NUMERICAL SIMULATIONS OF SOUND WAVE GENERATION IN A RANDOM MEDIUM

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Abstract: In turbulent media, both sound wave sources and the speed of sound can be stochastic variables. By means of numerical simulations of one-dimensional Euler equations with random source terms we have studied two cases in a homogeneous stochastic random medium for which the speed of sound and sound sources are: (1) correlated and (2) uncorrelated. The numerical simulations indicate that, if the source and the speed of sound fluctuations are uncorrelated, the acoustic field is incoherent, with a zero expectation value. The mean field is non-zero in the correlated case. The correlated and uncorrelated cases are clearly distinguishable by the mean field, but also – to some extent – in the power spectrum, which displays a modified Lorentzian profile with a shift in frequency.

Keywords: sound waves, random waves, frequency shift, amplitude alteration

1. Introduction

Motivated by the problem of the origin of waves, we discuss the generation of sound waves by random fields (*e.g.* Gough [1]). This subject has been investigated to understand the complex physical phenomena which occur in the presence of random fields and their influence on sound waves. From this point of view, simple models attract considerable attention as they allow one to separate and quantify the effects of various stochastic fields on frequencies and amplitudes of these oscillations. Indeed, the analytical and numerical study of sound waves in a space-dependent random mass density field reveals that these waves are accelerated and attenuated (Nocera *et al.* [2]). At the same time, the effect of a time-dependent random mass density field is acceleration and amplification of sound waves (Murawski *et al.* [3]).

The above-mentioned studies are made for sound waves which are launched in a random medium but their origin is not discussed. The problem of random wave generation was discussed recently by Skartlien [4], whose analytical conclusions were drawn on the basis of an assumption of singly and doubly scattered field components in cross power. The purpose of this paper is to verify and extend this first-order theory beyond the region of its validity. An ideal tool for such verification is numerical simulation. We assume 1D geometry for clarity of demonstration, but the general results will be valid for 2D and 3D as well.

The paper is organized as follows. Section 2 presents the numerical model. Section 3 contains results of the numerical simulations. This paper is concluded with Section 4.

2. Numerical model

We limit our discussion to magnetic-free one-dimensional plasma which is described by hydrodynamic equations:

$$\frac{\partial \varrho}{\partial t} + \frac{\partial(\varrho V)}{\partial x} = S_{\varrho}, \tag{1}$$

$$\frac{\partial(\varrho V)}{\partial t} + \frac{\partial(\varrho V^2 + p)}{\partial x} = V S_{\varrho} + \varrho S_V, \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial[V(E+p)]}{\partial x} = \frac{1}{2} V^2 S_{\varrho} + \varrho V S_V. \tag{3}$$

Here,

$$E = \frac{p}{\gamma - 1} + \frac{1}{2}\varrho V^2 \tag{4}$$

is the total energy, ϱ is mass density, V is flow velocity, p is pressure, γ is the adiabatic index, and S_{ϱ} and S_V are the source terms which can be used for seeding corresponding random fields (Murawski *et al.* [3]).

To perform numerical simulations for Equations (1)–(3) we had adapted the CLAWPACK code (LeVeque [5]), which is a packet of Fortran routines for solving hyperbolic equations (*e.g.* Murawski [6]). We performed the simulations with the use of homogeneous grids to discretize x and t. These simulations were carried out from $t = 0$ up to $t = 262.144$ with the time step $\Delta t = 0.001$. This means that we performed 262 144 iterations in time. A spatial interval was extended from $x = 0$ to $x = 16\pi$ with the length of numerical cell $\Delta x = 0.122718$. As a consequence of the application of the Fast Fourier Tranform (FFT) method we had to use a number of grid points which was a power of 2. The periodic boundary conditions were applied at the edges of the simulation region.

Initially, at $t = 0$, we set all the plasma quantities at their equilibrium values, *viz.*:

$$\varrho_e(x, t=0) = \varrho_0 + \varrho_r(x, t=0), \quad V_e = 0, \quad p_e = p_0, \tag{5}$$

where ϱ_0 and p_0 denote background mass density and pressure which are constant, and $\varrho_r(x,t)$ is a random mass density which is seeded with the use of the source term S_{ϱ} in Equation (1). The method of seeding has been described by Murawski *et al.* [3].

2.1. Random fields and sources

Random fields can be generated in various ways. The simplest case we can envisage is a factorised random mass density field:

$$\varrho_r(x,t) = \varrho_x(x)\varrho_t(t). \tag{6}$$

The source term S_V in Equation (2) is given as:

$$S_V(x,t) = S_x(x)S_t(t), \tag{7}$$

with

$$S_x(x) = \exp\left[-(x-x_0)^2\right], \qquad S_t(t) \sim \varrho_t(t). \tag{8}$$

As the cross-correlation:

$$\langle S_V \varrho_r \rangle = 0 \tag{9}$$

this case can be called the *uncorrelated case*.

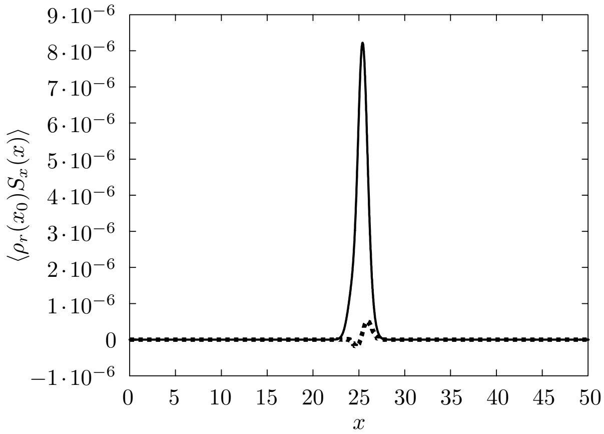

Figure 1. Numerically obtained cross-correlations for the correlated case (solid line) and the uncorrelated case (dashed line)

The second case we discuss is factorizations (6) and (7), but now:

$$S_x(x) \sim \exp\left[-(x-x_0)^2\right]\varrho_x(x), \qquad S_t(t) \sim \varrho_t(t). \tag{10}$$

This case is called the *correlated case* as the cross-correlation:

$$\langle S_V \varrho_r \rangle \neq 0. \tag{11}$$

We choose a random field $\varrho_r(x,t)$ to be Gaussian in the way that its Fourier transform $E(k,\omega)$ is:

$$E(k,\omega) = \frac{\sigma^2 l_x l_t}{\pi^2} e^{-k^2 l_x^2} e^{-\omega^2 l_t^2}, \tag{12}$$

where σ corresponds to a strength of the random field, l_x and l_t are the correlation length and the correlation time, respectively.

The cross-correlation functions which are given by Equations (9) and (11) are presented in Figure 1 both for the uncorrelated (dashed line) and the correlated (solid line) cases. These functions are obtained numerically by ensemble averaging over 90 realizations of a random field ϱ_r. This number of realizations is sufficient to discuss the results but it is not enough to represent the analytical Gaussian profile exactly. It is discernible that the cross-correlation is much lower in the case of the uncorrelated case than in the correlated case.

We have also tried the unfactorized choices of $\varrho_r(x,t)$ and $S_V(x,t)$, with no essential difference in the numerical results. So, we limit our discussion to the above simple cases.

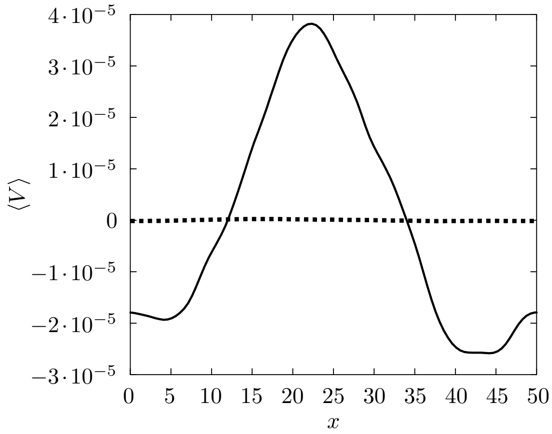

Figure 2. Numerically obtained averaged signal for uncorrelated (dashed line) and correlated (solid line) mass density fields

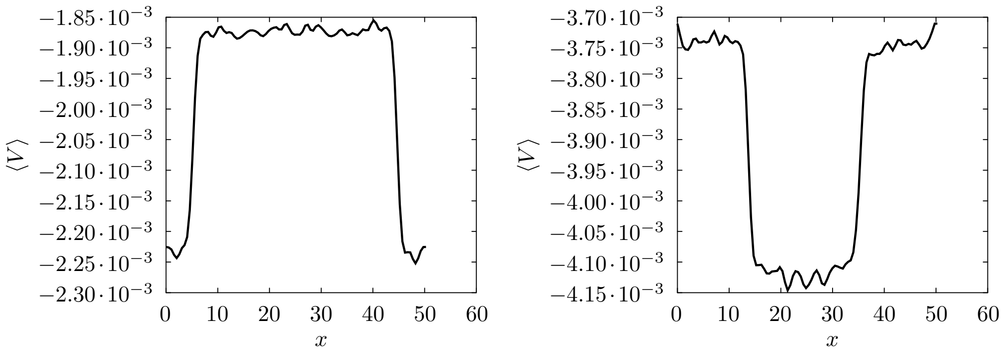

Figure 3. Numerically obtained averaged signal in the correlated case at $t_{\mathrm{end}}/2$ (left) and t_{end} (right)

3. Numerical results

In this part of the paper we present the results of numerical simulations. We choose and hold fixed an amplitude of the oscillator of $\sigma = 0.05$.

The wave generation mechanism will be modeled by setting the source term $S_V(x,t)$ in the momentum equation of Equation (2). Analytical results for weak random fields in the linear approximation show that a source term S_V correlated with ϱ_r generates a coherent wave field in the system (Skartlien [4]). Let us remind that our intention is to verify this finding.

The dashed line in Figure 2 shows that a random source S_V that is uncorrelated with ϱ_r generates a low averaged signal which dies with a number of realizations over which the ensamble averaging is performed. Consequently, we claim that such a random source is unable to generate a coherent field. The solid line in Figure 2 shows a similar field but for correlated $\varrho_r(x,t)$ and $S_V(x,t)$. As the averaged signal is about 2 orders of magnitude higher in the case of correlated random fields, we claim that the analytical results (Skartlien [4]) have been confirmed by these numerical findings.

Figure 3 shows the time evolution of averaged signals corresponding to the correlated case. It is interesting that a significant amount of energy propagates in the form of waves which possess steep wave fronts and almost rectangular profiles.

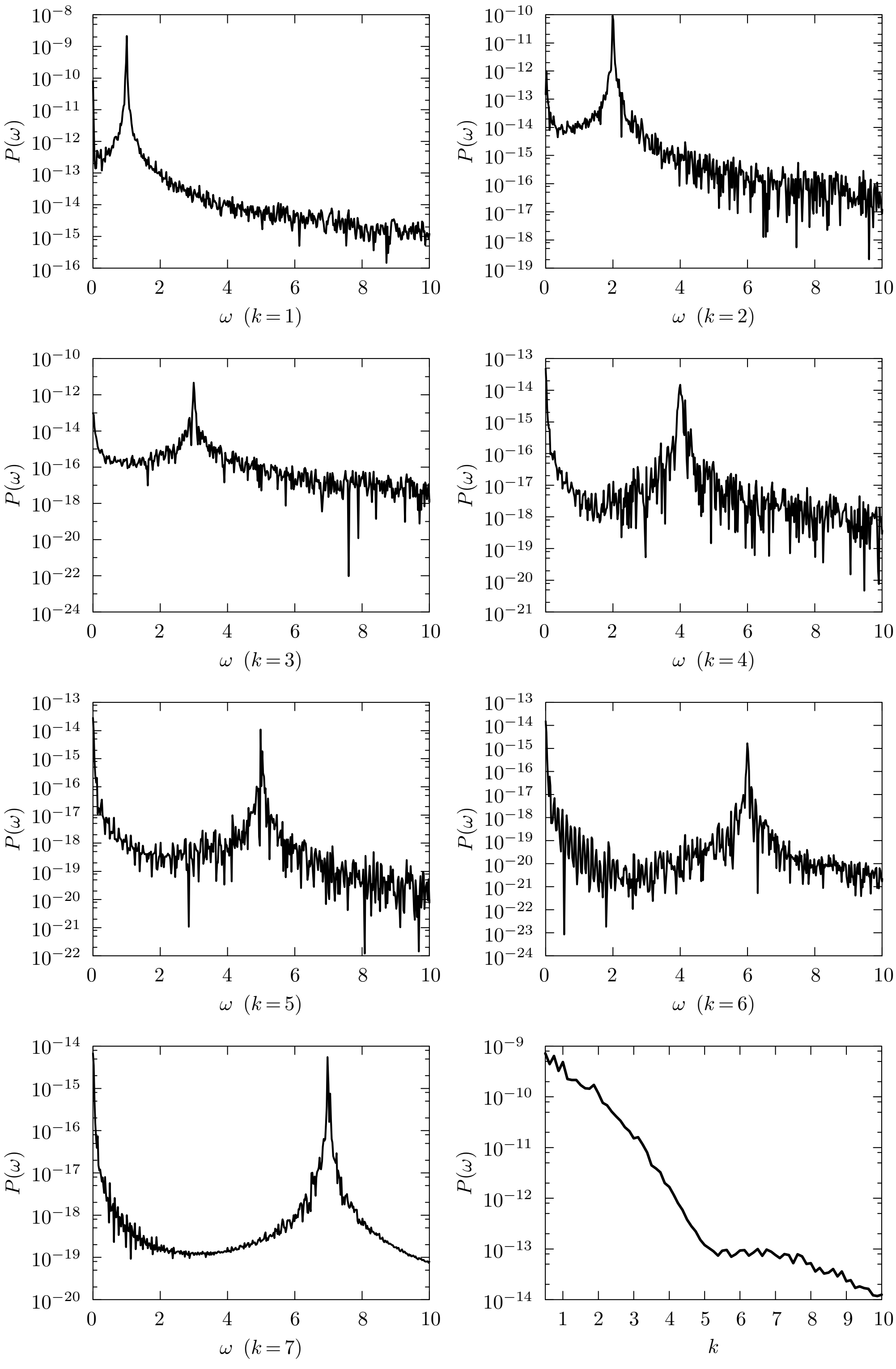

Figure 4. Numerically obtained power spectra for the correlated case

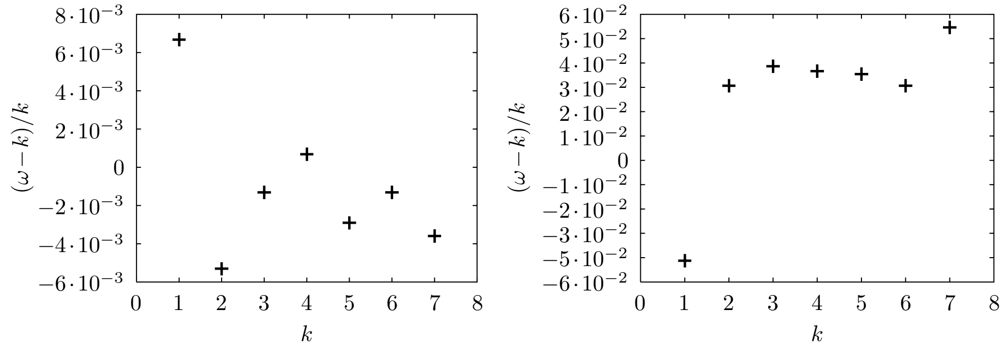

Figure 5. Relative frequency correction for the correlated case (for which $S_V \neq 0$, left) and for a free medium (for which $S_V = 0$, right)

Figure 4 shows wave spectra both in wavenumber k (the bottom right panel) and frequency ω (for different k) become wider for a narrower source term $S_x(x)$ (a lower value of ω). As there is a number of characteristic lines in the frequency spectra, we claim that these spectra are discrete while the wavenumber spectra are continuons. The power spectrum displays a modified Lorentzian profile in both cases, with enhanced width and a shift in wavenumber and frequency. Indeed, the left panel of Figure 5 shows that frequencies of the excited sound waves are essentially reduced in comparison to the sound wave in a deterministic medium. It is noteworthy that free random sound waves (for which $S_V = 0$) experience a frequency increase in agreement with Fermat's principle.

4. Summary

In this paper we have studied numerically the stochastic excitation of sound waves in a random medium in which both sound wave sources and the speed of sound can be random variables. We considered numerically two cases in a homogeneous one-dimensional stochastic medium, in which the speed of sound and sound sources are either correlated or uncorrelated. The main conclusion of our investigation is that if the source and the speed-of-sound fluctuations are uncorrelated, the acoustic field is incoherent, with a zero expectation value (a zero mean field) but the mean field is nonzero in the correlated case. The two cases are clearly distinguishable by the mean field, but also – to some extent – in the power spectrum. In both cases the power spectrum displays a modified Lorentzian profile, with enhanced width and a shift in wavenumber and frequency.

Acknowledgements

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