers and deciding of the capacity Q_{Mv} of volumetric losses in the working chambers as well as on the motor capacity Q_M directly deciding of the pressure losses Δp_{Mp} in the motor channels.

9. The motor overall efficiency η_M, as a function of the motor shaft torque M_M and speed n_M and as a function of the working fluid viscosity ν, is a product of the motor mechanical efficiency η_{Mm}, volumetric efficiency η_{Mv} and pressure efficiency η_{Mp}:

$$\eta_M = f(M_M, n_M, \nu) = \frac{P_{Mu}}{P_{Mc}} = \eta_{Mm}\, \eta_{Mv}\, \eta_{Mp}$$

Each of the three efficiencies, as a factor of the product describing the overall efficiency, is evaluated as a function of parameters having a direct impact on the respective losses and as a function of a parameter to which those losses are „added".

10. The mechanical efficiency:

$$\eta_{Mm} = \frac{P_{Mu}}{P_{Mi}} = \frac{M_M}{M_M + M_{Mm}} = f(M_M, n_M, \nu)$$

must be evaluated as a function of parameters which have a direct impact on the torque $M_{Mm} = f(M_M, n_M, \nu)$ of mechanical losses in the „shaft – working chambers" assembly, i.e. a function of the required motor shaft torque M_M and a function of the required shaft rotational speed n_M as well as a function of the working fluid viscosity ν. At the same time, the mechanical efficiency η_{Mm} is directly a function of the shaft torque M_M because the torque M_{Mm} of mechanical losses is „added" to the torque M_M, causing decrease of power transmission efficiency in the assembly.

11. The volumetric efficiency:

$$\eta_{Mv} = \frac{P_{Mi}}{P_{Mci}} = \frac{q_{Mt}\, n_M}{q_{Mt}\, n_M + Q_{Mv}} = f(\Delta p_{Mi}, n_M, \nu)$$

must be evaluated as a function of parameters which have a direct impact on the intensity Q_{Mv} of volumetric losses in the working chambers, i.e. a function of pressure decrease Δp_{Mi} indicated in the chambers and as a function of the required motor shaft rotational speed n_M as well as a function of the working fluid viscosity ν. At the same time, the volumetric efficiency η_{Mv} is directly a function of the shaft rotational speed n_M, because the intensity
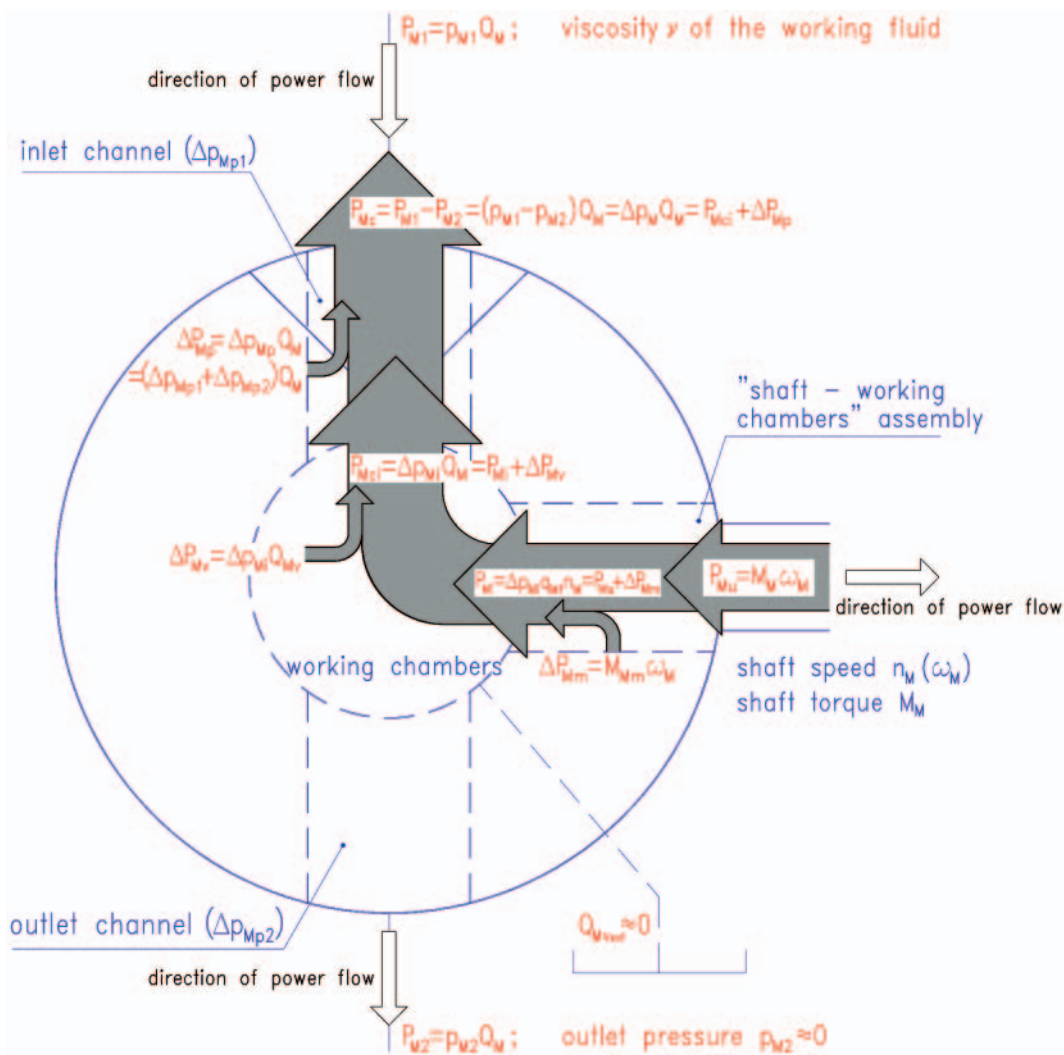
Fig. 1. *Diagram of the direction of increasing power stream in a rotational hydraulic motor; direction of the increase of power stream is opposite to the direction of power flow in the motor. Power stream increases from the motor useful power P_{Mu} required on the motor shaft by the driven machine (device) to power P_{Mc} consumed and required by the motor from the working fluid. The increase of power stream is an effect of the power of losses in the motor: power ΔP_{Mm} of mechanical losses in the "shaft – working chambers" assembly, power ΔP_{Mv} of volumetric losses in the working chambers and power ΔP_{Mp} of pressure losses in the motor channels. Powers ΔP_{Mm}, ΔP_{Mv} and ΔP_{Mp} of the losses are functions of the output parameters of the motor assembly where the losses occur and diversified functions of the working fluid viscosity ν: power ΔP_{Mm} of mechanical losses is a function of torque M_M and shaft speed n_M (ω_M) required of the motor by the driven machine (device) and a function of the working fluid viscosity ν, power ΔP_{Mv} of volumetric losses is a function of the decrease Δp_{Mi} of pressure indicated in working chambers (torque M_{Mi} indicated in the chambers) and of the shaft rotational speed n_M as well as a function of the working fluid viscosity ν, power ΔP_{Mp} of pressure losses is a function of motor capacity Q_M and of the working fluid viscosity ν. Power P_{Mi} indicated in the working chambers: $P_{Mi} = P_{Mu} + \Delta P_{Mm}$, power P_{Mci} of the working fluid consumed in the working chambers: $P_{Mci} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv}$, power P_{Mc} of the working fluid consumed by the motor: $P_{Mc} = P_{Mu} + \Delta P_{Mm} + \Delta P_{Mv} + \Delta P_{Mp}$. The diagram replaces the Sankey diagram of distribution of power in transmission systems, causing incorrect loss evaluation during the hydraulic motor energy investigations.*

Q_{Mv} of volumetric losses is "added" to the product of theoretical capacity q_{Mt} per one shaft revolution and speed n_M, causing decrease of power transmission efficiency in the chambers.

If we wish to present the motor volumetric efficiency η_{Mv} as a factor in the product $\eta_{Mm}\,\eta_{Mv}\,\eta_{Mp}$ describing the motor overall efficiency η_M, i.e. to present η_{Mv} as a complex dependence on the parameters (M_M, n_M, ν) describing the overall efficiency η_M and as a dependence on the mechanical losses in the motor, the intensity $Q_{Mv} = f\,(\Delta p_{Mi}, n_M, \nu)$ of volumetric losses in the chambers should be determined with:

$$\Delta p_{Mi} = \frac{2\Pi\,(M_M + M_{Mm})}{q_{Mt}}$$

and with torque M_{Mm} of mechanical losses in the "shaft – working chambers" assembly as a function of $M_{Mm} = f\,(M_M, n_M, \nu)$.

12. The pressure efficiency:

$$\eta_{Mp} = \frac{P_{Mci}}{P_{Mc}} = \frac{\Delta p_{Mi}}{\Delta p_{Mi} + \Delta p_{Mp}} = f\,(\Delta p_{Mi}, Q_M, \nu)$$

must be evaluated as a function of parameters which have a direct impact on the pressure losses Δp_{Mp} in the motor channels, i.e. as a function of the motor capacity Q_M and a function of the working fluid viscosity ν. At the same time, the pressure efficiency η_{Mp} is directly a function of the pressure decrease Δp_{Mi} indicated in the motor working chambers, because pressure losses Δp_{Mp} in the motor channels are "added" to the pressure decrease Δp_{Mi}, causing decrease of power transmission efficiency in the channels.

If we wish to present the motor pressure efficiency η_{Mp} as a factor in the product $\eta_{Mm}\,\eta_{Mv}\,\eta_{Mp}$ describing the motor overall efficiency η_M, i.e. to present η_{Mp} as a complex dependence on the parameters (M_M, n_M, ν) describing the

overall efficiency η_M and as a dependence on the mechanical and volumetric losses in the motor, the pressure losses $\Delta p_{Mp} = f(Q_M, \nu)$ in the channels should be determined with $Q_M = q_{Mt} n_M + Q_{Mv}$, the intensity $Q_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$ of volumetric losses in the chambers should be determined with:

$$\Delta p_{Mi} = \frac{2\Pi(M_M + M_{Mm})}{q_{Mt}}$$

and with torque M_{Mm} of mechanical losses in the „shaft – working chambers" assembly determined as a function of $M_{Mm} = f(M_M, n_M, \nu)$.

13. Therefore, the picture of the characteristics of overall efficiency as a product $\eta_{Mm} \eta_{Mv} \eta_{Mp}$ of efficiencies correctly described by the characteristics of mechanical efficiency $\eta_M = f(M_M, n_M, \nu)$, volumetric efficiency $\eta_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$ and pressure efficiency $\eta_{Mp} = f(\Delta p_{Mi}, Q_M, \nu)$ is complex.

CONCLUSIONS

1. The methods of investigation of the rotational hydraulic motor losses and energy efficiency, used in the scientific research and in industrial practice, give incorrect results because:
   – losses and efficiencies are evaluated as functions of parameters which depend on those losses or which have no direct impact on the losses,
   – the mechanical, volumetric and pressure losses and the corresponding efficiencies are presented as directly dependent on the same parameters, although each of those losses is a function of different parameters and is a different function of the working fluid viscosity ν.

2. In the investigations of the hydraulic motor (pump and a hydrostatic transmission system) losses and energy efficiency it is necessary to use as a guide the diagram of the direction of increasing power stream from the hydraulic motor shaft to the pump shaft.

3. The complex method of evaluation of the motor overall efficiency $\eta_M = f(M_M, n_M, \nu)$ as a product $\eta_{Mm} \eta_{Mv} \eta_{Mp}$ of three efficiencies correctly described by the characteristics of mechanical efficiency $\eta_{Mm} = f(M_M, n_M, \nu)$, volumetric efficiency $\eta_{Mv} = f(\Delta p_{Mi}, n_M, \nu)$ and pressure efficiency $\eta_{Mp} = f(\Delta p_{Mi}, Q_M, \nu)$ should be replaced by a method of evaluation of the motor efficiency based on the defined coefficients k_i of the motor and the motor driving system energy losses. The proposed motor efficiency evaluation is performed as a part of the energy efficiency evaluation of the hydrostatic driving system where the motor is used.

4. The evaluation method of the hydraulic motor (and also of the pump and of the hydrostatic driving system) energy efficiency is based on the mathematical models of losses where each type of losses is a function of parameters influencing directly the losses and independent of those losses.
   Evaluated are the loss coefficients k_i relating the hydraulic motor (pump and system) mechanical, volumetric and pressure losses to the reference values: driving system nominal pressure p_n, pump theoretical capacity Q_{Pt}, motor theoretical rotational speed n_{Mt} and theoretical torque M_{Mt}. The loss coefficient k_i are determined at the working fluid reference viscosity ν_n. Also the impact is determined of the

viscosity ratio ν/ν_n (viscosity changing in the $\nu_{min} \le \nu \le \nu_{max}$ range) on the value of loss coefficients k_i.
The method allows to evaluate the values and proportions of mechanical, volumetric and pressure losses in the motor (pump, driving system) and their dependence on the fluid viscosity ν.
The knowledge of coefficients k_i of the mechanical, volumetric and pressure losses allows to obtain, by applying a numerical method, a picture of the overall efficiency $\eta_M = f(\overline{\omega}_M, \overline{M}_M)$ of the motor (pump and system) in the $(0 \le \overline{\omega}_M \langle \overline{\omega}_{Mmax}, 0 \le \overline{M}_M \langle \overline{M}_{Mmax})$ motor operating field and for the selected ratio ν/ν_n of fluid viscosity.
Simultaneously, the $(0 \le \overline{\omega}_M \langle \overline{\omega}_{Mmax}, 0 \le \overline{M}_M \langle \overline{M}_{Mmax})$ motor (pump and system) operating field is determined for a selected ν/ν_n ratio of the working fluid viscosity to the reference viscosity.
It is assumed that the method is precise and simple in use. It reduces the necessary laboratory investigations of pumps and hydraulic motors. Il allows to seek energy saving displacement machine designs. It also allows to evaluate the drive energy efficiency and to seek energy saving structures of hydrostatic transmission systems.

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CONTACT WITH THE AUTHOR

Prof. Zygmunt Paszota
Faculty of Ocean Engineering
and Ship Technology
Gdansk University of Technology
Narutowicza 11/12
80-233 Gdansk, POLAND
e-mail: zpaszota@pg.gda.pl