

# Multi-objective optimization of high speed vehicle-passenger catamaran by genetic algorithm

## Part II Computational simulations

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### ABSTRACT



*Real ship structural design problems are usually characterized by presence of many conflicting objectives. Simultaneously, a complete definition of the optimum structural design requires a formulation of size-topology-shape-material optimization task unifying the optimization problems of the four areas and giving an effective solution of the problem. So far, a significant progress towards the solution of the problem has not been obtained. An objective of the present paper was to develop an evolutionary algorithm for multi-objective optimization of structural elements of large spatial sections of ships. Selected elements of the multi-criteria optimization theory have been presented in detail. Methods for solution of the multi-criteria optimization problems have been discussed with the focus on the evolutionary optimization algorithms. In the paper an evolutionary algorithm where selection takes place based on the aggregated objective function combined with domination attributes as well as distance to the asymptotic solution, is proposed and applied to solve the problem of optimizing structural elements with respect to their weight and surface area on a high speed vehicle-passenger catamaran structure, with several design variables, such as plate thickness, scantlings of longitudinal stiffeners and transverse frames, and spacing between longitudinal and transversal members, taken into account. Details of the computational models were at the level typical for conceptual design. Scantlings were analyzed by using selected rules of a classification society. The results of numerical experiments with the use of the developed algorithm, are presented. They show that the proposed genetic algorithm can be an efficient tool for multi-objective optimization of ship structures.*

*The paper is published in three parts: Part I: Theoretical background on evolutionary multi-objective optimization, Part II: Computational investigations, and Part III: Analysis of the results.*

**Keywords:** ship structure; multi-objective optimization; evolutionary algorithm; genetic algorithm; Pareto domination, set of non-dominated solutions

### SEAGOING SHIP HULL STRUCTURE MODEL FOR MULTI-OBJECTIVE OPTIMIZATION

#### *General*

Effectiveness of the developed evolutionary algorithm for the multi-objective optimization of seagoing ship structures has been verified by solving the multi-objective optimization problem for the midship segment of the passenger-car catamaran ferry, based on the Austal Auto Express 82 design developed by Austal [HANSA (1997)], [Significant Ships (1997)], Figure 14. Models developed for the multi-objective optimization are: (1) ship structural model, (2) optimization model and (3) genetic model.



**Fig. 14.** The Auto Express 82 high speed vehicle-passenger catamaran "Boomerang"

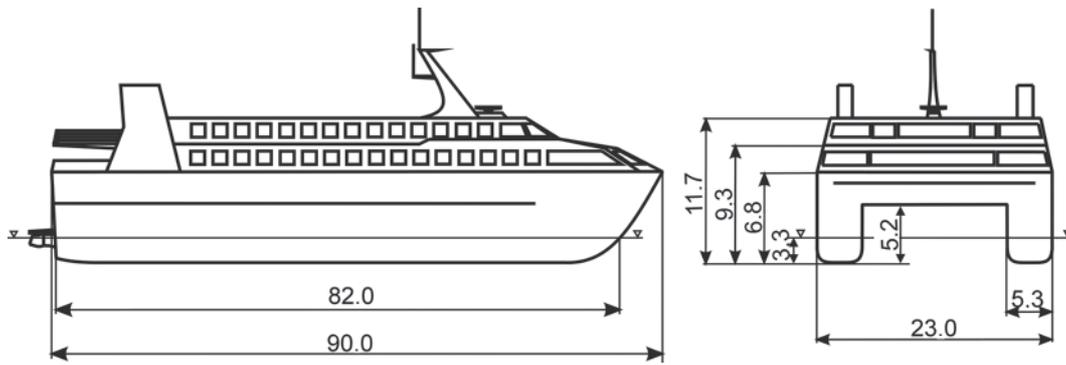


Fig. 15. Main particulars of the Auto Express 82 high-speed vehicle-passenger catamaran

### Structural model of the ship hull structure

Main particulars of the Austal Auto Express 82 vessel are given in Fig. 15. The general arrangement of the ship and her corresponding cross- and longitudinal sections are shown in Fig. 16. For seagoing ships structural design in its initial stage concerns the cylindrical and prismatic zone amidships. For this reason the analysis of the midship block-section (17.5 x 23.0 x 11.7 m) was assumed. Bulkheads form boundaries of the block in the longitudinal direction. In the block nine structural regions can be distinguished. All regions are longitudinally stiffened with stiffeners; their spacing being different in each structural region. The transverse web frame spacing is common for all the regions. Both types of spacing, i.e. of stiffeners and transverse frames, are considered the design variables. The transverse bulkheads were disregarded to minimize the number of design variables.

The structural materials are aluminium alloys of the properties given in Tab. 1. The 5083-H111 aluminium alloys are used for plate elements while 6082-T6 aluminium alloys are used for bulb extrusions. The plate thicknesses and the bulb and T-bulb extruded stiffener sections are assumed according to the commercial standards and given in Tab. 2, 3 and 4. The bulb extrusions are used as longitudinal stiffeners while the T-bulb extrusions are used as web frame profiles. Practically, the web frames are produced by welding the elements cut out of the metal sheets. Dimensions of the prefabricated T-bar elements are described by the four following design variables: web height and thickness as well as and flange breadth and thickness. In the case of extruded bulb a single variable is sufficient to identify the profile, its dimensions and geometric properties. It reduces the computational problem and accelerates analysis.

Tab. 1. Properties of structural material – aluminium alloys

No.	Property	Value
1	Yield stress $R_{0.2}$	125 (for 5083-H111 alloy) [N/mm <sup>2</sup> ] 250 (for 6082-T6 alloy) [N/mm <sup>2</sup> ]
2	Young modulus E	70,000 N/mm <sup>2</sup>
3	Poisson ratio $\nu$	0.33
4	Density $\rho$	26.1 kN/m <sup>3</sup>

The strength criteria for calculation of plate thicknesses and section moduli of stiffeners and web frames are taken in accordance to the classification rules [UNITAS (1995)]. It was assumed that bottom, wet deck, outer side and superstructure I and II are subject to the pressure of water depending on the speed and the navigation region. The main deck was loaded by the weight of the trucks transmitted through the tires, the mezzanine deck the – weight of the cars, while the upper deck

– the weight of equipment and passengers. Values of pressure were calculated according to the procedures taken from the classification rules.

In the study a minimum structural weight (volume of structure) and total outer area of structural elements intended for maintenance (cleaning, painting, etc.) were taken as the criteria and introduced to the definition of the objective functions and constraints defined on the basis of the classification rules. When structural weight and surface area are chosen as the objective functions, their values depend only on the geometrical properties of the structure (if structural material is fixed). The assumed optimization task is rather simple but the main objective of the study was to build the computational method, verify the computer code and prove its application to the multi-objective optimization of a ship hull.

Tab. 2. Thickness of plates

No.	Thickness t [mm]
1	3.00
2	4.00
3	5.00
4	6.00
5	7.00
6	8.00
7	10.00
8	12.00
9	15.00
10	20.00
11	30.00
12	40.00
13	50.00
14	60.00

Tab. 3. Dimensions of bulb extrusions

No.	Dimensions (h, b, s, s <sub>1</sub> ) <sup>1)</sup> [mm]	Cross-sectional area [cm <sup>2</sup> ]
1	80 x 19 x 5 x 7.5	5.05
2	100 x 20.5 x 5 x 7.5	6.16
3	120 x 25 x 8 x 12	11.64
4	140 x 27 x 8 x 12	13.64
5	150 x 25 x 6 x 9	10.71
6	160 x 29 x 7 x 10.5	13.51
7	200 x 38 x 10 x 15	24.20

<sup>1)</sup> h – cross-section height; b - flange width; s - web thickness; s<sub>1</sub> - flange thickness.

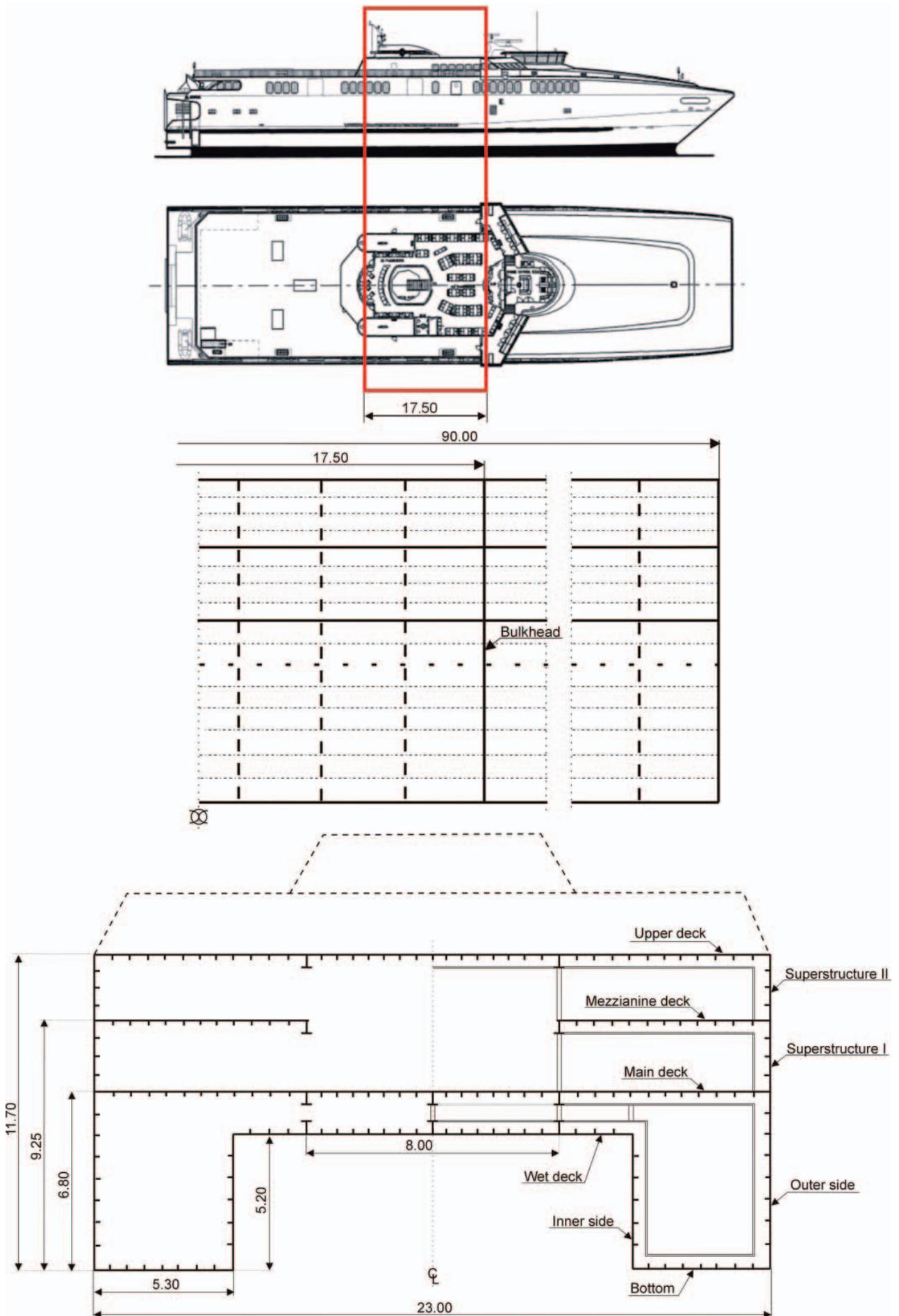


Fig. 16. Assumed model of craft structure – midship block-section, frame system and structural regions

Tab. 4. Dimensions of T-bulb extrusions

No.	Dimensions (h, b, s, s <sub>1</sub> ) <sup>2)</sup> [mm]	Cross-sectional area [cm <sup>2</sup> ]
1	200 x 100 x 8 x 15	29.80
2	200 x 140 x 8 x 5	35.80
3	200 x 60 x 10 x 12	22.50
4	200 x 50 x 8 x 9.5	21.04
5	210 x 50 x 5 x 16	14.78
6	216 x 140 x 7.6 x 8	37.60
7	220 x 80 x 5 x 8	17.00
8	230 x 80 x 10 x 8	28.60
9	230 x 80 x 5.8 x 8	19.28
10	235 x 170 x 8 x 10	35.00
11	240 x 140 x 6 x 10	27.80
12	260 x 90 x 5 x 9.5	21.08
13	275 x 150 x 9 x 12	41.67
14	280 x 100 x 5 x 8	21.60
15	280 x 100 x 8 x 10	31.60
16	300 x 60 x 15 x 15	51.75
17	310 x 100 x 7 x 16	36.58
18	310 x 123 x 5 x 8	24.94
19	350 x 100 x 8 x 10	37.20
20	350 x 100 x 5 x 8	25.10
21	390 x 150 x 6 x 8	34.92
22	390 x 150 x 6 x 12	40.68
23	400 x 140 x 5 x 8	30.80
24	410 x 100 x 6 x 8	32.12
25	420 x 15 x 5 x 10	35.10
26	420 x 15 x 8 x 10	47.80
27	450 x 100 x 9 x 10	49.60
28	450 x 150 x 10 x 12	61.80

<sup>2)</sup> h – cross-section height, b - flange width,  
s - web thickness, s<sub>1</sub> - flange thickness.

### Multi-objective optimization model of the ship hull structure

In the most general formulation to solve ship structural multi-objective optimization problem means to find a combination of values of the vector of design variables  $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T$  defining the structure which optimizes vector of the objective function  $\mathbf{f}(\mathbf{x})$ . The design variables should also meet complex set of constraints imposed on their values. The constraints formulate the set of feasible solutions. It is assumed that all functions of the multi-objective optimization problem are real and a number of constraints is finite. When considering computational costs an additional requirement may also be formulated that they should be as small as possible.

For the multi-objective optimization problem a substitute scalar objective function may be formulated, on the basis of components of vector objective function, in the following form:

$$F(\mathbf{x}) = F(f_1(\mathbf{x}), f_2(\mathbf{x})) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}) \rightarrow \min! \quad (15)$$

where:

- $f_1(\mathbf{x})$  – a structural weight of midship block-section taken to optimization,

- $f_2(\mathbf{x})$  – an area of the outer surface of structural members subjected to cleaning and painting operations (surface area for maintenance) in the section,
- $w_1$  and  $w_2$  – weight coefficients used for partial optimization criteria.

Taking into consideration the operational loads as well as the constraints imposed on the design variables especially those resulting from conditions of local and global strength formulated in the approved rules of a classification society the substitute scalar objective function can be expressed as augmented objective function of unconstrained minimization problem:

$$\begin{aligned} f(\mathbf{x}) &= F(\mathbf{x}) + \sum_{k=1}^{n_c} w_k P_k(\mathbf{x}) = \\ &= w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}) + \sum_{k=1}^{n_c} w_k P_k(\mathbf{x}) \rightarrow \min! \end{aligned} \quad (16)$$

where all symbols are described before.

The augmented objective function expression (Eq. 16) has been extended by components corresponding to dominance attributes and distance to the asymptotic solution. As a consequence the following form of combined objective function has been adopted:

$$\begin{aligned} f(\mathbf{x}) &= w_1 u_1(\mathbf{x}) + w_2 u_2(\mathbf{x}) + \\ &w_{\text{rank}} R_{\text{fi}}(\mathbf{x}) + w_{\text{count}} C_{\text{fi}}(\mathbf{x}) + \\ &+ w_{\text{distance}} [1 - d_{\text{fi}}(\mathbf{x})] + \sum_{k=1}^{n_c} w_k P_k(\mathbf{x}) \end{aligned} \quad (17)$$

where:

- $w_1$  and  $w_2$  – weight coefficients used for partial optimization criteria,
- $w_{\text{rank}}$  – dominance rank coefficient,
- $w_{\text{count}}$  – dominance count weight coefficient,
- $w_{\text{distance}}$  – distance from asymptotic solution weight coefficient,
- $u_1$  – utility function for structural weight:

$$u_1(\mathbf{x}) = \left( \frac{f_{1,\text{max}} - f_1(\mathbf{x})}{f_{1,\text{max}}} \right) \rightarrow \max! \quad (18a)$$

- $u_2$  – an utility function for area of the outer surface of structural members subjected to cleaning and painting operations:

$$u_2(\mathbf{x}) = \left( \frac{f_{2,\text{max}} - f_2(\mathbf{x})}{f_{2,\text{max}}} \right) \rightarrow \max! \quad (18b)$$

where:

- $f_1(\mathbf{x})$  – current value of the first optimization criterion,
- $f_{1,\text{max}}$  – maximum value of the first criterion,
- $f_2(\mathbf{x})$  – current value of the second optimization criterion,
- $f_{2,\text{max}}$  – maximum value of the second criterion and all the remaining symbols are as outlined before.

As it has already been stated earlier, three aggregation-based multi-objective evolutionary strategies for taking account of the partial optimization criteria  $f_1(\mathbf{x})$  and  $f_2(\mathbf{x})$  are used in the scalar objective function (Eq. 4) calculation, and therefore also in the fitness function value calculation (Eq. 16):

- selection of variants by using the scalar objective function (Eq. 16) with the values of weight coefficients  $w_1$  and  $w_2$  set by the user ( $w\_strategy = 2$ ),
- selection of variants by using the scalar objective function (Eq. 16) with the values of weight coefficients  $w_1$  and  $w_2$  randomly and independently generated in the range  $[0, 1]$  ( $w\_strategy = 4$ ),
- selection of variants by using the randomly selected single partial optimization criterion  $F(\mathbf{x}) = w_1 f_1(\mathbf{x})$  or  $F(\mathbf{x}) = w_2 f_2(\mathbf{x})$  ( $w\_strategy = 3$ ) which is implemented by the random selection of a single nonzero weight criterion. Additionally, it is also possible to have:
- selection of variants without ( $w_{rank} = 0$ ) or with ( $w_{rank} \neq 0$ ) (Eq. 17) by taking into account the dominance rank of feasible solutions,
- selection of variants without ( $w_{count} = 0$ ) or with ( $w_{count} \neq 0$ ) (Eq. 17) by taking into account the dominance count of feasible solutions,
- selection of variants without ( $w_{distance} = 0$ ) or with ( $w_{distance} \neq 0$ ) (Eq. 17) taking into account the distance of feasible solutions to the asymptotic solution.

In the present formulation a set of 37 design variables is applied, cf. Tab. 5 and Fig. 17. Introduction of the design variable representing the number of transverse frames in a considered section:  $x_4$ , and numbers of longitudinal stiffeners in the regions:  $x_5, x_9, x_{13}, x_{17}, x_{21}, x_{25}, x_{29}, x_{33}, x_{37}$ , enables to perform simultaneous optimization of both topology and scantlings within the topology-scantling optimization model.

Tab. 5. Simplified specification of bit representation of design variables

No. i	Symbol $x_i$	Item	Substring length (no of bits)	Value	
				Lower limit $x_{i,min}$	Upper limit $x_{i,max}$
1	$x_1$	serial No. of mezzanine deck plate	4	1	10
2	$x_2$	serial No. of mezzanine deck bulb	3	1	7
3	$x_3$	serial No. of mezzanine deck T-bulb	4	42	52
4	$x_4$	number of web frames	3	10	16
5	$x_5$	number of mezzanine deck stiffeners	4	25	40
6	$x_6$	serial No. of superstructure I plate	4	1	10
7	$x_7$	serial No. of superstructure I bulb	3	1	7
8	$x_8$	serial No. of superstructure I T-bulb	4	42	52
9	$x_9$	number of superstructure I stiffeners	3	4	11
10	$x_{10}$	serial No. of inner side plate	4	1	10
11	$x_{11}$	serial No. of inner side bulb	3	1	7
12	$x_{12}$	serial No. of inner side T-bulb	4	42	52
13	$x_{13}$	number of inner side stiffeners	3	18	25
14	$x_{14}$	serial No. of bottom plate	4	1	12
15	$x_{15}$	serial No. of bottom bulb	3	1	7
16	$x_{16}$	serial No. of bottom T-bulb	4	42	52
17	$x_{17}$	number of bottom stiffeners	4	15	25
18	$x_{18}$	serial No. of outer side plate	4	1	12
19	$x_{19}$	serial No. of outer side bulb	3	1	7
20	$x_{20}$	serial No. of outer side T-bulb	4	42	52
21	$x_{21}$	number of outer side stiffeners	4	18	33
22	$x_{22}$	serial No. of wet deck plate	4	1	12
23	$x_{23}$	serial No. of wet deck bulb	3	1	7
24	$x_{24}$	serial No. of wet deck T-bulb	4	42	52
25	$x_{25}$	number of wet deck stiffeners	4	25	40
26	$x_{26}$	serial No. of main deck plate	4	2	12
27	$x_{27}$	serial No. of main deck bulb	3	1	7
28	$x_{28}$	serial No. of main deck T-bulb	4	42	52
29	$x_{29}$	number of main deck stiffeners	4	25	40
30	$x_{30}$	serial No. of superstructure II plate	4	1	10
31	$x_{31}$	serial No. of superstructure II bulb	3	1	7
32	$x_{32}$	serial No. of superstructure II T-bulb	4	42	52
33	$x_{33}$	number of superstructure II stiffeners	3	4	11
34	$x_{34}$	serial No. of upper deck plate	4	1	10
35	$x_{35}$	serial No. of upper deck bulb	3	1	7
36	$x_{36}$	serial No. of upper deck T-bulb	4	42	52
37	$x_{37}$	number of upper deck stiffeners	4	25	40
Multivariable string length (chromosome length)			135		

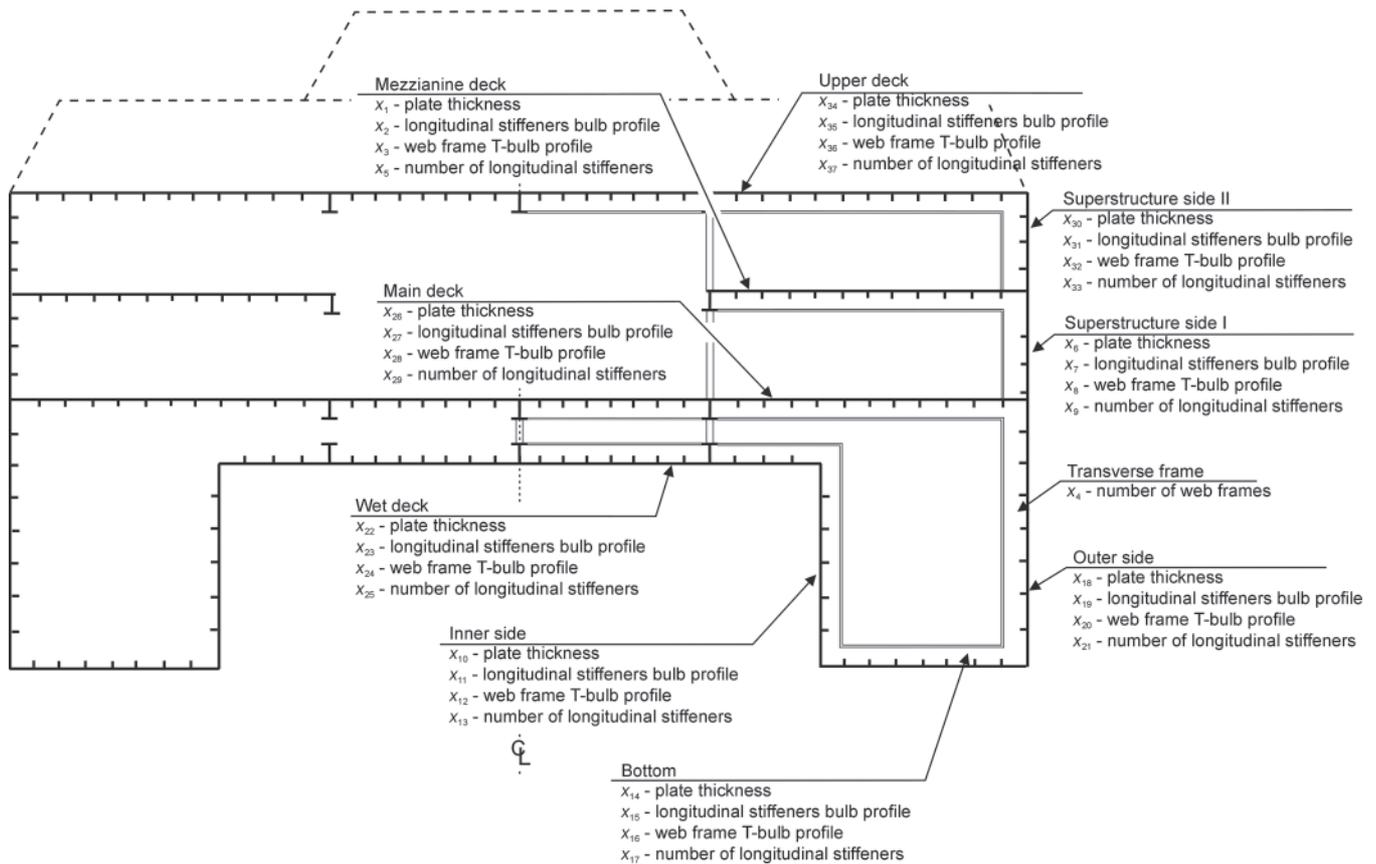


Fig. 17. Assumed model of craft – specification of design variables

Numbers of stiffeners and transverse web frames, varying throughout the processes of optimization, determine corresponding spacings. Scantlings and weights of the structural elements: plating, stiffeners and frames are directly depending on the stiffeners and frames spacings – topological properties of the structure.

When optimizing the structural topology of the ship, a difficult dilemma is to be solved concerning a relation between the number of structural elements in longitudinal and transverse directions, and their dimensions, influencing the structural weight. And, should be also considered constraints related to the manufacturing process and functional requirements of the ship, e.g. transportation corridors, container supporting seats on the container ships (usually realized by longitudinal girders and floors in the double bottom) or positioning supports on the girders in the distance enabling entry of cars on ro-ro vessels.

The behaviour constraints, ensuring that the designed structure is on the safe side, were formulated for each region according to the classification rules [UNITAS (1995)] constituting a part of the set of inequality constraints  $g_j(\mathbf{x})$ :

- the required plate thicknesses  $t_{j,rule}$ , based on the permissible bending stress:

$$t_j - t_{j,rule} \geq 0 \quad (19)$$

where:

$t_j$  – actual value of plate thickness in j-th region,

- the required section moduli of stiffeners  $Z_{s,j,rule}$ :

$$Z_{s,j} - Z_{s,j,rule} \geq 0 \quad (20)$$

where:

$Z_{s,j}$  – actual value of the section modulus of stiffeners in j-th region,

- the required section moduli of web frames  $Z_{f,j,rule}$ :

$$Z_{f,j} - Z_{f,rule} \geq 0 \quad (21)$$

where:

$Z_{f,j}$  – actual value of the section modulus of web frames in j-th region,

- the required shear areas of stiffeners  $A_{t,s,j,rule}$ :

$$A_{t,s,j} - A_{t,s,j,rule} \geq 0 \quad (22)$$

where:

$A_{t,s,j}$  – actual value of shear area of stiffeners in j-th region,

- the required shear areas of web frames  $A_{t,f,j,rule}$ :

$$A_{t,f,j} - A_{t,f,j,rule} \geq 0 \quad (23)$$

where:

$A_{t,f,j}$  – actual value of the shear area of web frames in j-th region.

He side constraints  $h_k(\mathbf{x})$ , mathematically defined as equilibrium constraints, for design variables are given in Tab. 5. They correspond to the limitations of the range of the profile set. Some of them are formulated according to the author's experience in improving the calculation convergence.

The additional geometrical constraints were introduced due to the “good practice” rules:

- the assumed relation between the plate thickness and web frame thickness:

$$t_j - t_{f,wj} \geq 0 \quad (24)$$

where:

$t_j$  – actual value of the plate thickness in j-th region,  
 $t_{f,wj}$  – actual value of web frame thickness in j-th region,

- the assumed relation between the plate thickness and stiffener web thickness:

$$t_j - t_{s,w,j} \geq 0 \quad (25)$$

where:

- $t_j$  – actual value of the plate thickness in j-th region,
- $t_{s,w,j}$  – actual value of stiffener web thickness in j-th region,

- the assumed minimum distance between the edges of frame flanges:

$$l(x_{4+1}) - b_{f,j} \geq 0.3 \text{ m} \quad (26)$$

where:

- $b_{f,j}$  – actual value of frame flange breadth in j-th region.

The relationships supplement the set of inequality constraints  $g_j(\mathbf{x})$ .

After formulating constraints it is necessary to formulate mathematical form of the penalty function  $P_{ii}$ . The choice of mathematical form of penalty function is actually free, however certain basic requirements must be met:

- promoting (preferring, awarding) the solutions which do not violate constraints, increasing in this manner the value of fitness function (selection probability) of the solutions,
- penalizing the solutions which do not fulfil (violate) the constraints, decreasing, in this manner, the value of fitness function and consequently selection probability,
- normalizing the value of fitness function to one.

After conducting series of test computations, the identical mathematical exponential form of penalty function was assumed for all constraints, Eq. 9. Weight coefficients  $w_k$  allow for implementing comparative, in relation to the others, meaning of a given constraint identified by coefficient  $k$ .

Finally by taking into consideration all the specified assumptions, the evolutionary multi-objective optimization model can be written as follows:

- find the vector of design variables  $\mathbf{x} = [x_1 \dots x_i \dots x_n]^T$ ,  $x_i$ ,  $i = 1, \dots, 37$  as shown in Tab. 5,
- optimize the combined objective function  $f(\mathbf{x}) \rightarrow \min!$  given by Eq. 17,
- subject to behaviour constraints given by Eq. 19÷23, side constraints given in Ta. 5 and geometrical constraints given by Eq. 24÷26, build a set of equality  $h_j(\mathbf{x})$  and inequality  $g_j(\mathbf{x})$  constraints,
- exponential forms of penalty functions for representing of constraints violation, Eq. 9.

## Genetic model of the ship hull structure

### General

The ship structural multi-objective optimization problem described earlier contains a large number of discrete design variables and also a large number of constraints. In such a case the GA seems to be especially useful. Solving of the optimization problem by using GA requires to formulate an appropriate optimization model. The optimization model specified earlier was reformulated into an optimization model according to requirements of the GA, which was further used to develop suitable procedures and define search parameters to be used in the computer code.

The genetic type model should cover:

- specification of chromosome structure,
- specification of fitness function:  $\text{fitness} \rightarrow \max!$ ,

- specification of genetic operators suitable for the defined chromosome structures and optimization task,
- specification of the searching control parameters.

### Chromosome structure

In this work, the space of possible solutions is the space of possible structural variants of the assumed model. The ship hull structural model is described (identified) as a collection of 37 design variables,  $x_i$ , described above. Each of them can be represented by a string of bits. For example the deck structural model is described (identified) as a collection of five design variables,  $x_i$ , presented on Fig. 18. As we mentioned above the bit string is used as chromosome in GAs. Defining the chromosome structure consists in assuming:

- a sequence in which variables in chromosome will be coded,
- a number of genes for recording every variable,
- resolution capability of coding actual variable values,
- the lowest and the highest values of the variables.

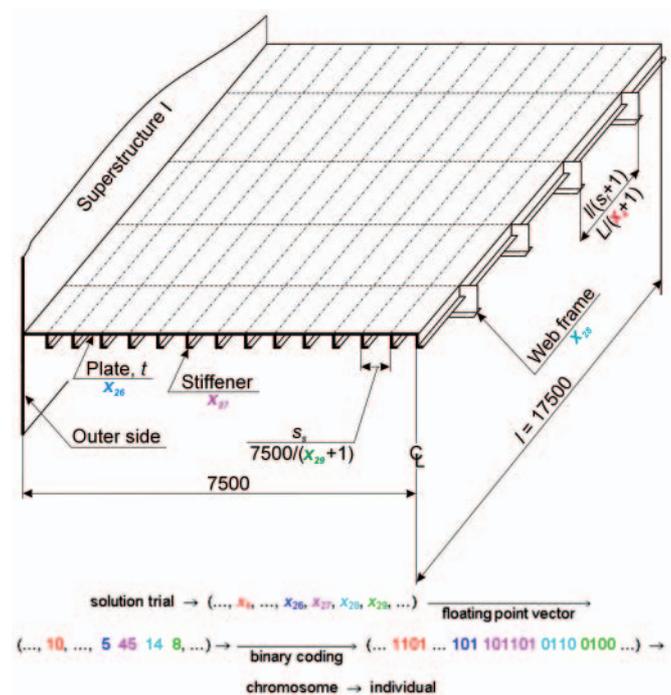


Fig. 18. Design variables and chromosome structure: main deck

Coding sequence is free to choose and it does not affect algorithm computational characteristics, but this knowledge is essential for proper genotype decoding to decisive variables values - phenotypes.

Number of genes in chromosome requires more attention. Generally, the higher gene number the greater demand for memory and increased computing time. Therefore number of genes cannot be too high. On the other hand, greater chromosome length allows for examination of objective space in greater detail. It cannot be too small otherwise mutation and crossover operators would not be able to function effectively on extremely short chromosomes and the resolution will be lower. The compromise is set by the user according to his own experience aiming at required resolution and acceptable time of computing. In this case memory size is of secondary significance. Bibliography review and the author's own experience suggest that usually one variable is coded in a chromosome section containing 5 to 20 genes.

Resolution capability determines sampling density of objective space. Greater resolution capability enables to

examine more points in objective space which increases probability of detecting interesting extremes. On the other hand, increased resolution capability results in increasing a number of computations and extends simulation time. The author's opinion is that resolution capability should be set on the lowest acceptable level. In case the algorithm does not detect any interesting solutions the increasing of resolution capability should be considered.

The user sets the lowest and highest values of variables on the basis of his own experience, according to the task being solved and providing appropriate convergence of algorithm while leaving discretion in exploration of objective space.

A simplified specification for bit representation of all design variables is given in Tab. 5. The solution variant can be represented simply by a string of bits.

The space of possible solutions is a space of structural variants of the assumed model. The hull structural model is identified by the vector  $\mathbf{x}$  of 37 design variables,  $x_i$ . Each variable is represented by a string of bits used as chromosome substring in GA. The simple binary code was applied. Such coding implies that each variant of solution is represented by a bit string named chromosome. Length of chromosome which represents of structural variant is equal to the sum of all substrings. Number of possible solutions is equal the product of values of all variables. In the present work the chromosome length is equal to 135 bits making the number of possible solutions equal approximately to  $10^{38}$ .

### Fitness function

A fitness function is used to determine how much the ship structure is suitable for a given condition in the optimum design with a GA. Because the combined objective function  $f(\mathbf{x})$  expressed by the relation Eq. 13 is: (1) well defined, (2) single-valued, (3) ascending, having real values and positive in the search space, it has been adopted directly to serve as the combined fitness function:

$$\begin{aligned} f(\mathbf{x}) = & w_1 u_1(\mathbf{x}) + w_2 u_2(\mathbf{x}) + \\ & w_{\text{rank}} R_{\text{fi}}(\mathbf{x}) + w_{\text{count}} C_{\text{fi}}(\mathbf{x}) + \\ & + w_{\text{distance}} [1 - d_{\text{fi}}(\mathbf{x})] + \sum_{k=1}^{n_c} w_k P_k(\mathbf{x}) \end{aligned} \quad (27)$$

combined fitness = criteria + rank + count +  
+ distance + constraints

All symbols used in the above given equation were already described. The expression (27) representing inclusion into selection process of test solutions, in addition to the degree with which they adjust to the established optimization criteria and formulated constraints, domination attributes as well as distance to the asymptotic solution, is the key point of the proposed Combined Fitness Multi-Objective Genetic Algorithm (CFMOGA).

### Genetic operators

The basic genetic algorithm (Simple Genetic Algorithm - SGA) produces variants of the new population by using the three main operators which constitute the GA search mechanism: selection, mutation and crossover. In the present work the algorithm was extended by introduction of elitism and updating.

The basic genetic algorithm (Simple Genetic Algorithm -SGA) produces variants of the new population by using

the three main operators which constitute the GA search mechanism: selection, mutation and crossover. In the present work the algorithm was extended by introduction of elitism and updating. Many authors described the selection operators responsible for chromosome selection due to their fitness function value [Goldberg and Deb (1991)], [De Jong (1995)], [Back (1996)], [Michalewicz (1996)]. After the analysis of the selection operators a roulette concept was applied for proportional selection. The roulette wheel selection is a process in which individual chromosomes (strings) are chosen according to their fitness function values; it means that strings with higher fitness value have higher probability of reproducing new strings in the next generation. In this selection strategy the greater fitness function value makes the individuals more important in a process of population growth and causes transmission of their genes to the next generations.

The mutation operator which introduces random changes of the chromosome, was also described [Back (1996)], [Michalewicz (1996)]. Mutation is a random modification of the chromosome. It gives new information to the population and adds diversity to the mate pool (pool of parents selected for reproduction). Without the mutation it is hard to reach solution point which is located far from the current direction of search, while due to introduction of the random mutation operator the probability of reaching any point in the search space never equals zero. This operator also prevents against the premature convergence of GA to one of the local optimum solutions, thus supporting exploration of the global search space.

The crossover operator combines the features of two parent chromosomes to create new solutions. The crossover allows to explore a local area in the solution space. Analysis of the features of the described operators [Goldberg and Deb (1991)], [Back (1996)], [Michalewicz (1996)] led to developing a new, n-point, random crossover operator. The crossover parameters in this case are: the lowest  $n\_x\_site\_min$  and the greatest  $n\_x\_site\_max$  number of the crossover points and the crossover probability  $p_c$ . The operator works automatically and independently for each pair being intersected (with probability  $p_c$ ), and it sets the number of crossover points  $n\_x\_site$ . The number of points is a random variable inside the set range [ $n\_x\_site\_min$ ,  $n\_x\_site\_max$ ]. The test calculations proved high effectiveness and quicker convergence of the algorithm in comparison with algorithm realizing single-point crossover. It was also found that the number of crossover points  $n\_x\_site\_max$  greater than 7 did not improve convergence of the algorithm. Therefore, the lowest and greatest values of the crossover points were set as follows:  $n\_x\_site\_min = 1$ ,  $n\_x\_site\_max = 7$ .

The effectiveness of the algorithm was improved with application of an additional updating operator as well as introduction of elitist strategy.

Random character of selection, mutation and crossing operators can have the effect that these are not the best fitting variants of the parental population which will be selected for crossing. Even in the case they will be selected, the result will be that progeny may have a lower adaptation level. Thus the efficient genome can be lost. Elitist strategy mitigates the potential effects of loss of genetic material copying certain number of best adapted parental individuals to progeny generation. In the most cases the elitist strategy increases the rate of dominating population by well-adapted individuals, accelerating the convergence of the algorithm. The algorithm selects fixed number of parental individuals  $n_p$  having the greatest values of the fitness function and the same number of descendant individuals having the least values of the fitness. Selected descendants are substituted by selected parents. In this way the operator increases exploitation of searching

space. Update operator with fixed probability of updating  $p_u$  introduces an individual, randomly selected from the parental population, to the progeny population, replacing a descendant less adapted individual. This operator enhances exploration of searching space at the expense of decreasing the search convergence. It also prevents the algorithm from converging to a local minimum. Both operators acts in opposite directions, and they should be well balanced; exploitation of attractive areas found in the searching spaces as well as exploration of searching space to find another attractive areas in the searching space depend on the user's experience.

### Control parameters

Single program run with the defined genetic model is characterized by values of eighteen control parameters. In this case the set of genetic model parameters set for each simulation run signed as symi includes 18 elements:

$$\text{symi} = (n_{dv}, l_{ch}, n_g, n_i, n_p, p_m, p_c, c\_strategy, n\_x\_site\_min, n\_x\_site\_max, p_u, elitist, w\_strategy, w_1, w_2, w_{rank}, w_{count}, w_{distance})$$

where:

$n_{dv}$	– number of design variables (number of genes),
$l_{ch}$	– chromosome length (number of bits),
$n_g$	– number of generations, $n_i$ - size of population,
$n_p$	– number of pretenders,
$p_m$	– mutation probability,
$p_c$	– crossover probability,
$c\_strategy$	– denotation of crossover strategy (0 for fixed, 1 for random number of crossover points),
$n\_x\_site\_min$	– the lowest number of crossover points,
$n\_x\_site\_max$	– the greatest number of crossover points,
$p_u$	– update probability,
$elitist$	– logical variable to switch on (elitism = yes) and off (elitism = no) the pretender selection strategy,
$w\_strategy$	– denotation of strategy for aggregation of objective function,
$w_1$	– weight coefficient of weight of structure,
$w_2$	– weight coefficient of surface area of structural element intended for cleaning and painting,
$w_{rank}$	– weight coefficient of solution dominance rank,
$w_{count}$	– weight coefficient of individual dominance count,
$w_{distance}$	– weight coefficient of distance of individual from asymptotical solution.

These 18 parameters control the successive simulation runs and identify them unambiguously for the adopted structure model.

For selection of more control parameters it is not possible to formulate quantitative premises because of the lack of an appropriate mathematical model for analysis of GA convergence in relation to control parameters. The control parameters were set due to test calculations results to achieve a required algorithm convergence.

All genetic parameters are specified by the user in advance of the calculations. This option is very important; the control of the parameter permits to perform search in the direction expected by the designer and in some cases it allows to find solution faster. The population size, number of variables and number of bits per variable, the total genome length, number

of individuals in the population are limited by the available computer memory.

### Conclusion

Finally, with taking into consideration all specified assumptions, the genetic model can be determined by the following:

- chromosome structure specified in Tab. 5,
- fitness function given by Eq. 28,
- genetic operators,
- control parameters.

## COMPUTATIONAL INVESTIGATIONS – THE SEARCH FOR SET OF NON-DOMINATED SOLUTIONS

### Computational investigations program

In order to verify the suitability of the proposed method and the computer code developed for the seeking of Pareto-optimal solutions of the formulated multi-objective seagoing ship structure optimization problem, a number of calculation experiments have been performed, Tab. 7, by using the ship structure models earlier formulated and discussed in Section 6.

From the multi-objective optimization point of view, the aim of the simulation was searching for non-dominated variants with respect to two optimization criteria with varying strategies for setting the values of weight coefficients for various criteria as well as dominance attributes:

**(1) Series 1.:** the simulations marked with symbols sym1-1, sym1-2 and sym1-3 having the following values of control parameters:

sym1-1: (37, 135, 10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 2, 0.5, 0.5, 0.0, 0.0, 0.0),

sym1-2: (37, 135, 10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 3, random in [0,1], random in [0,1], 0.0, 0.0, 0.0),

sym1-3: (37, 135, 10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 4, random 0 or 1, random 0 or 1, 0.0, 0.0, 0.0).

In the simulation marked as **sym1-1** the fixed values of weight coefficients are used for whole simulation:  $w_1 = 0.5$  and  $w_2 = 0.5$ ; which refers to a classical method of weighted criteria. In the simulation marked as **sym1-2** the values of weight coefficients  $w_1$  and  $w_2$  were generated by the computer code as **random variables in the range [0, 1]**, which was done independently for each variant whenever the value of fitness function is calculated. In the simulation marked as **sym1-3** the values of weight coefficients  $w_1$  and  $w_2$  were generated by the computer code as **random variables equal to either 0 or 1**, which was done independently for each variant whenever the value of fitness function is calculated; the value of 1 was used only for one, randomly selected criterion, with the remaining ones equal to 0.

**(2) Series 2.:** the simulations marked with symbols sym2-1, sym2-2 and sym2-3 having the following values of control parameters:

sym2-1: (37, 135, 10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 1, 0.0, 0.0, 3.0, 0.0, 0.0),

sym2-2: (37, 135, 10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 1, 0.0, 0.0, 0.0, 3.0, 0.0),

sym2-3: (37, 135, 10.000, 5.000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 1, 0.0, 0.0, 0.0, 0.0, 3.0).

Search for non-dominated variants while excluding the optimization criteria from the process of variant selection

Tab. 7. Control parameters of computational investigations

No.	Designation symi	Specification
		( $n_d, l_{ch}, n_g, n_i, n_p, p_m, p_c, c\_strategy, n\_x\_site\_min, n\_x\_site\_max, p_u, elitist, w\_strategy, w_1, w_2, w_{rank}, w_{count}, w_{distance}$ )
<b>Series 1.</b>		
1.	sym1-1	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 2, 0.5, 0.5, 0.0, 0.0, 0.0)
2.	sym1-2	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 4, random in [0,1], random in [0,1], 0.0, 0.0, 0.0)
3.	sym1-3	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 3, random 0 or 1, random 0 or 1, 0.0, 0.0, 0.0)
<b>Series 2.</b>		
4.	sym2-1	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 1, 0.0, 0.0, 3.0, 0.0, 0.0)
5.	sym2-2	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 1, 0.0, 0.0, 0.0, 3.0, 0.0)
6.	sym2-3	(37, 135, 10,000, 5,000, 10, 0.086, 0.800, 1, 1, 7, 0.33, yes, 1, 0.0, 0.0, 0.0, 0.0, 3.0)

$w_1 = w_2 = 0.0$  ( $w\_strategy = 1$ ) which was governed in particular simulations only by: (i) the value of the dominance rank of feasible solution,  $w_{rank} = 3.0$ ,  $w_{count} = 0.0$ ,  $w_{distance} = 0.0$ , in the simulation marked as **sym2-1**, (ii) the value of feasible variant dominance count,  $w_{count} = 3.0$ ,  $w_{rank} = 0.0$ ,  $w_{distance} = 0.0$ , in the simulation marked as **sym2-2**, (iii) the distance between the feasible variant and the asymptotic solution,  $w_{distance} = 3.0$ ,  $w_{rank} = 0.0$ ,  $w_{count} = 0.0$ , in the simulation marked as **sym2-3**. The purpose of the simulation series was to find out whether the developed tool is effective in case of evolution being governed only by (i) dominance rank, (ii) dominance count, or (iii) distance from a asymptotic solution. This refers to modern algorithms of evolutionary multi-objective optimization, where the evolution is governed only by dominance attributes.

In all the simulations the functions of penalties imposed in the violations of constraints were active,  $w_k \neq 0, k = 1, 2, \dots, n_c$ .

The computational investigations were carried out first of all for two-objective problems, as in this case it is possible to present obtained results graphically in a multitude of ways, which facilitate their interpretation and analysis.

### Results of computational investigations

#### Results of computational investigations – Series1: the simulation marked with symbols sym1-1, sym1-2 and sym1-3

The results of simulation sym1-1 are going to be discussed in the most detailed way by presenting the results characteristic

for the developed method. For other simulations only the most important results will be presented.

The Fig. 19 presents the evolution of macroscopic values characterizing the evolution of generated and evaluated population of ship structure solutions in the simulation **sym1-1**: (1) the greatest fitness function value  $f_{max}$ , (2) the lowest distance of the feasible solution from the asymptotic solution. Multi-objective optimization of ship structure was performed with regard to the structure weight  $f_1$  and the surface area for maintenance  $f_2$  in case of fixed values of weight coefficients  $w_1 = w_2 = 0.5$  used for optimization criteria. Dominance attributes: dominance rank, dominance count and distance from asymptotic solution, being excluded from selection. The figure shows a desired continuous rise of the greatest value of fitness function  $f_{max}$  indicating rising quality of the best generated test solutions. The highest values of fitness function saturate already in 1668 generation, which means that in the following generations no solutions were generated being better adapted in the sense of the fitness function used, and the computation resources were squandered replicating the non-dominated solutions set. The lowest distance between a non-dominated solution and the asymptotic solution changes during the evolution, but above the threshold of 87.38% of the highest value found during the simulation this takes place to a very small extent. For example, in the 857th generation the distance of the closest solution  $f_{857}^*(x)$  from the asymptotic solution is 1.114<sup>1)</sup>, see Fig. 20a, while in the successive 858th generation the distance of the closest solution  $f_{858}^*(x)$  to the asymptotic solution increased to 1.177, Fig. 20b. For the same

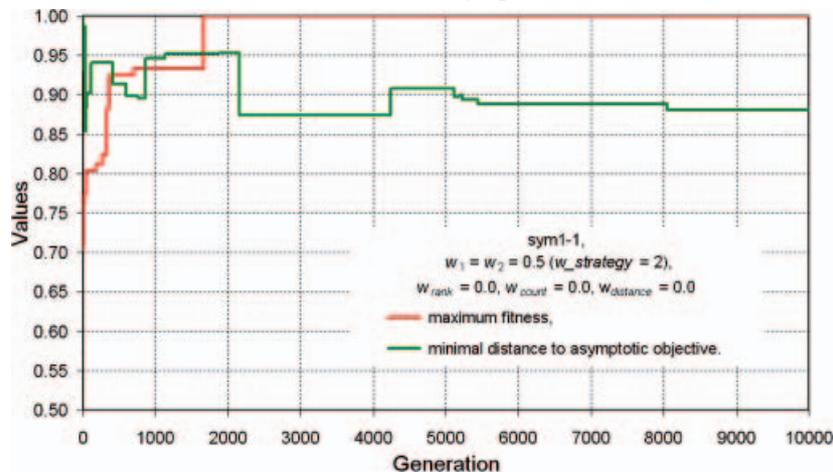
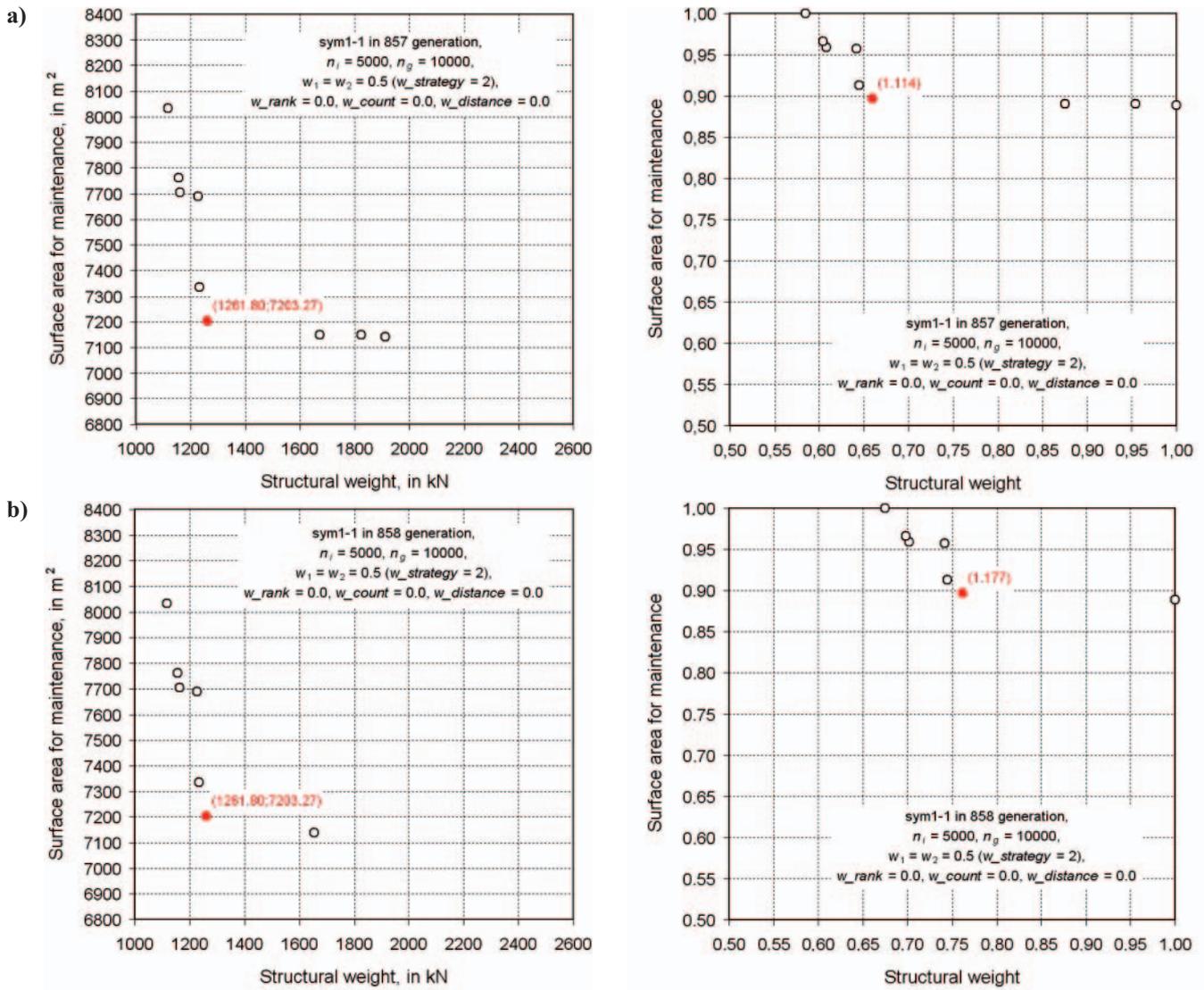


Fig. 19. The results of multi-objective genetic optimization of ship structure with respect to structure weight  $f_1$  and the area of element surface  $f_2$  in case of fixed values of optimization criteria weight coefficients  $w_1 = w_2 = 0.5$  with dominance attributes being excluded from selection (**sym1-1**); the curves present the evolution of a highest value of fitness function  $f_{max}$ , the lowest value of non-dominated solution distance from a asymptotic one; the values are dimensionless and standardized in  $[0,1]$  range in relation to the highest values found during the simulation

<sup>1)</sup> All values of distance from the asymptotic solution are calculated in normalized space because it is impossible to calculate the Euclidean distance in physical objective space when different objectives are measured in different units.



**Fig. 20.** Change of a structure of non-dominated solutions set when moving from generation 857 (a) to generation 858 (b); during the genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of the fixed values of optimization criteria weight coefficients  $w_1 = w_2 = 0.5$  with dominance attributes being excluded from selection (sym1-1); circles represent non-dominated solutions, red points represent non-dominated solutions closest from the asymptotic one; dimensionless values are normalized to the interval  $[0,1]$  in relation to the highest values in the set<sup>2)</sup>; change of non-dominated solution set structure and change of non-dimensional (normalized) of distance of the nearest solution from asymptotic solution caused not by change of values of partial optimization criteria but only change of set of non-dominated solutions set structure can be observed

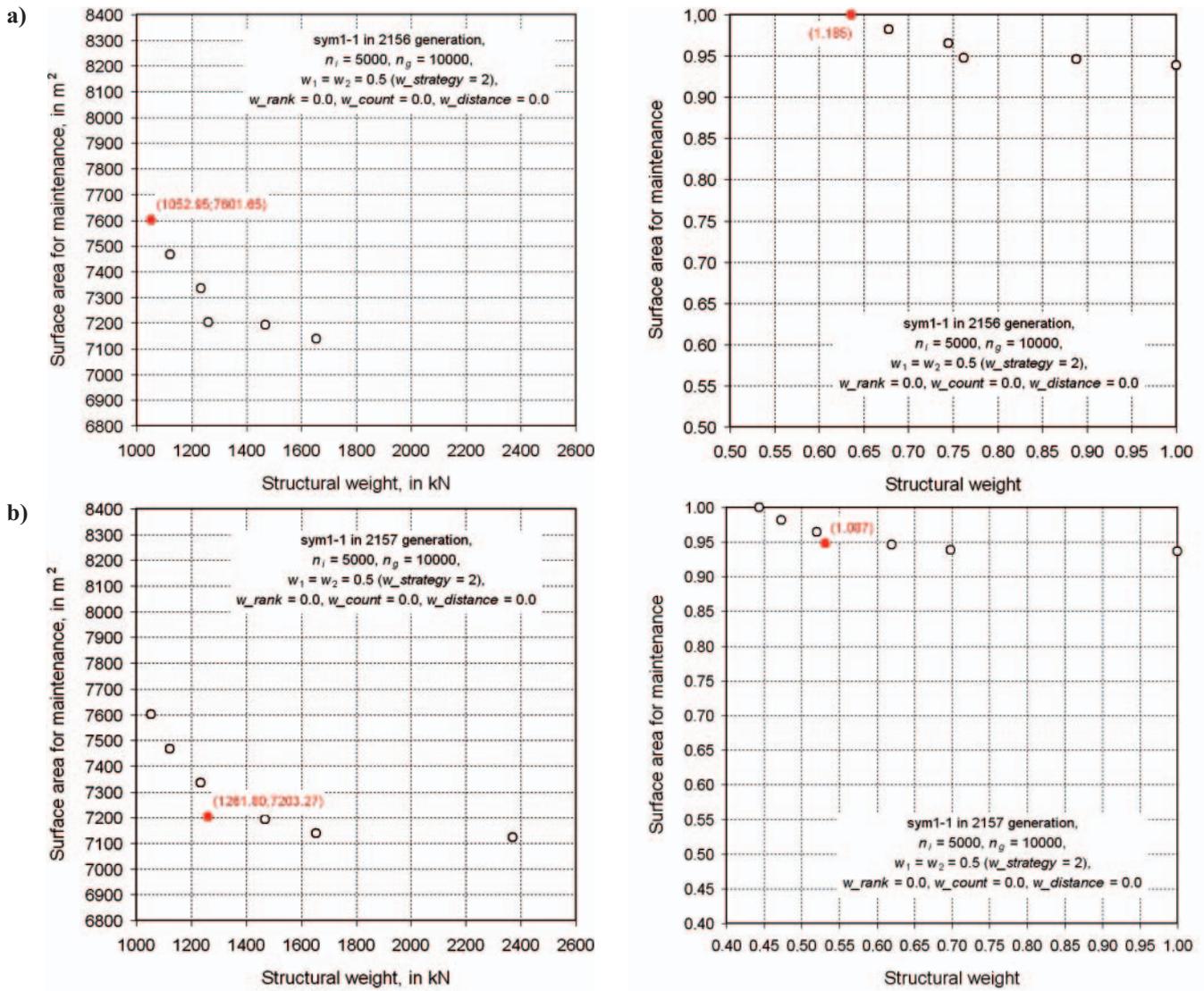
solution the change of the distance to the asymptotic solution occurs only due to the change of the structure of the set of non-dominated solutions. On the other hand, for example, in the 2156th generation the distance of the closest solution  $f_{2156}^{\approx}(\mathbf{x})$  from to the asymptotic solution is 1.187, Fig. 21a, while in the successive 2157th generation the distance of the closest solution  $f_{2157}^{\approx}(\mathbf{x})$  from the asymptotic solution decreased to 1.087, Fig. 21b. In this case the change of the set structure caused another solution, already present in the set, became the solution closest to the asymptotic solution.

In Fig. 22 the evolution of the structure of non-dominated solutions set in simulation sym1-1 is shown by using the selected time-based cross-sections of this set, e.g. for 1, 2000, 4000, 6000, 8000 and 10.000 generations as examples. A systematic growth of the size of non-dominated solutions set is apparent with 6, 6, 9, 12, 11 and 14 non-dominated solutions respectively in the consecutive time-based cross-sections as well as the desired evolution of this set in the direction of more advantageous values of partial optimization criteria.

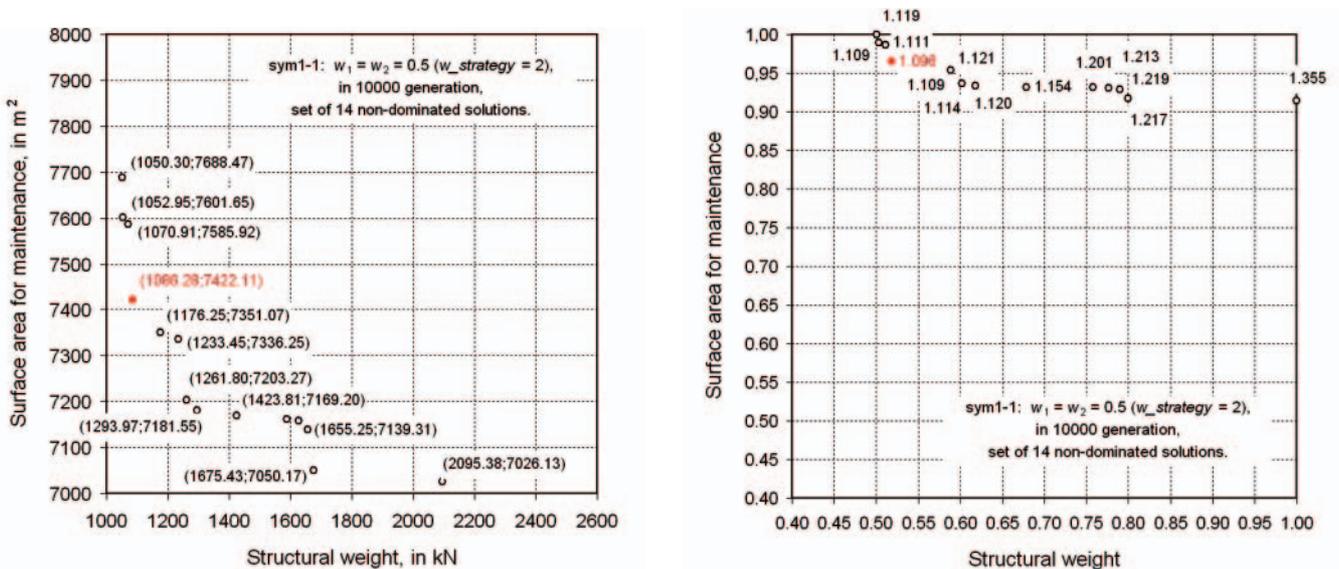
Fig. 23 presents a detailed structure of the non-dominated solution set of the last generation, presented in the physical space of objectives and normalized space of objectives. It can be seen that the set of non-dominated solutions including 14 variants of ship structure was found during the simulation. For each non-dominated variant the values of optimization criteria were specified as:  $f_1(\mathbf{x})$  – structural weight and  $f_2(\mathbf{x})$  – cleaned/painted surface area. The designer may select for further development one of these variants or a group of them deemed by him to be the best. For the variant closest from the asymptotic solution which was found in 5116 generation,  $f_{5116}^{\approx}(\mathbf{x})$ , the distance equals 1.096 in the normalized objective space, the structural weight is  $f_1(\mathbf{x}) = 1086.28$  kN and the cleaned/painted surface area is  $f_2(\mathbf{x}) = 7422.10$  m<sup>2</sup>. This variant may be recommended if there is a need to select a single solution for the formulated ship structure multi-objective optimization problem:

$$\begin{aligned} \mathbf{f}_{\text{sym1-1}}^{\approx} &= \mathbf{f}_{5116}^{\approx}(\mathbf{x}) = [f_{1;5116}^{\approx}(\mathbf{x}) \ f_{2;5116}^{\approx}(\mathbf{x})]^T = \\ &= [1086.28 \ 7422.10]^T \cdot [\text{kN} \ \text{m}^2] \end{aligned}$$

<sup>2)</sup> Normalization of the optimization objective values makes it possible to calculate the distance from asymptotic objective in the Euclidean sense in cases when the axes of the co-ordinate system represent the objectives denoted in various units.



**Fig. 21.** Change of a structure of non-dominated solutions set when moving from generation 2156 (a) to generation 2157 (b); during the genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of the fixed values of optimization criteria weight coefficients  $w_1 = w_2 = 0.5$  with dominance attributes being excluded from selection (sym1-1); circles represent non-dominated solutions, red points represent non-dominated solutions closest to the asymptotic one; dimensionless values are normalized to the interval [0,1] in relation to the highest values in the set; change of non-dominated solution set structure and change of non-dimensional (normalized) of distance of the nearest solution from asymptotic solution caused not by change of values of partial optimization criteria but only change of set of non-dominated solutions set structure can be observed



**Fig. 23.** Detailed specification of non-dominated solutions set obtained during the genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of fixed values of optimization criteria weight coefficients  $w_1 = w_2 = 0.5$  with dominance attributes being excluded from selection (sym1-1); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one; dimensionless values are normalized to the interval [0,1] in relation to the highest values in the set

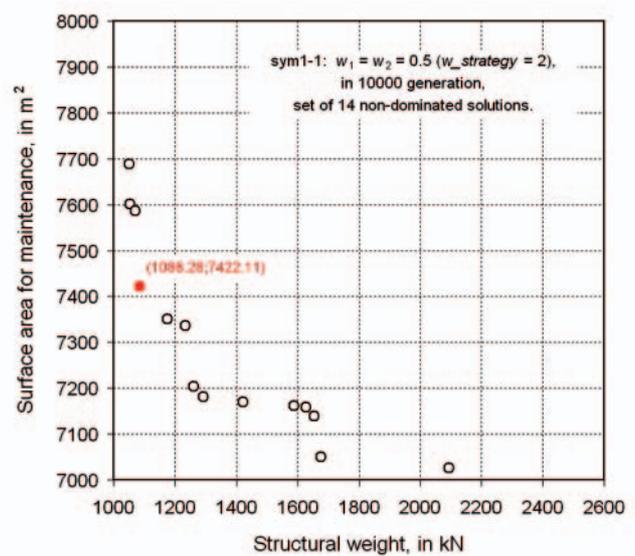
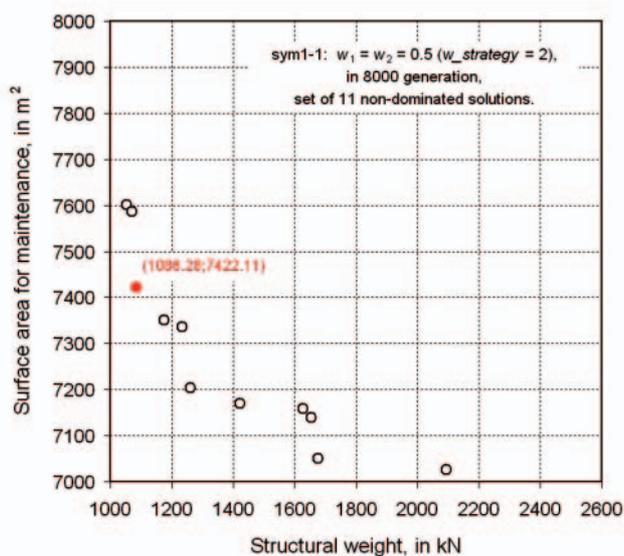
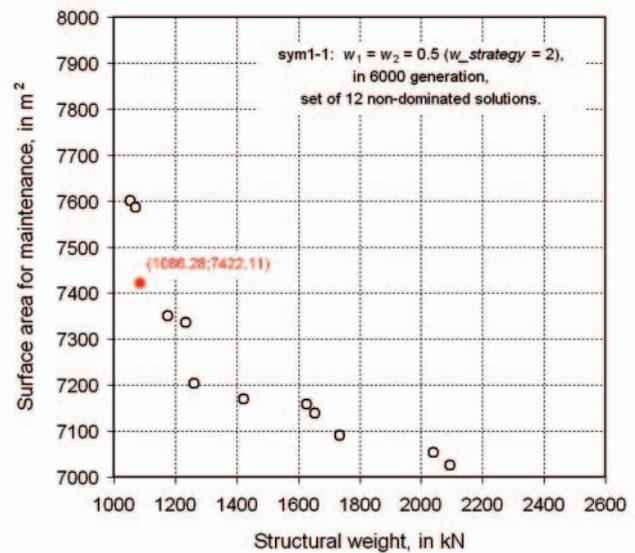
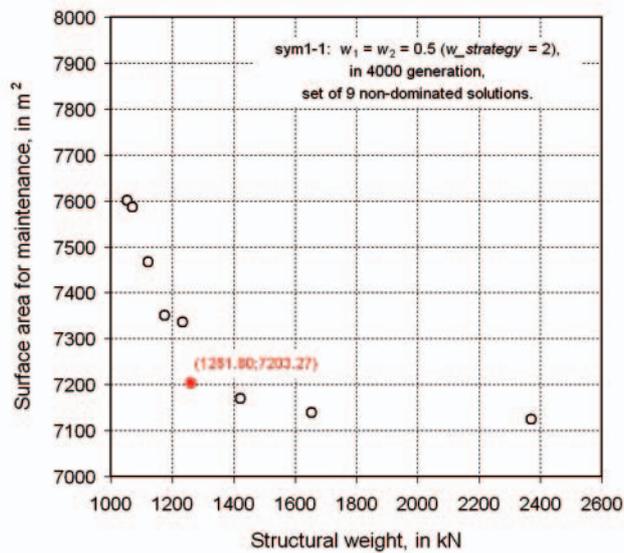
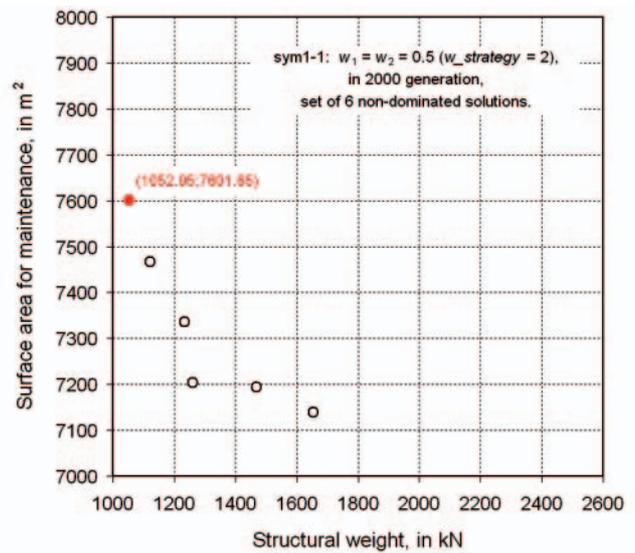
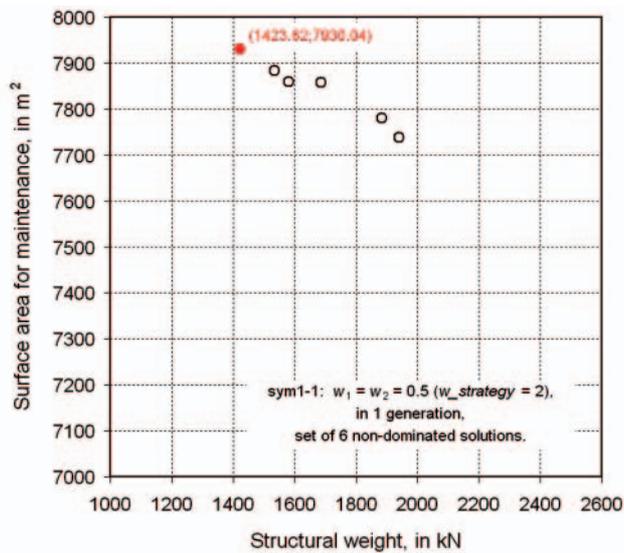


Fig. 22. History of the evolution of non-dominated solutions set during the genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of the fixed values of optimization criteria weight coefficients  $w_1 = w_2 = 0.5$  with dominance attributes being excluded from selection (sym1-1); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one

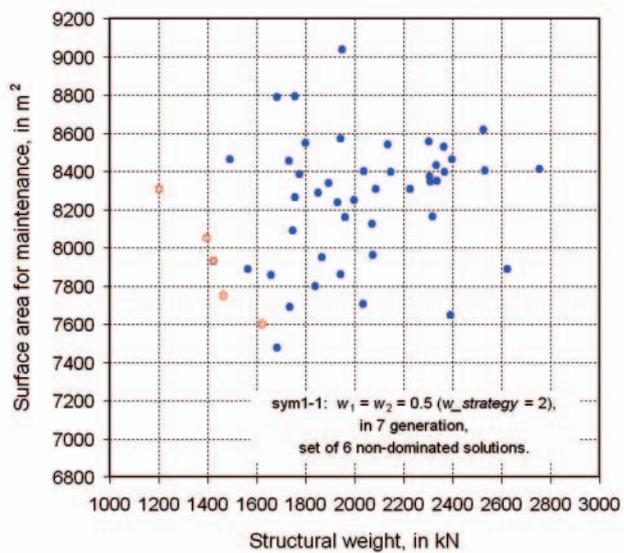
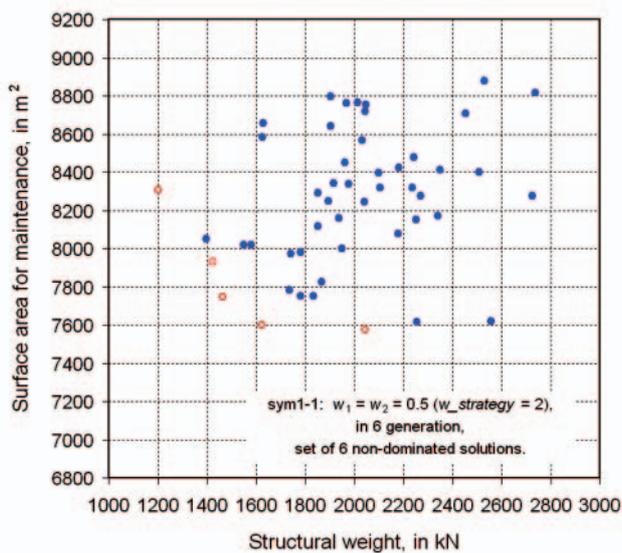
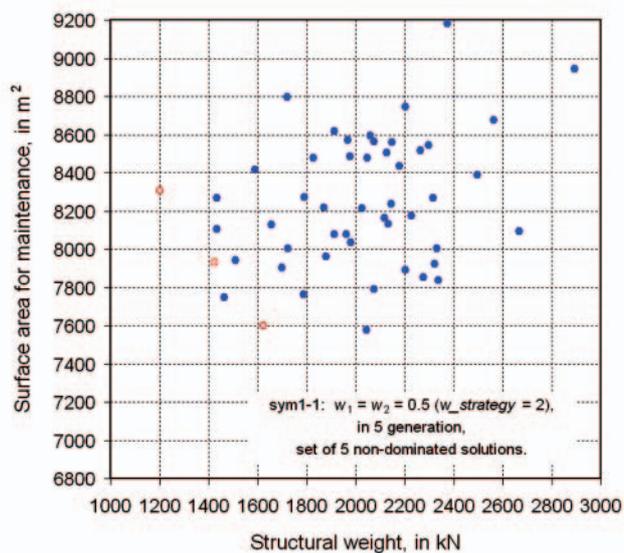
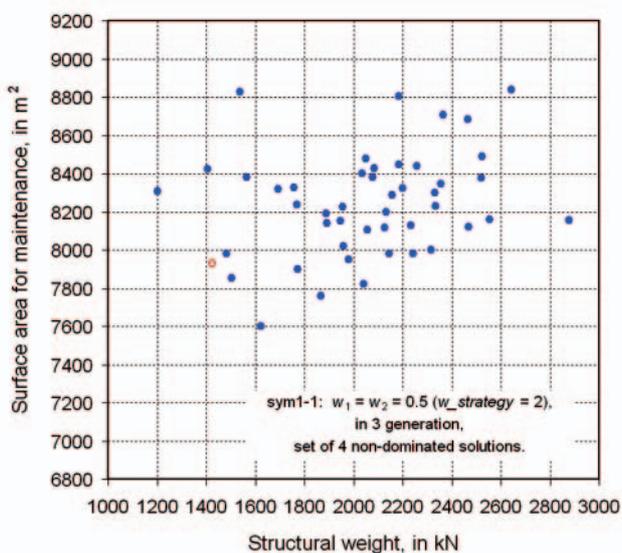
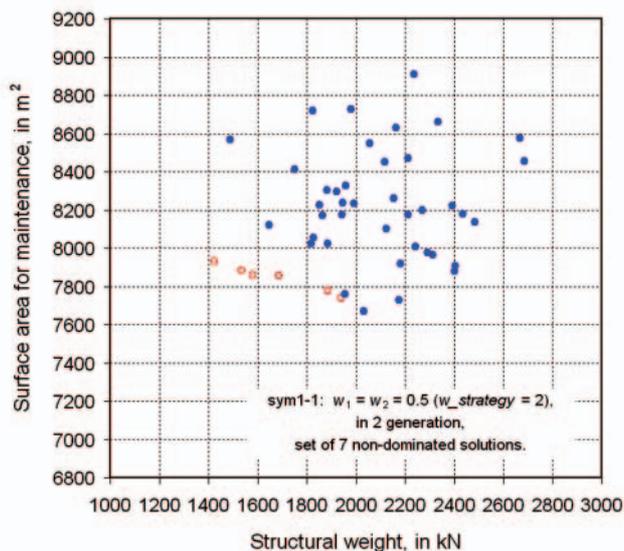
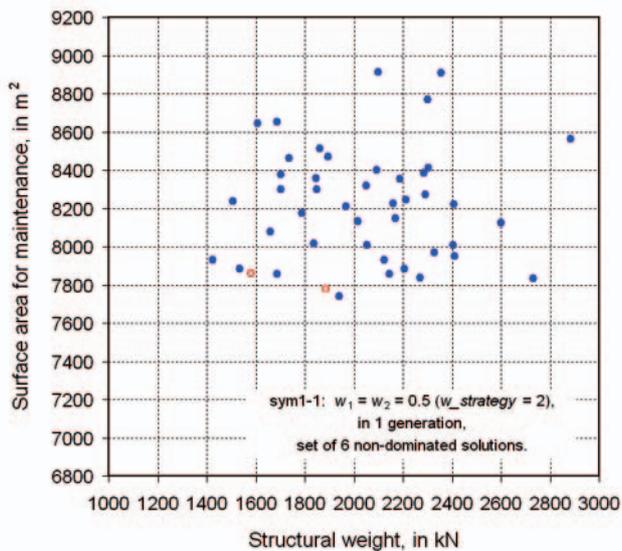


Fig. 24. History of creating a set of non-dominated solutions in the first six timeframes in which a change in structure of the set of non-dominated solutions has occurred during the genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of the fixed values of optimization criteria weight coefficients  $w_1 = w_2 = 0.5$  with dominance attributes being excluded from selection (sym1-1); red circles represent non-dominated solutions, blue dots represent feasible solutions in present generations; in case of the first generation two existed already non-dominated solutions are coming from initial generation

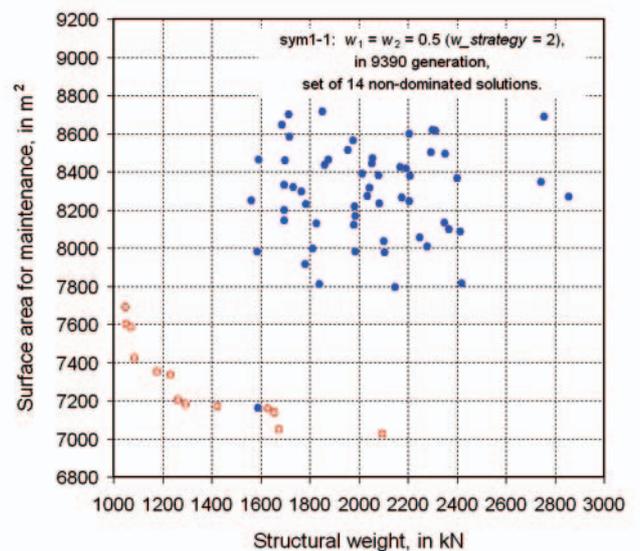
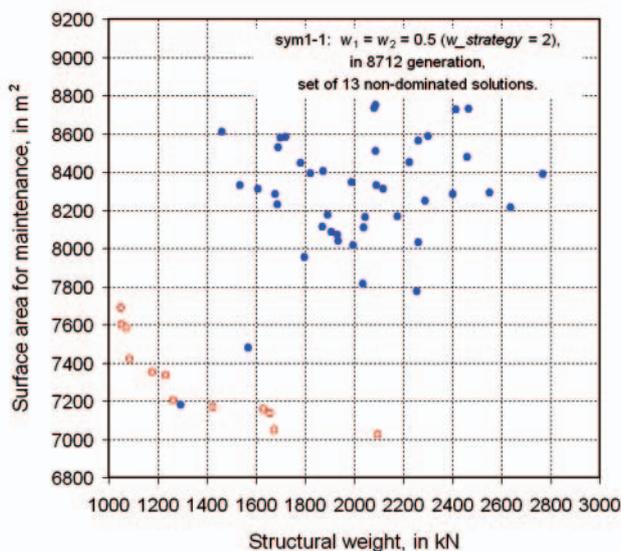
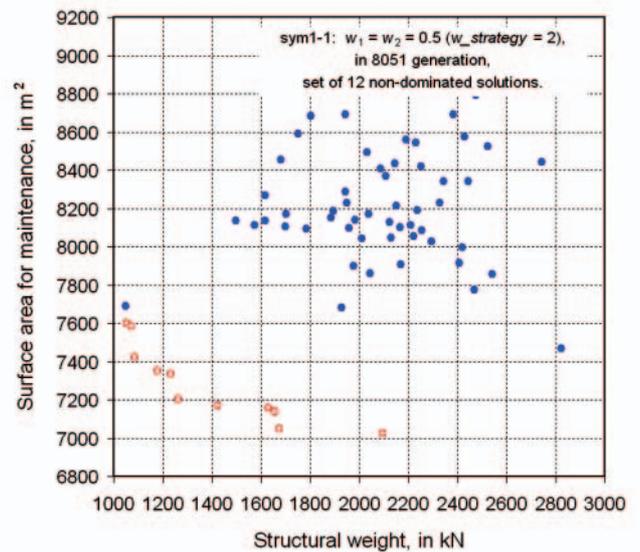
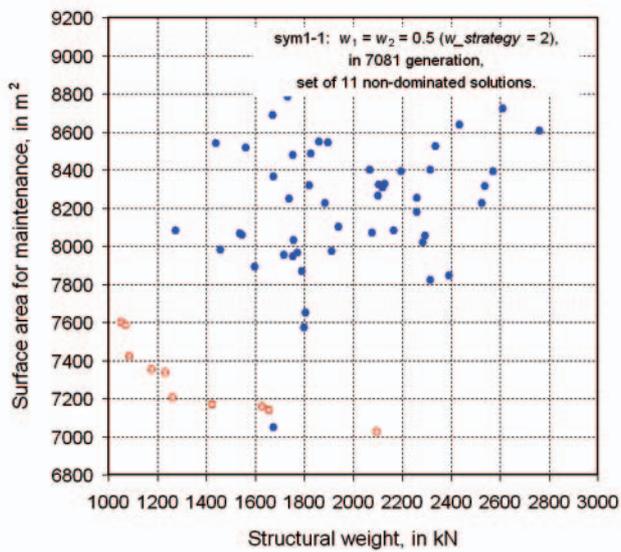
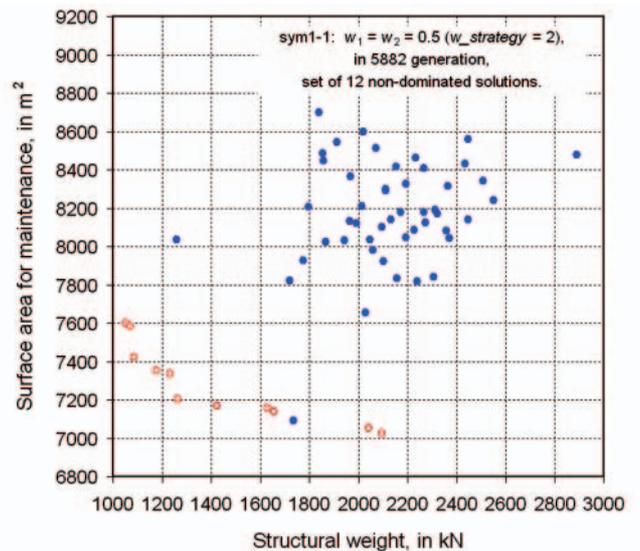
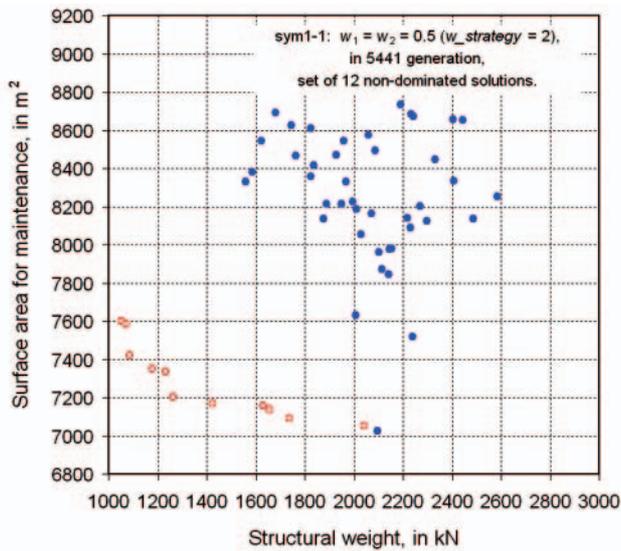


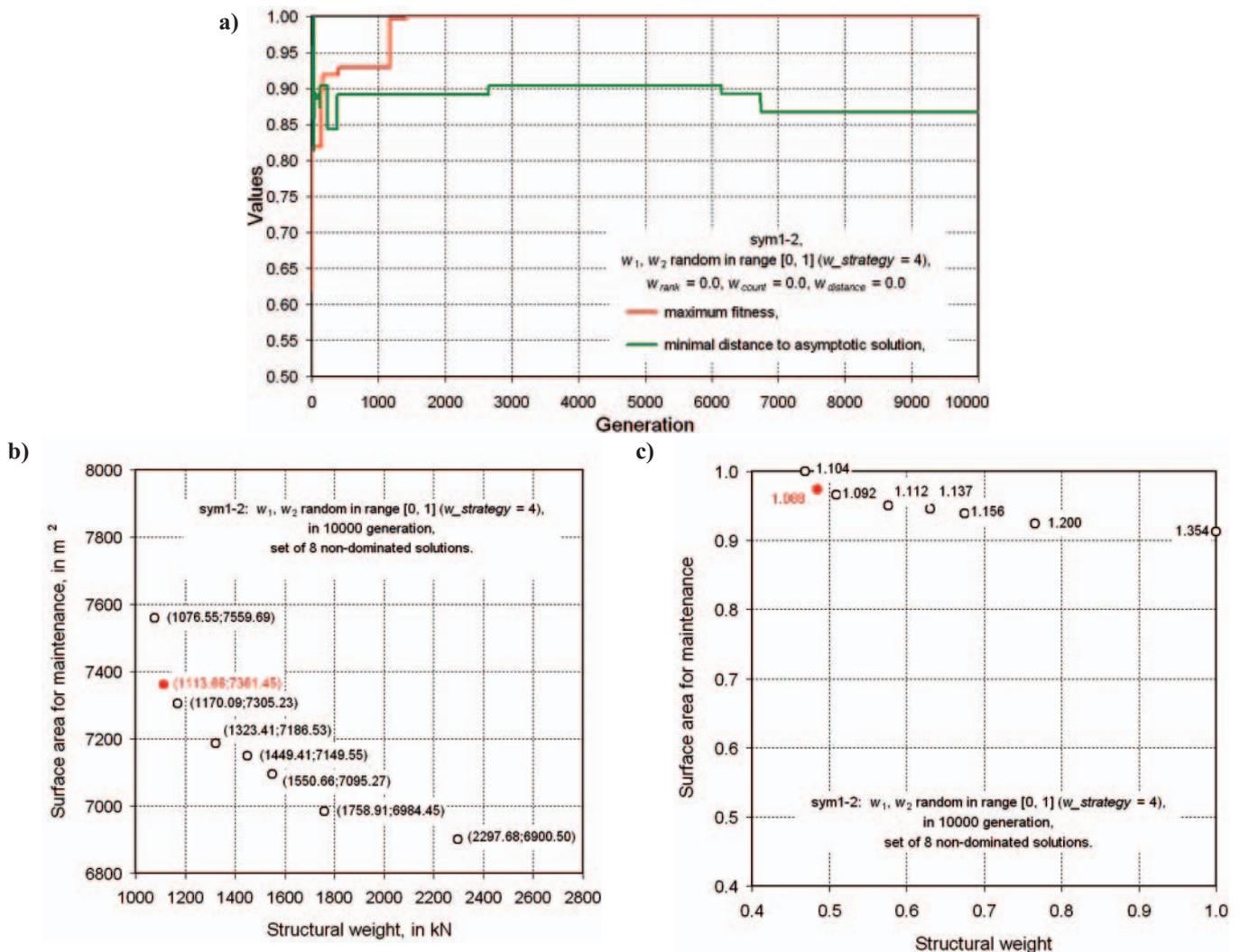
Fig. 25. History of creating a set of non-dominated solutions in the last six timeframes in which a change in structure of the set of non-dominated solutions has occurred during the genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of the fixed values of optimization criteria weight coefficients  $w_1 = w_2 = 0.5$  with dominance attributes being excluded from selection (*sym1-1*); red circles represent non-dominated solutions, blue dots represent feasible solutions in present generations

The author found it an interesting task to examine in which way the evolutionary optimization algorithm is builds a set of non-dominated solutions (approximation of Pareto set). Is the hypothesis correct, which states that during the evolution the set of feasible solutions as a whole shifts in direction of asymptotic solution, and is the set of non-dominated solutions approximating Pareto front is an edge of the set of feasible solutions? To verify the hypothesis, in Fig. 24 are presented feasible sets and approximation sets for time intersections corresponding to six first modifications of approximation set, i.e. 1, 2, 3, 5, 6 and 7 generations. In Fig. 25 are also presented approximation sets for time intersections corresponding six last modifications of approximation set, i.e. 5441, 5882, 7081, 8051, 8712 and 9390 generations. Modification of the approximation set took place by including, to the successive approximation set, selected solutions, non-dominated in the ongoing feasible set and non-dominated by existing approximation set simultaneously.

Figure 24 confirms, according to Fig. 18 that the algorithm builds the approximation set very intensively in early generations; in practice, in every following generation a new solution is being incorporated to the approximation set. The slow receding of the approximation set from the set of non-dominated solutions in the direction of the origin of the coordinate system

is visible. Set of feasible solutions maintains steady position in the objective space, preserving also similar range in this space. Fig. 25 confirms very slowly progressing building of the approximation set in final phases of the simulation; sequentially generated non-dominated variants are separated in spaces about 1000 generations of evolution simulated by the algorithm. The approximation set is already separated from the set of feasible solutions and sequentially incorporated to its non-dominated solutions already overcoming significant distance from the feasible set and approximation set in the objective space. To overcome so the large distance in the objective space is possible in the way of very high attractive mutations in the chromosomes of individuals. Location of the set of feasible solutions and its range in the objective space do not undergo significant changes in course of evolution. Slightly larger concentration of set of feasible solutions in the later generations is visible presented in the Fig. 25, however confirmation if it is an actual or apparent effect requires further research.

The presented results of the evolution of non-dominated solutions set indicate that non-dominated solutions set evolution can proceed also in such a way that the set of dominated feasible solutions preserves more less stable position in evaluation space, in turn discovered in sequence of non-dominated solutions are discovered sparse very good solutions found in course of



**Fig. 26.** Results of genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of random values of optimization criteria weight coefficients  $w_1$  and  $w_2$  in range of  $[0, 1]$  with dominance attributes being excluded from selection (sym1-2); **a)** the curves present the evolution of the highest value of fitness function  $f_{max}$ , the lowest value of non-dominated solution distance from a asymptotic one, **b)** detailed specification of final non-dominated solutions set in objective space, **c)** detailed specification of final non-dominated solutions set in normalized objective space; black circles represent non-dominated solutions, red circles represent non-dominated solution closest from the asymptotic one; dimensionless values are normalized to the interval  $[0,1]$  in relation to the highest values in the set

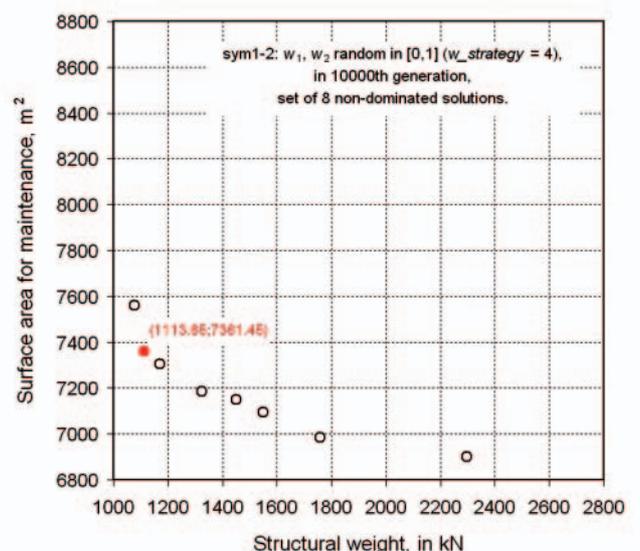
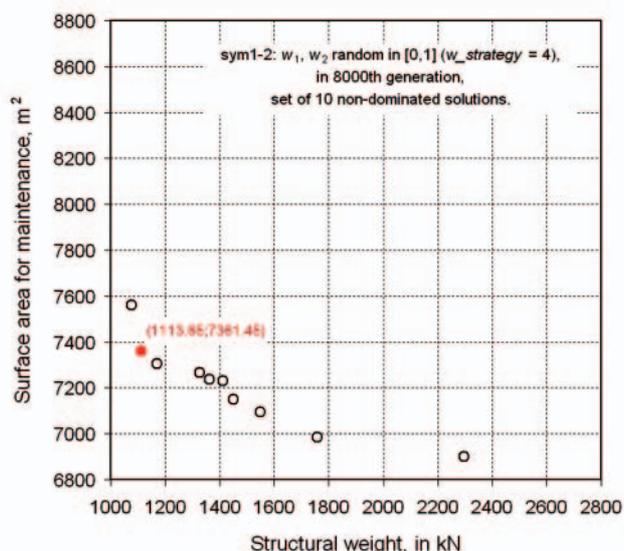
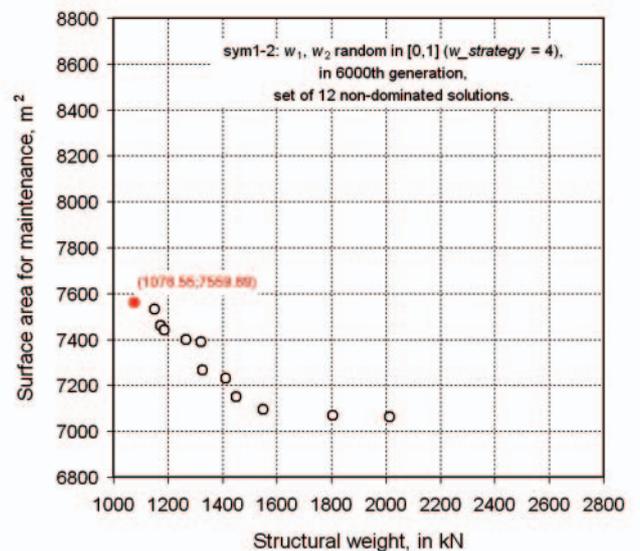
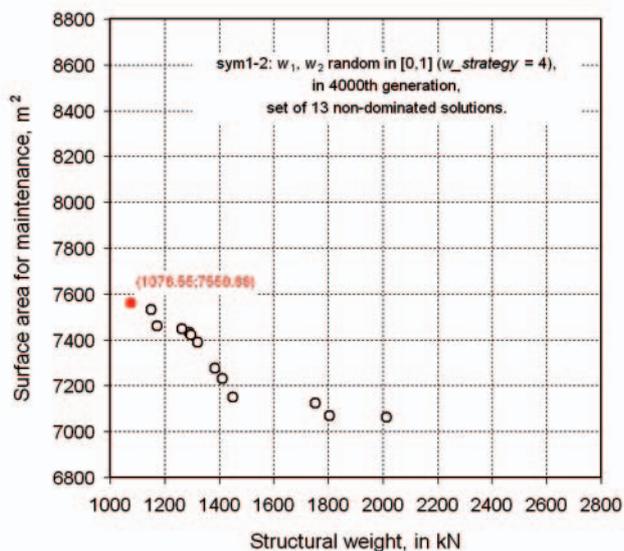
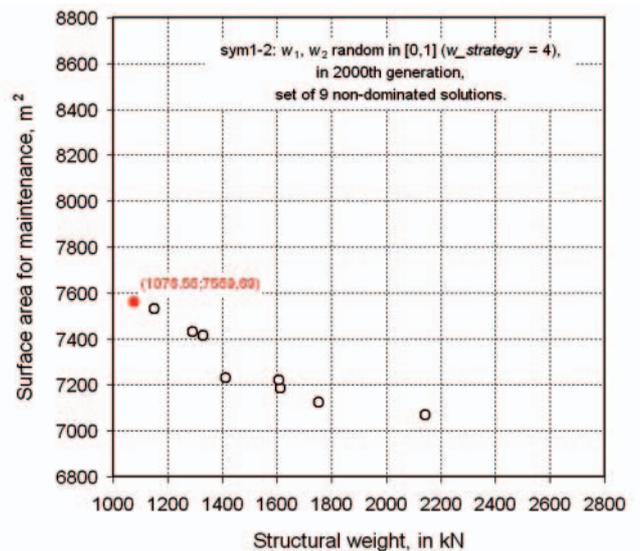
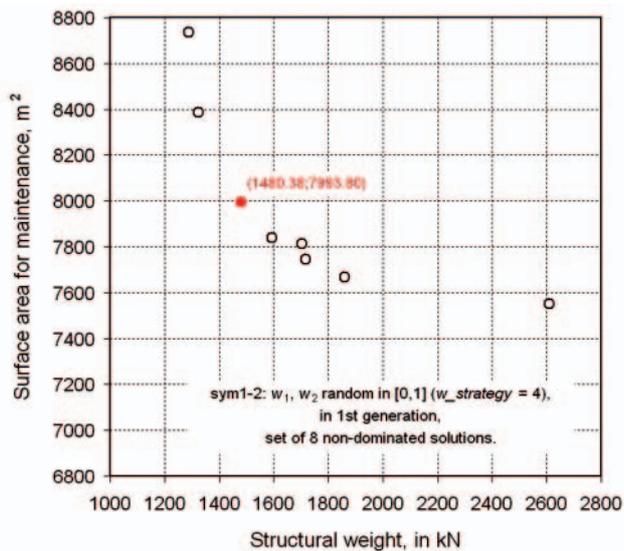


Fig. 27. History of the evolution of the non-dominated solutions set during the genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of random values of optimization criteria weight coefficients  $w_1$  and  $w_2$  in range of  $[0, 1]$  with dominance attributes being excluded from selection (sym1-2); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one

an algorithm simulated evolution; they create a set of non-dominated solutions receding in the act of the simulation from the set of feasible solutions to asymptotic ideal evaluations. This way of evolution the author suggests to call “drop by drop”, because it resembles rain drops falling on the ground; in this case the set of dominated feasible solutions represents a rain cloud while the set of non-dominated solutions, approximation set, represents drops that have fallen onto the ground.

Figure 26a presents the evolution of macroscopic values characterizing the evolution of generated and evaluated ship structure variants population in simulation **sym1-2** in case of **random values of weight coefficients  $w_1$  and  $w_2$  in range of  $[0, 1]$**  with dominance attributes being excluded from selection. The figure shows a desired continuous rise of the greatest value of fitness function  $f_{max}$  indicating rising quality of the best generated test variants. The highest values of fitness function saturate already in 1416 generation. The lowest distance between a non-dominated solution and the asymptotic solution changes during the evolution but above the threshold of 86,69% of the highest value found during the simulation is results in insignificant changes.

In Fig. 27 the evolution of the structure of the non-dominated solutions set in simulation **sym1-2** is shown by

using, as examples, the selected time-based cross-sections of this set, e.g. for 1, 2000, 4000, 6000, 8000 and 10,000 generations. The size of non-dominated solutions set in the selected generations is apparent with 8, 9, 13, 12, 10 and 8 non-dominated solutions and it does not change significantly in the course of simulation.

Figures 26b and 26c presents the detailed structure of the non-dominated solutions set of the last generation produced in simulation **sym1-2**. It can be seen that a set of non-dominated solutions including 8 variants of ship structure has been found during the simulation. For the solution closest from the asymptotic solution, which was found in 6145 generation,  $f_{6145}^{\approx}(\mathbf{x})$ , the distance equals 1.088 in the normalized objective space, the structure weight is  $f_1(\mathbf{x}) = 1113.66$  kN and the surface area for maintenance is  $f_2(\mathbf{x}) = 7361.45$  m<sup>2</sup>:

$$\begin{aligned} \mathbf{f}_{\text{sym1-2}}^{\approx} &= f_{6145}^{\approx}(\mathbf{x}) = [f_{1;6145}^{\approx}(\mathbf{x}) \ f_{2;6145}^{\approx}(\mathbf{x})]^T = \\ &= [1113.66 \ 7361.45]^T \cdot [\text{kN} \ \text{m}^2] \end{aligned}$$

Figure 28a presents the evolution of macroscopic values characterizing the evolution of generated and evaluated ship structure variants population in simulation **sym1-3** in case of **random values of weight coefficients  $w_1$  and  $w_2$  equal to 0 or 1**, used for optimization criteria with dominance attributes

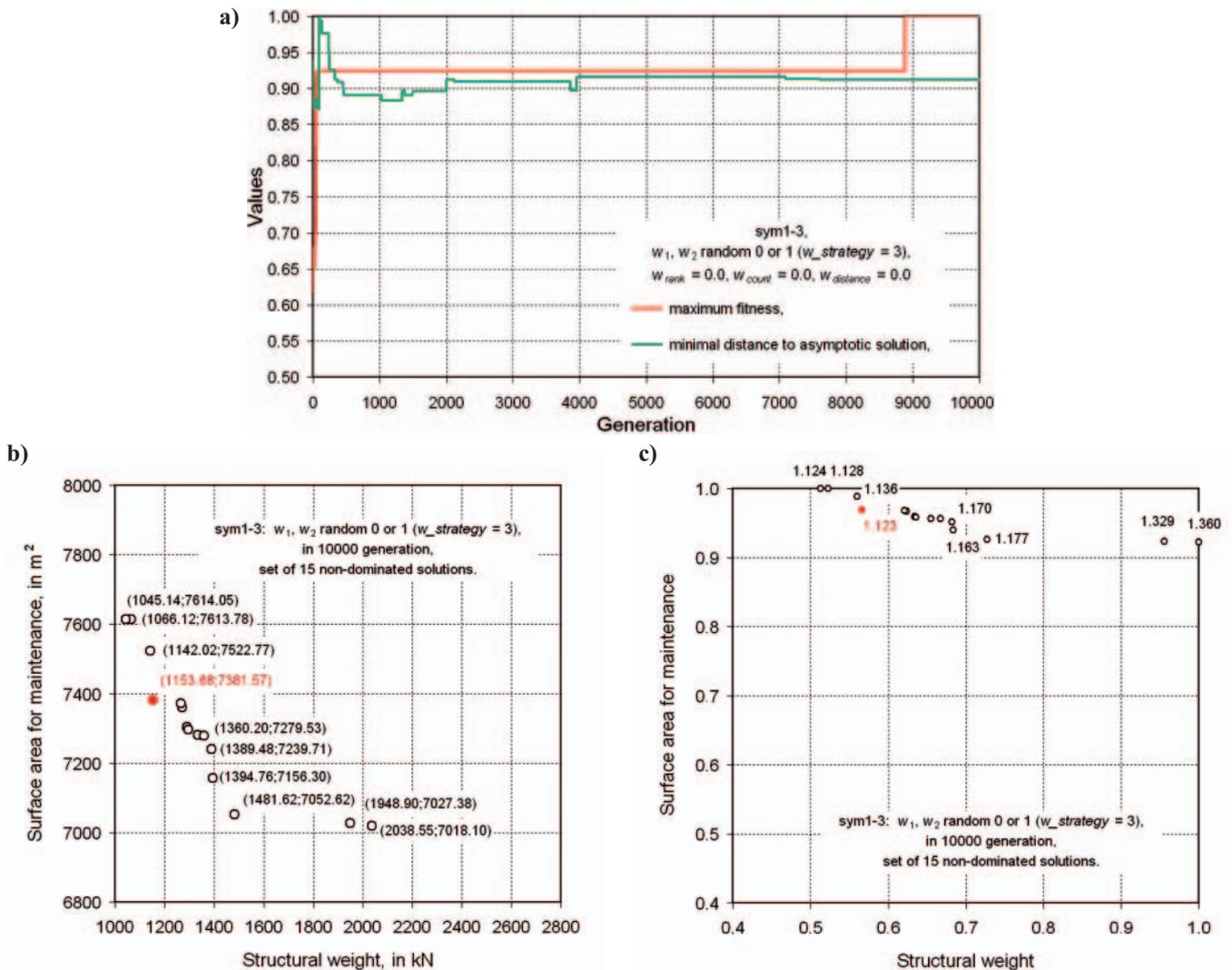


Fig. 28. Results of genetic multi-objective optimization of ship structure with respect to structural weight  $f_1$  and surface area  $f_2$  in case of **random values of weight coefficients  $w_1$  and  $w_2$  equal to 0 or 1** with dominance attributes being excluded from selection (**sym1-3**); **a)** the curves present the evolution of a highest value of fitness function  $f_{max}$ , the lowest value of non-dominated solution distance from a asymptotic one, **b)** detailed specification of final non-dominated solutions set in objective space, **c)** detailed specification of final non-dominated solutions set in normalized objective space; black circles represent non-dominated solutions, red dots represent non-dominated solution closest to the asymptotic one; dimensionless values are normalized to the interval  $[0,1]$  in relation to the highest values in the set

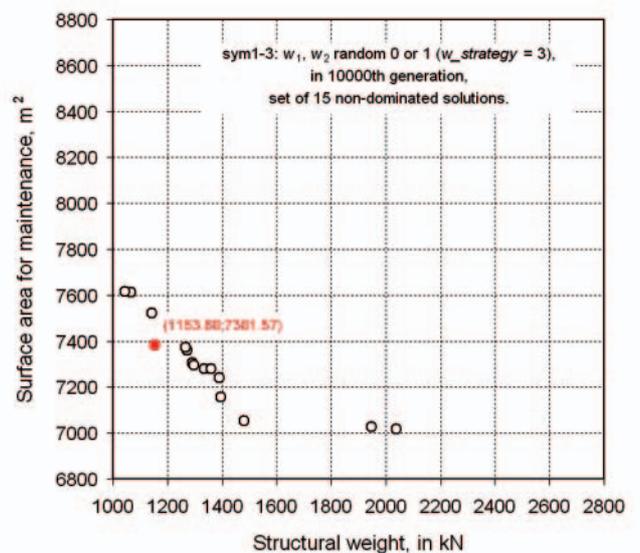
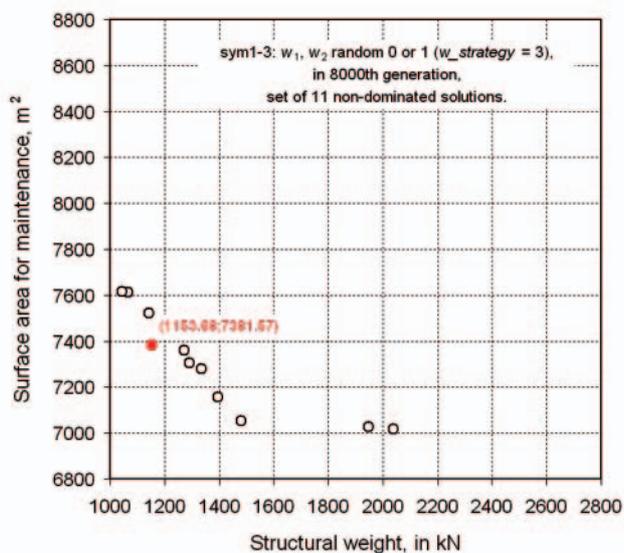
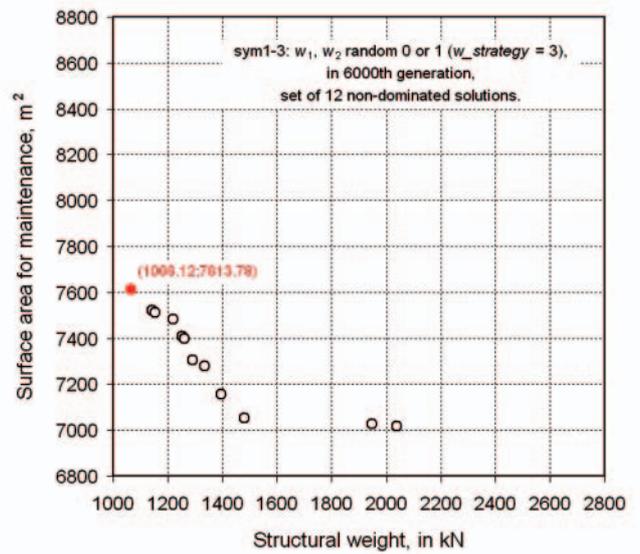
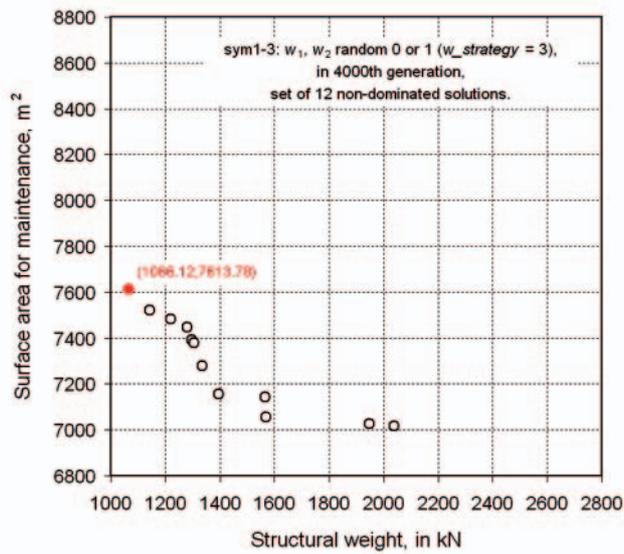
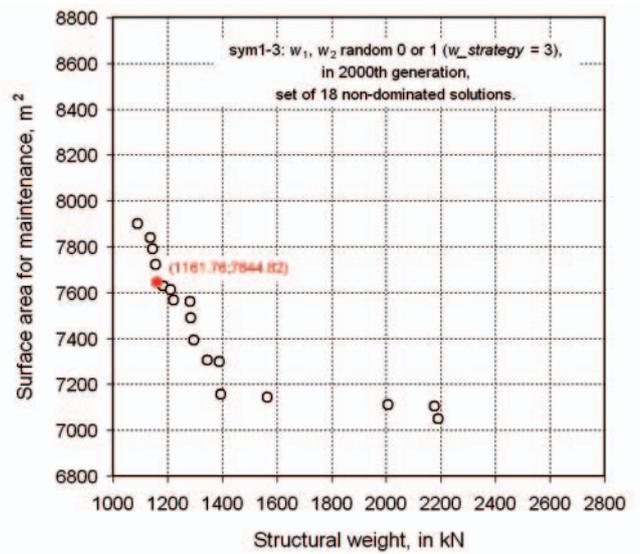
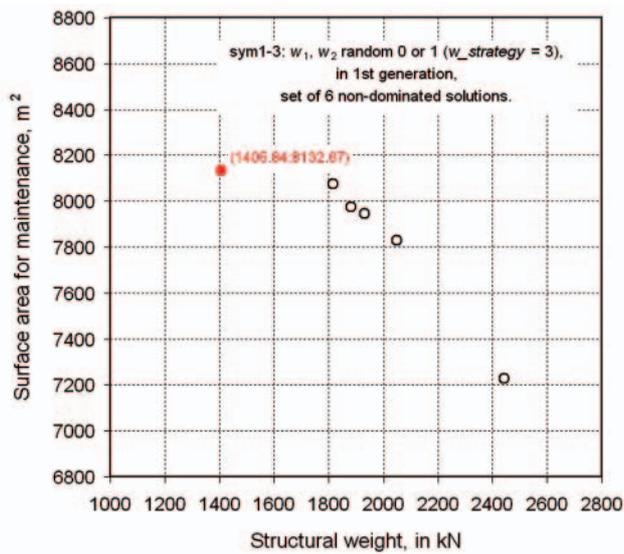


Fig. 29. History of the evolution of non-dominated solutions set during the genetic multi-objective optimization of ship structure with respect to structure weight  $f_1$  and surface area  $f_2$  in case of random values of weight coefficients  $w_1$  and  $w_2$  equal to 0 or 1 with dominance attributes being excluded from selection (sym1-3); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one

being excluded from selection. The figure shows a desired continuous rise of the greatest value of fitness function  $f_{\max}$  indicating rising quality of the best generated test solutions. The highest values of fitness function do not saturate before 8888 generation. i.e. very close to the end of simulation. The lowest distance between a non-dominated solution and the asymptotic solution changes during the evolution, but above the threshold of 88.30% of the highest value found during the simulation it takes place only to a very small extent.

In Fig. 29 the evolution of the structure of the non-dominated solutions set in simulation sym1-3 is shown by using, as examples, the selected time-based cross-sections of this set, e.g. for 1, 2000, 4000, 6000, 8000 and 10.000 generations. Cardinality of non-dominated solutions set in the selected generations amounts to 6, 18, 12, 12, 11 and 15, respectively, and it does not change significantly in the course of simulation.

Figures 28b and 28c present a detailed structure of a non-dominated solutions set of the last generation produced in simulation sym3. In can be seen that the set of non-dominated solutions including 15 ship structural variants has been found during the simulation. For the solution closest to the asymptotic solution which was found in 7611 generation,  $f_{7611}^{\approx}(\mathbf{x})$ , the distance of 1.123 in the normalized objective space, the structural weight is  $f_1(\mathbf{x}) = 1153.68$  kN and the surface area for maintenance is  $f_2(\mathbf{x}) = 7381.57$  m<sup>2</sup>:

$$\begin{aligned} \tilde{\mathbf{f}}_{\text{sym1-3}} &= \tilde{\mathbf{f}}_{7611}^{\approx}(\mathbf{x}) = [f_{1,7611}^{\approx}(\mathbf{x}) \ f_{2,7611}^{\approx}(\mathbf{x})]^T = \\ &= [1153.68 \ 7381.57]^T \cdot [\text{kN} \ \text{m}^2] \end{aligned}$$

### Results of computational investigations – Series2: the simulation marked with symbols sym2-1, sym2-2 and sym2-3

Figure 30 presents the evolution of macroscopic values characterizing the evolution of generated and evaluated ship structure variants population in simulation **sym2-1**: (1) the greatest fitness value  $f_{\max}$ , and (2) the lowest distance between the feasible variant and the asymptotic solution. Multi-objective optimization of ship structure with respect to structural weight  $f_1$  as well surface area for cleaning and painting,  $f_2$ , in the case of the values of optimization criteria weight coefficients,  $w_1 = w_2 = 0.0$ , dominance count,  $w_{\text{count}} = 0.0$  and the distance from asymptotic solution,  $w_{\text{distance}} = 0.0$ ; evolution of generations is governed by dominance rank,  $w_{\text{rank}} = 3.0$ , and constraints components,  $w_k \neq 0, k = 1, 2, \dots, n_c$ . The figure

shows a desired continuous rise of the greatest value of fitness function  $f_{\max}$  indicating rising quality of the best generated test variants. There can also be seen a very early, reached already in 585 generation, saturation of the fitness function maximum value  $f_{\max}$ . The smallest distance of the non-dominated solution from the asymptotic solution varies in course of evolution, but only in a small range over 79.42% of the greatest value found during the simulation.

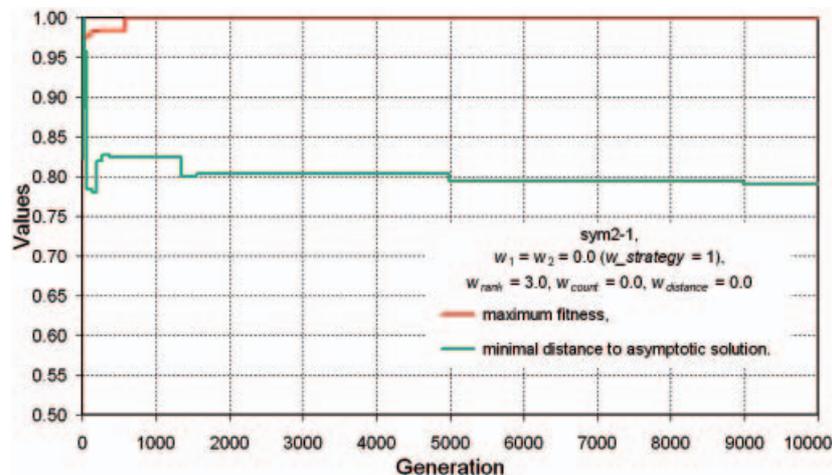
In Fig. 31 the evolution of the structure of the non-dominated solutions set in simulation sym2-1 is shown by using as examples the selected time-based cross-sections of this set, e.g. for 1, 2000, 4000, 6000, 8000 and 10.000 generations. Cardinality of non-dominated solutions set in the selected generations amounts to 4, 8, 6, 8, 8 and 10, respectively, and it does not change significantly in the course of simulation.

Figure 32 presents the detailed structure of the non-dominated solutions set of the last generation, presented in a physical space of objectives and the normalized space of objectives. In can be seen that a set of non-dominated solutions including 10 variants of ship structure has been found during the simulation. For each non-dominated variant the values of optimization criteria have been specified as:  $f_1(\mathbf{x})$  – structural weight and  $f_2(\mathbf{x})$  – cleaned/painted surface area. The designer may select, for further development, one of the variants or a group of them deemed by him to be the best. For the variant closest from the asymptotic solution, which was found already in 196 generation, the distance equals 1.064 in the normalized objective space, the structural weight is  $f_1(\mathbf{x}) = 1105.95$  kN and the cleaned/painted surface area is  $f_2(\mathbf{x}) = 7345.11$  m<sup>2</sup>:

$$\begin{aligned} \tilde{\mathbf{f}}_{\text{sym2-1}} &= \tilde{\mathbf{f}}_{196}^{\approx}(\mathbf{x}) = [f_{1,196}^{\approx}(\mathbf{x}) \ f_{2,196}^{\approx}(\mathbf{x})]^T = \\ &= [1105.95 \ 7345.11]^T \cdot [\text{kN} \ \text{m}^2] \end{aligned}$$

This variant may be recommended if there is a need to select a single solution for the formulated ship structure multi-objective optimization problem.

Figure 33 presents the evolution of macroscopic values characterizing the evolution of generated and evaluated ship structure variants population in simulation **sym2-2**. Multi-objective optimization of ship structure with respect to the structural weight  $f_1$  as well as surface for cleaning and painting  $f_2$  in case of the values of optimization criteria weight coefficients,  $w_1 = w_2 = 0$ , dominance rank,  $w_{\text{rank}} = 0.0$  as well as the distance from asymptotic solution,  $w_{\text{distance}} = 0.0$ ; evolution of generations is governed by dominance count,  $w_{\text{count}} = 3.0$ , and constraints components,  $w_k \neq 0, k = 1, 2, \dots, n_c$ . The figure



**Fig. 30.** The results of multi-objective genetic optimization of ship structure with respect to the structure weight  $f_1$  and the area of element surface  $f_2$ , in case of the fixed values of weight coefficients  $w_1 = w_2 = 0.0$ , with dominance count and the distance from a asymptotic solution being excluded from selection:  $w_{\text{count}} = 0.0, w_{\text{distance}} = 0.0$ , the evolution is governed by dominance rank,  $w_{\text{rank}} = 3.0$ , and constraints components, (**sym2-1**); the curves present the evolution of the highest value of fitness function  $f_{\max}$  and the lowest value of the non-dominated solution distance from a asymptotic one; the values are dimensionless and standardized in  $[0,1]$  range in relation to the highest values found during the simulation

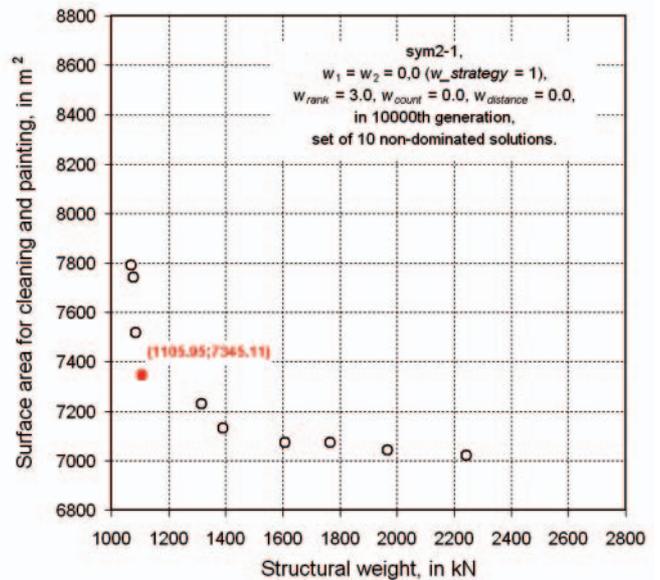
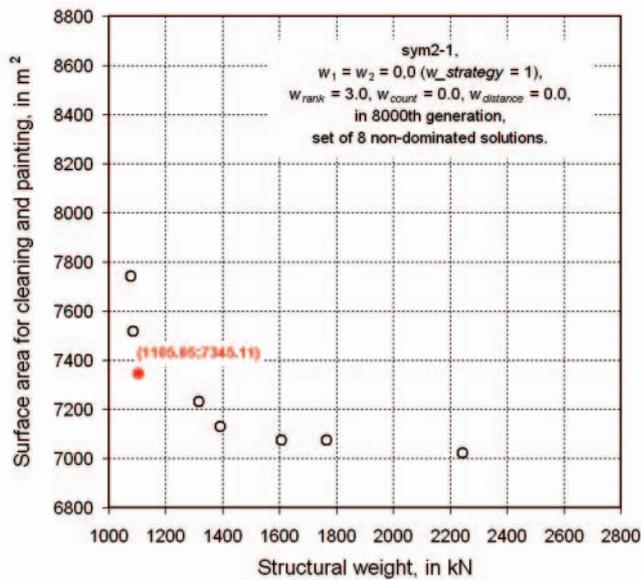
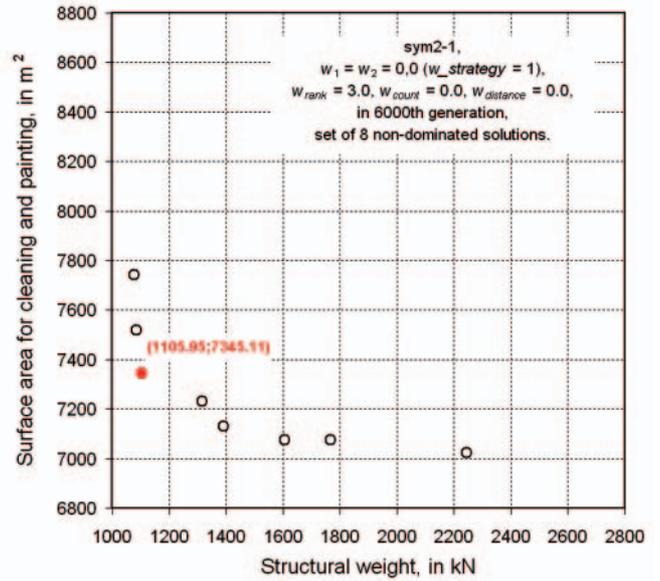
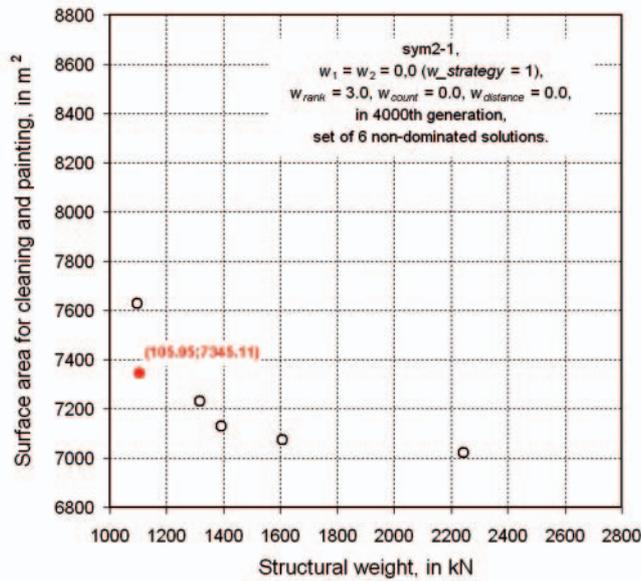
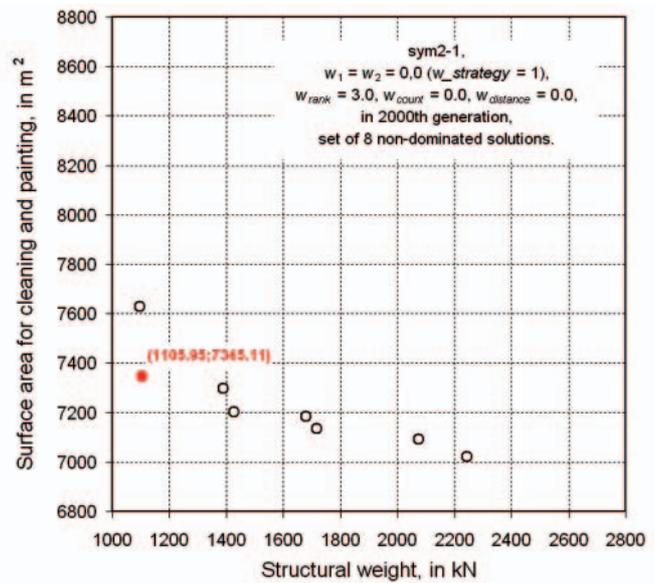
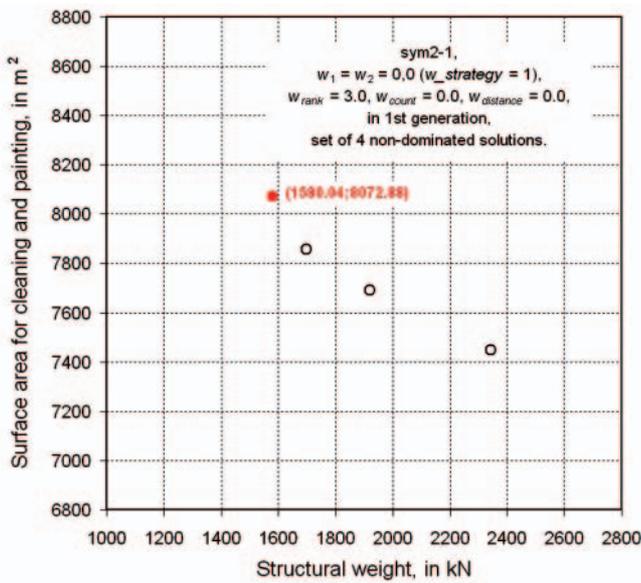
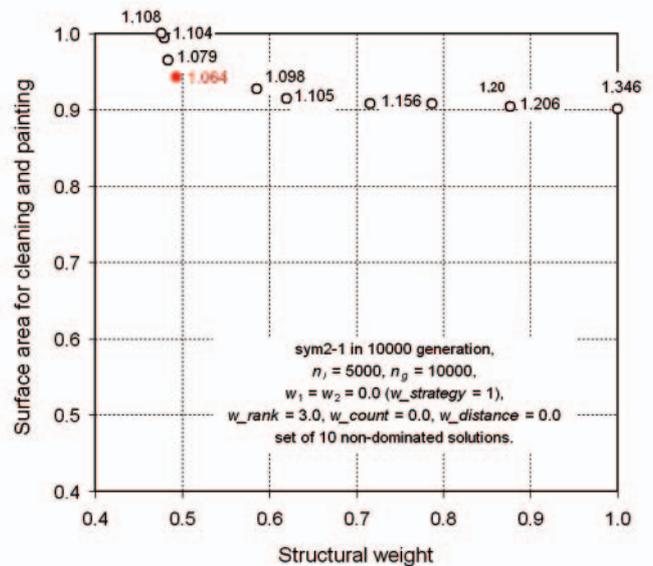
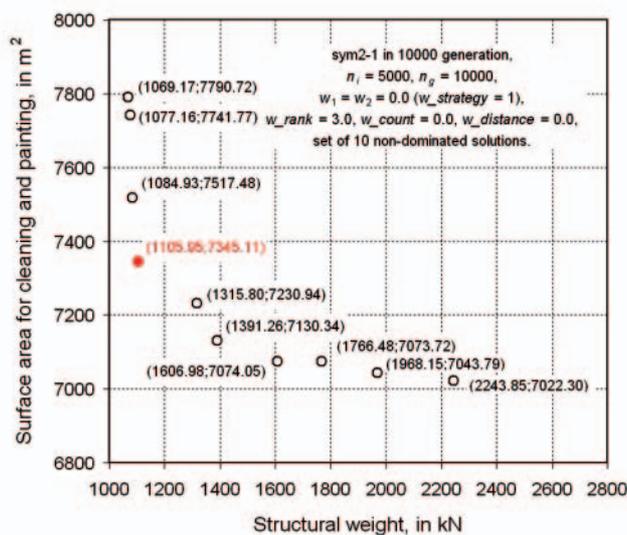
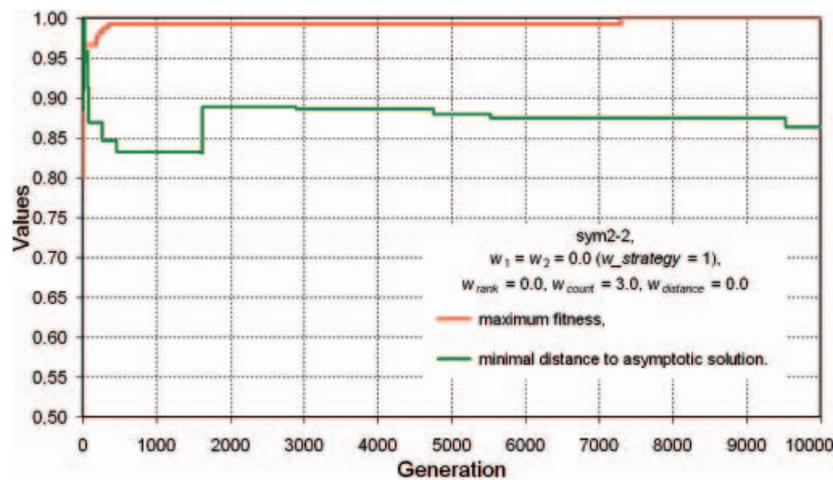


Fig. 31. History of the evolution of the non-dominated solutions set during the genetic multi-objective optimization of ship structure with respect to the structure weight  $f_1$  and surface area  $f_2$  in case of the fixed values of weight coefficients  $w_1 = w_2 = 0.0$ , with dominance count and the distance from a asymptotic solution being excluded from selection:  $w_{count} = 0.0$ ,  $w_{distance} = 0.0$ , the evolution is governed by dominance rank,  $w_{rank} = 3.0$ , and constraints components, (sym2-1); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest to the asymptotic one



**Fig. 32.** Detailed specification of the non-dominated solutions set obtained during the genetic multi-objective optimization of ship structure with respect to the structure weight  $f_1$  and surface area  $f_2$ , in case of exclusion from selection: optimization criteria,  $w_1 = w_2 = 0.0$ , dominance count,  $w_{count} = 0.0$  and distance from asymptotic solution,  $w_{distance} = 0.0$ ; the evolution is governed by dominance rank,  $w_{rank} = 3.0$ , and constraints components, (**sym2-1**); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one; dimensionless values are normalized to the interval  $[0,1]$  in relation to the highest values in the set



**Fig. 33.** The results of multi-objective genetic optimization of ship structure with respect to the structure weight  $f_1$  and the area of element surface  $f_2$  in case of the fixed values of weight coefficients  $w_1 = w_2 = 0.0$ , with dominance rank and the distance from a asymptotic solution being excluded from selection:  $w_{rank} = 0.0$ ,  $w_{distance} = 0.0$ , the evolution is governed by dominance count,  $w_{count} = 3.0$ , and constraints components, (**sym2-2**); the curves present the evolution of the highest value of fitness function  $f_{max}$  and the lowest value of non-dominated solution distance from a asymptotic one; the values are dimensionless and standardized in  $[0,1]$  range in relation to the highest values found during the simulation

shows a desired continuous rise of the greatest value of fitness function  $f_{max}$  indicating rising quality of the best generated test variants. The figure also shows fairly late, reached in 7301 generation, saturation of the maximum value of fitness function  $f_{max}$ . Minimum distance of the non-dominated solution from the asymptotic solution varies during the simulation but only in a small range over 83.18% of the greatest value found during the simulation.

In Fig. 34 the evolution of the structure of the non-dominated solutions set in simulation sym2-2 is shown by using as examples the selected time-based cross-sections of this set, e.g. for 1, 2000, 4000, 6000, 8000 and 10.000 generations. Cardinality of non-dominated solutions set in the selected generations amounts respectively to 4, 12, 12, 12, 13 and 13, and it does not change significantly in the course of simulation.

Figure 35 presents a detailed structure of the non-dominated solutions set of the last generation, presented in a physical space of objectives and the normalized space of objectives. In can be seen that a set of non-dominated solutions including 13

variants of ship structure has been found during the simulation. For each non-dominated variant the values of optimization criteria have been specified as:  $f_1(\mathbf{x})$  – structural weight and  $f_2(\mathbf{x})$  – cleaned/painted surface area. The designer may select, for further development, one of these variants or a group of them deemed by him to be the best. For the variant closest from the asymptotic solution, which was found in 5533 generation, the distance equals 1.047 in the normalized objective space, the structural weight is  $f_1(\mathbf{x}) = 1192.04$  kN and the cleaned/painted surface area is  $f_2(\mathbf{x}) = 7327.41$  m<sup>2</sup>:

$$\begin{aligned} \tilde{\mathbf{f}}_{\text{sym2-2}} &= \tilde{\mathbf{f}}_{5533}(\mathbf{x}) = [f_{1,5533}(\mathbf{x}) \ f_{2,5533}(\mathbf{x})]^T = \\ &= [1192.04 \ 7327.41]^T \cdot [\text{kN} \ \text{m}^2] \end{aligned}$$

This variant may be recommended if there is a need to select a single solution for the formulated ship structure multi-objective optimization problem.

Figure 36 presents the evolution of macroscopic values characterizing the evolution of generated and evaluated ship structure variants population in simulation **sym2-3**. Multi-objective optimization of ship structure with respect to the

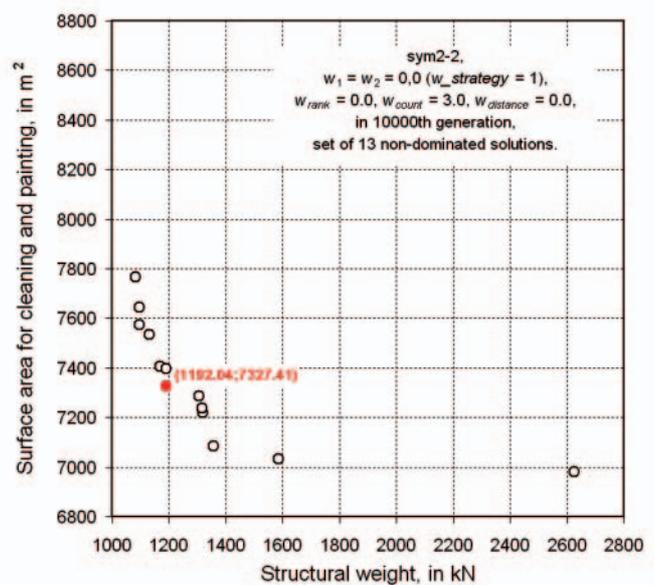
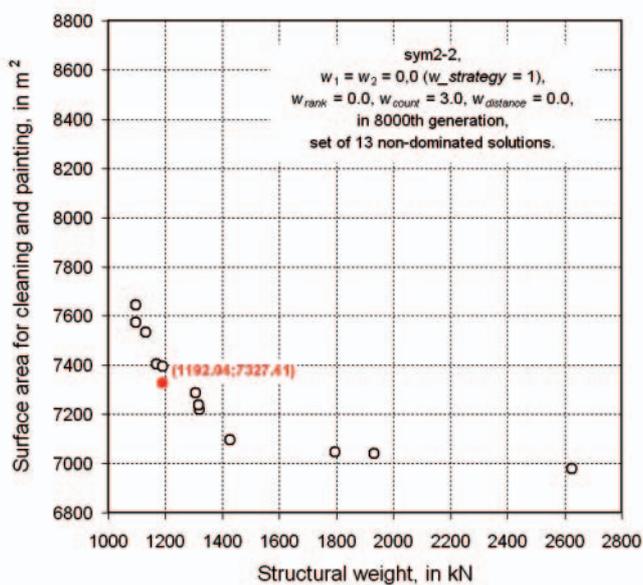
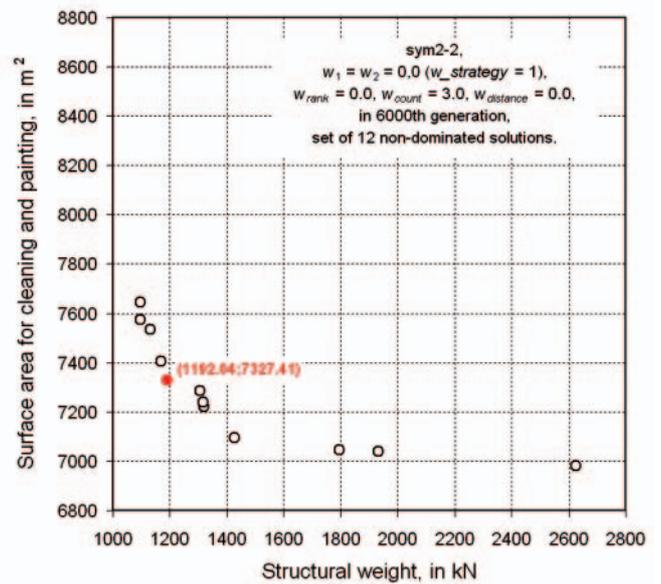
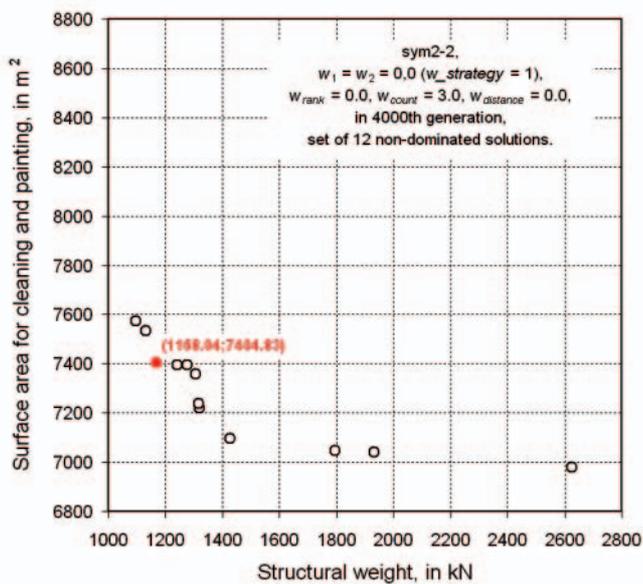
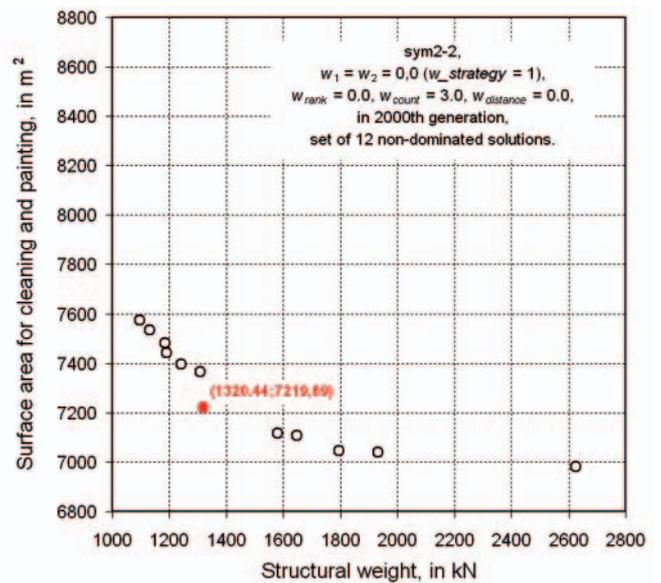
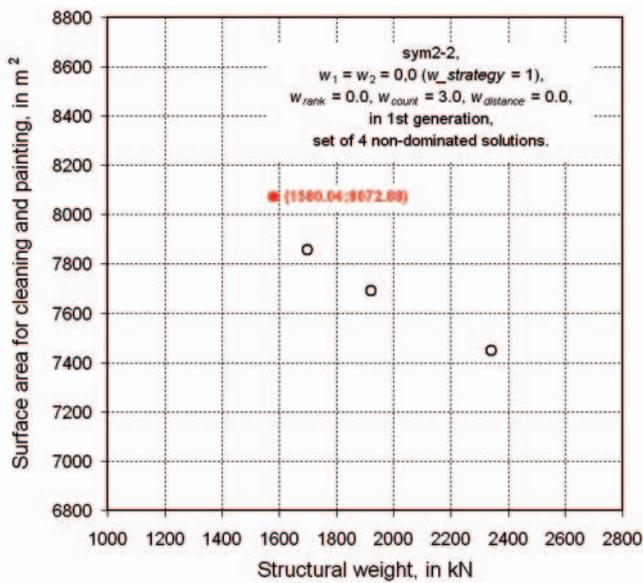


Fig. 34. History of the evolution of the non-dominated solutions set during the genetic multi-objective optimization of ship structure with respect to the structure weight  $f_1$  and surface area  $f_2$  in case of the fixed values of weight coefficients  $w_1 = w_2 = 0.0$ , with dominance rank and the distance from a asymptotic solution being excluded from selection:  $w_{rank} = 0.0, w_{distance} = 0.0$ , the evolution is governed by dominance count,  $w_{count} = 3.0$ , and constraints components, (sym2-2); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one

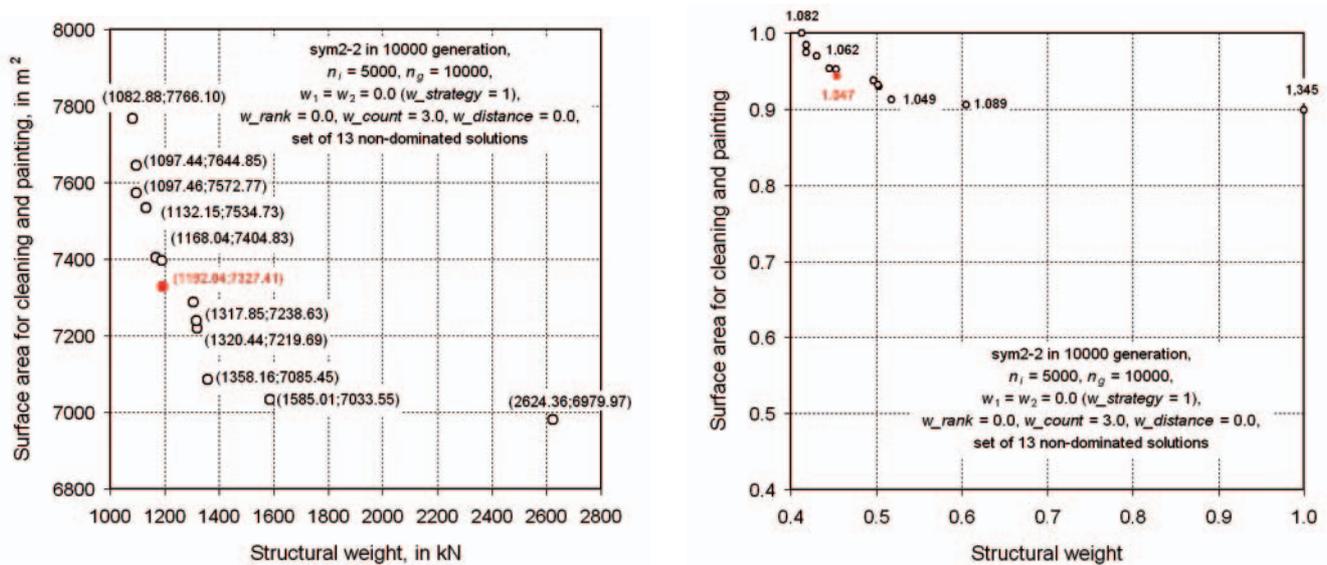


Fig. 35. Detailed specification of the non-dominated solutions set obtained during the genetic multi-objective optimization of ship structure with respect to the structure weight  $f_1$  and surface area  $f_2$  in case of exclusion from selection: optimization criteria,  $w_1 = w_2 = 0.0$ , dominance rank,  $w_{rank} = 0.0$  and distance from asymptotic solution,  $w_{distance} = 0.0$ ; evolution is governed by dominance count,  $w_{count} = 3.0$ , and penalty components, (sym2-2); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one; dimensionless values are normalized to the interval  $[0,1]$  in relation to the highest values in the set

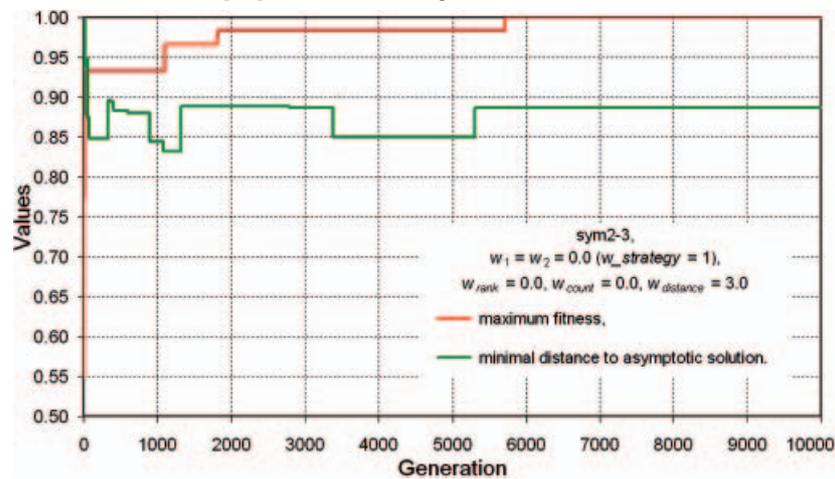


Fig. 36. The results of multi-objective genetic optimization of ship structure with respect to the structure weight  $f_1$  and the area of element surface  $f_2$  in case of the fixed values of weight coefficients  $w_1 = w_2 = 0.0$ , with dominance rank and the dominance count being excluded from selection:  $w_{rank} = 0.0$ ,  $w_{count} = 0.0$ , the evolution is governed by the distance from asymptotic solution,  $w_{distance} = 3.0$ , and constraints components, (sym2-3); the curves present the evolution of the highest value of fitness function  $f_{max}$  and the lowest value of non-dominated solution distance from an asymptotic one; the values are dimensionless and standardized in  $[0,1]$  range in relation to the highest values found during the simulation

structural weight  $f_1$  as well as surface area for cleaning and painting,  $f_2$ , in the weight coefficients values of optimization criteria  $w_1 = w_2 = 0$ , dominance rank,  $w_{rank} = 0.0$  and the dominance count,  $w_{count} = 0.0$ ; evolution of generations is governed by the distance from asymptotic solution,  $w_{distance} = 3.0$ , and constraints components,  $w_k \neq 0$ ,  $k = 1, 2, \dots, n_c$ . The figure shows a desired continuous rise of the greatest value of fitness function  $f_{max}$  indicating rising quality of the best generated test variants. The figure also shows fairly late, reached in 5714 generation, saturation of the maximum value of fitness function  $f_{max}$ . Minimum distance of the non-dominated solution from the asymptotic solution varies during the simulation but only in a small range over 83.25% of the greatest value found during the simulation.

In Figure 37 the evolution of the structure of the non-dominated solutions set in simulation sym2-3 is shown by using, as examples, the selected time-based cross-sections of this set, e.g. for 1, 2000, 4000, 6000, 8000 and 10,000 generations. Cardinality of non-dominated solutions set in these selected generations amounts to 4, 10, 12, 11, 14 and 13, respectively, and it does not change significantly in the course of simulation.

Figure 38 presents the detailed structure of a non-dominated solutions set of the last generation, presented in a physical space of objectives and the normalized space of objectives. In can be seen that a set of non-dominated solutions including 13 variants of ship structure has been found during the simulation. For each non-dominated variant the values of optimization criteria have been specified as:  $f_1(\mathbf{x})$  – structural weight and  $f_2(\mathbf{x})$  – cleaned/painted surface area. The designer may select, for further development, one of these variants or a group of them deemed by him to be the best. For the variant closest to the asymptotic solution, which was found in 5305 generation, the distance equals 1.085 in the normalized objective space, the structural weight is  $f_1(\mathbf{x}) = 1060.03$  kN and the cleaned/painted surface area is  $f_2(\mathbf{x}) = 7485.93$  m<sup>2</sup>:

$$\begin{aligned} \mathbf{f}_{\text{sym2-3}}^* &= \mathbf{f}_{5305}^*(\mathbf{x}) = [f_{1,5305}^*(\mathbf{x}) \ f_{2,5305}^*(\mathbf{x})]^T = \\ &= [1060.03 \ 7485.93]^T \cdot [\text{kN} \ \text{m}^2] \end{aligned}$$

This variant may be recommended if there is a need to select a single solution for the formulated ship structure multi-objective optimization problem.

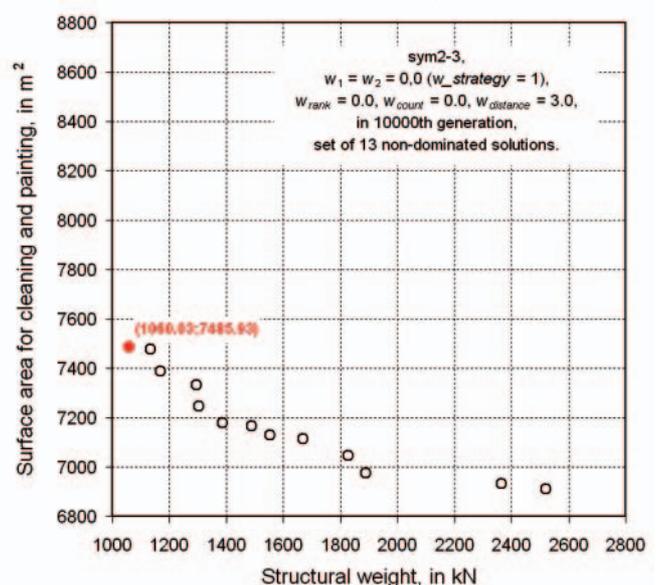
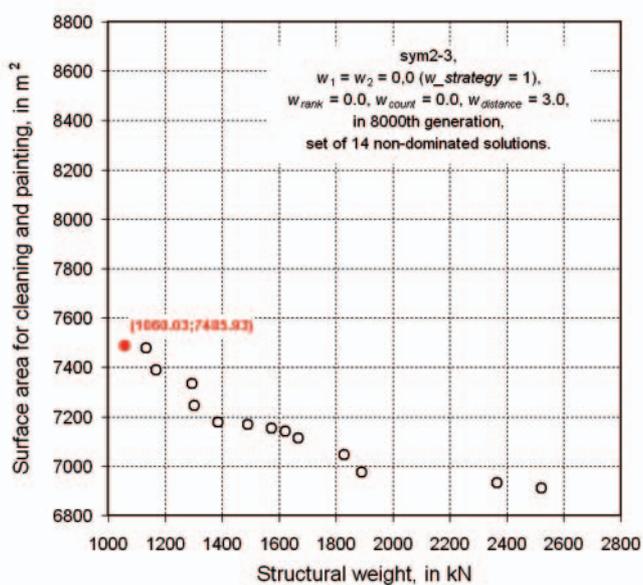
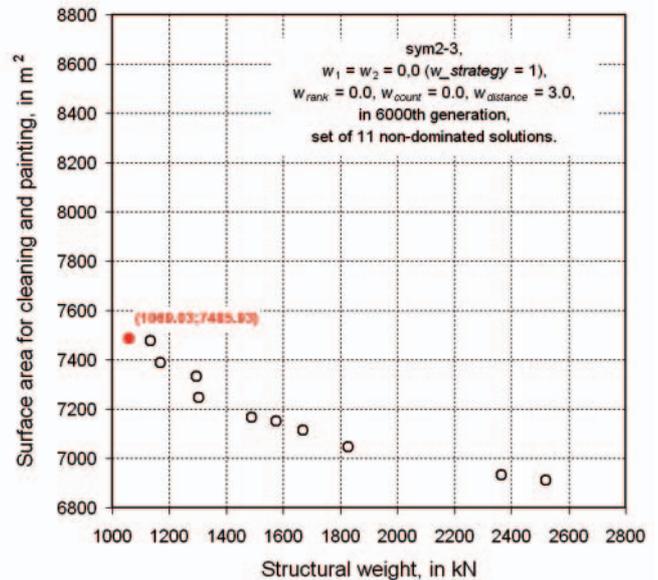
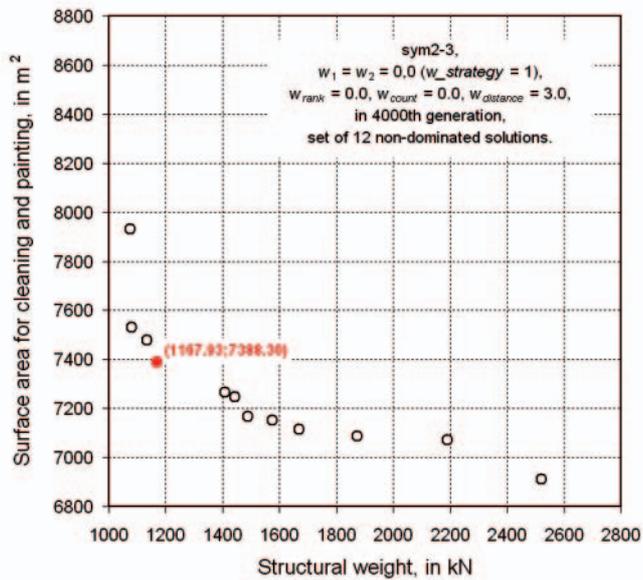
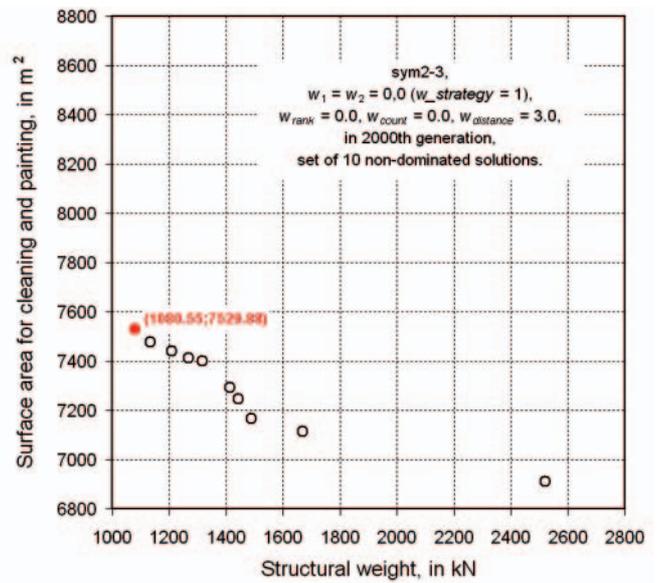
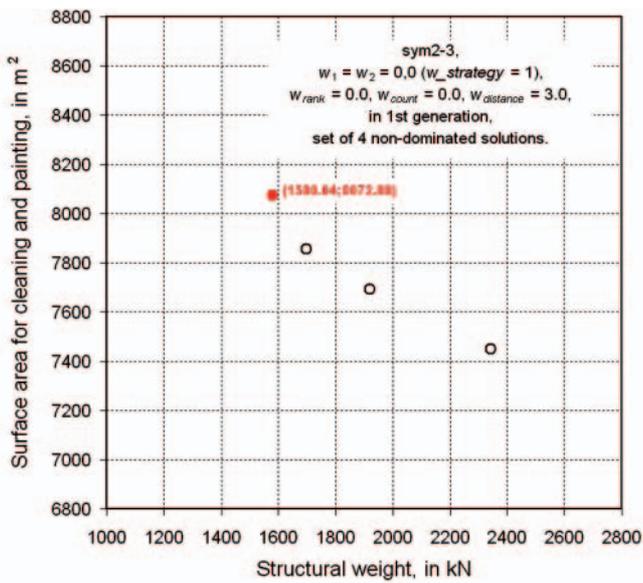


Fig. 37. History of the evolution of the non-dominated solutions set during the genetic multi-objective optimization of ship structure with respect to the structure weight  $f_1$  and surface area  $f_2$  in case of the fixed values of weight coefficients  $w_1 = w_2 = 0.0$ , with dominance rank and the dominance count being excluded from selection:  $w_{rank} = 0.0, w_{count} = 0.0$ , the evolution is governed by distance from asymptotic solution,  $w_{distance} = 3.0$ , and constraints components, (sym2-3); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one

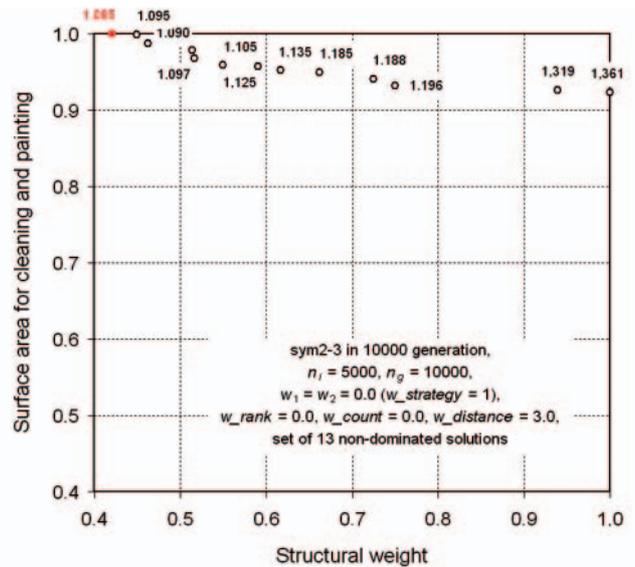
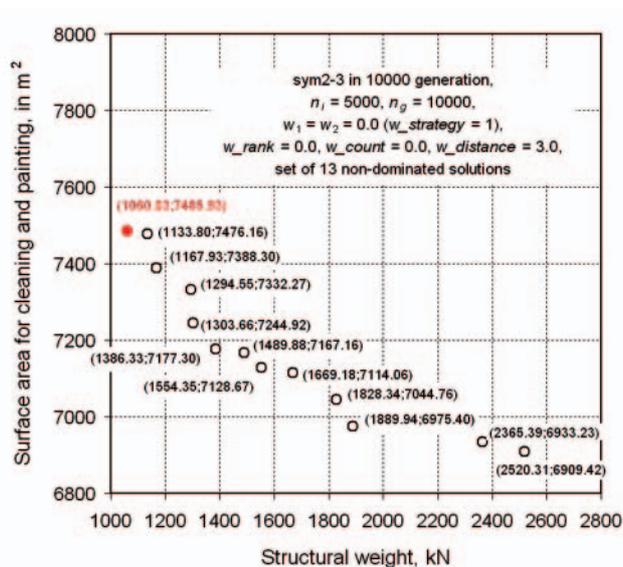


Fig. 38. Detailed specification of the non-dominated solutions set obtained during the genetic multi-objective optimization of ship structure with respect to the structure weight  $f_1$  and surface area  $f_2$  in case of exclusion from selection: optimization criteria,  $w_1 = w_2 = 0.0$ , dominance rank,  $w_{rank} = 0.0$  and dominance count,  $w_{count} = 0.0$ ; evolution is governed by distance from the asymptotic solution,  $w_{distance} = 3.0$ , and constraints components, (sym2-3); black circles represent non-dominated solutions, red dots represent non-dominated solutions closest from the asymptotic one; dimensionless values are normalized to the interval  $[0,1]$  in relation to the highest values in the set

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