APPLICATION OF WEIGHTED MYRIAD FILTERS TO SUPPRESS IMPULSIVE NOISE IN BIOMEDICAL SIGNALS

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Abstract: Biomedical signals are commonly recorded with many kinds of noise. One of these is a waveform of the electrical activity of muscles. This "natural" distortion is usually modelled with a white Gaussian noise. In order to suppress such noise a weighted myriad filter is applied. The weighted myriad filter belongs to a class of non-linear filters and requires knowledge about noise impulsiveness. An impulsive noise can be described with α-stable distributions. One objective of this paper is to apply α-stable distribution as a model of real-life muscle noise in ECG signals. The other objective of the paper is to apply a weighted myriad filter to suppress impulsive noise in biomedical (ECG) signals. The reference filters have been the Savitzky-Golay smoothing filter and the median filter. The obtained results have shown that α-stable distributions can be applied to model muscle noise and that a weighted myriad filter with a Chebyshev weighted function can effectively suppress such noise.

Keywords: weighted myriad filter, α-stable distribution, biomedical signals

1. Introduction

When biomedical signals are recorded, many kinds of noise may appear. Of many different biomedical signals, the electrocardiogram (ECG) signal has been chosen, well-known for a long time and well described in literature. The ECG signal arises as a result of the electrical activity of the heart and is almost always disturbed by noise, such as: 50 Hz power line interference, baseline wander, an electromyogram (EMG), motion artifact [1]. The spectrum of ECG signals significantly overlaps, with the spectrum of muscle noise and motion artifacts. Muscle noise is the most difficult to suppress. In fact, most types of noise are not stationary, which means that noise power measured by noise variance is characterized by variability [2]. White Gaussian noise is usually used to model an EMG signal, according to the Central Limit Theory. But this assumption may lead to over-optimistic conclusions. Muscle noise frequently shows an impulsive nature, which means that the Gaussian model may fail. In order to model muscle noise more properly, an α-stable distribution model is introduced. Impulsive noise can be suppressed with non-linear filters such as the weighted myriad filter.

Linear filtering techniques are often used in applications of digital signal processing. Their popularity is due to their mathematical simplicity and their efficiency in the presence of additive Gaussian noise. For example, a moving average filter is the optimal filter (according to the maximum likelihood criterion) for additive Gaussian noise in terms of mean square error [3]. The assumption that noise is normally distributed is rational and can be justified using the Central Limit Theorem, making it a reasonable model for any physical phenomena. But in the real worlds of signals there are many signals that are not well described by the Gaussian model. For instance, a large number of noise processes are impulsive in their nature. Under such conditions linear filtering techniques involve a significant performance degradation, even for small levels of impulsiveness [4]. Non-gaussianity often results in significant performance degradation for systems optimized under the Gaussian assumption [5]. Impulsive noise cannot be suppressed sufficiently by linear filtering. One type of distribution that exhibits heavier tails better than the Gaussian distribution is the class of α-stable distributions, which are recently used to model the diverse phenomena mention above [5, 6].

Biomedical signal processing often requires the use of filters to shape the frequency content of the signal. Signal smoothing, enhancement or shape preservation when impulsive noise appears makes robust methods the only alternative to linear filtering. Linear filters tend to blur sharp edges, destroy lines and other fine image or signal details in the presence of heavy-tailed noise. However, there is an important class of smoothing applications that require careful treatment and preservation of signal edges [3, 7, 8]. Weighted myriad filters have recently been proposed as a robust filtering and estimation technique in impulsive environments. This class of filters have originated from the simple myriad, which is defined as the maximum likelihood of location of the Cauchy distribution [6].

The main aim of this paper is modelling EMG signals with α-stable distributions and suppression of their noise with a weighted myriad filter. An additional aim of this paper is to present the effect of weights (weighted function) selection on suppressing impulsive noise in biomedical signals. The organization of the paper is as follows. The paper begins with an introduction of α-stable distribution and its main properties. In the next section muscle noise is analyzed. A myriad filter and its variety – a weighted myriad filter – are presented in the next section. Then, weighted functions are presented. The next section presents testing procedures. Finally, a number of results and conclusions are presented.

2. α-stable distributions

There are many signals and noises which do not exhibit gaussianity, for example power switching transients, accidental pulses in telephone lines, spread-spectrum signals and man-made noise in communication channels, impulsive phenomena in underwater acoustics and others. In biomedical engineering, such phenomena occur in diathermia, when using surgical devices, or in electrocardiology (muscle noise), when a system is switched from one mode to another. Such signals can be characterized by their impulsiveness. Signals and noises in this class of impulsiveness are more likely to occur in spikes or accidental bursts of outlying observations than one would expect from normally distributed signals. Consequently, their probability density functions (PDF) decay in the tails less rapidly than the Gaussian density function. One of the main features of impulsive noise is short-duration interferences, attaining large amplitudes with probability much higher than the probability predicted by Gaussian distributions. These impulsive features can be well characterized using α-stable distributions [5, 9]. In this work, a symmetric α-stable distribution is described.

The class of symmetric α-stable distributions ($S\alpha S$) can be characterized by their distribution having the following characteristic function:

$$\varphi(t) = \exp(j\mu t - \gamma|t|^{\alpha}), \tag{1}$$

where α is the characteristic exponent restricted to the range $0 < \alpha \leq 2$, μ is the real-valued location parameter (corresponding to the mean for $1 < \alpha \leq 2$ and the median for $0 < \alpha \leq 1$), and γ is the dispersion of the distribution ($\gamma > 0$, it determines the spread of density around its location parameter; dispersion behaves similarly to the variance of Gaussian density and is equal to half the variance when $\alpha = 2$, the Gaussian case) [4, 5, 10]. The most important parameter of α-stable distributions is the characteristic exponent, because α controls the heaviness of distribution tails. If a stable random variable is observed, the larger the value of α, the less likely it is to observe values of the random variable which are far from its central location. A small value of α denotes a significant shift in the probability mass of distribution tails [5]. An interesting property of $S\alpha S$ is that there are no closed-form expressions for the general stable density and distribution functions, except for the Gaussian ($\alpha = 2$) and Cauchy ($\alpha = 1$) ones. But power series expansions of stable density functions are available. The standard $S\alpha S$ density function is given by:

$$f_{\alpha}(x) \hat{=} \begin{cases} \dfrac{1}{\pi x}\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k!}\Gamma(\alpha k+1)x^{-\alpha k}\sin\left[\frac{k\alpha\pi}{2}\right], & \text{for} \quad 0<\alpha<1, \\ \dfrac{1}{\pi(x^2+1)}, & \text{for} \quad \alpha=1, \\ \displaystyle\sum_{k=0}^{\infty}\frac{(-1)^{k}}{2k!}\Gamma\left(\frac{2k+1}{\alpha}\right)x^{2k}, & \text{for} \quad 1<\alpha<2, \\ \dfrac{1}{2\sqrt{\pi}}\exp\left(-\dfrac{x^2}{4}\right), & \text{for} \quad \alpha=2, \end{cases} \tag{2}$$

where $\Gamma(\cdot)$ is the usual gamma function defined by:

$$\Gamma(x) \hat{=} \int_0^{\infty} t^{x-1}e^{-t}dt. \tag{3}$$

The algorithm of generating a random variable with an $S\alpha S$ is presented in [11]. For more information about α-stable distributions see [5].

The class of α-stable distributions does not have finite second (or higher) moments. This means that, for example the use of variance as a measure of dispersion is meaningless. In fact, α-stable distributions with $\alpha \neq 2$ have finite moments only for an order p lower than α. Likewise, many standard signal processing tools (*e.g.* spectral analysis and all least-squares techniques) which are based on the assumption of finite variance will be significantly destabilized and may in fact give misleading results [5]. An infinite variance also means that a random variable may receive values very remote form the most expected ones. This may be used to ascertain whether noise has an impulsive character. Some algorithms need only information about the nature of noise (Gaussian or non-Gaussian), and an accurate value of the characteristic exponent is not required [5]. The simplest method is to directly test whether the population variance has a finite variance.

Given a set of samples $\{x(n)\}_{n=1}^{L}$, the population variance based on the first l observations is written as:

$$S_l^2 = \frac{1}{l-1}\sum_{n=1}^{l}(x(n)-\bar{x}_l)^2, \tag{4}$$

where

$$\bar{x}_l = \frac{1}{l}\sum_{n=1}^{l}x(n), \tag{5}$$

and plot the running population variance S_l^2 against l. An example series of impulsive noise and the change of population variance are presented in Figure 1.

The knowledge of the parameters of impulsive noise is required for the proper operation of a weighted myriad filter. Several methods have been proposed to estimate α-stable distribution's parameters (see [5, 9, 10, 12]). In this paper, the method described in [10] has been used. Estimation in the $\log|S\alpha S|$-process is characterized by low computational complexity, and the estimators are closed-form expressions. Let

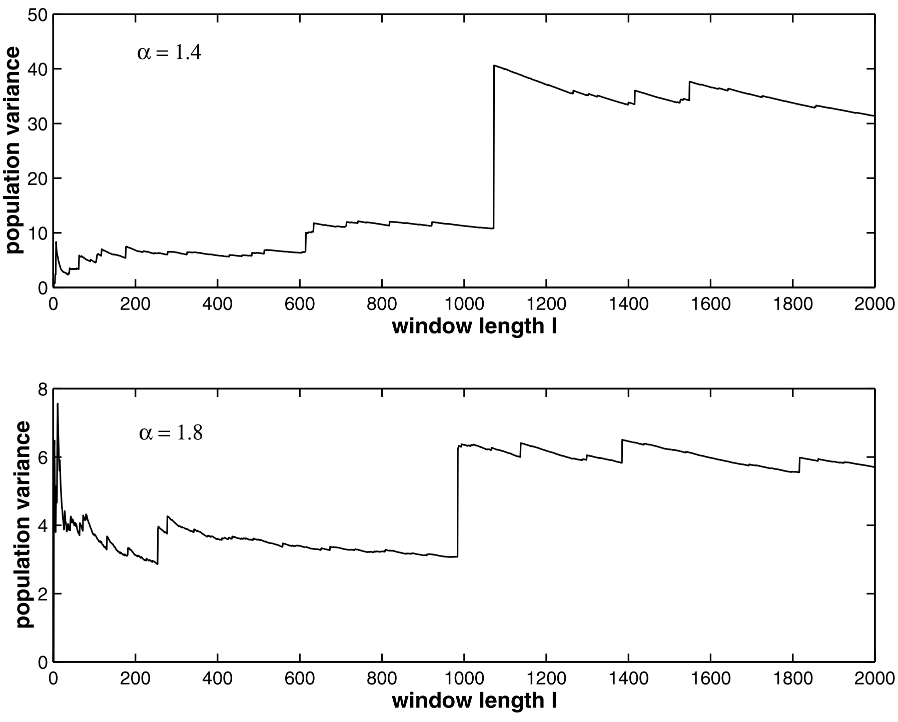


Figure 1. Running population variances for two values of α: $\alpha = 1.4$ and $\alpha = 1.8$

Y equal $\log|X|$, where X denotes a random variable of the $S\alpha S$ distribution. The first and second moment of Y can be determined as:

$$E(Y)=C_e\left(\frac{1}{\alpha}-1\right)+\frac{1}{\alpha}\log(\gamma), \tag{6}$$

$$Var(Y)=E\{[Y-E(Y)]^2\}=\frac{\pi^2}{6}\left(\frac{1}{\alpha^2}+\frac{1}{2}\right), \tag{7}$$

where $C_e = 0.57721566\ldots$ is the Euler constant and $E(\cdot)$ is the expected value operator.

The characteristic exponent α is calculated from Equation (7), while dispersion γ is obtained from Equation (6).

3. Modelling real-life muscle noise

Muscle noise is the result of superposition of a large number of action potentials which forms in muscles and such a bio-waveform usually coexists with ECG or other signals [13]. EMG signals are non-stationary and non-linear by nature and, as shown in [14], a part of an EMG signal in a finished time interval has sufficient stationary features.

Several methods have been proposed to mode muscle noise. White Gaussian noise is the most frequently used, but (as shown in [15]) only some parts of an EMG signal (10% of subsequences of an EMG signal of 2sec. duration, sampled 2 000 times per sec.) can be correctly modelled with white Gaussian noise. This means that there were no reasons to reject the hypothesis of a Gaussian distribution of muscle noise. Muscle noise can also be modelled using the auto-regressive (AR) model, but this model is characterized by a relatively high order [15]. It can be also modelled as muscle spikes, which are assumed to be an impulsive response of a second-order linear system [13].

While analyzing an EMG signal, one can observe a number of samples for which values are significantly removed from the average sample value. Such samples occur as spikes of large value. In this situation α-stable distribution can be applied as the model proper for muscle noise. An example of a real-life EMG signal and simulated muscle noise distributed with α-stable distribution is presented in Figure 2.

The application of α-stable distribution as a statistical model is theoretically justified by two important properties. The first is the stability property: the sum of two independent stable random variables with the same characteristic exponent is also stable and has the same characteristic exponent. The second is the Generalized Central Limit Theory: if the sum of an infinite number of independent and identically distributed (i.i.d.) random variables (with finite or infinite variance) converges in distribution, the limiting distribution is α-stable. Thus, α-stable random variables can arise in the physical world as the effects of a large number of independent contributing factors in the same way that Gaussian random variables do. If an observed signal or noise can be thought of as the sum or result of a large number of i.i.d. effects, then the Generalized Central Limit Theorem suggests that a stable model may be appropriate [5, 6, 9]. For example, α-stable distributions have been used to model multiple access interference in radio networks where the independent

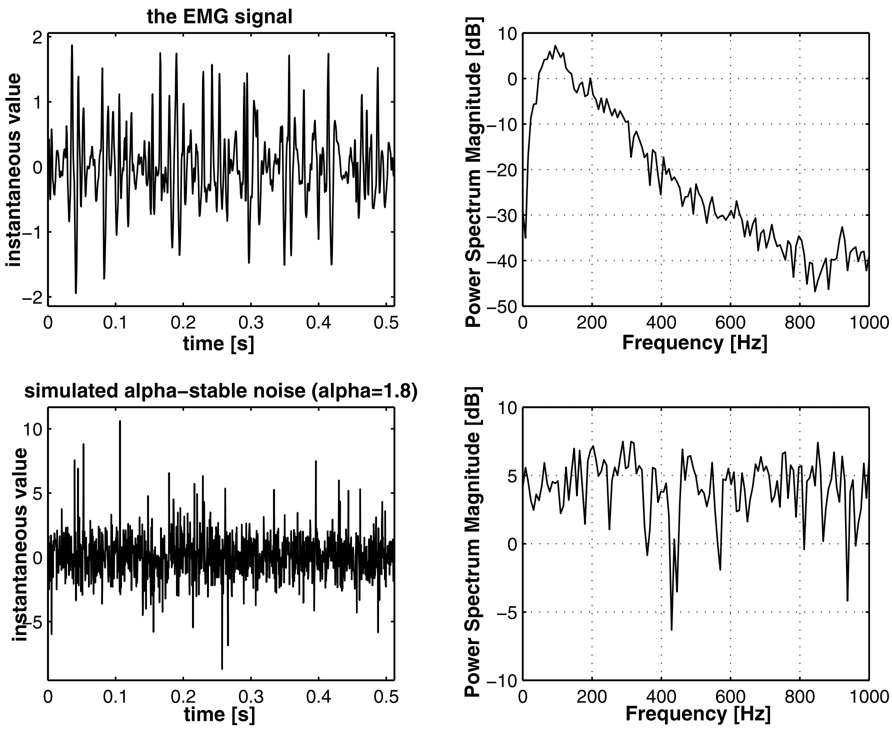


Figure 2. A real-life EMG signal (upper plot) and a realization of simulated alpha-stable noise ($\alpha = 1.8$, lower plot) and their power spectrum characteristics in the frequency domain

interfering sources are modelled as a Poisson field in space and the superposition of the interfering electromagnetic waves follows an α-stable distribution [6].

Figure 3 presents an EMG signal randomly chosen from the available base, consisting of 31 200 samples at a sampling rate of 2 000 samples/s. The plot is divided into 20 subsequences and the characteristic exponent α is estimated for each subsequence. It may be noted that for the 10^{th} and 16^{th} subsequences the value of α decreases to *ca.* 1.5, which indicates that the impulsiveness of this signal's sequence has increased. On the time-series plot, one can see the instantaneous values of these subsequences grow rapidly. The average value of the characteristic exponent α for the whole of this EMG signal equals *ca.* 1.74, which value suggests that this signal cannot be modelled with Gaussian distribution and the impulsiveness of the signal is not very high.

4. Myriad filters and weighted myriad filters

Myriad filters belong to a class of maximum likelihood estimators of location (M-estimators). The theory of M-estimators is a very general and mature field in robust statistics. For a sequence of samples $x_1, x_2, \ldots, x_N$ the M-estimator of location is defined as the value β which minimizes the sum:

$$\hat{\beta} = \arg\min_{\beta} \sum_{i=1}^{N} \rho(x_i - \beta). \tag{8}$$

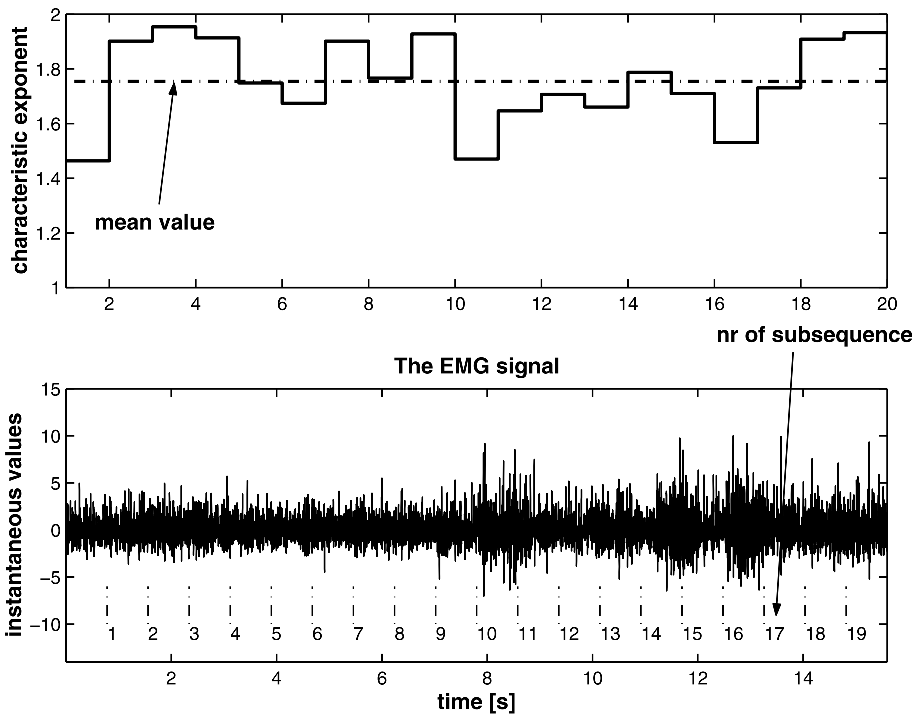


Figure 3. A real-life EMG signal (lower plot) and the characteristic exponent α estimated for each subsequence according to Equation (7)

The $\rho(\cdot)$ function is usually known as the cost function associated with the estimator. The term "cost" is explained on the basis of the engineering interpretation that a "penalty" of value $\rho(x_i-\beta)$ shall be paid for the estimator to be away from sample x_i. From this point of view, the M-estimator is the point β with the minimum sum of costs [8]. Table 1 shows M-estimator cost functions and filter outputs for various filter families [8, 16, 17].

Table 1. M-estimator cost function and filter output for various groups of filters

Filter	Cost function	Filter output	Density function
linear	$(x_i-\beta)^2$	$\mathrm{mean}\{x_1,x_2,\ldots,x_N\}=\sum_{i=1}^N \frac{x_i}{N}$	Gaussian
median	$\lvert x_i-\beta \rvert$	$\mathrm{median}\{x_1,x_2,\ldots,x_N\}$	Laplacian
myriad	$\log[k^2+(x_i-\beta)^2]$	$\mathrm{myriad}\{x_1,x_2,\ldots,x_N;k\}$	Cauchy
weighted mean	$w_i(x_i-\beta)^2$	$\sum_{i=1}^N \frac{w_i x_i}{N} / \sum_{i=1}^N w_i$	Gaussian
weighted median	$w_i \lvert x_i-\beta \rvert$	$\mathrm{median}\{x_i \diamond w_i\}_{i=1}^N$	Laplacian
weighted myriad	$\log[k^2+w_i(x_i-\beta)^2]$	$\mathrm{myriad}\{w_1 \circ x_1, w_2 \circ x_2, \ldots, w_N \circ x_N; k\}$	Cauchy

A sample myriad is defined using the cost function $\log(k^2+x_i^2)$, where the so-called linearity parameter k controls the impulse-resistance (outlier-rejection capability) of the estimator [16]. For small values of k, the filter tends to favor values near the most populated clusters of input samples. The $k \to 0$ case leads to a highly robust selection filter called a weighted mode-myriad filter, which means that the myriad becomes a mode-like estimator in the sense that it will always lie in one of the most repeated values [4]. When $k \to \infty$ and $\alpha \to 2$, the output of the myriad filter behaves like the output of a moving average filter [6, 17, 18]. For the weighted median (Table 1), $x_i \diamond w_i$ denotes the replication of sample x_i by a positive integer value, w_i. The filter output is then an unweighted median of a modified set of observations, where each sample x_i appears w_i times [16]. As the extension of the sample mean to the linear normalized FIR filter, a weighted myriad filter is defined by assigning weights to the sample in the maximum likelihood of location estimation. The weights reflect the different levels of reliability of the observed samples [6, 16, 17].

Let us consider a set of observations, $\{x_i\}_{i=1}^N$, and a set of filter weights, $\{w_i\}_{i=1}^N$. The output of the weighted myriad filter is defined as:

$$\begin{aligned}\hat{\beta} \hat{=} &\text{myriad } (x_1 \circ w_1, x_2 \circ w_2, \ldots, x_N \circ w_N; k) = \\ &= \arg\min_{\beta} \prod_{i=1}^{N} [k^2 + w_i (x_i - \beta)^2] = \\ &= \arg\min_{\beta} \sum_{i=1}^{N} \log[k^2 + w_i (x_i - \beta)^2],\end{aligned} \tag{9}$$

where $x_i \circ w_i$ denotes the weighting operation and the weights are restricted to non negative, $w_i \geq 0$, $i = 1, \ldots, N$, N is the window length. Weighted myriad filters may use negative weights, which results in their potential instability (the output may be $+\infty$ or $-\infty$) [6]. When all weights equal 1 ($w_i = 1$ for $i = 1, \ldots, N$), a sample myriad is obtained in dependence on the linearity parameter, k.

It is important to realize that the location estimation problem considered in this paper is related to the problem of filtering a time-series $x(n)$ using a sliding window. The output, $y(n)$, at time n, can be interpreted as an estimate of location based on the input samples:

$$y(n) = \text{myriad } \{x(n - N_1), \ldots, x(n-1), x(n), x(n+1), \ldots, x(n+N_2); k\}, \tag{10}$$

where N is window length. (If N is even then $N_1 = \frac{N}{2}$ and $N_2 = N_1 - 1$, if N is odd then $N_1 = \frac{N-1}{2}$ and $N_2 = N_1$.)

The presented model of independent but not identically distributed samples can acquire the temporal relationships which usually exist among input samples. As an estimation of location the output, $y(n)$, would expect more on sample $x(n)$ in comparison with samples that are further away in time. By assigning varying scale factors in modelling the input samples, leading to different weights (reliabilities), their temporal correlations can be effectively accounted for [17]. For more information about how to estimate the output of a myriad filter see [4, 16, 17].

The fundamental problem of weighted myriad filters, and weighted filters in general, is the proper choice of weights and window length N. Of course, there are special optimization procedures and it is possible to find the proper values of weights in the adaptive way. But this approach is rather time-consuming and when the characteristic of the signal changes the optimization procedure has to be repeated. A description of an adaptive weighted myriad filter can be found in [6]. In this paper, weights have been calculated using the well-known weighted functions, also known as window functions.

5. Weighted functions

Weighted (window) functions or data windows are well known in the literature on digital signal processing. Such functions are frequently used to improve the accuracy of discrete Fourier transform (DFT)-based spectrum analysis, to design digital filters, to simulate antenna radiation patterns, *etc.* [19]. In this paper, the Chebyshev, Kaiser and triangular windows functions have been chosen. Expressions defining these window functions are presented below:

- The Chebyshev window (also called the Dolph-Chebyshev or the Tchebyshev window): let $w_c(n)$ be the N-point inverse Discrete Fourier Transform (DFT) of

$$\frac{\cos\left\{N\cdot\arccos\left[\xi\cdot\cos(\pi\frac{m}{N})\right]\right\}}{\cosh\left[N\cdot\operatorname{arccosh}(\xi)\right]}, \tag{11}$$

  where

$$\xi=\cosh\left(\frac{1}{N}\operatorname{arccosh}(10^A)\right) \text{ and } m=0,1,2,\ldots,N-1.$$

  A is a parameter that allows to adjust side-lobe levels with respect to the main lobe level of the window. In this paper this parameter has been chosen arbitrarily as $A=25$. The window's length should be odd.

- The Kaiser window:

$$w_k(n)=\frac{I_0\left[\eta\sqrt{1-\left(\frac{n-p}{p}\right)^2}\right]}{I_0(\eta)}, \tag{12}$$

  where $n=0,1,2,\ldots,N-1$ and $p=(N-1)/2$. $I_0(\eta)$ is the first-kind Bessel function of the zero order,

$$I_0(x)\hat{=}1+\sum_{k=1}^{\infty}\left[\frac{(x/2)^k}{k!}\right]^2,$$

$$\eta\hat{=}\begin{cases}0, & \text{for } A<21,\\ 0.5842\cdot(A-21)^{0.4}+0.07886\cdot(A-21), & \text{for } 21\le A\le 50,\\ 0.1102\cdot(A-8.7), & \text{for } A>50.\end{cases}$$

  A is side-lobe attenuation [dB], η is the Kaiser window parameter that affects the side-lobe attenuation of the Fourier transform of the window. In this paper $\eta=5$.

- The triangular window:

$$w_t=1-\frac{2|n-(N-1)/2|}{N-1}, \tag{13}$$

  where $n=0,1,2,\ldots,N-1$.

Matlab procedures have been used. For more information about window functions see [19].

6. Numerical experiments

Weighted myriad filters with the weight functions presented in previous section were evaluated in a computer simulation procedure involving filtering an ECG signal corrupted by noise distributed with symmetric α-stable distribution and real-life muscle noise. The five, different, single ECG cycles used in this experiment were chosen randomly from the available database. All signals were sampled 2 000 times per second and consisted of 1 560 samples each. In order to obtain signals with a high signal-to-noise ratio (SNR), the signals were modelled by a linear combination of Hermite functions. These signals were corrupted with α-stable distributed noise for different values of α. The noisy ECG cycles were obtained by adding each ECG cycle to noise of various SNR. As mentioned above, the variance of a random variable of α-stable distribution does not exist and therefore a generalized signal-to-noise ratio (GSNR) was used, defined as follows:

$$\mathrm{GSNR} = \log_{10}\left(\frac{\sigma_s^2}{a\gamma}\right) \text{ [dB]}, \tag{14}$$

where σ_s^2 is the variance (power) of a clean signal, γ is the dispersion of impulsive noise calculated as per [10], and a is a scaling factor. The additive assumption was that the GSNR value was independent of time. Results were obtained for two values of GSNR, *viz.* 0dB and 20dB. For each value of α, 50 different realizations of impulsive noise were generated with known GSNR values. Then the results were averaged.

Generally, filtering is processing a signal in the time-domain resulting in a change of the signal's original spectral content. This change is usually a reduction of unwanted components of the input signal [19]. Biomedical signals, like the ECG signal, are corrupted with noise the spectral components of which lie in the range of the useful part of deterministic signals. For example, the accuracy of high-frequency ECG analysis is limited by the presence of noise, the precision of time alignment and non-stationary signals [20]. The filtering process should not deform the signal, but there exists a group of filters which introduce inadmissible deformations of the signal. One of the ways to overcome this disadvantage in a FIR filter is the requirement of a constant group delay. It means that no phase distortion is induced in the filter's desired output signal. All the frequency components of the filter input signal are delayed by an equal amount of time before they reach the filter's output [19]. Another is that the signal and the noise should be orthogonal.

This approach is difficult to apply for non-linear filters, hence in this simulation the relative power of the additive residual distortion introduced by the myriad filter has been measured. It shows how the myriad filter distorts the ECG signal while attempting to cancel the impulsive noise. Comparing the output of the myriad filter, $x_o(n)$, with the clean ECG cycle, $s(n)$, the additive residual distortion is defined as:

$$D = 10\log_{10}\frac{\sum_{n=1}^{L}[x_o(n)-s(n)]^2}{\sum_{n=1}^{L}[s(n)]^2} \text{ [dB]}, \tag{15}$$

where L is the signal's length. Signals $x_o(n)$ and $s(n)$ are aligned and they have the same time index.

The Savitzky-Golay smoothing filter, also called the digital smoothing polynomial or least-squares smoothing filter, has been used as a reference filter. The fundamental idea of using this filter is to fit a different polynomial to the data surrounding each signal sample. Smoothed samples are computed by replacing each sample with the value of its fitted polynomial [21].

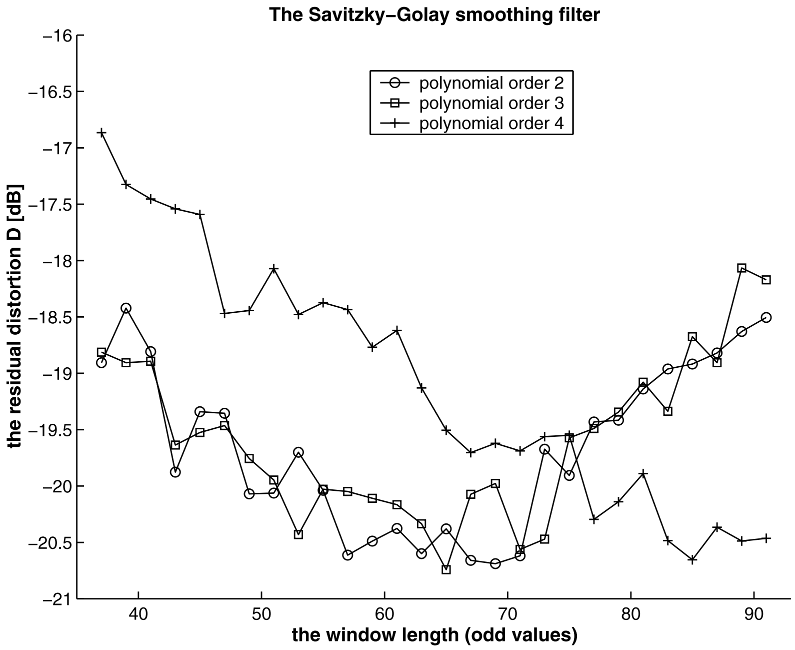


Figure 4. The distortion, D, of the Savitzky-Golay filter for an ECG cycle disturbed with noise at GSNR=10dB, modelled with $S\alpha S$ distribution for $\alpha = 1.8$

The Savitzky-Golay filter is determined by two parameters: the polynomial order k which must be less then the window size, N, which must be odd. An experiment was carried out in order to choose the proper values of these parameters. For a randomly chosen ECG cycle which was disturbed with noise of α-stable distribution at GSNR=10dB and $\alpha = 1.8$, the residual distortion, D, introduced by the Savitzky-Golay filter was measured. The results are presented in Figure 4. The minimum value of distortion for the Savitzky-Golay filter was introduced when the polynomial order equaled 3 and the window length was 65. These values were used in further analysis to be compared with the weighted myriad filters when different weighted functions were used.

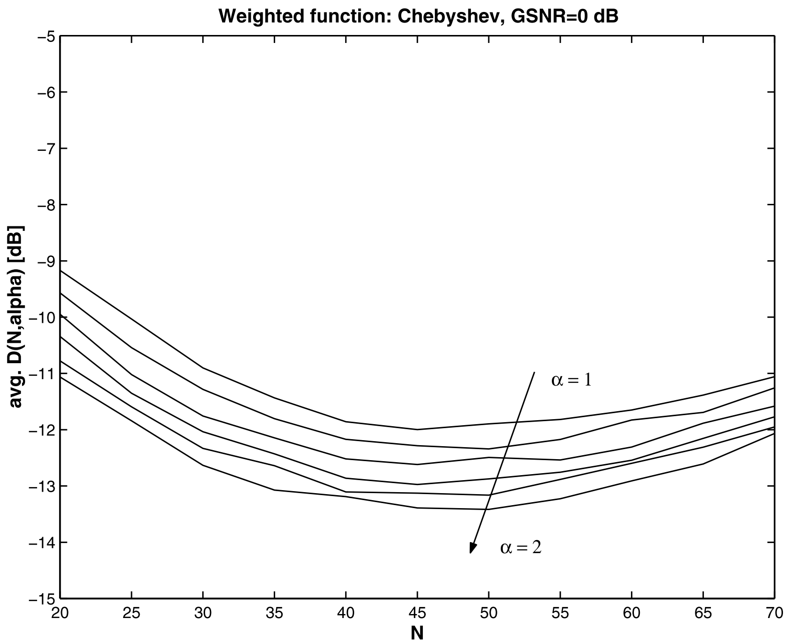
The other reference filter is a well-known non-linear median filter [8].

7. Quantitative results and discussion

Figures 5, 6 and 7 present results obtained for various window lengths (numbers of weights), N, and various values of the characteristic exponent, α.

On the basis of the obtained results, it may be concluded that the small window length of the weighted myriad filter for GSNR= 0dB and minimal residual distortion has been obtained for the Chebyshev weighted function, with the window length of $N \cong 45$ (see Figure 5). If the weights of the weighted myriad filter are selected according to the Kaiser or triangular weighted functions, the weighted myriad filter requires a longer window (window length > 60, see Figures 6 and 7).

When the value of GSNR is higher, the window length decreases for all the weighted functions (the minimum value of D is obtained for $25 \leq N \leq 35$ for all the weighted functions). But when α is in the $\langle 1.0, 1.4 \rangle$ range, the smallest residual



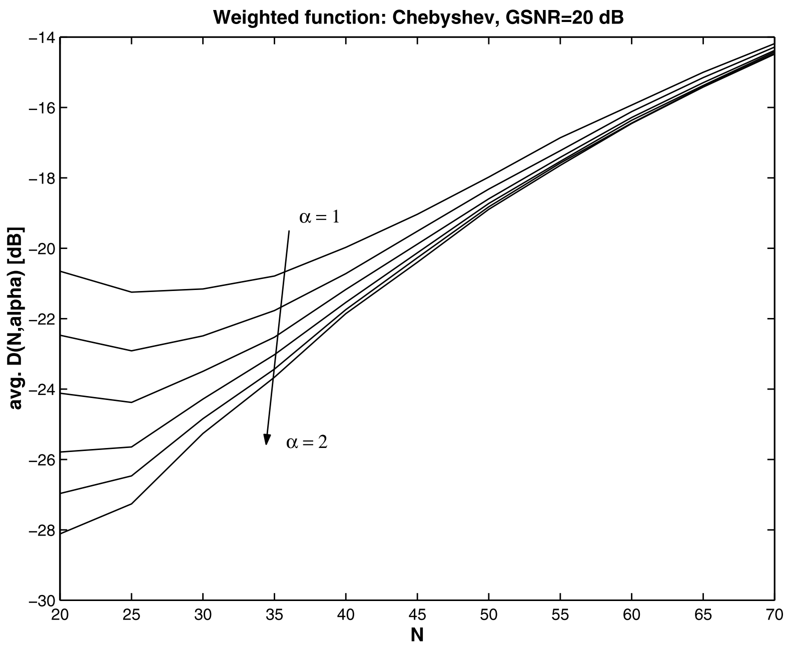


Figure 5. The average D in the function of window length, N, and the characteristic exponent, α, for GSNR= 0dB (upper graph) and 20dB (lower graph) for the Chebyshev weighted function

distortions are introduced by the Chebyshev function. For $\alpha \geq 1.6$, the Kaiser and triangular functions yield better results. Each of the evaluated weighted functions requires a different window length at which the filtering process leads to the smallest residual distortions of the investigated signal. The most useful is the Chebyshev function, especially when the GSNR value is low.

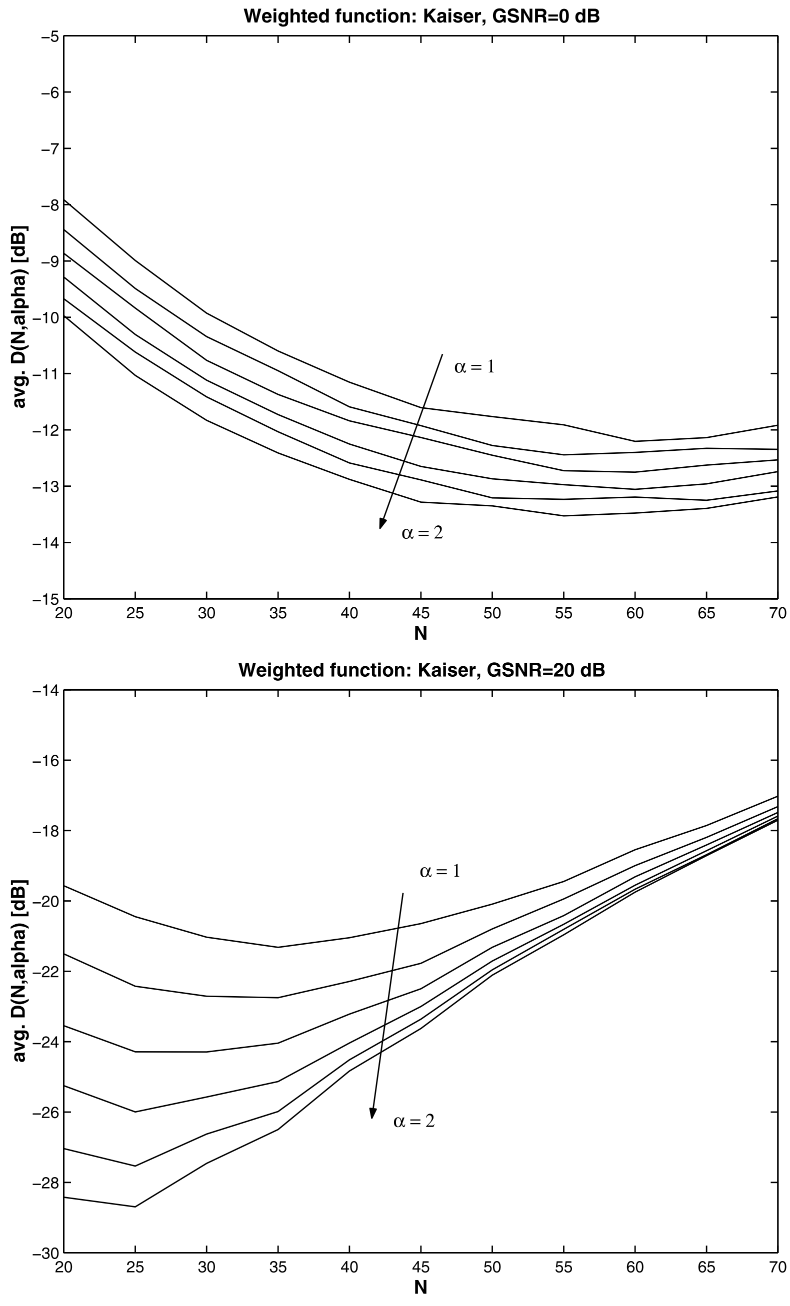
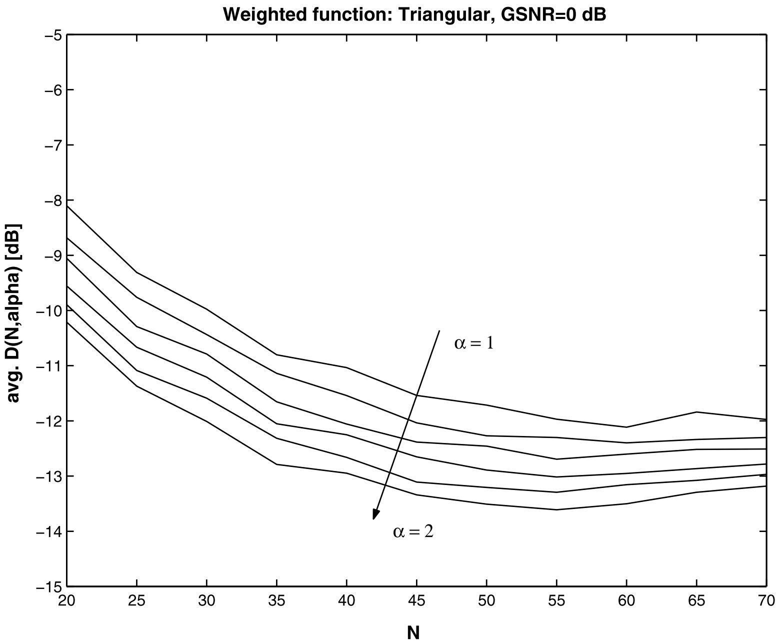


Figure 6. The average D in the function of window length, N, and the characteristic exponent, α, for GSNR= 0dB (upper graph) and 20dB (lower graph) for the Kaiser weighted function

In all cases, when the noise is very impulsive ($\alpha = 1$), distortion reaches its highest value above that for $\alpha = 2$, which corresponds to the Gaussian noise. This shows that, regardless of the choice of weights, the impulsiveness of noise has a strong effect on the filtering process, especially when the signal-to-noise ratio is low or extremely low.



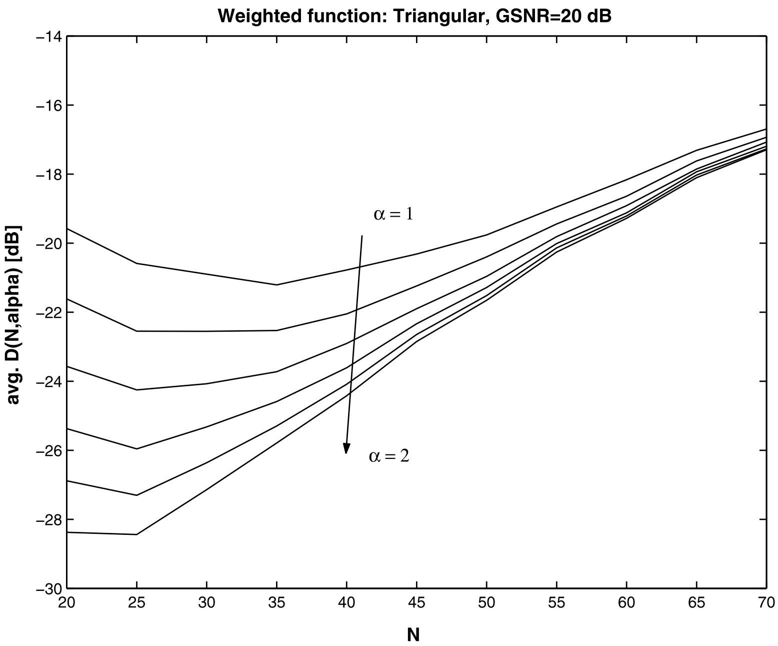


Figure 7. The average D in the function of window length, N, and the characteristic exponent, α, for GSNR= 0dB (upper graph) and 20dB (lower graph) for the triangular weighted function

A comparison of residual distortions has also been made for the different weighted functions (Chebyshev, Kaiser, triangular), uniform weights, *e.g.* $w_i = 1/N(i = 1,2,\ldots,N)$, the median filter and the Savitzky-Golay smoothing filter (a linear filter). In order to do this, one ECG cycle was chosen randomly. This ECG cycle was disturbed by an impulsive noise at GSNR= 0dB (window length for non-linear

Table 2. The residual distortion, D, of the various weighted functions for a randomly chosen ECG cycle disturbed by simulated noise of α-stable distribution (GSNR= 0dB)

α	Weighted myriad filter ($N=45$)				Median filter	The Savitzky-Golay filter
	Chebyshev $A=25$	Kaiser $\eta=5$	triangular	uniform	(non-linear) $N=45$	(linear), $N=65$, polynomial order of 3
1.0	−11.65	−11.05	−11.14	−6.50	−11.66	21.88
1.1	−11.80	−11.21	−11.32	−7.28	−11.57	14.21
1.2	−11.77	−11.24	−11.34	−7.88	−11.43	11.21
1.3	−11.77	−11.25	−11.37	−8.53	−11.32	5.27
1.4	−12.14	−11.60	−11.74	−9.41	−11.32	2.37
1.5	−12.41	−11.86	−12.02	−10.11	−11.49	−0.40
1.6	−12.30	−11.84	−12.00	−10.68	−11.31	−3.67
1.7	−12.43	−11.97	−12.14	−11.36	−11.27	−6.97
1.8	−12.52	−12.00	−12.19	−11.80	−11.16	−8.10
1.9	−12.91	−12.38	−12.59	−12.56	−11.51	−10.02
2.0	−12.86	−12.38	−12.58	−12.92	−11.25	−11.92

Table 3. The residual distortion, D, of the various weighted functions for a randomly chosen ECG cycle disturbed by simulated noise of α-stable distribution (GSNR= 10dB)

α	Weighted myriad filter ($N=35$)				Median filter	The Savitzky-Golay filter
	Chebyshev $A=25$	Kaiser $\eta=5$	triangular	uniform	(non-linear) $N=35$	(linear), $N=65$, polynomial order of 3
1.0	−16.97	−16.23	−16.30	−11.08	−18.91	11.88
1.1	−17.46	−16.80	−16.87	−12.27	−18.85	4.21
1.2	−17.80	−17.21	−17.29	−13.16	−18.98	1.21
1.3	−18.12	−17.61	−17.70	−14.39	−18.89	−4.73
1.4	−18.77	−18.30	−18.39	−15.41	−18.99	−7.61
1.5	−19.27	−18.85	−18.95	−16.49	−19.26	−10.37
1.6	−19.51	−19.25	−19.31	−17.18	−19.23	−13.62
1.7	−19.93	−19.70	−19.81	−18.19	−19.16	−16.89
1.8	−20.17	−19.96	−20.08	−18.85	−19.10	−17.99
1.9	−20.66	−20.53	−20.64	−19.60	−19.44	−19.83
2.0	−20.87	−20.85	−20.95	−20.10	−19.28	−21.65

filters $N=45$) and GSNR= 10dB (window length for non-linear filters $N=35$), for various values of α (from 1.0 to 2.0). This operation was repeated 50 times. Then the average residual distortion was calculated. The linearity parameter of the weighted myriad filter, k, was equal to 1. The results are presented in the Tables 2 and 3.

For GSNR= 0dB and $\alpha=1.0$, the median filter leads to the smallest distortion. But when $1.1 \leq \alpha \leq 1.9$, the weighted myriad filter with the Chebyshev weighted function is better than the median filter. The worst results have been obtained for the Savitzky-Golay filter, but as α increases, this filter produces better and better results. The uniformly weighted myriad filter also introduces large errors into the filtered signal, but for $\alpha=2$ this weighted function leads to the best result.

When GSNR increases to 10dB and α is in the $\langle 1.0, 1.4 \rangle$ range, the median filter produces the smallest residual distortion values. For $\alpha \geq 1.5$, the weighted

Table 4. The residual distortion, D, of the various weighted functions when ECG cycle is disturbed by real-life muscle noise (window length is constant)

GSNR [dB]	Weighted myriad filter ($N=35$) Chebyshev $A=25$	Kaiser $\eta=5$	triangular	uniform	Median filter (non-linear) $N=35$	The Savitzky-Golay filter (linear), $N=65$, polynomial order of 3
0	−9.26	−7.94	−8.32	−9.59	−8.75	−8.82
5	−13.87	−12.74	−13.06	−13.79	−13.25	−13.68
10	−17.73	−17.25	−17.42	−17.11	−17.46	−18.12
20	−22.72	−24.06	−23.78	−21.06	−23.79	−24.89

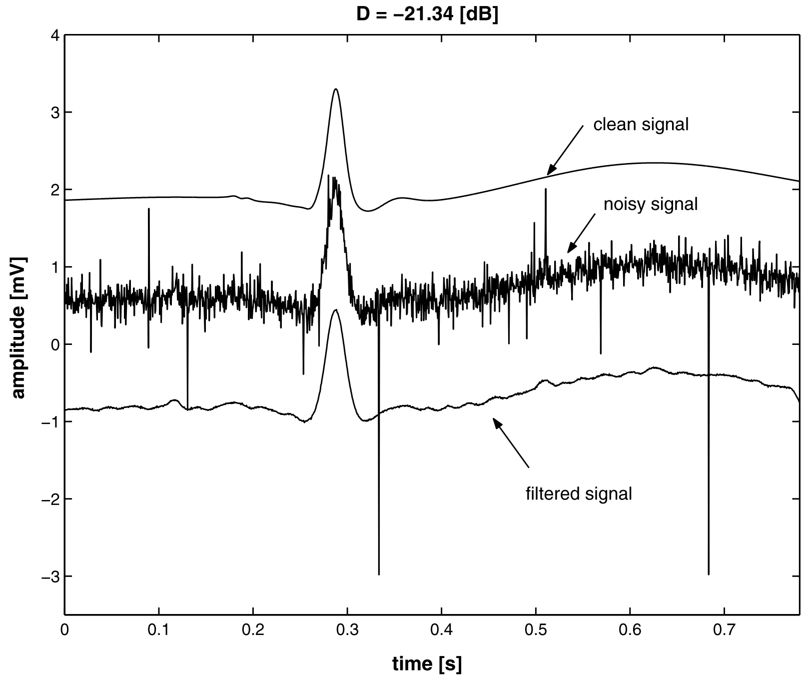


Figure 8. Results of the filtering of ECG cycle disturbed with simulated α-stable distributed noise (GSNR$=10$dB, $\alpha=1.7$) with the weighted myriad filter (linear parameter $k=1$) for weights calculated according to the Chebyshev weighted function, the window length is $N=35$

myriad filter with the Chebyshev weighted function yields the best results. But when $\alpha=2.0$, the Savitzky-Golay smoothing filter has the smallest residual distortion value.

Results obtained for a ECG cycle disturbed with real-life muscle noise are presented in Table 4. These results show that suppression of real muscle noise is a serious problem. When GSNR is low (0dB or 5dB), the residual distortion is small for the weighted myriad filter with the Chebyshev weighted function or uniform weights. For GSNR≥ 10dB, the Savitzky-Golay filter produces the best results.

An example of filtering an ECG cycle disturbed with noise of α-stable distribution with a weighted myriad filter and weights calculated according to the Chebyshev function is presented in Figure 8.

8. Conclusions

This paper has dealt with modelling real-life muscle noise with α-stable distributions and weighted myriad filters with various selection of weights.

Biomedical signals contain noise and outliers and thus require robust processing methods. The application of a weighted myriad filter is such a robust method. Biomedical signals are usually non-stationary, but for short time intervals an assumption of stationary features can be made. According to the Generalized Central Limit Theory, real-life muscle noise, which is produced by a great number of electrical sources, converges to α-stable distribution. The mean value of the characteristic exponent is usually *ca.* 1.7–1.9. α-stable distribution can be useful in modelling muscle noise, even though this approach has some restriction.

The optimal choice of weights for a weighted myriad filter is sometimes impossible. The application of weighted functions to weighted myriad filters produces good results in comparison with those for linear and median filters when the noise is characterized by their characteristic exponent α in the range from 1.5 to 1.9. This range is suitable for real-life muscle noise. The most useful is the Chebyshev weighted function: it requires short window lengths and yields the best results in the relevant range of α changes. This function introduces the smallest residual distortion into the filtered signal. For the Gaussian noise, linear filters almost always produce the best results. The proposed filtering process does not distort the signal in the significant mean and is able to effectively cancel impulsive types of noise.

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