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Comparative analysis of fatigue life calculation methods of C45 steel in conditions of variable amplitude loads in the low- and high-cycle fatigue ranges

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ABSTRACT

In the paper [1] assumptions for fatigue life calculations in the low- and high- cycle fatigue ranges of metal alloys, have been formulated. Three calculation methods: that expressed in stresses, strains and a mixed (stress-strain) one, have been there presented. In this paper results of fatigue life calculations of C45 steel performed in line with the above mentioned methods have been described, and their comparative analysis has been made. The calculations have been carried out for two-level and multi-level load programs of the varying parameters: maximum load within a load program and different values of spectrum filling factor. The comparative analysis of the results of fatigue life calculations in compliance with the above mentioned methods makes it possible to assess differences in the results and depenendence of the differences on values of load program parameters.

Keywords: fatigue life calculations; low- and high- cycle fatigue; C45 steel

INTRODUCTION

Operational loads of structural elements are generally random. Load spectra elaborated in accordance with relevant cycle-counting methods contain sets of sinusoidal cycles of different parameters, especially different values of the load amplitude S_a and mean value S_m [2]. In the case when the load $S_{max} = S_m + S_a \leq R_e$ fatigue life is calculated with the use of the method which applies Wöhler diagram (stress approach – the high-cycle fatigue range, HCF), whereas for $S_{max} > R_e$ the method using Manson-Coffin diagram is applied to fatigue calculations (strain approach – the low-cycle fatigue range, LCF) [3, 4].

Load spectrum usually contains cycles of different share of S_{max} values occurring both in the HCF and LCF range. Therefore in the authors' research project [5] has been introduced the theme of a hybrid calculation method in which fatigue damage due to cycles in the HCF range is calculated by means of the stress approach method whereas in the LCF range the strain approach method is used. Then the question appears whether applying either stress approach methods or strain approach ones for the entire range of loads (LCF + HCF) one obtains fatigue life results significantly different from those calculated by using the hybrid method. Significance of the so formulated problem, as shown in [6, 7], consists also in that criteria for qualifying the loads either to LCF or HCF range are ambiguous – blurred. This work is aimed at answering the above formulated question.

Assumptions for calculations of fatigue life of metal alloys in the conditions when loading occurs both in the LCF and HCF ranges, were presented in the paper [1]. In the calculation method described in the paper three paths were distinguished: 1st – based on application of the full (complete) Wöhler diagram (i.e. acc. stress approach) 2nd – based on application of Manson-Coffin diagram (i.e. acc. strain approach), and 3rd – hybrid one.

Fatigue life calculations in the varying amplitude load conditions require to know a load spectrum or program, cyclic properties of material, usually in the form of the above mentioned fatigue diagrams, as well as to assume an appropriate hypothesis for fatigue damage summation [8]. Out of many known hypotheses, for the calculations described in this paper the Palmgren-Miner linear summation hypothesis, the best experimentally verified one, was selected.

In accordance with the hypothesis the sum of damages resulting from the number of cycles, $n_{0.}$ during realization of loading program, is calculated from the formula:

- for 1st path:

$$D_0 = \sum_{i=1}^{k} \frac{n_{oi}}{N_i} \tag{1}$$

- for 2nd path:

$$D_0 = \sum_{i=1}^{k} \frac{n_{oi}}{N_{fi}} \tag{2}$$

- for 3rd path:

$$D_0 = \sum_{i=1}^{l} \frac{n_{oi}}{N_{fi}} + \sum_{i=(l+1)}^{k} \frac{n_{oi}}{N_i} \tag{3}$$

Fatigue fracture will occur when the sum of damages D = 1.0. which leads to the number of program repetitions:

$$\lambda = 1.0/D_0 \quad (4)$$

and to the fatigue life expressed by number of cycles:

$$N_c = \lambda \cdot n_o \quad (5)$$

DATA FOR CALCULATIONS OF C45 STEEL FATIGUE LIFE

Load programs

The calculations were performed for load programs of two kinds: two-level one of variable parameters, and multi-level one (Fig. 1).

In the two-level program the following parameters were assumed:

- four valuess of the maximum stresses S_{max} within the program: 650 MPa, 570 MPa, 460 MPa and 340 MPa,
- three ratios of stress values, S_{a2}/S_{a1}: 0.75; 0.5; 0.25,
- four ratios of numbers of load cycles, n01/n0: 0.75; 0.5; 0.25; 0.1.

In the further part of this work the assumed values of S_{max} loads are marked as follows: I – 650 MPa, II – 570 MPa, III – 460 MPa i IV – 340 MPa.

Such selection of the parameters makes it possible to realize a wide research program comprising 64 different cases.

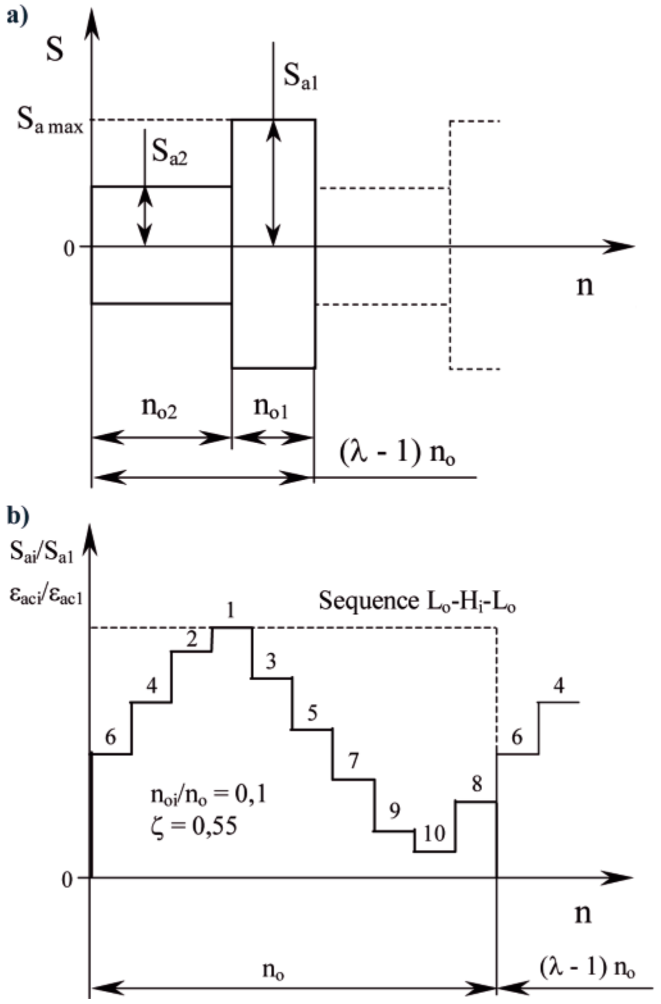

*Fig. 1. Schematic diagrams of the load programs assumed for calculations: **a)** two-level program, **b)** multi-level program*

In the tests according to 2nd path (i.e. strain approach) and 3rd path (i.e. hybrid one) values of the total strains ε_{ac} correspond to relevant values of the stresses S_a.

The parameter which characterizes load intensity is the spectrum filling factor described by the formula:

$$\zeta = \sum_{i-1}^{k} \frac{S_{ai}}{S_{a\,max}} \cdot \frac{n_{oi}}{n_o} \quad (6)$$

which, on transformation, for the two-level program, takes the following form:

$$\zeta = \frac{n_{o1}}{n_o} + \frac{S_{a2}}{S_{a1}}\left(1 - \frac{n_{o1}}{n}\right) \quad (7)$$

Values of the spectrum filling factor for the assumed parameters are collected in Tab. 1.

Tab. 1. Values of the spectrum filling factor ζ

		S_{a2}/S_{a1}			
		0.25	**0.50**	**0.75**	**1.0**
n_{o1}/n_o	**0.10**	0.325	0.550	0.775	1.0
	0.25	0.437	0.625	0.812	1.0
	0.50	0.625	0.750	0.875	1.0
	0.75	0.812	0.875	0.937	1.0

The data contained in Tab. 1 indicate the wide range of loading conditions for fatigue calculations, starting from $\zeta = 0.325$ which corresponds to e.g. the loading of track system elements of road vehicles, up to $\zeta = 1.0$ which corresponds to sinusoidal load of constant amplitude.

The multi-level load program was elaborated on the basis of the load spectrum of the mean value of the spectrum filling factor ζ, calculated from the corresponding data given in Tab. 1. And, the graphical form of the program is shown in Fig. 2 in the system of relative quantities.

The program assumed for the tests in question has 10 load levels (S_{ai} or ε_{aci}) of the same cycle capacity $n_{oi}/n_o = 0.1$ and the same load span S_{ai}-$S_{ai+1} = 0.1 \cdot S_{a1}$. The spectrum filling factor $\zeta = 0.55$.

Moreover, for the fatigue life tests and calculations were assumed maximum stress levels complying with those taken for fatigue life analyses in the two-level loading conditions.

Static and cyclic properties of C45 steel

Mechanical properties of C45 steel were determined on the basis of specimens prepared in compliance with the Polish standards: static properties – acc. PN-EN ISO 6892-1:2010. cyclic properties – acc. PN-84/H-04334. The values of the determined parameters are: $R_m = 682$ MPa, $R_e = 458$ MPa, $E = 2.15 \cdot 10^5$ MPa. The Wöhler diagram experimentally determined for R = -1.0 is described by the formula:

$$\log S_a = -0.1020 \log N + 2.9611 \quad (8)$$

for which the fatigue limit at $N_0 = 10^6$ is equal to $S_{f(-1)} = 223.5$ MPa, and the exponent $m_{(-1)} = 9.8$.

The Manson-Coffin diagram experimentally determined is described by the formula:

$$\varepsilon_{ac} = \frac{1204}{2.15 \cdot 10^5}(2N_f)^{-0.1033} + 0.2179(2N_f)^{-0.475} \quad (9)$$

To transform the load program from that formulated in stress approach to strain approach one, was used the experimentally determined Ramberg-Osgood diagram of the following form:

$$\varepsilon_{ac} = \frac{S_a}{2.15 \cdot 10^5} + \left(\frac{S_a}{1232}\right)^{5.06} \quad (10)$$

RESULTS OF CALCULATIONS FOR TWO-LEVEL LOADS

Results of calculations according the 1st path

The calculation results in the form of Gassner diagrams are shown in Fig. 2 on the Wöhler diagram background. As results from the data assumed for the calculations, the levels I, II and III correspond to the LCF condition, i.e. $S_{a1} > R_e$, similarly the values S_{a2} for $S_{a2}/S_{a1} = 0.75$ are contained within the low-cycle fatigue (LCF). The remaining values, S_{a1} and S_{a2} are smaller than R_e, hence in compliance with the assumed criterion they are rated to be in the high-cycle fatigue range (HCF).

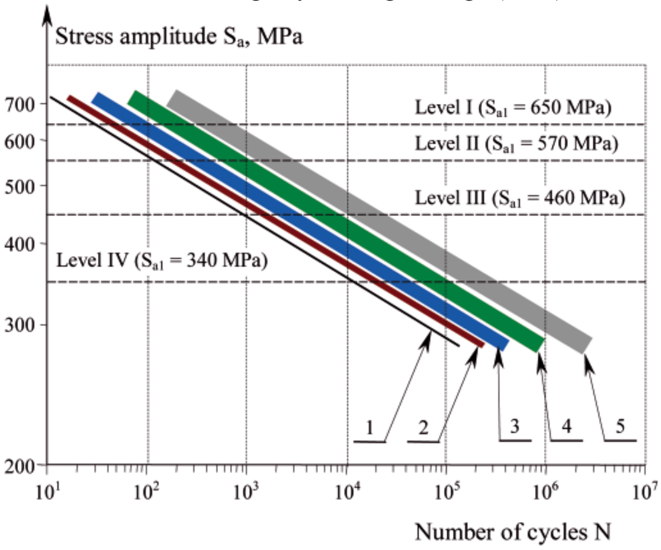

Fig. 2. *Fatigue life diagrams for C45 steel determined by calculations according to 1st path in the two-level loading conditions:* ***1)*** *Wöhler diagram for $n_{01}/n_0 = 1.0$;* ***2)*** *Gassner diagram for $n_{01}/n_0 = 0.75$;* ***3)*** *Gassner diagram for $n_{01}/n_0 = 0.5$;* ***4)*** *Gassner diagram for $n_{01}/n_0 = 0.25$;* ***5)*** *Gassner diagram for $n_{01}/n_0 = 0.1$*

Results of calculations according to the 2nd path

In compliance with the description given in p. 1. in the 2nd path calculations the load ranges LCF and HCF have to be related to Masnon-Coffin diagram. To this end, it is necessary, for description cyclic strain diagram, to transform loads expressed in stress units into those expressed in total strains by using the Ramberg-Osgood diagram (10). The total strain values corresponding to relevant stress values are given on the ordinate axis of the diagrams in Fig. 3. The number of $2N_f$ recurrence to fatigue fracture was calculated by using the Mason-Coffin formula (9). The calculation results are graphically presented in Fig. 3.

In Fig. 2 and 3 the diagrams are presented in the form of shadowed bands. Their left edges correspond to the ratio $S_{a2}/S_{a1} = 0.75$, right ones – the ratio $S_{a2}/S_{a1} = 0.25$, and the middle line of the band corresponds to the ratio $S_{a2}/S_{a1} = 0.5$.

Calculations in compliance with the hybrid method for two-level loading (the 3rd path)

According to the hybrid method (the 3rd path) complete damage corresponds to sum of the damage resulting from the loading within the low-cycle range (LCF), calculated by using Manson-Coffin fatigue diagram and the damage due to the loading within the high-cycle range (HCF) calculated by means of Wöhler fatigue diagram according to the formula (3).

The sums of damages calculated acc. (3) and the fatigue life calculated in compliance with the hybrid method, are presented in Tab. 2.

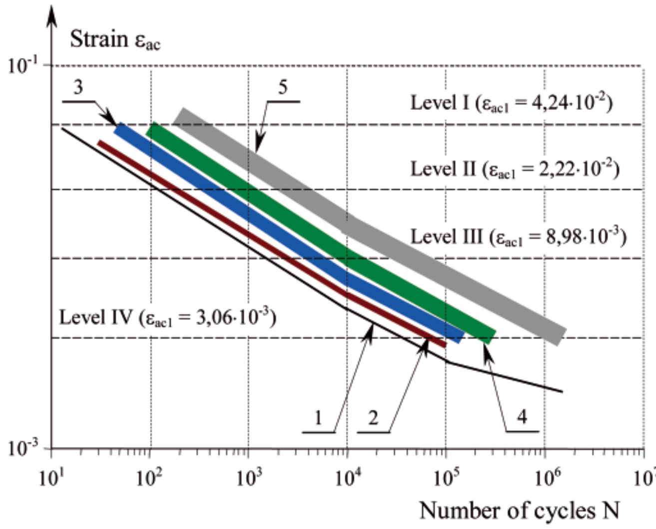

Fig. 3. *Fatigue life diagrams for C45 steel determined by calculations according to 2nd path in the two-level loading conditions:* ***1)*** *Manson-Coffin diagram;* ***2)*** *Gassner diagrams for $n_{01}/n_0 = 0.75$;* ***3)*** *Gassner diagrams for $n_{01}/n_0 = 0.5$;* ***4)*** *Gassner diagrams for $n_{01}/n_0 = 0.25$;* ***5)*** *Gassner diagrams for $n_{01}/n_0 = 0.1$*

In line with expectations, for high load values (the level I) the results of fatigue life calculated by using the hybrid method (the 3rd path) are closer to the results of calculations according to the 2nd path, and for low load values (the level IV) the results of fatigue life calculations according to the 3rd path are closer to those calculated in compliance with the 1st path.

The above given statement shows that in the case of higher values of the ratio the higher level, i.e. the amplitude S_{a1} or $\varepsilon_{ac1.}$ is significantly decisive of fatigue whereas influence of the lower level, i.e. S_{a2} or $\varepsilon_{ac2.}$ is not significant. This is confirmed by the data given in Tab. 3 and 4. Tab. 3 contains the numerical data concerning the ratio of the fatigue life calculated by applying stress approach (the 1st path) and that calculated by using strain approach (the 2nd path).

As results from the data contained in Tab. 3, for high stress values and strains corresponding to them (the load levels I and II) the more conservative results are achieved from the strain approach calculations (the 2nd path), whereas for lower load values (the levels II and IV) the results of stress approach calculations (the 1st path) are more conservative. Fig. 4 illustrates the conclusion.

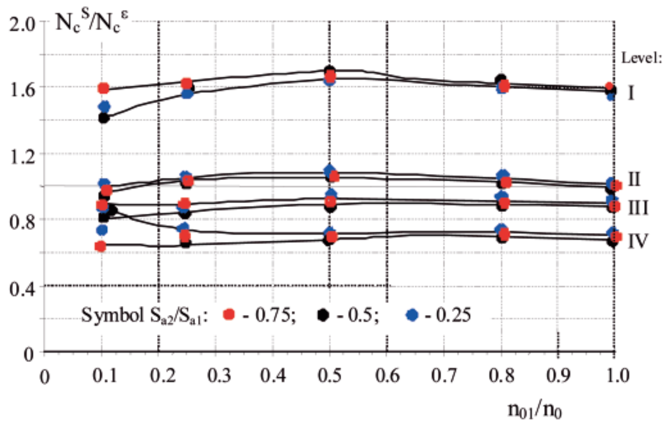

Fig. 4. *Dependence of the ratio of the fatigue life calculated acc. the 1st path and that calculated acc. the 2nd path on load level and the test program form described by the ratios S_{a2}/S_{a1} and n_{01}/n_0*

As results from the diagrams shown in Fig. 4, the influence of loading program form on difference in the calculation

Tab. 2. Values of damage parameter and fatigue life calculated in compliance with the hybrid method (the 3rd path)

No. of level	n_{01}/n_0							
	0.75		0.5		0.25		0.1	
	$D_0 = D_{01}+D_{02}$	N_cH	$D_0 = D_{01}+D_{02}$	N_cH	$D_0 = D_{01}+D_{02}$	N_cH	$D_0 = D_{01}+D_{02}$	N_cH
1	2	3	4	5	6	7	8	9
$n_0 = 10$								
I								
a	0.433540	23.0	0.297120	33.6	0.144466	69.0	0.077200	129.0
b	0.428100	23.4	0.286198	35.0	0.143098	70.0	0.057557	173.7
c	0.428000	23.4	0.286002	35.0	0.142800	70.0	0.572004	175.0
II								
a	0.084850	117.9	0.058500	170.9	0.032260	309.9	0.016330	612.3
b	0.083423	120.0	0.055655	179.7	0.027882	358.7	0.011198	893.0
c	0.083400	120.0	0.055600	180.0	0.027800	360.0	0.011100	900.9
III								
a	0.085180	1173.9	0.059100	1692.0	0.032950	3034.9	0.011743	8515.7
b	0.083434	1198.6	0.055667	1796.4	0.027701	3609.9	0.011112	8999.1
c	0.083400	1200.0	0.055600	1800.0	0.027600	3623.2	0.011100	9009.0
IV								
a	$4.76\cdot10^{-5}$	21000	$3.315\cdot10^{-5}$	30166	$1.805\cdot10^{-5}$	55400	$6.281\cdot10^{-6}$	159200
b	$4.69\cdot10^{-5}$	21300	$3.134\cdot10^{-5}$	31912	$1.667\cdot10^{-5}$	60000	$6.250\cdot10^{-6}$	160000
c	$4.69\cdot10^{-5}$	21300	$3.134\cdot10^{-5}$	31912	$1.667\cdot10^{-5}$	60000	$6.250\cdot10^{-6}$	160000
Notation: **a** – $S_{a2}/S_{a1} = 0.75$; **b** – $S_{a2}/S_{a1} = 0.5$; **c** – $S_{a2}/S_{a1} = 0.25$								

Tab. 3. The ratio of the fatigue life calculated by applying stress approach (the 1st path) and that calculated by using strain approach (the 2nd path)

No. of level	S_{a1} MPa	ε_{ac1}	S_{a2}/S_{a1}; $\varepsilon_{ac}/\varepsilon_{ac1}$	n_{01}/n_0				
				1.0	0.75	0.50	0.25	0.1
1	2	3	4	5	6	7	8	9
I	650	$4.24 \cdot 10^{-2}$	1.0	1.6				
			0.75		1.59	1.65	1.62	1.41
			0.50		1.58	1.59	1.61	1.44
			0.25		1.58	1.60	1.61	1.60
II	570	$2.22 \cdot 10^{-2}$	1.0	1.13				
			0.75		1.14	1.14	1.07	0.99
			0.50		1.14	1.14	1.13	1.13
			0.25		1.14	1.14	1.13	1.13
III	460	$8.93 \cdot 10^{-3}$	1.0	0.93				
			0.75		0.93	0.93	0.89	0.83
			0.50		0.93	0.93	0.92	0.92
			0.25		0.93	0.93	0.92	0.92
IV	340	$3.06 \cdot 10^{-3}$	1.0	0.71				
			0.75		0.71	0.67	0.67	0.88
			0.50		0.71	0.71	0.72	0.75
			0.25		0.71	0.71	0.72	0.71

Tab. 4. The ratio of the fatigue life calculated acc. stress approach (the 1st path) and acc. strain approach (the 2nd path), respectively, and the fatigue life calculated by using the hybrid method

No. of level	S_{a1} MPa	ε_{ac1}	S_{a2}/S_{a1}; $\varepsilon_{ac}/\varepsilon_{ac1}$	n_{01}/n_0									
				1.0		0.75		0.5		0.25		0.1	
				N_c^S/N_c^H	N_c^ε/N_c^H	N_c^S/N_c^H	N_c^ε/N_c^H	N_c^S/N_c^H	N_c^ε/N_c^H	N_c^S/N_c^H	N_c^ε/N_c^H	N_c^S/N_c^H	N_c^ε/N_c^H
1	2	3	4	5	6	7	8	9	10	11	12	13	14
I	650	$4.24 \cdot 10^{-2}$	1.0	1.6	1.0								
			0.75			1.56	1.00	1.65	1.00	1.60	1.00	1.42	1.00
			0.50			1.56	1.00	1.60	1.00	1.60	1.00	1.43	1.00
			0.25			1.56	1.00	1.60	1.00	1.60	1.00	1.60	1.00
II	570	$2.22 \cdot 10^{-2}$	1.0	1.13	1.0								
			0.75			1.16	1.00	1.13	1.01	1.12	1.04	1.08	1.09
			0.50			1.14	1.00	1.13	0.99	1.13	1.00	1.13	1.00
			0.25			1.14	1.00	1.13	0.99	1.13	1.00	1.13	1.00
III	460	$8.98 \cdot 10^{-3}$	1.0	0.93	1.0								
			0.75			0.93	1.00	0.93	1.00	0.93	1.05	0.63	0.76
			0.50			0.92	0.99	0.92	0.99	0.92	0.99	0.91	0.99
			0.25			0.92	0.99	0.92	0.99	0.92	0.99	0.93	0.99
IV	340	$3.06 \cdot 10^{-3}$	1.0	1.0	0.71								
			0.75			1.0	1.41	1.0	1.49	1.0	1.50	1.0	1.13
			0.50			1.0	1.40	1.0	1.41	1.0	1.38	1.0	1.33
			0.25			1.0	1.40	1.0	1.41	1.0	1.49	1.0	1.40

results according to the 1st path and 2nd path, expressed by the fatigue life ratio (N_c^S/N_c^ε), is not significant and observed only for the programs of $n_{01}/n_0 = 0.1$. However in accordance with expectations, the load level influence on the ratio (N_c^S/N_c^ε) is significant and amounts to: (N_c^S/N_c^ε) ≈ 1.6 for the level I, (N_c^S/N_c^ε) = 1.4 for the level II, (N_c^S/N_c^ε) = 0.92 for the level III and (N_c^S/N_c^ε) = 0.71 for the level IV.

On assumption that the reference point is the fatigue life value N_c^H, calculated in accordance with the hybrid method (the 3rd path), the fatigue life ratios N_c^S/N_c^H and N_c^ε/N_c^H were calculated. The data are collected in Tab. 4 and illustrated by the diagram shown in Fig. 5a and 5b.

Like in the case of the data shown in Fig. 4, load level shows significant influence on the conformity between the results of calculations according to the 1st, 2nd and 3rd path, and a lower influence is produced by load program form described by the ratios S_{a2}/S_{a1} and $n_{01}/n_{0.}$ the latter - with the exception for $n_{01}/n_0 = 0.1$. Comapring the diagrams in Fig. 5a with the relevant diagrams in Fig. 5b one states that a greater conformity is shown by the calculation results according to the 1st and 3rd path for low load levels (obviously, full conformity is observed for the load levels III and IV) as well as the 2nd and 3rd path for high load levels (obviously, full conformity is observed for the load levels I, II and III).

The maximum difference between fatigue life calculated according to the 1st and 3rd path amounts to δ = 39%, whereas the relative difference between fatigue life calculated according to the 2nd and 3rd path amounts to δ = 33%.

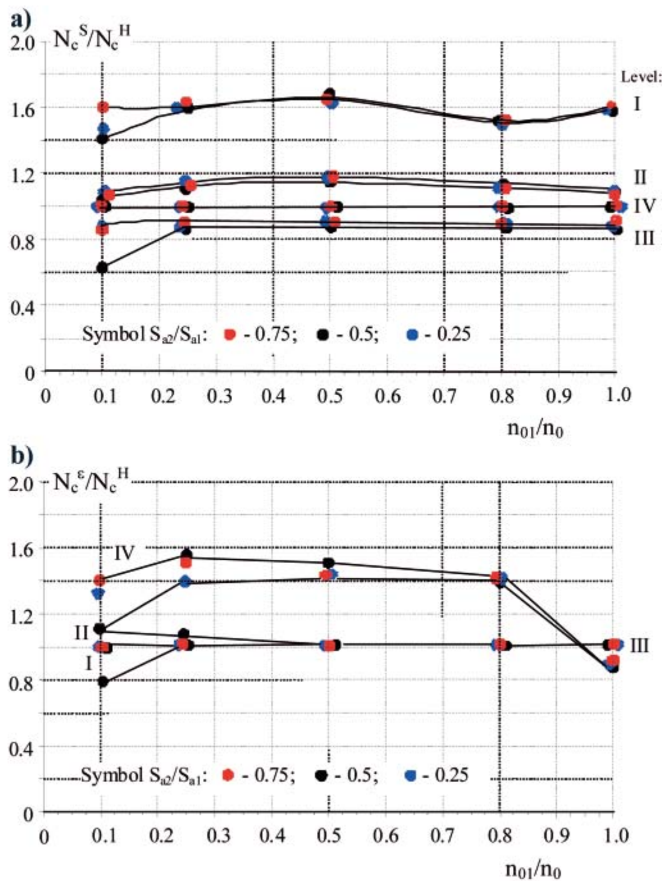

*Fig. 5. Dependence of the ratio of the fatigue life: **a)** calculated acc. the 1st path and that calculated acc. the hybrid method (the 3rd path), **b)** calculated acc. the 2nd path and that calculated acc. the hybrid method (the 3rd path), on load level (I, II, III and IV, respectively) and the test program form described by the ratios S_{a2}/S_{a1} and n_{01}/n_0*

CALCULATIONS FOR C45 STEEL IN MULTI-LEVEL LOAD CONDITIONS

Results of calculations according to the 1st path

The multi-level load program has been already described in p. 2.1. In Fig. 6 is presented Gassner diagram (2) on the background of Wöhler diagram (1), moreover for comparison, by means of the dotted line (3) is shown the Gassner diagram for the same two-level loads and the same spectrum filling factor ζ,.

As results from the comparison the fatigue life determined in the two-level load conditions differs from that determined in the multi-level load conditions within the entire variability range of S_{a1}.

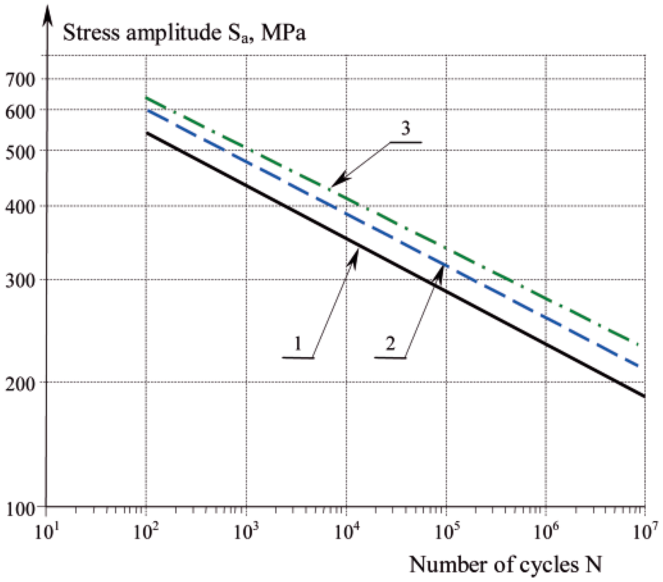

*Fig. 6. Fatigue diagrams for C45 steel in stress approach: **1)** Wöhler diagram, **2)** fatigue life diagram (acc.Gassner) for the multi-level load program of the spectrum filling factor ζ = 0.55, **3)** fatigue life diagram (acc.Gassner) for the two-level load program of the spectrum filling factor ζ = 0.55*

In accordance with the criterion of LCF assumed in the program for the load level I (S_{a1} = 650 MPa) four first levels of the program are situated within LCF range. Similarly, in the program for the load level II (S_{a1} = 570 MPa) three first levels of the program are situated within LCF range, whereas in the program for the load level III (S_{a1} = 460 MPa) only the first level is contained within LCF range. The remaining levels in all the programs are situated in HCF range.

Results of calculations according to the 2nd path

The Gassner fatigue life diagram (the line marked 2) on the background of the Manson-Coffin fatigue diagram (the line marked 1) is presented in Fig. 7. On the discussed diagram is presented for comparison the fatigue life diagram for two-level load program (the line marked 3). Both the two-level and multi-level program are of the same value of the spectrum filling factor ζ. The mutual location of the diagrams 2 and 3 shows small differences in fatigue life values calculated for multi-level and two-level program.

Like in the case of calculations according to the 1st path, some part of the programs is located in LCF range and the remaining in HCF range. Specification of the levels has been given in the discription of the 1st path.

Results of calculations according to the 3rd path

The hybrid method (the 3rd path) consists in summing damages in LCF range in accordance with the method based on the concept of application of Manson-Coffin fatigue diagram (the 2nd path), and in HCF range in accordance of the method based on the concept of application of Wöhler diagram (the 1st path).

In Fig. 8 is presented the comparison of fatigue life diagrams for C45 steel, determined in the conditions of multi-level load program. The diagrams were elaborated in the system: the

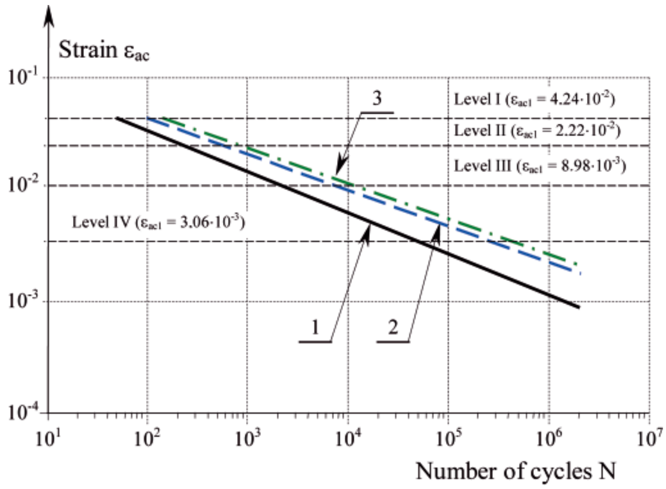

*Fig. 7. The fatigue life diagram for C45 steel in the conditions of the multi-level load program **(line 2)** on the background of the Manson-Coffin fatigue diagram **(line 1)**, as well as the diagram for the two-level load program **(line 3)***

load level S_a (I, II, III, IV) versus fatigue life expressed by the number of cycles N_c. As results from the comparison of the diagrams, the results of the stress approach calculations (the 1st path) are closest to the values of fatigue life determined by using the hybrid method (the 3rd path). Only for the level I (650 MPa) the results of the strain approach calculations (the 2nd path) show a higher conformity with those calculated by using the hybrid method (the 3rd path). The above given observation is in compliance with the conclusion formulated in the case of analysis of the data achieved from the calculations in the conditions of two-level load programs with the exception that in the case of multi-level load programs differences in calculation results are much smaller and amount, in extreme cases, to:

- for the level I (S_{a1} = 650 MPa), between the results acc. the 3rd and 1st path:

$$\delta^{I}_{H1} = \frac{N_c^{(H)} - N_c^{(1)}}{N_c^{(H)}} = -0.42$$

- for the level I (S_{a1} = 650 MPa), between the results acc. the 3rd and 2nd path:

$$\delta^{I}_{H2} = \frac{N_c^{(H)} - N_c^{(2)}}{N_c^{(H)}} = 0.008$$

- for the level IV (S_{a1} = 340 MPa), between the results acc. the 3rd and 1st path:

$$\delta^{IV}_{H1} = \frac{N_c^{(H)} - N_c^{(1)}}{N_c^{(H)}} = 0$$

- for the level IV (S_{a1} = 340 MPa), between the results acc. the 3rd and 2nd path:

$$\delta^{IV}_{H1} = \frac{N_c^{(H)} - N_c^{(2)}}{N_c^{(H)}} = -0.66$$

ANALYSIS OF RESULTS AND SUMMARY

The basic analysis of fatigue life calculation methods for C45 steel concerns programmed two-level loads whose parameters assumed in the program: S_{a1}; S_{a2}/S_{a1} and n_{01}/n_0 made it possible to perform calculations for 64 cases. The so wide range of the

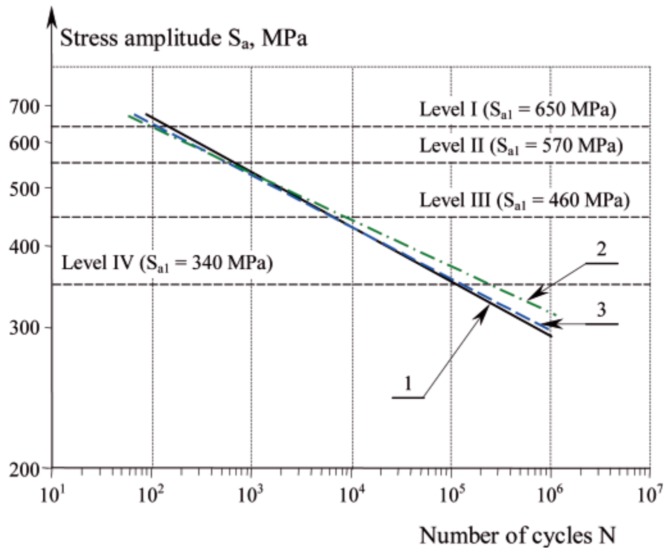

Fig. 8. *Gassner fatigue life diagrams for C45 steel in the multi-level load conditions, determined with the use of the following calculation methods:* ***1)*** *acc. stress approach (the 1st path)* ***2)*** *acc. strain approach (the 2nd path),* ***3)*** *hybrid one (the 3rd path)*

two-level load cases makes it possible to quantitatively assess influence of program's parameters on fatigue life calculated in accordance with all the calculation paths.

As expected, results of the calculations demonstrate:

- prevailing influence of the load level: S_{a1} or $\varepsilon_{ac1.}$
- much lower influence of the program's form: S_{a2}/S_{a1} and $n_{01}/n_{0.}$ which decreases along with spectrum filling factor increasing.

Purposefulness of assuming, in the calculation program, the loads of high values of the spectrum filling factor ζ (Tab. 1) results from the necessity of stressing significant contribution of loads from LCF range. This is the case which corresponds to trends in the proposed calculation methods. The programs in question differ, as to the range of ζ -values, from the known programs elaborated on the basis of random operational loads of structural elements. For instance [9] for the track system elements of road vehicles the ζ factor takes values from the interval of 0.25 ÷ 0.35.

As results from the data contained in Fig. 2 (the 1st path), Fig. 3 (the 2nd path) and Tab. 2 (the 3rd path), influence of fatigue damages due to cycles of the 2nd degree amplitude (S_{a2}; ε_{ac2}) on fatigue life is insignificant, with the exception of the program of $n_{01}/n_0 = 0.1$.

As expected, is observed a high conformity of the calculation results according to the assumed paths on the load level $S_{a1} \approx R_e$ (Fig. 4 and 5) for which the fatigue life ratios N^S_c/N^ε_c, N^S_c/N^H_c and N^ε_c/N^H_c are close to 1.

The wide range of analysis of calculation results of fatigue life for the two-level load programs made it possible to elaborate an appropriate multi-level load program (of 10 levels). The selected ten-level load program (Fig. 1b) is characteristic of the same number of cycles on particular levels ($n_{0i} = n_0/10$) and the same load increase on particular levels ($\Delta S_a = S_{ai}$-$S_{a(i+1)} = S_{a1}/10$). It leads to the spectrum filling factor ζ = 0.55, close, as for value, to the mean counted from Tab. 1 for two-level load programs. Moreover the same load levels S_{a1} and $\varepsilon_{ac1.}$ were maintained, that makes it possible to conduct comparative analysis of calculation results of fatigue life for multi-level programs and those for two-level programs.

As results from the data contained in Fig. 6, the low values of the stress amplitude S_{ai} corresponding to the program levels from i = 6 through i = 10, are of insignificant influence on the summing of fatigue damages (less than 0.001 D_{01}) and therefore may be neglected in fatigue life analyses. The above formulated statement is highly important for programmed fatigue tests since exclusion, from the program, of load levels which insignificantly affect fatigue life, shortens testing duration time considerably.

The similar conclusion results from the calculations according to the 2nd path (Fig. 7) and those according to the 3rd path (Fig.8). From comparison of the fatigue life diagrams determined in accordance with the analyzed paths it results, like in the case of two-level load programs, that the results of the strain approach calculations (the 2nd path) for high load levels (the level I) are closest to those obtained from the hybrid method (the 3rd path) whereas for low load levels (the level IV) the results of the stress approach calculations (the 1st path) are closest to the results of the calculations according to the hybrid method (the 3rd path). For mean values of loads (the level II and III) the results of the fatgue life calculations according to the assumed paths (1st, 2nd and 3rd one) are close to each other. The relative maximum differences in calculation results amount, in extreme cases, to: between those acc. the 3rd and 1st path – the relative difference $\delta^{I}_{H1} = -0.42$; between those acc. the 3rd and 2nd path – the relative difference $\delta^{IV}_{H2} = -0.66$, and are significantly smaller than the corresponding differences observed in two-level programs.

In Fig. 6 are collected fatigue life diagrams determined by means of calculations in accordance with the 1st path for the multi-level load program (the line 2) and two-level program (the line 3) of similar values of the spectrum filling factor ζ. From their comparison rather low relative difference in calculated fatigue life values, amounting to 0.40 in the extreme case (for the load level IV), results. In the case of the calculations according to the 2nd path relative difference in calculated fatigue life values (for the load level IV), amounts to 0.26. The above formulated statements indicate that to simplify load program form by reducing number of program levels is possible, that is very important for programmed experimental tests.

The problems described in this work find application to fatigue calculations of ship structures and significantly widen the calculation methods described in the subject-matter literature [10, 11 and 12] to cover the load range common for both low-cycle and high-cycle fatigue. The above mentioned load case - a sum of regularly changeable loads and high overloads imposed on them - is that commonly occurring in ship structures.

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NOMENCLATURE OF MAJOR NOTATIONS

D_0 – fatigue damage due to realization of n_0 cycles of loading,
N – number of cycles to fatigue damage – general notation (fatigue life),
N_c – fatigue life expressed by number of cycles, determined in conditions of programmed loading,
N_{fi} – number of cycles to fatigue fracture read from Manson-Coffin fatigue diagram for the total strain ε_{aci},
N_i – number of cycles to fatigue fracture read from Wöhler diagram for the stress amplitude S_{ai},
N_0 – basic number of cycles corresponding to fatigue limit,
S – general notation of stress, [MPa],
S_{max} – maximum stress value in sinusoidal cycle, [MPa],
S_{min} – minimum stress value in sinusoidal cycle, [MPa],

S_a – stress amplitude in sinusoidal cycle, ($S_a = 0.5(S_{max}-S_{min})$), [MPa],
S_m – mean stress value in sinusoidal cycle, ($S_a = 0.5(S_{max}-S_{min})$), [MPa],
R – stress ratio ($R = S_{min}/S_{max}$),
R_e – yield point [MPa],
R_m – tensile strength of a material [MPa],
k – number of load levels in a loading program,
l – number of load levels in a loading program of LCF range,
n_0 – number of cycles in a loading program,
n_{01} – number of cycles on 1st level of a loading program,
n_{0i} – number of cycles on i-th level of a loading program,
$m_{(-1)}$ – exponent in Wöhler diagram formula,
δ – relative difference between results of tests and calculations,
ε_{ac} – total strain value,
ζ – spectrum filling factor,
λ – number of repetitions of a program to fatigue fracture,
LCF – low-cycle fatigue,
HCF – high-cycle fatigue.

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