A MATHEMATICAL MODEL OF THE LEFT VENTRICLE SURFACE AND A PROGRAM FOR VISUALIZATION AND ANALYSIS OF CARDIAC VENTRICLE FUNCTIONING

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(Received 12 September 2003; revised manuscript received 14 February 2004)

Abstract: The left heart chamber's contractibility is an important part of heart diagnostics. Ultrasonographic pictures are very often used as the imaging method, as they are widely available, inexpensive and non-invasive. However, ultrasonographic pictures are very unclear, blurred and noisy, and thus very difficult for automatic analysis. To obtain a quick and useful analysis of ventricle performance, a special mathematical model has been created. The model can be used in contour detection, visualization of the heart's motion and even in automatic surface analysis. We hope that in the future such programs could be incorporated into a general medical expert system.

Keywords: cardiac diagnostics, pattern recognition, contour detection, expert system, image processing, computer graphics

1. Introduction

A package of computer programs has been developed to support diagnostics of the left cardiac ventricle. A mathematical model, called an *ABCD* model, has been introduced in the examination of the surface of the left ventricle. This model, described in [1–3], has been modified many a time to optimally match the situation with ultrasonographic registration. The operation of the applied programs can be divided into four parts. The first part constitutes the most interesting and challenging problem, which is an attempt to create an appropriate instrument for automatic detection of ventricular contours on a series of USG images. The second part is a spatial reconstruction of the entire surface of the ventricle, at any moment, based on a pair of its sections made at different angles of projection. The third part includes various techniques for visualization of the surface of the ventricle, which have didactic and diagnostic applications. The fourth part includes the development of a system for automatic detection of suspect cases. At present, it is based on techniques associated with surface geometry and dynamics. All these stages have been continuously developed. We believe that in the future these programs could be included into a major expert system dealing with the health of a patient as a whole.

2. Contour detection of the surface of the left ventricle

Automatic detection of the ventricular contour, based on the examined series of USG images, is the basic process of the analysis. The edge of the cardiac ventricle seen in an ultrasonogram is considered to constitute the ventricular contour. Mathematically, it coincides with the intersection of the ventricle surface and the projection plane of the ultrasonographic sound. These methods are based on a very low-precision imaging method, so automatic contour detection is very difficult. However, the mathematical surface representation and visualization programs used in the described system are relatively simple. This system is fast, useful and capable of further development. Contour detection is a very interesting and challenging task. USG images are very sophisticated from the point of view of automatic analysis. These images are blurred and accompanied by a high level of noise, with interference and diffraction of waves. There are many artefacts and other disturbances. This results in significant image defects. The ventricle contour can be seen only in some places, its edge having wide gaps. Moreover, in some images (to be found in every cycle) the contour is hardly discernible at all. In Figure 1 three USG images are presented, with the ventricle contour only slightly discernible in the image in the middle.

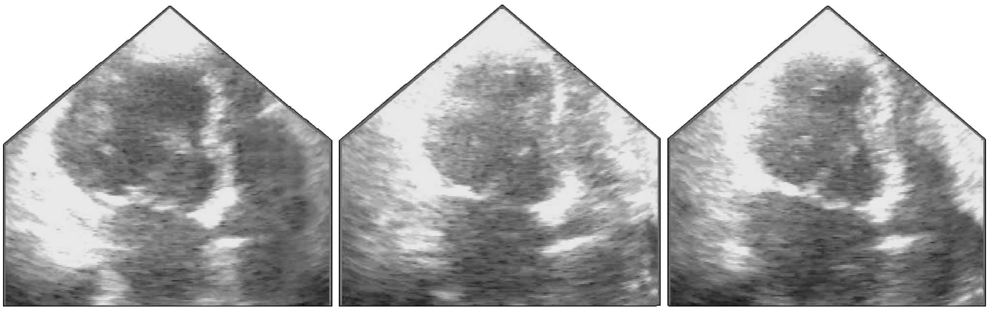

Figure 1. Three examples of USG images from the same series

When USG images are observed in sequence, it becomes clear that all the images together presenting, one after another, the ventricle in operation are necessary for the proper functioning of the contour-detecting system. So, to detect at least one contour in a series, the entire series of images is necessary. When only one image is examined statically, often not much can be seen and the computer program is unable to provide adequate information on the basis of just one image. Similarly, in visual recognition by an observer, the ventricle cannot be seen in a single still image, while the entire series of images observed dynamically can better demonstrate the ventricle in operation. This is so because one image fails to present one fragment of the contour, while another image fails to present another fragment of the contour, and these data are supplementary to each other. By examining such images together dynamically, a more complete image of the ventricle in operation can be discerned. That is why a third dimension connected with time is also taken into account. If time is treated as a spatial dimension, a contour moving in time resembles a bent pipe immersed in a time plane.

This time plane is a spatial image of USG, with time as the third dimension, while the USG image itself is only two-dimensional. Two examples of ventricular movement of the contour as "a pipe" in a time plane are shown in Figure 2.

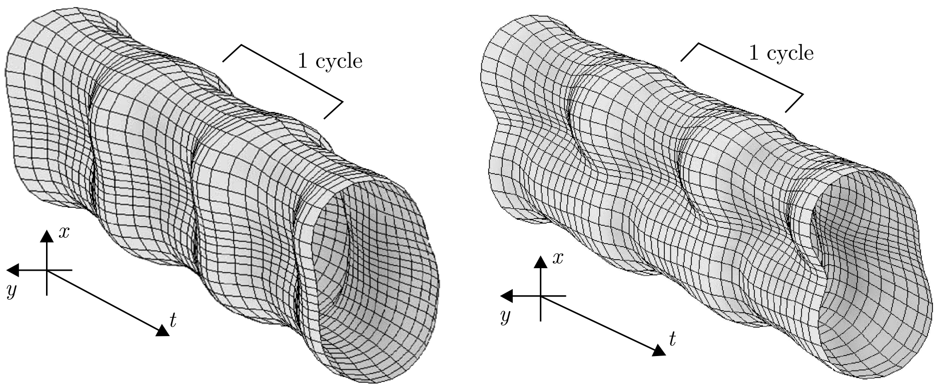

Figure 2. Two examples of the ventricular contour movement

Besides, it is taken into account that the contour does not change much from image to image. To put it more mathematically, the ventricle contour is quite regular both in respect of time and in respect of space. In an adequate system for automatic detection of the contour, all the data from the whole series of images should be taken into account simultaneously. It is this system that satisfies all these requirements, and these methods are being developed and tested. Recently, two new contour detection methods have been investigated, both based on dynamic systems [4]. These are not typical techniques that should be effective just in such quite specific situations. One of these contour detection techniques involves an appropriate dynamic system. A sequence of closed curves of a certain time interval is an element of this dynamic system. This object, being an element of a dynamic system, can be represented with a certain vector of coefficients. The loops are parameterized through the system of coefficients, in accordance with the mathematical model of ventricle surface. In this case, time is regarded as a discrete variable. The object of the system moves under the influence of two factors, the natural factor being associated with the free dynamics of the system and influenced by certain effects associated with USG images. In operation, this process looks like the simultaneous movement of the loops of the dynamic system and ventricle movement in the USG image series. To put it in a simplified way, the system is designed so that just a series of well-detected contours in movement should be its attractor. When this algorithm works properly, it looks like the loops approach the real contours of the ventricle in movement in the USG images and then begin to move together, with the loops superimposed upon the ventricle contours. Three parameters are calculated for each loop: integral along the curve with the special function of the curvature, special integral of the brightness gradient along the entire curve, and the integral of the appropriate function of darkness on the entire area inside the closed curve. These parameters control the dynamics of the loop system movement. It is still difficult to find out what may be decisive for the effectiveness of this algorithm. In some cases the loops of the dynamic system almost completely fail to approach the contour of the ventricle, while sometimes they happen to superimpose on the ventricle contour afterwards and move together. This technique appears to have a chance to be a success. It is important, that the parameters of the loop associated with USG images, as well as those associated with the geometry of the loop and those associated with the interrelations of the successive loops are taken into account. The same data are taken into account in the other technique.

The other contour technique consists in fitting an elastic surface in the time plane to the edges of the ventricle in movement (Figure 3). The process of adjustment of the pipe to the edges of the ventricle depends upon many parameters. Mathematically, in this model the pipe is a surface designated with nodes of a network. Certain elasticity is assumed to be characteristic of the pipe, which is reflected in the corresponding equations. To put it figuratively, the surface of the pipe is quite resistant to bending by external forces, including the effect of the ultrasonographic image contents on the surface of the pipe. It is also a dynamic process; the pipe grows wider, evolves to get adjusted in the best possible way to the ventricle contour motion in the time plane. The pipe evolves under the influence of an intrinsic force associated with its shape and an extrinsic force associated with the USG images. At the beginning the center of the ventricle has to be established, which corresponds to a curve in the time plane. The initial condition is a thin pipe around the center of the ventricle in the time plane. Then the evolving pipe hidens until it reaches the ventricle contours in motion and the problem is solved. In this case, it is important that the conditions of elasticity of the pipe both in respect of time and space, as well as the way the USG images affect the evolution of the pipe. All the USG images in the studied series have been compiled into a cube through an interpolation of the luminosity function with respect to time, which yields a certain function in the $\mathbb{R}^3$ space.

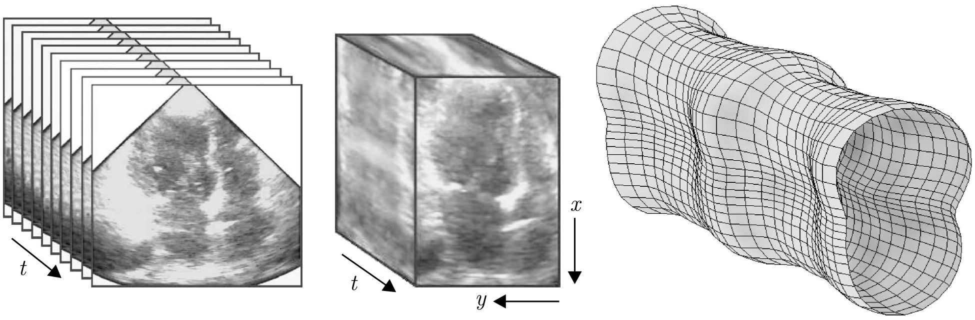

Figure 3. The series of USG images, the brightness function in the time plane and the elastic pipe which is fitted to the contours of the ventricle in motion

The function is a spatial image of luminosity in the time plane; it is this function that effects the dynamics of the pipe as an extrinsic force. Thus USG images influence the pipe's evolution. The intrinsic forces which affect the pipe's evolution depend on geometrical features of the bent pipe in the time plane. The effectiveness of this method, which is still developed, depends on many parameters.

On the one hand, these two techniques are quite different, but on the other hand, they are based upon very similar assumptions.

3. The matematical model of ventricle surface

In this model the surface of a ventricle is mathematically treated as a two-dimensional surface embedded in three-dimensional space. The whole surface at one moment is defined by a special function of two arguments. This function, called f in this model, determines the distance between the center of the coordinate system and the point on the surface related to angles α and ε. The whole ventricle surface is defined in three-dimensional space by parameterization of the F function:

$$F(\alpha,\varepsilon)=f(\alpha,\varepsilon)\cdot\begin{bmatrix}\cos(\alpha)\\ \sin(\alpha)\cos(\varepsilon)\\ -\sin(\alpha)\sin(\varepsilon)\end{bmatrix}. \qquad (1)$$

The values of the F function are the points on the surface in space. This F function is very convenient for ventricle surface representation in this case (Figure 4).

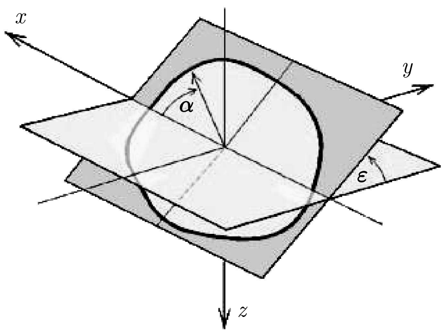

Figure 4. A surface section corresponding to the formula of surface parameterization (1)

The acceptance of the above assumptions requires the acceptance of special conditions on the f function. The surface parameterization implies special conditions on function f, because the ventricle surface should be continuous.

These conditions must be taken into account in the course of surface parameterization. Functions used for construction of function f have been so selected that they satisfy not only continuous conditions but also smoothing conditions. So, we have the following conditions of the f function:

$$\begin{aligned}\forall\alpha &\quad f(\alpha,-\tfrac{1}{2}\pi)=f(2\pi-\alpha,\tfrac{1}{2}\pi),\\ \forall\varepsilon &\quad f(0,\varepsilon)=f(2\pi,\varepsilon)=\mathrm{const}_1,\\ \forall\varepsilon &\quad f(\pi,\varepsilon)=\mathrm{const}_2.\end{aligned} \qquad (2)$$

These continuous conditions must be incorporated into the reconstruction process of the whole surface in space. The ventricle surface in this mathematical model should be continuous and smooth.

We approximate the f function by a trigonometric polynomial of angles α and ε. Naturally, for a polynomial of two arguments, there are four types of coefficients, a, b, c and d, and therefore the model has been called an $ABCD$ model of ventricle surface. An additional component Φ is introduced to satisfy the continuity conditions of the f function. The Φ function is a special smoothing function, necessary to correct polynomial values in order to obtain continuity of the f function on the entire surface:

$$f(\alpha,\varepsilon)=A(\alpha,\varepsilon)+B(\alpha,\varepsilon)+C(\alpha,\varepsilon)+D(\alpha,\varepsilon)+\Phi(\alpha,\varepsilon),$$
$$A(\alpha,\varepsilon)=\sum_{j=1}^{K}\sum_{i=1}^{M_1}a_{ij}\sin(i\alpha)\sin(j\varepsilon),\qquad B(\alpha,\varepsilon)=\sum_{j=1}^{K}\sum_{i=1}^{M_2}b_{ij}\sin(i\alpha)\cos(j\varepsilon),\tag{3}$$
$$C(\alpha,\varepsilon)=\sum_{j=1}^{K}\sum_{i=1}^{M_1}c_{ij}\cos(i\alpha)\sin(j\varepsilon),\qquad D(\alpha,\varepsilon)=\sum_{j=1}^{K}\sum_{i=1}^{M_2}d_{ij}\cos(i\alpha)\cos(j\varepsilon).$$

This solution turns out to be optimal and useful for implementation in this case. We would like to have a fast and simple application for this surface reconstruction. The functions of the $ABCD$ model are described in detail in [1, 5–7].

4. The reconstruction of whole surface motion in space

First, the contour of ventricle surface for some different sections is found, so we have a few sections of ventricle surface. Each surface section has a different elevation angle and projection plane. Each elevation angle ε determinates one cutting plane. If the elevation angle, ε, is equal to the angle of a projection plane, then this surface section must exactly fit the ventricle contour for this elevation angle. Each projection plane is indexed by number k. We can say that we know the values of function f for these sections, that is for arguments (α,ε_k), and we want to find the values of function f for all arguments (α,ε), which represent the entire surface. For each section, we have the following formula:

$$f(\alpha,\varepsilon_k)=r_k(\alpha)=\sum_{1}^{N}p_{i,k}\sin(i\alpha)+\sum_{0}^{N}q_{i,k}\cos(i\alpha).\tag{4}$$

Using this coefficient we calculate information about the whole surface model at one moment, thus creating a reconstruction of the whole surface in three-dimensional space, shown in Figure 5.

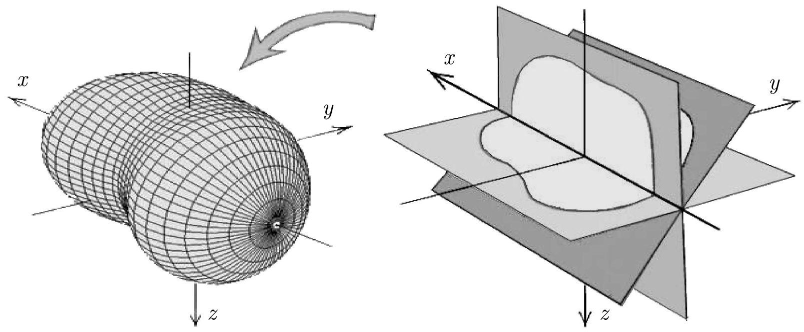

Figure 5. The whole surface and a few surface sections at different angles

Such a solution is obviously not unique; a lot of functions can satisfy such conditions. We can also say that it is an interpolation problem. It is necessary to decide how to construct the f function, and what interpolation method to apply. We use a special interpolation method of the p_{ik} q_{jk} coefficients and, additionally, the application of the Φ component. This space reconstruction of ventricle surface is exactly described in [5]. If we have coefficients of the surface model for each moment when an ultrasonographic image was made, we then have information about the surface motion. It is possible to create a smooth surface model by interpolation of the model coefficients related to time. We assume that its dynamics is periodic in time, so that we can interpolate the coefficients by a trigonometric polynomial. Then, each coefficient of the surface model is a continuous and smooth function of time. For instance the a_{ij} coefficient is described by the following formula:

$$a_{ij} = \sum_{0}^{M} \tilde{a}_{ijk} \cos(\omega k t) + \sum_{1}^{M} \hat{a}_{ijk} \sin(\omega k t). \tag{5}$$

The time interpolation of the vetricle surface is presented in [8]. In this dynamic model F is a function of three arguments: α, ε and t. This function is defined by many parameters obtained from all ultrasonographic images.

5. Surface visualization and automatic ventricle analysis

Having calculated the model coefficients based on a series of USG images, we have a surface model which can be used for diagnostic purposes. The system can support a physician's diagnosis by various surface visualizations in three-dimensional space. Such visualizations obviously offer other new possibilities (Figure 6). The surface could be observed by many different methods. It can be turned in space, which helps to observe its shape, or it can be observed in motion, highlighting the dynamic aspect of ventricle performance. The system can calculate many parameters describing ventricle characteristics and present them in surface picture. For example, it is possible to calculate and present ventricle volume, cardiac output, velocity of a selected point on the surface and other parameters. An important advantage of this visualization system is the possibility to present scalar function values with a surface picture, *e.g.* with a color scale. Surface visualization is realized with computer programs, which are continuously improved.

Another important application of this model is the creation of an automatic system for ventricle surface examination, a much more ambitious and difficult problem. It could be considered a part of an automatic medical expert system. As such, it would be a system to detect clear irregularities of ventricle surface motion, which would select suspicious cases for further investigation. If we have calculated the model coefficients properly, we can apply various mathematical tools to examine the left ventricle's performance. This system could detect clear irregularities of surface shape, for example when the surface is untypically diffracted, distorted or stretched. The system could detect irregularities of ventricle dynamics, as well. To investigate surface deformations we apply tools from the differential geometry of surfaces, such as the first surface fundamental form, the second surface fundamental form and the Gaussian curvature. These mathematical problems are widely presented in [9–12] These control functions represent geometrical features of a surface, like distortions, diffractions and other geometrical irregularities. To investigate surface dynamics we use functions based on time derivatives.

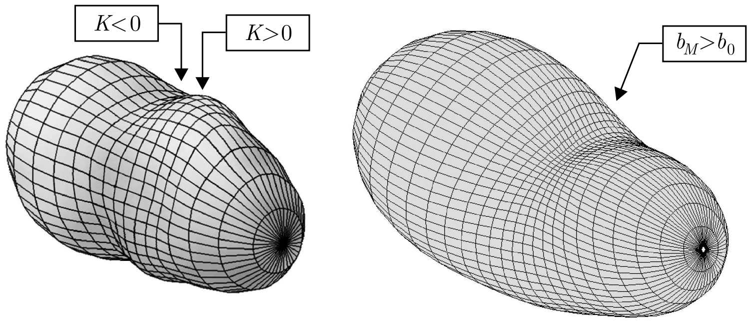

Figure 6. Calculated values showing geometrical irregularities of surface

A properly working ventricle surface is quite regular. If the surface is clearly deflected or stretched at any point, the control values should clearly demonstrate this. If the calculated values exceed established limits at any point of the surface, it means that the surface around this point is not typical. Our system can operate in this way: if the examinated values exceed the established limits, the system signals irregular behavior. However, the system is unable to perfectly differentiate proper and improper activities of working ventricles; it should only highlight suspicious cases for further analysis.

6. Conclusions

Automatic analysis of the surface of the ventricle can be a very useful auxiliary instrument applied in diagnostic examinations. Detection of the contour of the ventricle in USG images is the basic element of the problem. It is an interesting and challenging task that can also be regarded from the point of view of pattern recognition and image processing. The contour of the ventricle can be adequately detected when a series of USG images is treated as a whole. The contour having been recognized, the ventricle surface in motion is reconstructed spatially according to a mathematical model. In the future, such programs may be included into a major expert system. Then they could be applied in the preliminary analysis of USG images associated with the functioning of the left ventricle. In the opinion of the authors, research in contour detection in such an unusual and demanding context is most valuable. Another interesting aspect is attempted automatic analysis of the ventricle's functioning, based on examination of the geometrical characteristics of its surface. The attempts at contour detection described in this paper may be applicable in many other contexts of pattern recognition and analysis of moving images.

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